

# MHT-CET Mathematics Sample Paper-8

Duration: 90 Minutes

Maximum Marks: 100

## Instructions

- This paper contains a total of **50** Multiple Choice Questions.
- Each correct answer carries **+2 marks**.
- No negative marking for incorrect questions.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.
- No marks will be deducted for questions that are left unattempted.

**Q1.** If the function  $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$  is continuous at  $x = 0$ , then  $f(0)$  is equal to:

- (A)  $a - b$
- (B)  $a + b$
- (C)  $b - a$
- (D)  $\log(a + b)$

**Q2.** If  $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ 3, & x = \frac{\pi}{2} \end{cases}$  is continuous at  $x = \frac{\pi}{2}$ , then the value of  $k$  is:

- (A) 3
- (B) 6
- (C) 9
- (D) 12

**Q3.** The value of  $\lim_{x \rightarrow 0} \frac{1 - \cos(4x)}{x^2}$  is:

- (A) 4
- (B) 8
- (C) 16



(D) 2

**Q4.** If  $y = \tan^{-1} \left( \frac{\sin x + \cos x}{\cos x - \sin x} \right)$ , then  $\frac{dy}{dx}$  is:

(A) 1

(B) 0

(C)  $\frac{1}{2}$

(D) -1

**Q5.** If  $x = a \cos^3 \theta$  and  $y = a \sin^3 \theta$ , then  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{4}$  is:

(A)  $\frac{4\sqrt{2}}{3a}$

(B)  $\frac{3a}{4\sqrt{2}}$

(C)  $\frac{-4\sqrt{2}}{3a}$

(D) 0

**Q6.** If  $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}}$ , then  $(2y - 1) \frac{dy}{dx}$  is equal to:

(A)  $\sin x$

(B)  $\cos x$

(C)  $-\cos x$

(D) 1

**Q7.** If  $f(x) = x^x$ , then  $f(x)$  has a stationary point at  $x = \underline{\hspace{2cm}}$ .

(A)  $e$

(B)  $1/e$

(C) 1

(D)  $\sqrt{e}$

**Q8.** Derivative of  $\log_{10} x$  with respect to  $x^2$  is:

(A)  $\frac{1}{2x^2 \log_e 10}$



- (B)  $\frac{2x^2}{\log_e 10}$
- (C)  $\frac{1}{x^2}$
- (D)  $\frac{\log_{10} e}{x}$

**Q9.** The equation of the tangent to the curve  $y = 2x^2 + 3 \sin x$  at  $x = 0$  is:

- (A)  $3x - y = 0$
- (B)  $3x + y = 0$
- (C)  $x - 3y = 0$
- (D)  $y = 0$

**Q10.** A spherical balloon is being inflated at the rate of 35 cc/min. The rate of increase of its surface area when its radius is 10 cm is:

- (A) 7 sq.cm/min
- (B) 3.5 sq.cm/min
- (C) 14 sq.cm/min
- (D) 2 sq.cm/min

**Q11.** The maximum value of  $f(x) = \frac{\log x}{x}$  in  $(0, \infty)$  is:

- (A)  $e$
- (B)  $1/e$
- (C) 1
- (D)  $2/e$

**Q12.** The function  $f(x) = 2x^3 - 15x^2 + 36x + 1$  is increasing in the interval:

- (A)  $(2, 3)$
- (B)  $(-\infty, 2) \cup (3, \infty)$
- (C)  $(-\infty, 3)$
- (D)  $(2, \infty)$



**Q13.** The value of the Rolle's constant  $c$  for  $f(x) = x^2 - 4x + 3$  in  $[1, 3]$  is:

- (A) 1
- (B) 2
- (C) 3
- (D) 0

**Q14.**  $\int \frac{1}{x(x^5+1)} dx =$

- (A)  $\frac{1}{5} \log \left| \frac{x^5}{x^5+1} \right| + C$
- (B)  $\log \left| \frac{x^5}{x^5+1} \right| + C$
- (C)  $5 \log \left| \frac{x^5+1}{x^5} \right| + C$
- (D)  $\frac{1}{5} \log \left| \frac{x^5+1}{x^5} \right| + C$

**Q15.**  $\int e^x \left( \frac{1+\sin x}{1+\cos x} \right) dx =$

- (A)  $e^x \tan(x/2) + C$
- (B)  $e^x \cot(x/2) + C$
- (C)  $e^x \sin(x/2) + C$
- (D)  $e^x \cos(x/2) + C$

**Q16.** The value of  $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \sqrt{\cos x}}} dx$  is:

- (A)  $\pi$
- (B)  $\pi/2$
- (C)  $\pi/4$
- (D) 0

**Q17.**  $\int \frac{dx}{\sqrt{1-e^{2x}}} =$

- (A)  $\log |e^{-x} + \sqrt{e^{-2x} - 1}| + C$
- (B)  $-\log |e^{-x} + \sqrt{e^{-2x} - 1}| + C$
- (C)  $\sin^{-1}(e^x) + C$



(D)  $\cos^{-1}(e^x) + C$

**Q18.** The value of  $\int_{-1}^1 |x| dx$  is:

(A) 0

(B) 1

(C) 2

(D)  $1/2$

**Q19.** The area bounded by the curve  $y^2 = 4x$  and the line  $x = 3$  is:

(A)  $4\sqrt{3}$  sq. units

(B)  $8\sqrt{3}$  sq. units

(C)  $16\sqrt{3}$  sq. units

(D)  $2\sqrt{3}$  sq. units

**Q20.** Area bounded by the curve  $y = \sin x$  between  $x = 0$  and  $x = 2\pi$  is:

(A) 2 sq. units

(B) 4 sq. units

(C) 0 sq. units

(D) 1 sq. unit

**Q21.** The order and degree of the differential equation  $\frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^3} = 0$  are respectively:

(A) 2, 2

(B) 2, 3

(C) 3, 2

(D) 1, 2

**Q22.** The solution of  $\frac{dy}{dx} + y \cot x = 2 \cos x$  is:

(A)  $y \sin x = \sin^2 x + C$



(B)  $y \sin x = -\cos 2x + C$

(C)  $y \cos x = \sin^2 x + C$

(D)  $y \sin x = -\sin^2 x + C$

**Q23.** The integrating factor of the differential equation  $(1 + x^2)\frac{dy}{dx} + 2xy = \cos x$  is:

(A)  $\log(1 + x^2)$

(B)  $1 + x^2$

(C)  $e^{x^2}$

(D)  $\tan^{-1} x$

**Q24.** If  $z = 1 + i\sqrt{3}$ , then amplitude (argument) of  $z$  is:

(A)  $30^\circ$

(B)  $60^\circ$

(C)  $120^\circ$

(D)  $45^\circ$

**Q25.** The value of  $(1 + i)^{10} + (1 - i)^{10}$  is:

(A) 0

(B)  $2^{10}$

(C)  $2^5$

(D)  $2^6$

**Q26.** If  $\omega$  is a complex cube root of unity, then  $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5 =$

(A) 32

(B) -64

(C) 64

(D) 0

**Q27.** If the roots of  $x^2 - px + q = 0$  differ by unity, then:



- (A)  $p^2 = 4q + 1$
- (B)  $p^2 = 4q - 1$
- (C)  $q^2 = 4p + 1$
- (D)  $q^2 = 4p - 1$

**Q28.** The value of  $k$  for which the quadratic equation  $kx(x - 2) + 6 = 0$  has two equal roots is:

- (A) 6
- (B) 0
- (C) 4
- (D) 2

**Q29.** If  $\alpha, \beta$  are the roots of  $x^2 + x + 1 = 0$ , then  $\alpha^{19} + \beta^{19}$  is:

- (A) 1
- (B) -1
- (C)  $i\sqrt{3}$
- (D) 0

**Q30.** The  $n^{\text{th}}$  term of the series  $5 + 11 + 19 + 29 + \dots$  is:

- (A)  $n^2 + 3n + 1$
- (B)  $n^2 + 2n + 2$
- (C)  $n^2 + n + 3$
- (D)  $2n^2 + 3$

**Q31.** If the  $5^{\text{th}}$  term of an A.P. is 30 and the  $12^{\text{th}}$  term is 65, then the sum of first 20 terms is:

- (A) 1150
- (B) 1160
- (C) 1170



(D) 1180

**Q32.** The sum to infinity of the series  $1 + \frac{2}{3} + \frac{4}{9} + \dots$  is:

(A) 3

(B) 2

(C)  $\frac{3}{2}$

(D)  $\frac{5}{3}$

**Q33.** The coefficient of  $x^4$  in the expansion of  $(1 + 2x)^{10}$  is:

(A)  $210 \times 2^4$

(B)  $210 \times 2^6$

(C)  $120 \times 2^4$

(D)  $45 \times 2^4$

**Q34.** The middle term in the expansion of  $(x + \frac{1}{x})^{10}$  is:

(A)  ${}^{10}C_5$

(B)  ${}^{10}C_6$

(C)  ${}^{10}C_4$

(D)  ${}^{10}C_5x$

**Q35.** How many different words can be formed using all the letters of the word "MAHARASHTRA"?

(A)  $\frac{11!}{4!2!2!}$

(B)  $\frac{11!}{4!2!}$

(C)  $\frac{11!}{3!2!2!}$

(D)  $11!$

**Q36.** A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random, the probability that exactly 2 are red is:



- (A)  $15/28$
- (B)  $15/56$
- (C)  $30/56$
- (D)  $5/28$

**Q37.** If  $P(A) = 0.4$ ,  $P(B) = 0.8$  and  $P(B|A) = 0.6$ , then  $P(A \cup B)$  is:

- (A) 0.24
- (B) 0.96
- (C) 0.48
- (D) 0.72

**Q38.** Two dice are thrown simultaneously. The probability of getting a total score of 7 is:

- (A)  $1/6$
- (B)  $1/12$
- (C)  $5/36$
- (D)  $7/36$

**Q39.** The distance between the parallel lines  $3x + 4y + 5 = 0$  and  $3x + 4y + 15 = 0$  is:

- (A) 2 units
- (B) 10 units
- (C) 5 units
- (D) 4 units

**Q40.** The acute angle between the lines  $y = 3x + 7$  and  $y = \frac{1}{2}x + 2$  is:

- (A)  $45^\circ$
- (B)  $30^\circ$
- (C)  $60^\circ$



(D)  $\tan^{-1}(2)$

**Q41.** The equation of the circle passing through the origin and having intercepts 4 and 6 on the x and y axes respectively is:

(A)  $x^2 + y^2 - 4x - 6y = 0$

(B)  $x^2 + y^2 + 4x + 6y = 0$

(C)  $x^2 + y^2 - 2x - 3y = 0$

(D)  $x^2 + y^2 - 8x - 12y = 0$

**Q42.** The center and radius of the circle  $x^2 + y^2 - 6x + 4y - 12 = 0$  are:

(A)  $(3, -2), 5$

(B)  $(-3, 2), 5$

(C)  $(3, -2), 25$

(D)  $(6, -4), 5$

**Q43.** The focus of the parabola  $y^2 = -12x$  is:

(A)  $(3, 0)$

(B)  $(0, 3)$

(C)  $(-3, 0)$

(D)  $(0, -3)$

**Q44.** The eccentricity of the ellipse  $\frac{x^2}{25} + \frac{y^2}{16} = 1$  is:

(A)  $3/5$

(B)  $4/5$

(C)  $9/25$

(D)  $3/4$

**Q45.** The equation of the hyperbola with vertices  $(\pm 3, 0)$  and foci  $(\pm 5, 0)$  is:

(A)  $\frac{x^2}{9} - \frac{y^2}{16} = 1$



(B)  $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(C)  $\frac{x^2}{9} - \frac{y^2}{25} = 1$

(D)  $\frac{x^2}{25} - \frac{y^2}{9} = 1$

**Q46.** If  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ , then  $|\vec{a} \times \vec{b}|$  is:

(A)  $\sqrt{35}$

(B)  $\sqrt{25}$

(C) 5

(D)  $\sqrt{15}$

**Q47.** If the vectors  $2\hat{i} - 3\hat{j} + 4\hat{k}$  and  $a\hat{i} + 6\hat{j} - 8\hat{k}$  are collinear, then  $a =$  \_\_\_\_\_.

(A) 4

(B) -4

(C) 3

(D) -3

**Q48.** The direction cosines of a line which makes equal angles with the coordinate axes are:

(A) (1, 1, 1)

(B)  $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

(C)  $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

(D) (0, 0, 0)

**Q49.** The angle between the lines whose direction ratios are (1, 1, 2) and  $(\sqrt{3} - 1, -\sqrt{3} - 1, 4)$  is:

(A)  $30^\circ$

(B)  $45^\circ$

(C)  $60^\circ$



(D)  $90^\circ$

**Q50.** If the points  $(1, 1, p)$  and  $(-3, 0, 1)$  are equidistant from the plane  $3x + 4y - 12z + 13 = 0$ , then  $p =$  \_\_\_\_\_.

(A) 1

(B)  $7/3$

(C) 0

(D)  $1/2$



## Detailed Solutions

Q1.

## Solution

**Concept:**

A function  $f(x)$  is continuous at a point  $x = a$  if the limit of the function as  $x$  approaches  $a$  is equal to the value of the function at that point. Mathematically, this is expressed as  $\lim_{x \rightarrow a} f(x) = f(a)$ . For problems involving logarithmic terms like  $\log(1+x)$ , we frequently utilize the standard limit result:

$$\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$$

This property allows us to evaluate the indeterminate form of type  $0/0$  by manipulating the expression into a sum of standard limits.

**Solution:**

1. Since  $f(x)$  is continuous at  $x = 0$ , we must have:

$$f(0) = \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{\log(1+ax) - \log(1-bx)}{x}$$

2. We can split the limit into two separate parts to match the standard logarithmic limit form:

$$f(0) = \lim_{x \rightarrow 0} \left[ \frac{\log(1+ax)}{x} - \frac{\log(1-bx)}{x} \right]$$

3. To apply the formula  $\lim_{u \rightarrow 0} \frac{\log(1+u)}{u} = 1$ , we multiply and divide the first term by  $a$  and the second term by  $-b$ :

$$f(0) = \lim_{x \rightarrow 0} \left[ a \cdot \frac{\log(1+ax)}{ax} + b \cdot \frac{\log(1-bx)}{-bx} \right]$$

4. Applying the limit to each term:

As  $x \rightarrow 0$ , both  $ax \rightarrow 0$  and  $-bx \rightarrow 0$ . Therefore:

$$f(0) = a(1) + b(1)$$

5. Simplifying the expression:

$$f(0) = a + b$$

**Final Answer:**

$$\boxed{a + b}$$

**Answer: (B)**

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Q2.

**Solution****Concept:**

For a function to be continuous at  $x = \frac{\pi}{2}$ , the limit as  $x \rightarrow \frac{\pi}{2}$  must exist and be equal to  $f(\frac{\pi}{2})$ . When dealing with trigonometric limits where the variable approaches a non-zero value, a common strategy is substitution, such as  $h = \frac{\pi}{2} - x$ . This transforms the limit into a standard trigonometric limit as  $h \rightarrow 0$ . Alternatively, L'Hospital's Rule can be applied if the limit results in an indeterminate form of type  $0/0$ .

**Solution:**

1. Given  $f(\frac{\pi}{2}) = 3$ . For continuity:

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x} = 3$$

2. Let  $x = \frac{\pi}{2} - h$ . As  $x \rightarrow \frac{\pi}{2}$ , it follows that  $h \rightarrow 0$ . Substituting these into the limit:

$$\lim_{h \rightarrow 0} \frac{k \cos(\frac{\pi}{2} - h)}{\pi - 2(\frac{\pi}{2} - h)} = 3$$

3. Using the trigonometric identity  $\cos(\frac{\pi}{2} - h) = \sin h$ :

$$\lim_{h \rightarrow 0} \frac{k \sin h}{\pi - \pi + 2h} = 3$$

4. Simplify the denominator and extract constants:

$$\frac{k}{2} \lim_{h \rightarrow 0} \frac{\sin h}{h} = 3$$

5. We know that the standard limit  $\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$ :

$$\frac{k}{2}(1) = 3$$

$$k = 6$$

**Final Answer:**

6

**Answer: (B)**[Go Back to Question 2](#)

Q3.

**Solution****Concept:**

Evaluating trigonometric limits of the form  $\frac{1-\cos\theta}{\theta^2}$  is a fundamental skill in calculus. The core identity used here is the half-angle formula:  $1 - \cos(2\theta) = 2 \sin^2 \theta$ . By transforming the numerator using this identity, the limit can be reduced to the standard form  $\lim_{u \rightarrow 0} \frac{\sin u}{u} = 1$ . This specific pattern is very common in competitive exams like MHT-CET.

**Solution:**

1. Identify the given limit:

$$L = \lim_{x \rightarrow 0} \frac{1 - \cos(4x)}{x^2}$$

2. Apply the trigonometric identity  $1 - \cos(2\theta) = 2 \sin^2 \theta$ , where  $2\theta = 4x$ , hence  $\theta = 2x$ :

$$L = \lim_{x \rightarrow 0} \frac{2 \sin^2(2x)}{x^2}$$

3. To use the standard limit property, the argument of the sine function must match the denominator. Here the argument is  $2x$ , so we need  $(2x)^2 = 4x^2$  in the denominator:

$$L = 2 \lim_{x \rightarrow 0} \left[ \frac{\sin(2x)}{x} \right]^2$$

4. Multiply and divide inside the square by 2:

$$L = 2 \lim_{x \rightarrow 0} \left[ 2 \cdot \frac{\sin(2x)}{2x} \right]^2$$

5. Since  $\lim_{2x \rightarrow 0} \frac{\sin(2x)}{2x} = 1$ , we have:

$$L = 2 \times (2 \times 1)^2$$

$$L = 2 \times 4 = 8$$

**Final Answer:**

8

**Answer: (B)**

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Q4.

**Solution****Concept:**

In problems involving inverse trigonometric functions like  $y = \tan^{-1}(\dots)$ , the goal is usually to simplify the inner expression using trigonometric identities to match the outer function. Specifically, we aim to transform the bracket into  $\tan(A + B)$  or  $\tan(A - B)$ . The relevant identity is  $\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan x}{1 - \tan x}$ . Once simplified, the derivative becomes straightforward as  $\tan^{-1}(\tan \theta) = \theta$ .

**Solution:**

1. Start with the given expression:

$$y = \tan^{-1} \left( \frac{\sin x + \cos x}{\cos x - \sin x} \right)$$

2. To transform the expression into a tangent form, divide both the numerator and the denominator by  $\cos x$ :

$$y = \tan^{-1} \left( \frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}}{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}} \right)$$

3. Simplify the terms inside:

$$y = \tan^{-1} \left( \frac{\tan x + 1}{1 - \tan x} \right) = \tan^{-1} \left( \frac{1 + \tan x}{1 - \tan x} \right)$$

4. Recognize the identity  $\frac{1 + \tan x}{1 - \tan x} = \tan\left(\frac{\pi}{4} + x\right)$ :

$$y = \tan^{-1} \left[ \tan \left( \frac{\pi}{4} + x \right) \right]$$

5. Simplify the expression for  $y$ :

$$y = \frac{\pi}{4} + x$$

6. Differentiate with respect to  $x$ :

$$\frac{dy}{dx} = \frac{d}{dx} \left( \frac{\pi}{4} \right) + \frac{d}{dx} (x)$$

$$\frac{dy}{dx} = 0 + 1 = 1$$

**Final Answer:**

1

**Answer: (A)**

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Q5.

**Solution****Concept:**

Parametric differentiation is used when  $x$  and  $y$  are functions of a third variable, often called a parameter (here  $\theta$ ). The first derivative is found using  $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ . To find the second derivative  $\frac{d^2y}{dx^2}$ , we must differentiate  $\frac{dy}{dx}$  with respect to  $\theta$  and then multiply by  $\frac{d\theta}{dx}$  using the chain rule:

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta} \left( \frac{dy}{dx} \right) \cdot \frac{d\theta}{dx}$$

**Solution:**

1. Given  $x = a \cos^3 \theta$  and  $y = a \sin^3 \theta$ . Calculate the derivatives with respect to  $\theta$ :

$$\frac{dx}{d\theta} = 3a \cos^2 \theta (-\sin \theta) = -3a \cos^2 \theta \sin \theta$$

$$\frac{dy}{d\theta} = 3a \sin^2 \theta (\cos \theta) = 3a \sin^2 \theta \cos \theta$$

2. Find  $\frac{dy}{dx}$ :

$$\frac{dy}{dx} = \frac{3a \sin^2 \theta \cos \theta}{-3a \cos^2 \theta \sin \theta} = -\frac{\sin \theta}{\cos \theta} = -\tan \theta$$

3. Calculate the second derivative  $\frac{d^2y}{dx^2}$ :

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta} (-\tan \theta) \cdot \frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = (-\sec^2 \theta) \cdot \frac{1}{-3a \cos^2 \theta \sin \theta}$$

4. Simplify the expression:

$$\frac{d^2y}{dx^2} = \frac{1}{3a \cos^4 \theta \sin \theta}$$

5. Evaluate at  $\theta = \frac{\pi}{4}$ :

$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}, \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\frac{d^2y}{dx^2} = \frac{1}{3a \left(\frac{1}{\sqrt{2}}\right)^4 \left(\frac{1}{\sqrt{2}}\right)} = \frac{1}{3a \left(\frac{1}{4}\right) \left(\frac{1}{\sqrt{2}}\right)} = \frac{4\sqrt{2}}{3a}$$

**Final Answer:**

$$\boxed{\frac{4\sqrt{2}}{3a}}$$

**Answer: (A)**

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Q6.

**Solution****Concept:**

This is an infinite series problem involving a radical expression. For a function defined as  $y = \sqrt{f(x) + \sqrt{f(x) + \dots \infty}}$ , we can utilize the recursive nature of the expression to simplify it. Since the series is infinite, the part under the first radical is also equal to  $y$ . This transforms the transcendental equation into a simple algebraic equation  $y^2 = f(x) + y$ , which can then be differentiated implicitly.

**Solution:**

1. Write the given equation in its recursive form:

$$y = \sqrt{\sin x + y}$$

2. Square both sides to remove the radical:

$$y^2 = \sin x + y$$

3. Rearrange the equation to group the  $y$  terms:

$$y^2 - y = \sin x$$

4. Differentiate both sides with respect to  $x$  using the chain rule:

$$\frac{d}{dx}(y^2) - \frac{d}{dx}(y) = \frac{d}{dx}(\sin x)$$

$$2y \frac{dy}{dx} - \frac{dy}{dx} = \cos x$$

5. Factor out  $\frac{dy}{dx}$  on the left-hand side:

$$(2y - 1) \frac{dy}{dx} = \cos x$$

**Final Answer:**

$$\boxed{\cos x}$$

**Answer: (B)**[Go Back to Question 6](#)

Q7.

**Solution****Concept:**

A stationary point of a function  $f(x)$  occurs where its first derivative is zero ( $f'(x) = 0$ ). For functions of the form  $x^x$ , logarithmic differentiation is required. By taking the natural logarithm of both sides, we convert the exponentiation into a product, making it easier to apply the product rule. This specific function  $x^x$  is a classic example in MHT-CET for finding minima or stationary points.

**Solution:**

1. Let  $y = x^x$ . Take the natural logarithm on both sides:

$$\log y = \log(x^x) = x \log x$$

2. Differentiate both sides with respect to  $x$  using the product rule on the right:

$$\frac{1}{y} \frac{dy}{dx} = \frac{d}{dx}(x) \cdot \log x + x \cdot \frac{d}{dx}(\log x)$$

$$\frac{1}{y} \frac{dy}{dx} = 1 \cdot \log x + x \cdot \frac{1}{x}$$

$$\frac{1}{y} \frac{dy}{dx} = \log x + 1$$

3. Solve for  $f'(x)$  (which is  $\frac{dy}{dx}$ ):

$$f'(x) = y(1 + \log x) = x^x(1 + \log x)$$

4. Set the derivative to zero to find the stationary point:

$$x^x(1 + \log x) = 0$$

Since  $x^x$  is always positive for  $x > 0$ , we have:

$$1 + \log x = 0$$

5. Solve for  $x$ :

$$\log x = -1$$

$$x = e^{-1} = \frac{1}{e}$$

**Final Answer:**

$$\boxed{1/e}$$

**Answer: (B)**

[Go Back to Question 7](#)



Q8.

**Solution****Concept:**

The derivative of a function  $u$  with respect to another function  $v$  is calculated as  $\frac{du}{dv} = \frac{du/dx}{dv/dx}$ . It is also important to remember the base conversion formula for logarithms:  $\log_a b = \frac{\log_e b}{\log_e a}$ . Most differentiation rules for logarithms are derived using base  $e$ , so converting from base 10 is a mandatory first step.

**Solution:**

- Let  $u = \log_{10} x$  and  $v = x^2$ .
- Rewrite  $u$  using the base change formula:

$$u = \frac{\log_e x}{\log_e 10}$$

- Differentiate  $u$  with respect to  $x$ :

$$\frac{du}{dx} = \frac{1}{\log_e 10} \cdot \frac{1}{x} = \frac{1}{x \log_e 10}$$

- Differentiate  $v$  with respect to  $x$ :

$$\frac{dv}{dx} = 2x$$

- Apply the formula  $\frac{du}{dv} = \frac{du/dx}{dv/dx}$ :

$$\frac{du}{dv} = \frac{\frac{1}{x \log_e 10}}{2x}$$

$$\frac{du}{dv} = \frac{1}{2x^2 \log_e 10}$$

**Final Answer:**

$$\frac{1}{2x^2 \log_e 10}$$

**Answer: (A)**[Go Back to Question 8](#)

Q9.

**Solution****Concept:**

The equation of a tangent to a curve at a given point  $(x_1, y_1)$  is found using the point-slope form:  $y - y_1 = m(x - x_1)$ . The slope  $m$  is the value of the first derivative  $\frac{dy}{dx}$  evaluated at  $x = x_1$ . This problem requires finding both the  $y$ -coordinate and the derivative at the point where  $x = 0$ .

**Solution:**

1. Find the  $y$ -coordinate when  $x = 0$ :

$$y = 2(0)^2 + 3 \sin(0) = 0 + 0 = 0$$

So the point is  $(0, 0)$ .

2. Differentiate the curve equation with respect to  $x$ :

$$\frac{dy}{dx} = \frac{d}{dx}(2x^2 + 3 \sin x) = 4x + 3 \cos x$$

3. Calculate the slope  $m$  at  $x = 0$ :

$$m = \left. \frac{dy}{dx} \right|_{x=0} = 4(0) + 3 \cos(0) = 0 + 3(1) = 3$$

4. Use the point-slope form  $y - y_1 = m(x - x_1)$  with  $(0, 0)$  and  $m = 3$ :

$$y - 0 = 3(x - 0)$$

$$y = 3x$$

5. Rearrange into standard form:

$$3x - y = 0$$

**Final Answer:**

$$\boxed{3x - y = 0}$$

**Answer: (A)**[Go Back to Question 9](#)

## Q10.

**Solution****Concept:**

This is a Related Rates problem from the Applications of Derivatives. We use the formulas for the volume ( $V$ ) and surface area ( $S$ ) of a sphere:  $V = \frac{4}{3}\pi r^3$  and  $S = 4\pi r^2$ . Given the rate of change of volume ( $\frac{dV}{dt}$ ), we first find the rate of change of radius ( $\frac{dr}{dt}$ ), and then use it to find the rate of change of surface area ( $\frac{dS}{dt}$ ).

**Solution:**

1. Given  $\frac{dV}{dt} = 35$  cc/min and  $r = 10$  cm. 2. From  $V = \frac{4}{3}\pi r^3$ , differentiate with respect to  $t$ :

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Substitute the known values:

$$35 = 4\pi(10)^2 \frac{dr}{dt} = 400\pi \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{35}{400\pi}$$

3. From  $S = 4\pi r^2$ , differentiate with respect to  $t$ :

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

4. Substitute  $r = 10$  and  $\frac{dr}{dt} = \frac{35}{400\pi}$ :

$$\frac{dS}{dt} = 8\pi(10) \left( \frac{35}{400\pi} \right)$$

5. Simplify the expression:

$$\frac{dS}{dt} = 80\pi \cdot \frac{35}{400\pi} = \frac{80 \cdot 35}{400} = \frac{35}{5} = 7$$

**Final Answer:**

$$7 \text{ sq.cm/min}$$

**Answer: (A)**

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Q11.

**Solution****Concept:**

To find the maximum value of a function  $f(x)$ , we follow the first derivative test. A critical point exists where  $f'(x) = 0$ . Once the critical point  $x = c$  is found, we verify if it is a maximum by checking if  $f''(c) < 0$ . For the function  $f(x) = \frac{\log x}{x}$ , the domain is  $x > 0$  due to the logarithmic term. This specific function is a standard result in calculus often tested for its unique properties near  $x = e$ .

**Solution:**

1. Given function:

$$f(x) = \frac{\log x}{x}$$

2. Use the quotient rule to find  $f'(x)$ :

$$f'(x) = \frac{x \cdot \frac{d}{dx}(\log x) - \log x \cdot \frac{d}{dx}(x)}{x^2}$$

$$f'(x) = \frac{x \cdot \frac{1}{x} - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2}$$

3. Set  $f'(x) = 0$  to find stationary points:

$$1 - \log x = 0 \implies \log x = 1 \implies x = e$$

4. Determine the maximum value by substituting  $x = e$  back into  $f(x)$ :

$$f(e) = \frac{\log e}{e}$$

5. Since  $\log e = 1$ , we get:

$$f(e) = \frac{1}{e}$$

**Final Answer:**

$$\boxed{1/e}$$

**Answer: (B)**[Go Back to Question 11](#)

Q12.

**Solution****Concept:**

A function  $f(x)$  is said to be strictly increasing in an interval if its first derivative  $f'(x) > 0$  for all  $x$  in interval. For polynomial functions, this involves finding the derivative, factoring it, and solving the resulting quadratic inequality. We typically use the "Wavy Curve" method (sign scheme) to identify the regions where the derivative maintains a positive sign.

**Solution:**

1. Given the polynomial:

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

2. Differentiate with respect to  $x$ :

$$f'(x) = 6x^2 - 30x + 36$$

3. Factor out the constant 6:

$$f'(x) = 6(x^2 - 5x + 6)$$

4. Factor the quadratic expression:

$$f'(x) = 6(x - 2)(x - 3)$$

5. For  $f(x)$  to be increasing,  $f'(x) > 0$ :

$$6(x - 2)(x - 3) > 0$$

By the sign scheme (Wavy Curve), the expression  $(x - 2)(x - 3)$  is positive when  $x < 2$  or  $x > 3$ .  
In interval notation:  $x \in (-\infty, 2) \cup (3, \infty)$ .

**Final Answer:**

$$(-\infty, 2) \cup (3, \infty)$$

**Answer: (B)**

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Q13.

**Solution****Concept:**

Rolle's Theorem states that if a function  $f(x)$  is continuous on  $[a, b]$ , differentiable on  $(a, b)$ , and  $f(a) = f(b)$ , then there exists at least one  $c \in (a, b)$  such that  $f'(c) = 0$ . The constant  $c$  represents the point where the tangent to the curve is horizontal. For quadratic functions,  $c$  always lies exactly at the midpoint of the interval if  $f(a) = f(b)$ .

**Solution:**

1. Given  $f(x) = x^2 - 4x + 3$  on  $[1, 3]$ .

2. Verify  $f(a) = f(b)$ :  $f(1) = 1^2 - 4(1) + 3 = 0$   $f(3) = 3^2 - 4(3) + 3 = 9 - 12 + 3 = 0$  Since  $f(1) = f(3)$ , Rolle's Theorem is applicable.

3. Differentiate  $f(x)$ :

$$f'(x) = 2x - 4$$

4. Set  $f'(c) = 0$ :

$$2c - 4 = 0$$

$$2c = 4$$

$$c = 2$$

5. Check if  $c \in (1, 3)$ : Since 2 lies between 1 and 3, the value is valid.

**Final Answer:**

2

**Answer: (B)**

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## Q14.

**Solution****Concept:**

To integrate expressions of the form  $\frac{1}{x(x^n+1)}$ , a common algebraic manipulation is used. We multiply and divide the integrand by  $x^{n-1}$ . This substitution method transforms the denominator into a function of  $x^n$ , allowing us to use  $t = x^n$  as a variable. This effectively reduces a complex rational function into a simple logarithmic integral.

**Solution:**

1. Given integral:

$$I = \int \frac{1}{x(x^5+1)} dx$$

2. Multiply and divide by  $x^4$ :

$$I = \int \frac{x^4}{x^5(x^5+1)} dx$$

3. Let  $t = x^5$ , then  $dt = 5x^4 dx \implies x^4 dx = \frac{1}{5} dt$ :

$$I = \frac{1}{5} \int \frac{dt}{t(t+1)}$$

4. Use partial fractions or the identity  $\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$ :

$$I = \frac{1}{5} \int \left( \frac{1}{t} - \frac{1}{t+1} \right) dt$$

$$I = \frac{1}{5} [\log |t| - \log |t+1|] + C$$

5. Apply logarithmic property  $\log A - \log B = \log \frac{A}{B}$  and substitute  $t = x^5$ :

$$I = \frac{1}{5} \log \left| \frac{t}{t+1} \right| + C = \frac{1}{5} \log \left| \frac{x^5}{x^5+1} \right| + C$$

**Final Answer:**

$$\boxed{\frac{1}{5} \log \left| \frac{x^5}{x^5+1} \right| + C}$$

**Answer: (A)**

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Q15.

**Solution****Concept:**

This integral follows the special form  $\int e^x [f(x) + f'(x)] dx = e^x f(x) + C$ . To solve this, we must manipulate the trigonometric expression inside the brackets to identify a function and its derivative. Key identities involved are the half-angle formulas:  $1 + \cos x = 2 \cos^2(x/2)$  and  $\sin x = 2 \sin(x/2) \cos(x/2)$ .

**Solution:**

1. Start with the integrand:

$$\frac{1 + \sin x}{1 + \cos x} = \frac{1 + 2 \sin(x/2) \cos(x/2)}{2 \cos^2(x/2)}$$

2. Split the fraction into two parts:

$$= \frac{1}{2 \cos^2(x/2)} + \frac{2 \sin(x/2) \cos(x/2)}{2 \cos^2(x/2)}$$

3. Use reciprocal and tangent identities:

$$= \frac{1}{2} \sec^2(x/2) + \tan(x/2)$$

4. Identify  $f(x)$  and  $f'(x)$ : Let  $f(x) = \tan(x/2)$ . Then  $f'(x) = \frac{d}{dx} [\tan(x/2)] = \sec^2(x/2) \cdot \frac{1}{2} = \frac{1}{2} \sec^2(x/2)$ .

5. The integral is now in the form  $\int e^x [f'(x) + f(x)] dx$ :

$$I = e^x \tan(x/2) + C$$

**Final Answer:**

$$e^x \tan(x/2) + C$$

**Answer: (A)**

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## Q16.

## Solution

**Concept:**

This problem utilizes the fundamental property of definite integrals:

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

This is often referred to as the "King's Property." When the integrand is a rational function involving  $\sin x$  and  $\cos x$  with limits from 0 to  $\pi/2$ , applying this property usually results in a complementary form that, when added to the original integral, simplifies to a constant.

**Solution:**

1. Let the given integral be  $I$ :

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \dots (1)$$

2. Apply the property  $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ :

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)} + \sqrt{\cos(\pi/2-x)}} dx$$

3. Use the identities  $\sin(\pi/2-x) = \cos x$  and  $\cos(\pi/2-x) = \sin x$ :

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \dots (2)$$

4. Add equations (1) and (2):

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$2I = \int_0^{\pi/2} 1 \cdot dx$$

5. Integrate and evaluate the limits:

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$$

$$I = \frac{\pi}{4}$$

**Final Answer:**

$$\boxed{\pi/4}$$

**Answer: (C)**

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Q17.

**Solution****Concept:**

To integrate expressions involving  $e^x$  within a square root in the denominator, the substitution  $t = e^{-x}$  or multiplying numerator and denominator by  $e^{-x}$  is a powerful technique. This transforms the integral into a standard inverse trigonometric form, specifically  $\int \frac{-du}{\sqrt{1-u^2}}$ , which is  $\cos^{-1}(u) + C$  or  $-\sin^{-1}(u) + C$ .

**Solution:**

1. Given integral:

$$I = \int \frac{dx}{\sqrt{1-e^{2x}}}$$

2. Divide numerator and denominator by  $e^x$  (or multiply by  $e^{-x}$ ):

$$I = \int \frac{e^{-x} dx}{e^{-x} \sqrt{1-e^{2x}}} = \int \frac{e^{-x} dx}{\sqrt{(e^{-x})^2 - 1}}$$

\*Correction\*: Let's use the substitution  $u = e^{-x}$ . Then  $du = -e^{-x} dx$ . However, a cleaner approach is to factor out  $e^{2x}$  from the square root:

$$I = \int \frac{dx}{e^x \sqrt{e^{-2x} - 1}} = \int \frac{e^{-x} dx}{\sqrt{(e^{-x})^2 - 1}}$$

3. Let  $u = e^{-x}$ , then  $du = -e^{-x} dx \implies -du = e^{-x} dx$ . The integral becomes:

$$I = \int \frac{-du}{\sqrt{u^2 - 1}}$$

4. Using the standard formula  $\int \frac{du}{\sqrt{u^2-1}} = \log |u + \sqrt{u^2-1}|$ :

$$I = -\log |u + \sqrt{u^2-1}| + C$$

5. Substitute  $u = e^{-x}$  back:

$$I = -\log |e^{-x} + \sqrt{e^{-2x} - 1}| + C$$

**Final Answer:**

$$-\log |e^{-x} + \sqrt{e^{-2x} - 1}| + C$$

**Answer: (B)**

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Q18.

**Solution****Concept:**

The absolute value function  $|x|$  is defined as  $x$  for  $x \geq 0$  and  $-x$  for  $x < 0$ . When integrating  $|x|$  over an interval that spans both positive and negative values, we must split the integral at the point where the expression inside the modulus becomes zero ( $x = 0$  in this case). Geometrically, this represents the sum of the areas of two identical triangles formed by the line  $y = |x|$  and the x-axis.

**Solution:**

1. Identify the splitting point for  $|x|$ : The function changes at  $x = 0$ .
2. Split the integral:

$$\int_{-1}^1 |x| dx = \int_{-1}^0 (-x) dx + \int_0^1 x dx$$

3. Integrate each part:

$$\text{Part 1: } \left[ -\frac{x^2}{2} \right]_{-1}^0 = -(0) - \left( -\frac{(-1)^2}{2} \right) = \frac{1}{2}$$

$$\text{Part 2: } \left[ \frac{x^2}{2} \right]_0^1 = \frac{1^2}{2} - 0 = \frac{1}{2}$$

4. Sum the results:

$$I = \frac{1}{2} + \frac{1}{2} = 1$$

**Final Answer:**

1
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Answer: (B)
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## Q19.

**Solution****Concept:**

The area bounded by a parabola  $y^2 = 4ax$  and a vertical line  $x = h$  is given by the integral  $2 \int_0^h \sqrt{4ax} dx$ . The factor of 2 accounts for the symmetry of the parabola about the x-axis (both upper and lower halves). In MHT-CET, remembering the direct formula  $Area = \frac{8}{3} \sqrt{a} \cdot h^{3/2}$  can save significant time.

**Solution:**

1. Given the parabola  $y^2 = 4x$ . Here,  $4a = 4 \implies a = 1$ . The line is  $x = 3$ .
2. The area is symmetric about the x-axis. We calculate the area in the first quadrant and double it:

$$A = 2 \int_0^3 y dx = 2 \int_0^3 \sqrt{4x} dx$$

3. Simplify the integrand:

$$A = 2 \cdot 2 \int_0^3 x^{1/2} dx = 4 \int_0^3 x^{1/2} dx$$

4. Integrate using the power rule:

$$A = 4 \left[ \frac{x^{3/2}}{3/2} \right]_0^3 = 4 \cdot \frac{2}{3} [x^{3/2}]_0^3$$

$$A = \frac{8}{3} [3\sqrt{3} - 0]$$

5. Final simplification:

$$A = 8\sqrt{3}$$

**Final Answer:**

$$8\sqrt{3} \text{ sq. units}$$

**Answer: (B)**

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Q20.

**Solution****Concept:**

When calculating the "Area" (not the definite integral) of a trigonometric function like  $\sin x$  over a full period  $[0, 2\pi]$ , we must treat the areas above and below the  $x$ -axis as positive. While the definite integral  $\int_0^{2\pi} \sin x dx = 0$ , the physical area is the sum of the absolute values:  $\int_0^{\pi} |\sin x| dx + \int_{\pi}^{2\pi} |\sin x| dx$ .

**Solution:**

1. The function  $y = \sin x$  is positive on  $[0, \pi]$  and negative on  $[\pi, 2\pi]$ . The total area  $A$  is:

$$A = \int_0^{\pi} \sin x dx + \left| \int_{\pi}^{2\pi} \sin x dx \right|$$

2. Evaluate the first part:

$$\int_0^{\pi} \sin x dx = [-\cos x]_0^{\pi} = -(\cos \pi - \cos 0) = -(-1 - 1) = 2$$

3. Evaluate the second part:

$$\int_{\pi}^{2\pi} \sin x dx = [-\cos x]_{\pi}^{2\pi} = -(\cos 2\pi - \cos \pi) = -(1 - (-1)) = -2$$

4. Take the absolute value of the negative area and add:

$$A = 2 + |-2| = 4$$

**Final Answer:**

4 sq. units

Answer: (B)

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Q21.

**Solution****Concept:**

The **Order** of a differential equation is the order of the highest derivative present. The **Degree** is the power of the highest order derivative after the equation is made free from radical signs and fractions. To find the degree correctly, one must eliminate any square roots, cube roots, or fractional exponents involving derivatives.

**Solution:**

1. Given equation:

$$\frac{d^2y}{dx^2} + \sqrt{1 + \left(\frac{dy}{dx}\right)^3} = 0$$

2. Identify the highest order derivative: The highest derivative is  $\frac{d^2y}{dx^2}$ . Therefore, the **Order** = 2.

3. To find the degree, isolate the radical and square both sides:

$$\frac{d^2y}{dx^2} = -\sqrt{1 + \left(\frac{dy}{dx}\right)^3}$$

$$\left(\frac{d^2y}{dx^2}\right)^2 = \left(-\sqrt{1 + \left(\frac{dy}{dx}\right)^3}\right)^2$$

$$\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^3$$

4. Identify the power of the highest order derivative: The highest order derivative is  $\frac{d^2y}{dx^2}$ , and its power is 2. Therefore, the **Degree** = 2.

**Final Answer:**

2, 2
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Answer: (A)
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Q22.

**Solution****Concept:**

This is a **Linear Differential Equation** of the first order, which follows the general form  $\frac{dy}{dx} + Py = Q$ , where  $P$  and  $Q$  are functions of  $x$ . The solution is found using the formula:

$$y \cdot (\text{I.F.}) = \int Q \cdot (\text{I.F.}) dx + C$$

where the Integrating Factor (I.F.) is  $e^{\int P dx}$ .

**Solution:**

1. Identify  $P$  and  $Q$ :  $P = \cot x$ ,  $Q = 2 \cos x$ .
2. Find the Integrating Factor (I.F.):

$$\int P dx = \int \cot x dx = \log(\sin x)$$

$$\text{I.F.} = e^{\log(\sin x)} = \sin x$$

3. Apply the general solution formula:

$$y \cdot \sin x = \int (2 \cos x \cdot \sin x) dx + C$$

4. Integrate the right side: Using the identity  $2 \sin x \cos x = \sin 2x$ :

$$y \sin x = \int \sin 2x dx + C$$

$$y \sin x = -\frac{\cos 2x}{2} + C$$

Note: Another form of  $\int 2 \sin x \cos x dx$  is  $\sin^2 x + C'$ . Comparing with options, we use the  $\sin^2 x$  form.

$$y \sin x = \sin^2 x + C$$

**Final Answer:**

$$y \sin x = \sin^2 x + C$$

**Answer: (A)**

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Q23.

**Solution****Concept:**

To find the **Integrating Factor (I.F.)**, the differential equation must first be written in the standard form  $\frac{dy}{dx} + Py = Q$ . The I.F. is then computed as  $e^{\int P dx}$ . This factor is essential as it makes the left side of the equation a perfect derivative of  $(y \cdot \text{I.F.})$ .

**Solution:**

1. Rewrite the given equation  $(1 + x^2)\frac{dy}{dx} + 2xy = \cos x$  in standard form: Divide the whole equation by  $(1 + x^2)$ :

$$\frac{dy}{dx} + \left( \frac{2x}{1 + x^2} \right) y = \frac{\cos x}{1 + x^2}$$

2. Identify  $P$ :

$$P = \frac{2x}{1 + x^2}$$

3. Calculate  $\int P dx$ : Using the substitution  $u = 1 + x^2$ ,  $du = 2x dx$ :

$$\int \frac{2x}{1 + x^2} dx = \log(1 + x^2)$$

4. Calculate the I.F.:

$$\text{I.F.} = e^{\int P dx} = e^{\log(1+x^2)}$$

Using the identity  $e^{\log f(x)} = f(x)$ :

$$\text{I.F.} = 1 + x^2$$

**Final Answer:**

$$\boxed{1 + x^2}$$

**Answer: (B)**[Go Back to Question 23](#)

Q24.

**Solution****Concept:**

The **Amplitude** or **Argument** of a complex number  $z = x + iy$  is the angle  $\theta$  that the line representing  $z$  makes with the positive x-axis in the Argand plane. It is calculated as  $\theta = \tan^{-1} \left( \frac{y}{x} \right)$ .

The quadrant in which  $z$  lies must be checked to determine the correct principal argument.

**Solution:**

1. Identify  $x$  and  $y$  from  $z = 1 + i\sqrt{3}$ :  $x = 1$ ,  $y = \sqrt{3}$ .
2. Determine the quadrant: Since both  $x > 0$  and  $y > 0$ , the point lies in the **First Quadrant**.
3. Calculate the argument  $\theta$ :

$$\theta = \tan^{-1} \left( \frac{\sqrt{3}}{1} \right)$$

$$\theta = \tan^{-1}(\sqrt{3})$$

4. Find the angle: We know that  $\tan 60^\circ = \sqrt{3}$ . Therefore,  $\theta = 60^\circ$  (or  $\frac{\pi}{3}$  radians).

**Final Answer:**

$$60^\circ$$

**Answer: (B)**[Go Back to Question 24](#)

Q25.

**Solution****Concept:**

When dealing with high powers of  $(1 \pm i)$ , it is often easier to simplify the squared term first. Note that  $(1 + i)^2 = 1 + 2i + i^2 = 2i$  and  $(1 - i)^2 = 1 - 2i + i^2 = -2i$ . This conversion reduces the problem from complex binomial expansion to simple exponential arithmetic with  $i$ .

**Solution:**

1. Simplify  $(1 + i)^2$  and  $(1 - i)^2$ :

$$(1 + i)^2 = 1 + 2i - 1 = 2i$$

$$(1 - i)^2 = 1 - 2i - 1 = -2i$$

2. Rewrite the given expression:

$$(1 + i)^{10} + (1 - i)^{10} = [(1 + i)^2]^5 + [(1 - i)^2]^5$$

3. Substitute the simplified values:

$$(2i)^5 + (-2i)^5$$

4. Expand the terms:

$$2^5 i^5 + (-2)^5 i^5$$

$$32i^5 - 32i^5$$

5. Calculate the result:

$$32i^5 - 32i^5 = 0$$

Note:  $i^5 = i$ , so  $32i - 32i = 0$ .

**Final Answer:**

$$\boxed{0}$$

**Answer: (A)**

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Q26.

**Solution****Concept:**

When working with the complex cube roots of unity ( $\omega$ ), two fundamental properties are used: 1.  $1 + \omega + \omega^2 = 0$  2.  $\omega^3 = 1$  These identities allow us to simplify complex expressions by replacing pairs of terms (like  $1 + \omega^2$ ) with a single term (like  $-\omega$ ). This significantly reduces the power to be calculated.

**Solution:**

- From  $1 + \omega + \omega^2 = 0$ , we can derive:  $1 + \omega^2 = -\omega$  and  $1 + \omega = -\omega^2$ .
- Substitute these into the given expression:

$$\begin{aligned} & (1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5 \\ &= ((1 + \omega^2) - \omega)^5 + ((1 + \omega) - \omega^2)^5 \end{aligned}$$

- Substitute the values from step 1:

$$\begin{aligned} &= (-\omega - \omega)^5 + (-\omega^2 - \omega^2)^5 \\ &= (-2\omega)^5 + (-2\omega^2)^5 \end{aligned}$$

- Expand the powers:

$$= -32\omega^5 - 32\omega^{10}$$

- Simplify using  $\omega^3 = 1$ :  $\omega^5 = \omega^3 \cdot \omega^2 = \omega^2$   
 $\omega^{10} = (\omega^3)^3 \cdot \omega = \omega$

$$= -32\omega^2 - 32\omega$$

- Factor out  $-32$ :

$$= -32(\omega^2 + \omega)$$

Since  $1 + \omega + \omega^2 = 0 \implies \omega + \omega^2 = -1$ :

$$= -32(-1) = 32$$

**Final Answer:**

32

**Answer: (A)**[Go Back to Question 26](#)

Q27.

**Solution****Concept:**

For a quadratic equation  $ax^2 + bx + c = 0$ , let the roots be  $\alpha$  and  $\beta$ . We know that  $\alpha + \beta = -b/a$  and  $\alpha\beta = c/a$ . The difference of roots is given by  $|\alpha - \beta| = \frac{\sqrt{D}}{|a|}$ , where  $D = b^2 - 4ac$  is the discriminant. If the roots differ by unity, then  $|\alpha - \beta| = 1$ .

**Solution:**

1. Given the equation  $x^2 - px + q = 0$ . Here,  $a = 1, b = -p, c = q$ .
2. Sum of roots  $\alpha + \beta = p$ . Product of roots  $\alpha\beta = q$ .
3. It is given that the roots differ by unity:

$$|\alpha - \beta| = 1$$

4. Square both sides:

$$(\alpha - \beta)^2 = 1^2 = 1$$

5. Rewrite the squared difference in terms of sum and product:

$$(\alpha + \beta)^2 - 4\alpha\beta = 1$$

6. Substitute the values of  $p$  and  $q$ :

$$p^2 - 4q = 1$$

$$p^2 = 4q + 1$$

**Final Answer:**

$$p^2 = 4q + 1$$

**Answer: (A)**[Go Back to Question 27](#)

Q28.

**Solution****Concept:**

A quadratic equation  $Ax^2 + Bx + C = 0$  has equal roots if and only if its discriminant  $D$  is zero ( $D = B^2 - 4AC = 0$ ). In this problem, we first need to expand the given expression into the standard quadratic form before identifying the coefficients.

**Solution:**

1. Expand the given equation  $kx(x - 2) + 6 = 0$ :

$$kx^2 - 2kx + 6 = 0$$

2. Identify the coefficients:  $A = k, B = -2k, C = 6$ .  
3. For equal roots, set the discriminant to zero:

$$B^2 - 4AC = 0$$

$$(-2k)^2 - 4(k)(6) = 0$$

4. Solve the resulting equation:

$$4k^2 - 24k = 0$$

$$4k(k - 6) = 0$$

5. Determine the possible values of  $k$ :  $k = 0$  or  $k = 6$ . However, if  $k = 0$ , the equation is no longer quadratic ( $6 = 0$ , which is impossible). Therefore,  $k$  must be 6.

**Final Answer:**

6
---

Answer: (A)
-------------

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Q29.

**Solution****Concept:**

The roots of the equation  $x^2 + x + 1 = 0$  are the non-real cube roots of unity,  $\omega$  and  $\omega^2$ . This is a standard result that appears frequently in competitive exams. Since these roots satisfy  $\omega^3 = 1$  and  $1 + \omega + \omega^2 = 0$ , we can evaluate high powers of  $\alpha$  and  $\beta$  by reducing them modulo 3.

**Solution:**

1. The roots of  $x^2 + x + 1 = 0$  are  $\alpha = \omega$  and  $\beta = \omega^2$ .

2. We need to find  $\alpha^{19} + \beta^{19}$ :

$$\alpha^{19} + \beta^{19} = \omega^{19} + (\omega^2)^{19}$$

3. Simplify  $\omega^{19}$ : Since  $\omega^3 = 1$ , divide the power 19 by 3. The remainder is 1.

$$\omega^{19} = (\omega^3)^6 \cdot \omega^1 = (1)^6 \cdot \omega = \omega$$

4. Simplify  $\omega^{38}$ : Divide the power 38 by 3. The remainder is 2.

$$\omega^{38} = (\omega^3)^{12} \cdot \omega^2 = (1)^{12} \cdot \omega^2 = \omega^2$$

5. Combine the results:

$$\alpha^{19} + \beta^{19} = \omega + \omega^2$$

6. Use the identity  $1 + \omega + \omega^2 = 0$ :

$$\omega + \omega^2 = -1$$

**Final Answer:**

$$\boxed{-1}$$

**Answer: (B)**

[Go Back to Question 29](#)



Q30.

**Solution****Concept:**

In a sequence where the differences between consecutive terms form an Arithmetic Progression (A.P.), the sequence itself is a quadratic sequence of the form  $T_n = an^2 + bn + c$ . To find the general term, we can set up a system of linear equations by substituting  $n = 1, 2, 3$  or use the method of differences.

**Solution:**

1. Given series: 5, 11, 19, 29, ... First Differences:  $11 - 5 = 6, 19 - 11 = 8, 29 - 19 = 10, \dots$   
The differences (6, 8, 10, ...) are in A.P.

2. Since the first differences are in A.P., let  $T_n = an^2 + bn + c$ . For  $n = 1 : a(1)^2 + b(1) + c = 5 \implies a + b + c = 5$

For  $n = 2 : a(2)^2 + b(2) + c = 11 \implies 4a + 2b + c = 11$

For  $n = 3 : a(3)^2 + b(3) + c = 19 \implies 9a + 3b + c = 19$

3. Solve for  $a, b, c$ :  $(4a + 2b + c) - (a + b + c) \implies 3a + b = 6$

$(9a + 3b + c) - (4a + 2b + c) \implies 5a + b = 8$

4. Subtracting  $(3a + b = 6)$  from  $(5a + b = 8)$ :  $2a = 2 \implies a = 1$ . Substitute  $a = 1$  into  $3a + b = 6 \implies 3(1) + b = 6 \implies b = 3$ . Substitute  $a = 1, b = 3$  into  $a + b + c = 5 \implies 1 + 3 + c = 5 \implies c = 1$ .

5. The  $n^{\text{th}}$  term is:

$$T_n = 1n^2 + 3n + 1 = n^2 + 3n + 1$$

**Final Answer:**

$$n^2 + 3n + 1$$

**Answer: (A)**

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Q31.

**Solution****Concept:**

For an Arithmetic Progression (A.P.), the  $n^{\text{th}}$  term is given by  $T_n = a + (n - 1)d$  and the sum of the first  $n$  terms is given by  $S_n = \frac{n}{2}[2a + (n - 1)d]$ . Here,  $a$  is the first term and  $d$  is the common difference. To solve this, we establish two linear equations based on the given terms to find  $a$  and  $d$ , then apply the sum formula.

**Solution:**

1. Given:  $T_5 = a + 4d = 30 \quad \dots (1)$   $T_{12} = a + 11d = 65 \quad \dots (2)$
2. Subtract equation (1) from (2):  $(a + 11d) - (a + 4d) = 65 - 30$   $7d = 35 \implies d = 5$
3. Substitute  $d = 5$  into equation (1):  $a + 4(5) = 30$   $a + 20 = 30 \implies a = 10$
4. Calculate the sum of the first 20 terms ( $S_{20}$ ):

$$S_{20} = \frac{20}{2}[2(10) + (20 - 1)5]$$

$$S_{20} = 10[20 + 19 \times 5]$$

5. Simplify:

$$S_{20} = 10[20 + 95] = 10[115] = 1150$$

**Final Answer:**

1150
------

Answer: (A)
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Q32.

**Solution****Concept:**

A Geometric Progression (G.P.) has a sum to infinity if the absolute value of the common ratio  $|r|$  is less than 1. The formula for the sum to infinity is  $S_{\infty} = \frac{a}{1-r}$ , where  $a$  is the first term. This formula is derived from the limit of the finite sum formula as  $n \rightarrow \infty$ .

**Solution:**

1. Identify the first term and common ratio: First term  $a = 1$ . Common ratio  $r = \frac{2/3}{1} = \frac{2}{3}$ .
2. Check the condition for convergence: Since  $|r| = |2/3| < 1$ , the sum to infinity exists.
3. Apply the  $S_{\infty}$  formula:

$$S_{\infty} = \frac{a}{1-r}$$

$$S_{\infty} = \frac{1}{1-2/3}$$

4. Simplify the denominator:  $1 - 2/3 = 1/3$ .
5. Final calculation:

$$S_{\infty} = \frac{1}{1/3} = 3$$

**Final Answer:**

3
---

Answer: (A)
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Q33.

**Solution****Concept:**

In the binomial expansion of  $(a + b)^n$ , the general term is given by  $T_{r+1} = {}^n C_r \cdot a^{n-r} \cdot b^r$ . To find the coefficient of a specific power of  $x$ , we identify the value of  $r$  that results in that power and then evaluate the corresponding term.

**Solution:**

1. Given the expression  $(1 + 2x)^{10}$ . Here,  $a = 1, b = 2x, n = 10$ .
2. Write the general term  $T_{r+1}$ :

$$T_{r+1} = {}^{10} C_r \cdot (1)^{10-r} \cdot (2x)^r$$

$$T_{r+1} = {}^{10} C_r \cdot 2^r \cdot x^r$$

3. We need the coefficient of  $x^4$ , so set  $r = 4$ .
4. Calculate the term: Coefficient =  ${}^{10} C_4 \cdot 2^4$ .
5. Evaluate  ${}^{10} C_4$ :

$${}^{10} C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 10 \times 3 \times 7 = 210$$

6. Final form: Coefficient =  $210 \times 2^4$ .

**Final Answer:**

$$210 \times 2^4$$

**Answer: (A)**[Go Back to Question 33](#)

Q34.

**Solution****Concept:**

The middle term in the expansion of  $(x + y)^n$  depends on whether  $n$  is even or odd. If  $n$  is even, there is only one middle term, which is the  $(\frac{n}{2} + 1)^{th}$  term. If  $n$  is odd, there are two middle terms. In this problem,  $n = 10$  (even), so the middle term is the  $6^{th}$  term ( $r = 5$ ).

**Solution:**

1. Given  $(x + \frac{1}{x})^{10}$ , where  $n = 10$ . 2. Since  $n$  is even, the middle term is:

$$\text{Middle term} = T_{\frac{10}{2}+1} = T_6$$

3. For  $T_6$ , we use  $r = 5$  in the general term formula:

$$T_{r+1} = {}^n C_r \cdot a^{n-r} \cdot b^r$$

4. Substitute the values:

$$T_6 = {}^{10} C_5 \cdot x^{10-5} \cdot \left(\frac{1}{x}\right)^5$$

$$T_6 = {}^{10} C_5 \cdot x^5 \cdot \frac{1}{x^5}$$

5. Simplify the expression:

$$T_6 = {}^{10} C_5$$

**Final Answer:**

$$\boxed{{}^{10} C_5}$$

**Answer: (A)**

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Q35.

**Solution****Concept:**

The number of permutations of  $n$  objects, where  $p$  objects are of one kind,  $q$  objects are of another kind, and  $r$  objects are of a third kind, is given by  $\frac{n!}{p!q!r!}$ . This concept is used to find the number of arrangements of letters in a word where some letters repeat.

**Solution:**

1. Identify the total number of letters in "MAHARASHTRA": M - 1, A - 4, H - 2, R - 2, S - 1, T - 1. Total letters ( $n$ ) = 1 + 4 + 2 + 2 + 1 + 1 = 11.
2. Identify the repeating letters: 'A' repeats 4 times. 'H' repeats 2 times. 'R' repeats 2 times.
3. Apply the permutation formula: Number of words =  $\frac{11!}{4!2!2!}$
4. This matches option (A).

**Final Answer:**

$$\frac{11!}{4!2!2!}$$

**Answer: (A)**[Go Back to Question 35](#)

Q36.

**Solution****Concept:**

This problem involves the **Hypergeometric Distribution** principle, commonly used in selection problems without replacement. The probability is calculated by taking the number of favorable ways to choose the specific objects divided by the total number of ways to choose the same number of objects from the entire set. The formula for combinations,  ${}^n C_r$ , is essential here.

**Solution:**

1. Identify the total number of balls and the number of balls to be drawn: Total balls = 5 Red + 3 Blue = 8 balls. Number of balls drawn = 3. Total ways to draw 3 balls:  $n(S) = {}^8 C_3$ .
2. Calculate  $n(S)$ :

$${}^8 C_3 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

3. Identify the favorable event: We need exactly 2 Red balls and 1 Blue ball (since 3 are drawn in total). Ways to choose 2 Red from 5:  ${}^5 C_2$ . Ways to choose 1 Blue from 3:  ${}^3 C_1$ .  $n(E) = {}^5 C_2 \times {}^3 C_1$ .
4. Calculate  $n(E)$ :  ${}^5 C_2 = \frac{5 \times 4}{2 \times 1} = 10$ .  ${}^3 C_1 = 3$ .  $n(E) = 10 \times 3 = 30$ .
5. Calculate Probability:

$$P(E) = \frac{n(E)}{n(S)} = \frac{30}{56}$$

**Final Answer:**

$$30/56$$

**Answer: (C)**[Go Back to Question 36](#)

Q37.

**Solution****Concept:**

This problem tests the understanding of **Conditional Probability** and the **Addition Theorem of Probability**. The conditional probability formula is  $P(B|A) = \frac{P(A \cap B)}{P(A)}$ , and the addition theorem is  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ . We must find the intersection first to determine the union.

**Solution:**

1. Given values:  $P(A) = 0.4$ ,  $P(B) = 0.8$ , and  $P(B|A) = 0.6$ .
2. Find  $P(A \cap B)$  using the conditional probability formula:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \implies P(A \cap B) = P(B|A) \times P(A)$$

$$P(A \cap B) = 0.6 \times 0.4 = 0.24$$

3. Use the Addition Theorem to find  $P(A \cup B)$ :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

4. Substitute the values:

$$P(A \cup B) = 0.4 + 0.8 - 0.24$$

$$P(A \cup B) = 1.2 - 0.24 = 0.96$$

**Final Answer:**

0.96
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Answer: (B)
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[Go Back to Question 37](#)

Q38.

**Solution****Concept:**

When two dice are thrown, the total number of outcomes in the sample space is  $6 \times 6 = 36$ . For sum-related problems, it is helpful to list the pairs  $(x, y)$  where  $x + y = S$ . In MHT-CET, knowing that the most frequent sum is 7 (occurring 6 times) is a useful shortcut for speed.

**Solution:**

1. Total outcomes in sample space  $n(S)$ :

$$n(S) = 6^2 = 36$$

2. Identify the favorable outcomes where the sum is 7: The pairs are: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1).

3. Count the favorable outcomes  $n(E)$ :  $n(E) = 6$ .

4. Calculate the probability:

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36}$$

5. Simplify the fraction:

$$P(E) = \frac{1}{6}$$

**Final Answer:**

$$\boxed{1/6}$$

**Answer: (A)**[Go Back to Question 38](#)

Q39.

**Solution****Concept:**

The distance between two parallel lines  $ax + by + c_1 = 0$  and  $ax + by + c_2 = 0$  is given by the formula:

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

It is crucial to ensure that the coefficients of  $x$  and  $y$  are identical in both equations before applying this formula. If they are not, one equation must be scaled accordingly.

**Solution:**

1. Identify the coefficients and constants from the lines: Line 1:  $3x + 4y + 5 = 0 \implies a = 3, b = 4, c_1 = 5$ . Line 2:  $3x + 4y + 15 = 0 \implies a = 3, b = 4, c_2 = 15$ .

2. Apply the distance formula:

$$d = \frac{|5 - 15|}{\sqrt{3^2 + 4^2}}$$

3. Calculate the numerator:  $|5 - 15| = |-10| = 10$ .

4. Calculate the denominator:  $\sqrt{9 + 16} = \sqrt{25} = 5$ .

5. Final result:

$$d = \frac{10}{5} = 2 \text{ units}$$

**Final Answer:**

2 units

**Answer: (A)**

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Q40.

**Solution****Concept:**

The acute angle  $\theta$  between two lines with slopes  $m_1$  and  $m_2$  is found using the formula:

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

The absolute value ensures that we obtain the acute angle. If the denominator  $1 + m_1 m_2 = 0$ , the lines are perpendicular ( $90^\circ$ ). If  $m_1 = m_2$ , the lines are parallel.

**Solution:**

1. Identify the slopes of the given lines: Line 1:  $y = 3x + 7 \implies m_1 = 3$ . Line 2:  $y = \frac{1}{2}x + 2 \implies m_2 = 1/2$ .
2. Substitute into the angle formula:

$$\tan \theta = \left| \frac{3 - 1/2}{1 + 3(1/2)} \right|$$

3. Simplify the numerator:  $3 - 0.5 = 2.5$  or  $5/2$ .
4. Simplify the denominator:  $1 + 1.5 = 2.5$  or  $5/2$ .
5. Calculate  $\tan \theta$ :

$$\tan \theta = \left| \frac{5/2}{5/2} \right| = 1$$

6. Find the angle  $\theta$ : Since  $\tan \theta = 1$ ,  $\theta = 45^\circ$ .

**Final Answer:**

$45^\circ$
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<b>Answer: (A)</b>
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Q41.

**Solution****Concept:**

When a circle passes through the origin  $(0, 0)$  and has intercepts  $a$  and  $b$  on the  $x$  and  $y$  axes respectively, it passes through the points  $(a, 0)$  and  $(0, b)$ . The general equation of a circle is  $x^2 + y^2 + 2gx + 2fy + c = 0$ . Since it passes through the origin,  $c = 0$ . The intercepts on the axes are given by  $-2g$  and  $-2f$ .

**Solution:**

1. Since the circle passes through the origin  $(0, 0)$ , the constant term  $c = 0$ . Let the equation be  $x^2 + y^2 + 2gx + 2fy = 0$ .

2. The  $x$ -intercept is 4. The point  $(4, 0)$  lies on the circle:  $4^2 + 0^2 + 2g(4) + 2f(0) = 0$   
 $16 + 8g = 0 \implies g = -2$ .

3. The  $y$ -intercept is 6. The point  $(0, 6)$  lies on the circle:  $0^2 + 6^2 + 2g(0) + 2f(6) = 0$   
 $36 + 12f = 0 \implies f = -3$ .

4. Substitute  $g = -2$  and  $f = -3$  back into the equation:  $x^2 + y^2 + 2(-2)x + 2(-3)y = 0$   
 $x^2 + y^2 - 4x - 6y = 0$ .

**Final Answer:**

$$x^2 + y^2 - 4x - 6y = 0$$

**Answer: (A)**[Go Back to Question 41](#)

Q42.

**Solution****Concept:**

For a circle given in the general form  $x^2 + y^2 + 2gx + 2fy + c = 0$ , the center is located at  $(-g, -f)$  and the radius  $r$  is calculated using the formula:

$$r = \sqrt{g^2 + f^2 - c}$$

This is a fundamental skill in conic sections, requiring careful identification of signs when extracting  $g$  and  $f$  from the coefficients of  $x$  and  $y$ .

**Solution:**

1. Given equation:  $x^2 + y^2 - 6x + 4y - 12 = 0$ . Compare this with  $x^2 + y^2 + 2gx + 2fy + c = 0$ .
2. Identify coefficients:  $2g = -6 \implies g = -3$   $2f = 4 \implies f = 2$   $c = -12$ .
3. Find the center: Center =  $(-g, -f) = (-(-3), -(2)) = (3, -2)$ .
4. Calculate the radius  $r$ :

$$r = \sqrt{(-3)^2 + (2)^2 - (-12)}$$

$$r = \sqrt{9 + 4 + 12} = \sqrt{25} = 5$$

**Final Answer:**

$$(3, -2), 5$$

**Answer: (A)**[Go Back to Question 42](#)

Q43.

**Solution****Concept:**

Standard parabolas take four primary forms. The parabola  $y^2 = -4ax$  opens to the left. For this form, the vertex is at  $(0, 0)$ , the focus is at  $(-a, 0)$ , and the equation of the directrix is  $x = a$ . Identifying the correct orientation and the value of  $a$  is the first step in solving any parabola problem.

**Solution:**

1. Given the parabola  $y^2 = -12x$ . This is of the form  $y^2 = -4ax$ .
2. Find the value of  $a$ :  $4a = 12 \implies a = 3$ .
3. Identify the focus: For  $y^2 = -4ax$ , the focus is at  $(-a, 0)$ . Since  $a = 3$ , the focus is  $(-3, 0)$ .
4. The point  $(-3, 0)$  lies on the negative  $x$ -axis, consistent with a parabola opening to the left.

**Final Answer:**

$$(-3, 0)$$

**Answer: (C)**[Go Back to Question 43](#)

Q44.

**Solution****Concept:**

For an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  where  $a > b$ , the eccentricity  $e$  measures how "flat" the ellipse is. It is calculated using the relation:

$$b^2 = a^2(1 - e^2)$$

Rearranging this formula gives  $e = \sqrt{1 - \frac{b^2}{a^2}}$ . The eccentricity of an ellipse is always less than 1.

**Solution:**

1. Given the ellipse  $\frac{x^2}{25} + \frac{y^2}{16} = 1$ . Identify  $a^2$  and  $b^2$ :  $a^2 = 25$  and  $b^2 = 16$ .
2. Since  $a^2 > b^2$ , the major axis is along the x-axis.
3. Use the eccentricity formula:

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$e^2 = 1 - \frac{16}{25}$$

4. Simplify the fraction:

$$e^2 = \frac{25 - 16}{25} = \frac{9}{25}$$

5. Calculate  $e$ :

$$e = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

**Final Answer:**

$$\boxed{\frac{3}{5}}$$

**Answer: (A)**[Go Back to Question 44](#)

Q45.

**Solution****Concept:**

A hyperbola centered at the origin with vertices on the x-axis has the standard equation  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ . The distance from the center to the vertices is  $a$ , and the distance to the foci is  $c = ae$ . The relationship between the parameters is  $c^2 = a^2 + b^2$ .

**Solution:**

1. Given vertices are  $(\pm 3, 0)$ . This implies  $a = 3 \implies a^2 = 9$ .
2. Given foci are  $(\pm 5, 0)$ . This implies  $c = 5 \implies c^2 = 25$ .
3. Use the hyperbola relation  $c^2 = a^2 + b^2$  to find  $b^2$ :  $25 = 9 + b^2$   $b^2 = 25 - 9 = 16$ .
4. Write the equation using  $a^2 = 9$  and  $b^2 = 16$ :

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

**Final Answer:**

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

**Answer: (A)**[Go Back to Question 45](#)

Q46.

**Solution****Concept:**

The cross product of two vectors  $\vec{a}$  and  $\vec{b}$  results in a vector perpendicular to both. The magnitude  $|\vec{a} \times \vec{b}|$  represents the area of a parallelogram with sides  $\vec{a}$  and  $\vec{b}$ . To calculate it, we use the determinant of a  $3 \times 3$  matrix with unit vectors  $\hat{i}, \hat{j}, \hat{k}$  in the first row.

**Solution:**

1. Given  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ . Calculate  $\vec{a} \times \vec{b}$  using the determinant:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & -2 \end{vmatrix}$$

2. Expand the determinant along the first row:  $\hat{i}[(-1)(-2) - (1)(1)] - \hat{j}[(2)(-2) - (1)(1)] + \hat{k}[(2)(1) - (-1)(1)]$

$$= \hat{i}[2 - 1] - \hat{j}[-4 - 1] + \hat{k}[2 + 1]$$

$$= \hat{i} + 5\hat{j} + 3\hat{k}$$

3. Find the magnitude:

$$|\vec{a} \times \vec{b}| = \sqrt{1^2 + 5^2 + 3^2}$$

$$= \sqrt{1 + 25 + 9}$$

$$= \sqrt{35}$$

**Final Answer:**

$$\boxed{\sqrt{35}}$$

**Answer: (A)**[Go Back to Question 46](#)

Q47.

**Solution****Concept:**

Two vectors  $\vec{u}$  and  $\vec{v}$  are collinear if one is a scalar multiple of the other ( $\vec{u} = \lambda\vec{v}$ ). This implies that their corresponding components must be in the same ratio:

$$\frac{x_1}{x_2} = \frac{y_1}{y_2} = \frac{z_1}{z_2}$$

Using this proportionality is the most efficient way to find unknown variables in collinearity problems.

**Solution:**

1. Let  $\vec{u} = 2\hat{i} - 3\hat{j} + 4\hat{k}$  and  $\vec{v} = a\hat{i} + 6\hat{j} - 8\hat{k}$ . 2. Since the vectors are collinear, equate the ratios of their components:

$$\frac{2}{a} = \frac{-3}{6} = \frac{4}{-8}$$

3. Simplify the known ratios:  $\frac{-3}{6} = -\frac{1}{2}$   $\frac{4}{-8} = -\frac{1}{2}$

4. Solve for  $a$ :

$$\frac{2}{a} = -\frac{1}{2}$$

$$a = 2 \times (-2) = -4$$

**Final Answer:**

-4

**Answer: (B)**[Go Back to Question 47](#)

Q48.

**Solution****Concept:**

The direction cosines (DCs) of a line, denoted as  $l, m, n$ , satisfy the identity  $l^2 + m^2 + n^2 = 1$ . If a line makes angles  $\alpha, \beta, \gamma$  with the coordinate axes, then  $l = \cos \alpha, m = \cos \beta, n = \cos \gamma$ . If the angles are equal, then  $l = m = n$ .

**Solution:**

1. Since the line makes equal angles with the axes, we have:  $\alpha = \beta = \gamma$

$$\Rightarrow \cos \alpha = \cos \beta = \cos \gamma$$

$$\Rightarrow l = m = n$$

2. Substitute into the fundamental identity:

$$l^2 + m^2 + n^2 = 1$$

$$l^2 + l^2 + l^2 = 1$$

$$3l^2 = 1$$

3. Solve for  $l$ :

$$l^2 = 1/3 \Rightarrow l = \pm \frac{1}{\sqrt{3}}$$

4. Since  $l = m = n$ , the direction cosines are  $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ .

**Final Answer:**

$$\left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$$

**Answer: (B)**

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Q49.

**Solution****Concept:**

The angle  $\theta$  between two lines with direction ratios  $(a_1, b_1, c_1)$  and  $(a_2, b_2, c_2)$  is given by the cosine formula:

$$\cos \theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

This formula is derived from the dot product of two vectors lying along those lines.

**Solution:**

1. Identify direction ratios: Line 1:  $a_1 = 1, b_1 = 1, c_1 = 2$ . Line 2:  $a_2 = \sqrt{3} - 1, b_2 = -\sqrt{3} - 1, c_2 = 4$ .

2. Calculate the numerator:  $1(\sqrt{3} - 1) + 1(-\sqrt{3} - 1) + 2(4)$   
 $= \sqrt{3} - 1 - \sqrt{3} - 1 + 8 = 6$

3. Calculate the magnitudes:  $\sqrt{a_1^2 + b_1^2 + c_1^2} = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$

$$\sqrt{a_2^2 + b_2^2 + c_2^2} = \sqrt{(\sqrt{3} - 1)^2 + (-\sqrt{3} - 1)^2 + 4^2}$$

$$= \sqrt{(3 - 2\sqrt{3} + 1) + (3 + 2\sqrt{3} + 1) + 16}$$

$$= \sqrt{4 + 4 + 16} = \sqrt{24} = 2\sqrt{6}$$

4. Apply the formula:

$$\cos \theta = \frac{6}{\sqrt{6} \times 2\sqrt{6}} = \frac{6}{2 \times 6} = \frac{1}{2}$$

5. Since  $\cos \theta = 1/2, \theta = 60^\circ$ .

**Final Answer:**

$$\boxed{60^\circ}$$

**Answer: (C)**

[Go Back to Question 49](#)



Q50.

**Solution****Concept:**

The perpendicular distance from a point  $(x_1, y_1, z_1)$  to a plane  $Ax + By + Cz + D = 0$  is:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

When two points are equidistant from a plane, their calculated distances  $d_1$  and  $d_2$  are equal. This setup results in an equation where we can solve for an unknown coordinate.

**Solution:**

1. Plane:  $3x + 4y - 12z + 13 = 0$ . Denominator for distance:  $\sqrt{3^2 + 4^2 + (-12)^2} = \sqrt{9 + 16 + 144} = \sqrt{169} = 13$ .

2. Distance from  $(1, 1, p)$ :  $d_1 = \frac{|3(1) + 4(1) - 12(p) + 13|}{13} = \frac{|20 - 12p|}{13}$

3. Distance from  $(-3, 0, 1)$ :  $d_2 = \frac{|3(-3) + 4(0) - 12(1) + 13|}{13} = \frac{|-9 - 12 + 13|}{13} = \frac{|-8|}{13} = \frac{8}{13}$

4. Equate  $d_1$  and  $d_2$ :  $|20 - 12p| = 8$

5. Solve the modular equation: Case 1:  $20 - 12p = 8 \implies 12p = 12 \implies p = 1$ . Case 2:  $20 - 12p = -8 \implies 12p = 28 \implies p = 7/3$ .

6. Checking the options,  $p = 1$  is present.

**Final Answer:**

1

**Answer: (A)****Go Back to Question 50**

## Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	B	4	A	5	A
6	B	7	B	8	A	9	A	10	A
11	B	12	B	13	B	14	A	15	A
16	C	17	B	18	B	19	B	20	B
21	A	22	A	23	B	24	B	25	A
26	A	27	A	28	A	29	B	30	A
31	A	32	A	33	A	34	A	35	A
36	C	37	B	38	A	39	A	40	A
41	A	42	A	43	C	44	A	45	A
46	A	47	B	48	B	49	C	50	A

