MHT-CET Mathematics Sample Paper- 9

Duration: 90 Minutes **Maximum Marks: 100**

Instructions

- This paper contains a total of **50** Multiple Choice Questions.
- Each correct answer carries **+2 marks**.
- No negative marking for incorrect questions.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.
- No marks will be deducted for questions that are left unattempted.

Q1. If $f(x) = \lim_{n\to\infty} \frac{\log(2+x)-x^{2n}\sin x}{1+x^{2n}}$, then the value of $f(1)$ so that $f(x)$ is continuous at $x = 1$ is:

(A) $\log 3$

(B) $\frac{1}{2}(\log 3 - \sin 1)$

(C) $\frac{1}{2}(\sin 1 - \log 3)$

(D) 0

Q2. The value of $\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$ is:

(A) e

(B) $1/e$

(C) e^2

(D) 1

Q3. If $f(x) = \frac{\cos(\frac{\pi}{2}[x-k])}{1+(x-k)^{2n}}$ (where $[\cdot]$ is GIF), then $f(x)$ is continuous at $x = k$ only if:

(A) $k \in \mathbb{I}$

(B) k is any real number

(C) $k = 0$

(D) No value of k

Q4. If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$, then $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$ is:

(A) $\frac{8\sqrt{2}}{a\pi}$

(B) $\frac{4\sqrt{2}}{a\pi}$

(C) $\frac{8}{a\pi}$

(D) $\frac{4}{a\pi}$

Q5. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$, then $\frac{dy}{dx}$ is:

(A) $\frac{x}{\sqrt{1-x^4}}$

(B) $\frac{-x}{\sqrt{1-x^4}}$

(C) $\frac{1}{\sqrt{1-x^4}}$

(D) $\frac{-1}{\sqrt{1-x^4}}$

Q6. If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{2}\right)$ is:

(A) 1

(B) -1

(C) 0

(D) $\sqrt{2}$

Q7. If $y = \sin(m\sin^{-1}x)$, then $(1-x^2)y_2 - xy_1 + m^2y$ is: ł7belq7

(A) 0

(B) m^2

(C) 1

(D) -1

Q8. Derivative of $\ln(\sec x + \tan x)$ with respect to $\sec x$ at $x = \frac{\pi}{4}$ is:

(A) $\sqrt{2}$

(B) 1

(C) $1/\sqrt{2}$

(D) 2

Q9. The side of an equilateral triangle is increasing at the rate of 2 cm/s. The rate at which the area increases when the side is 10 cm is:

(A) $10\,\text{cm}^2/\text{s}$

(B) $10\sqrt{3}\,\text{cm}^2/\text{s}$

(C) $20\,\text{cm}^2/\text{s}$

(D) $20\sqrt{3}\,\text{cm}^2/\text{s}$

Q10. The function $f(x) = x^x$ has a stationary point at:

(A) $x = e$

(B) $x = 1/e$

(C) $x = 1$

(D) $x = \sqrt{e}$

Q11. The equation of the normal to the curve $y = \sin x$ at $(0, 0)$ is:

(A) $x + y = 0$

(B) $x - y = 0$

(C) $x = 0$

(D) $y = 0$

Q12. The maximum value of $\left(\frac{1}{x}\right)^x$ is:

(A) e

(B) $e^{1/e}$

(C) e^e

(D) $(1/e)^e$

Q13. The interval in which $f(x) = 2x^3 - 3x^2 - 12x + 1$ is strictly decreasing is:

(A) $(-1, 2)$

(B) $(-\infty, -1)$

(C) $(2, \infty)$

(D) $(-\infty, \infty)$

Q14. The integral $\int \frac{dx}{\cos^6 x+\sin^6 x}$ is equal to:

(A) $\tan^{-1}(\tan x - \cot x) + C$

(B) $\tan^{-1}(\tan x + \cot x) + C$

(C) $\sin^{-1}(\sin 2x) + C$

(D) None

Q15. The value of $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}} dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) $\pi/3$

(D) π

Q16. $\int e^x \left(\frac{2+\sin 2x}{1+\cos 2x}\right) dx$ is:

(A) $e^x \tan x + C$ item $e^x \cot x + C$

(B) $e^x \sec x + C$

(C) $e^x \cos x + C$

Q17. The value of $\int_{-1}^{1} |x \sin \pi x| dx$ is:

(A) $2/\pi$

(B) $1/\pi$

(C) $4/\pi$

(D) 0

Q18. $\int \frac{1}{x(x^n+1)} dx$ is equal to:

(A) $\frac{1}{n}\ln\left|\frac{x^n}{x^n+1}\right| + C$

(B) $\ln\left|\frac{x^n}{x^n+1}\right| + C$

(C) $\frac{1}{n}\ln\left|\frac{x^n+1}{x^n}\right| + C$

(D) None

Q19. The area bounded by the curves $y = \ln x$, y-axis and the lines $y = 1, y = 2$ is:

(A) $e^2 - e$

(B) $e^2 + e$

(C) $e - 1$

(D) e^2

Q20. The area of the region bounded by $y^2 = 2x$ and $y = x$ is:

(A) $2/3$

(B) $4/3$

(C) $1/3$

(D) 1

Q21. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ is:

(A) $y - x = \ln(x + y) + C$

(B) $y + x = \ln(x + y) + C$

(C) $y - x = \ln(x + y + C)$

(D) None

Q22. The integrating factor of $\cos x\frac{dy}{dx} + y\sin x = 1$ is:

(A) $\cos x$

(B) $\sec x$

(C) $\tan x$

(D) $\sin x$

Q23. The order and degree of $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ are:

(A) 2, 2

(B) 2, 3

(C) 3, 2

(D) 2, 1

Q24. If ω is a complex cube root of unity, then the value of $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$ is:

(A) 32

(B) −32

(C) 64

(D) 0

Q25. If $|z-4| < |z-2|$, its solution is:

(A) $\text{Re}(z) > 3$

(B) $\text{Re}(z) < 3$

(C) $\text{Im}(z) > 3$

(D) $\text{Im}(z) < 3$

Q26. The amplitude of $\sin\frac{\pi}{5} + i(1-\cos\frac{\pi}{5})$ is:

(A) $\pi/5$

(B) $\pi/10$

(C) $\pi/15$

(D) $\pi/20$

Q27. If α, β are roots of $x^2 - px + r = 0$ and $\alpha/2, 2\beta$ are roots of $x^2 - qx + r = 0$, then the value of r is:

(A) $\frac{2}{9}(p-q)(2q-p)$

(B) $\frac{2}{9}(q-p)(2p-q)$

(C) $\frac{2}{9}(q-2p)(2q-p)$

(D) $\frac{2}{9}(2p-q)(2q-p)$

Q28. The number of real solutions of the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ is:

(A) 1

(B) 2

(C) Infinite

(D) 0

Q29. If the roots of $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c$ is equal to:

(A) 0

(B) 1

(C) 2

(D) 4

Q30. If a, b, c are in H.P., then the value of $\frac{b+a}{b-a} + \frac{b+c}{b-c}$ is:

(A) 2

(B) 1

(C) 0

(D) 4

Q31. The sum of the series $1 + \frac{1+2}{2} + \frac{1+2+3}{4} + \frac{1+2+3+4}{8} + \ldots \infty$ is:

(A) 4

(B) 6

(C) 8

(D) 10

Q32. If three positive numbers a, b, c are in A.P. such that $abc = 8$, the minimum possible value of b is:

(A) 2

(B) 4

(C) 1

(D) $\sqrt[3]{4}$

Q33. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is:

(A) 144

(B) -144

(C) 132

(D) -132

Q34. If the sum of coefficients in the expansion of $(x + y)^n$ is 4096, then the greatest coefficient in the expansion is:

(A) 924

(B) 792

(C) 1584

(D) 462

Q35. The number of ways in which 10 identical apples can be distributed among 3 boys such that each boy gets at least one apple is:

(A) 36

(B) 45

(C) 55

(D) 66

Q36. Five digit numbers are formed using digits $\{1, 2, 3, 4, 5, 6, 7, 8\}$ without repetition. The probability that a number chosen is divisible by 5 is:

(A) $1/8$

(B) $1/5$

(C) $5/8$

(D) $1/40$

Q37. If $P(A) = 0.4$, $P(B|A) = 0.3$ and $P(B^c|A^c) = 0.2$, then $P(A|B)$ is:

(A) $1/2$

(B) $1/5$

(C) $3/15$

(D) $12/60$

Q38. The number of ways to seat 5 boys and 5 girls around a circular table so that no two girls are together is:

(A) $4! \times 5!$

(B) $5! \times 5!$

(C) $4! \times 4!$

(D) $2 \times 5!$

Q39. The distance of the point $(1, 2)$ from the line $x + y + 5 = 0$ measured along the line making an angle of 45° with x-axis is:

(A) $4\sqrt{2}$

(B) 8

(C) $6\sqrt{2}$

(D) $5\sqrt{2}$

Q40. If the lines $x + y = |a|$ and $ax - y = 1$ are perpendicular, then a is:

(A) 1

(B) -1

(C) 0

(D) No real value

Q41. The length of the tangent from $(1, 1)$ to the circle $x^2 + y^2 + 4x + 6y + 1 = 0$ is:

(A) $\sqrt{13}$

(B) 13

(C) 4

(D) $\sqrt{15}$

Q42. The circle $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ touch each other:

(A) Externally

(B) Internally

(C) Not at all

(D) At two points

Q43. The area of the triangle formed by the tangent at $(a, 2a)$ to the parabola $y^2 = 4ax$ and the coordinate axes is:

(A) a^2

(B) $2a^2$

(C) $a^2/2$

(D) $4a^2$

Q44. If the latus rectum of an ellipse is equal to half of its minor axis, then its eccentricity is:

(A) $1/2$

(B) $\sqrt{3}/2$

(C) $\sqrt{2}/3$

(D) $1/\sqrt{2}$

Q45. The equation of the hyperbola with foci $(\pm 5, 0)$ and eccentricity $e = 5/4$ is:

(A) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(B) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(C) $\frac{x^2}{25} - \frac{y^2}{9} = 1$

(D) $\frac{x^2}{16} - \frac{y^2}{25} = 1$

Q46. The locus of the point of intersection of perpendicular tangents to the parabola $y^2 = 4ax$ is:

(A) $x = a$

(B) $x = -a$

(C) $y = a$

(D) $y = -a$

Q47. If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) 30°

(B) 45°

(C) 60°

(D) 90°

Q48. The volume of a parallelepiped whose edges are represented by $\vec{a} = 2\hat{i} - 3\hat{j}, \vec{b} = \hat{i} + \hat{j} - \hat{k}, \vec{c} = 3\hat{i} - \hat{k}$ is:

(A) 4

(B) 7

(C) 10

(D) 13

Q49. The distance of the point $(1, 2, 3)$ from the plane $x + 2y - 2z + 5 = 0$ is:

(A) $4/3$ units

(B) 2 units

(C) $1/3$ units

(D) 3 units

Q50. The angle between the lines whose direction cosines are $(1, 2, 1)$ and $(-1, 1, -2)$ is:

(A) $\cos^{-1}(1/6)$

(B) $\pi/2$

(C) $\pi/3$

(D) $\cos^{-1}(-1/6)$

Detailed Solutions

Q1.

Solution

Concept: Direct evaluation of limit defining function at a point.

Solution:

Step 1: Substitute $x = 1$ in the expression inside limit

$$f(1) = \lim_{n\to\infty} \frac{\log(2+1) - 1^{2n}\sin 1}{1 + 1^{2n}}$$

Step 2: Simplify powers

$$1^{2n} = 1$$

So,

$$f(1) = \frac{\log 3 - \sin 1}{2}$$

Step 3: Continuity condition Since $f(x)$ is defined via limit expression, this value ensures continuity at $x = 1$.

Final Answer: $\boxed{\frac{1}{2}(\log 3 - \sin 1)}$

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept: Standard limit using logarithm expansion.

Solution:

Step 1: Let

$$L = \left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x - \sin x}}$$

Take log:

$$\ln L = \frac{\sin x}{x - \sin x} \ln\left(\frac{\sin x}{x}\right)$$

Step 2: Use standard approximations

$$\frac{\sin x}{x} = 1 - \frac{x^2}{6} + O(x^4)$$

$$\ln\left(\frac{\sin x}{x}\right) \approx -\frac{x^2}{6}$$

Also,

$$x - \sin x \approx \frac{x^3}{6} \Rightarrow \frac{\sin x}{x - \sin x} \approx \frac{x}{x^3/6} = \frac{6}{x^2}$$

Step 3: Compute limit

$$\ln L \approx \frac{6}{x^2} \cdot \left(-\frac{x^2}{6}\right) = -1$$

Step 4: Exponentiate

$$L = e^{-1} = \frac{1}{e}$$

Final Answer: $\boxed{\frac{1}{e}}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: Continuity with floor function and piecewise behavior.

Solution:

Step 1: Evaluate at $x = k$

$$f(k) = \frac{\cos\left(\frac{\pi}{2}[0]\right)}{1+0} = 1$$

Step 2: Check behavior near $x = k$

For $x > k$: $[x-k] = 0 \Rightarrow \cos(0) = 1$

For $x < k$: $[x-k] = -1 \Rightarrow \cos(-\pi/2) = 0$

Step 3: Left and right limits

$$\lim_{x \to k^-} f(x) = 0, \quad \lim_{x \to k^+} f(x) = 1$$

Step 4: Continuity condition Limits are unequal for every real k, hence discontinuity always exists.

Final Answer: No value of k

Answer: (D)

Go Back to Question 3

Q4.

Solution

Concept: Parametric differentiation.

Solution:

Step 1: Differentiate x and y

$$x = a(\cos\theta + \theta\sin\theta) \Rightarrow \frac{dx}{d\theta} = a(-\sin\theta + \sin\theta + \theta\cos\theta) = a\theta\cos\theta$$

$$y = a(\sin\theta - \theta\cos\theta) \Rightarrow \frac{dy}{d\theta} = a(\cos\theta - \cos\theta + \theta\sin\theta) = a\theta\sin\theta$$

Step 2: First derivative

$$\frac{dy}{dx} = \frac{\theta\sin\theta}{\theta\cos\theta} = \tan\theta$$

Step 3: Second derivative

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}(\tan\theta)\cdot\frac{1}{dx/d\theta} = \frac{\sec^2\theta}{a\theta\cos\theta}$$

Step 4: Substitute $\theta = \frac{\pi}{4}$

$$\cos\theta = \frac{\sqrt{2}}{2}, \quad \sec^2\theta = 2$$

$$\frac{d^2y}{dx^2} = \frac{2}{a(\pi/4)(\sqrt{2}/2)^3} = \frac{8\sqrt{2}}{a\pi}$$

Final Answer: $\boxed{\frac{8\sqrt{2}}{a\pi}}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: Trigonometric simplification and differentiation.

Solution:

Step 1: Let

$$A = \sqrt{1+x^2}, \quad B = \sqrt{1-x^2}$$

Step 2: Known simplification

$$\frac{A-B}{A+B} \Rightarrow y = \tan^{-1}(\text{expression})$$

This standard form reduces to:

$$y = -\frac{1}{2}\sin^{-1}(x^2)$$

Step 3: Differentiate

$$\frac{dy}{dx} = -\frac{1}{2} \cdot \frac{2x}{\sqrt{1-x^4}}$$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^4}}$$

Final Answer: $\boxed{\frac{-x}{\sqrt{1-x^4}}}$

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept: Differentiation of modulus function.

Solution:

Step 1: Define inside function

$$g(x) = \cos x - \sin x \Rightarrow f(x) = |g(x)|$$

Step 2: Check sign at $x = \frac{\pi}{2}$

$$g\left(\frac{\pi}{2}\right) = 0 - 1 = -1 < 0$$

So near the point:

$$f'(x) = -g'(x)$$

Step 3: Differentiate

$$g'(x) = -\sin x - \cos x$$

Step 4: Substitute $x = \frac{\pi}{2}$

$$g'\left(\frac{\pi}{2}\right) = -1 - 0 = -1$$

$$f'\left(\frac{\pi}{2}\right) = -(-1) = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: Verification of standard differential equation of inverse sine composition.

Solution:

Step 1: Given function

$$y = \sin(m \sin^{-1} x)$$

This is a standard form satisfying:

$$(1 - x^2)y'' - xy' + m^2 y = 0$$

Step 2: Substitute expression Directly using known identity, the expression equals zero.

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: Chain rule with respect to different variable.

Solution:

Step 1: Let

$$y = \ln(\sec x + \tan x)$$

Step 2: Differentiate w.r.t. x

$$\frac{dy}{dx} = \sec x$$

Step 3: Differentiate $\sec x$

$$\frac{d(\sec x)}{dx} = \sec x \tan x$$

Step 4: Required derivative

$$\frac{dy}{d(\sec x)} = \frac{dy/dx}{d(\sec x)/dx} = \frac{\sec x}{\sec x \tan x} = \frac{1}{\tan x}$$

Step 5: Substitute $x = \frac{\pi}{4}$

$$\tan\frac{\pi}{4} = 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept: Related rates for area of equilateral triangle.

Solution:

Step 1: Area formula

$$A = \frac{\sqrt{3}}{4}a^2$$

Step 2: Differentiate w.r.t. time

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2}a\frac{da}{dt}$$

Step 3: Substitute values

$$a = 10, \quad \frac{da}{dt} = 2$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2} \cdot 10 \cdot 2 = 10\sqrt{3}$$

Final Answer: $\boxed{10\sqrt{3}\text{ cm}^2/\text{s}}$

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: Stationary point using logarithmic differentiation.

Solution:

Step 1: Function

$$f(x) = x^x$$

Step 2: Take log

$$\ln f = x \ln x$$

Step 3: Differentiate

$$\frac{f'}{f} = \ln x + 1$$

$$f' = x^x(\ln x + 1)$$

Step 4: Stationary point condition

$$x^x(\ln x + 1) = 0 \Rightarrow \ln x + 1 = 0$$

$$x = \frac{1}{e}$$

Final Answer: $\boxed{\frac{1}{e}}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: Equation of normal using slope of tangent.

Solution:

Step 1: Given curve

$$y = \sin x$$

Step 2: Slope of tangent

$$\frac{dy}{dx} = \cos x$$

At $x = 0$:

$$\cos 0 = 1$$

So slope of tangent = 1

Step 3: Slope of normal

$$m_n = -1$$

Step 4: Equation of normal through (0,0)

$$y - 0 = -1(x - 0) \Rightarrow y = -x$$

Final Answer: $\boxed{x + y = 0}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: Maximum of exponential-type function using logarithm.

Solution:

Step 1: Let

$$y = \left(\frac{1}{x}\right)^x$$

Take log:

$$\ln y = x \ln\left(\frac{1}{x}\right) = -x \ln x$$

Step 2: Differentiate

$$\frac{1}{y}\frac{dy}{dx} = -(\ln x + 1)$$

Step 3: Stationary point

$$\ln x + 1 = 0 \Rightarrow x = \frac{1}{e}$$

Step 4: Value at $x = \frac{1}{e}$

$$y = (e)^{1/e} = e^{1/e}$$

Final Answer: $\boxed{e^{1/e}}$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: Monotonicity using derivative test.

Solution:

Step 1: Function

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

Step 2: First derivative

$$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x - 2)(x + 1)$$

Step 3: Sign analysis

$$f'(x) < 0 \text{ when } (x - 2)(x + 1) < 0$$

This occurs in:

$$(-1, 2)$$

Final Answer: $\boxed{(-1, 2)}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: Trigonometric simplification before integration.

Solution:

Step 1: Use identity

$$\cos^6 x + \sin^6 x = (\cos^2 x + \sin^2 x)^3 - 3\sin^2 x \cos^2 x(\sin^2 x + \cos^2 x)$$

$$= 1 - 3\sin^2 x \cos^2 x$$

$$= 1 - \frac{3}{4}\sin^2 2x$$

Step 2: Standard form leads to non-elementary inverse simplification mismatch

Given options do not match correct closed form.

Final Answer: None

Answer: (D)

Go Back to Question 14

Q15.

Solution

Concept: Symmetry property of definite integrals.

Solution:

Step 1: Let

$$I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$$

Step 2: Substitute $x \to \frac{\pi}{2} - x$

$$I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$$

Step 3: Add both forms

$$2I = \int_0^{\pi/2} 1\, dx = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Final Answer: $\frac{\pi}{4}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: Trigonometric simplification before integration.

Solution:

Step 1: Simplify the expression

$$\frac{2 + \sin 2x}{1 + \cos 2x}$$

Use identities:

$$\sin 2x = 2 \sin x \cos x, \quad 1 + \cos 2x = 2 \cos^2 x$$

So,

$$\frac{2 + 2 \sin x \cos x}{2 \cos^2 x} = \frac{2(1 + \sin x \cos x)}{2 \cos^2 x} = \frac{1 + \sin x \cos x}{\cos^2 x}$$

$$= \sec^2 x + \tan x \sec x$$

Step 2: Integral becomes

$$\int e^x (\sec^2 x + \tan x \sec x)\, dx$$

Step 3: Recognize derivative

$$\frac{d}{dx}(e^x \tan x) = e^x \tan x + e^x \sec^2 x$$

Final Answer: $\boxed{e^x \tan x + C}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: Even function and symmetry in definite integrals.

Solution:

Step 1: Check symmetry

$$|x \sin \pi x| \text{ is even}$$

So,

$$\int_{-1}^{1} |x \sin \pi x| dx = 2 \int_{0}^{1} x \sin \pi x \, dx$$

Step 2: Integration by parts

Let:

$$u = x, \quad dv = \sin \pi x \, dx$$

$$v = -\frac{\cos \pi x}{\pi}$$

$$\int_{0}^{1} x \sin \pi x dx = \left[-\frac{x \cos \pi x}{\pi} \right]_{0}^{1} + \frac{1}{\pi} \int_{0}^{1} \cos \pi x dx$$

$$= \frac{1}{\pi}$$

Step 3: Final value

$$I = 2 \cdot \frac{1}{\pi} = \frac{2}{\pi}$$

Final Answer: $\boxed{\frac{2}{\pi}}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: Standard logarithmic substitution.

Solution:

Step 1: Let

$$t = x^n \Rightarrow dt = nx^{n-1}dx$$

Rewrite:

$$\int \frac{1}{x(x^n+1)}dx$$

Step 2: Substitute directly

$$= \frac{1}{n}\int \frac{1}{t(t+1)}dt$$

Step 3: Partial fractions

$$\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$$

Step 4: Integrate

$$\frac{1}{n}\ln\left|\frac{t}{t+1}\right| + C$$

Step 5: Back substitute

$$= \frac{1}{n}\ln\left|\frac{x^n}{x^n+1}\right| + C$$

Final Answer: $\boxed{\frac{1}{n}\ln\left|\frac{x^n}{x^n+1}\right| + C}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: Area between curve and axis using inverse function.

Solution:

Step 1: Given region

$$y = \ln x \Rightarrow x = e^y$$

Step 2: Area in terms of y

$$A = \int_1^2 e^y dy$$

Step 3: Integrate

$$A = [e^y]_1^2 = e^2 - e$$

Final Answer: $\boxed{e^2 - e}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: Area between parabola and line.

Solution:

Step 1: Curves

$$y^2 = 2x \Rightarrow x = \frac{y^2}{2}, \quad y = x$$

Step 2: Intersection points

$$y = \frac{y^2}{2} \Rightarrow y(y-2) = 0 \Rightarrow y = 0, 2$$

Step 3: Area between curves

$$A = \int_0^2 \left(y - \frac{y^2}{2}\right) dy$$

Step 4: Integrate

$$A = \left[\frac{y^2}{2} - \frac{y^3}{6}\right]_0^2$$

$$= 2 - \frac{8}{6} = 2 - \frac{4}{3} = \frac{2}{3}$$

Final Answer: $\boxed{\frac{2}{3}}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: Substitution method for differential equations.

Solution:

Step 1: Let

$$t = x + y \Rightarrow \frac{dt}{dx} = 1 + \frac{dy}{dx}$$

Step 2: Given

$$\frac{dy}{dx} = \frac{x+y+1}{x+y-1} = \frac{t+1}{t-1}$$

So,

$$\frac{dt}{dx} = 1 + \frac{t+1}{t-1} = \frac{t-1+t+1}{t-1} = \frac{2t}{t-1}$$

Step 3: Separate variables

$$\frac{t-1}{t}dt = 2dx$$

$$\left(1 - \frac{1}{t}\right) dt = 2dx$$

Step 4: Integrate

$$t - \ln|t| = 2x + C$$

Step 5: Substitute back $t = x + y$

$$x + y - \ln(x+y) = 2x + C$$

$$y - x = \ln(x+y) + C$$

Final Answer: $\boxed{y - x = \ln(x+y) + C}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: Linear differential equation in standard form.

Solution:

Step 1: Rewrite equation

$$\cos x \frac{dy}{dx} + y \sin x = 1$$

Divide by $\cos x$:

$$\frac{dy}{dx} + y \tan x = \sec x$$

Step 2: Identify integrating factor

$$I.F. = e^{\int \tan x \, dx} = e^{-\ln(\cos x)} = \sec x$$

Final Answer: $\boxed{\sec x}$

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept: Order and degree of differential equation.

Solution:

Step 1: Given equation

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$$

Step 2: Order Highest derivative is $\frac{d^2y}{dx^2}$

$$\Rightarrow \text{Order} = 2$$

Step 3: Degree Make free from radicals:

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

Highest power of highest order derivative = 2

$$\Rightarrow \text{Degree} = 2$$

Final Answer: $\boxed{2, 2}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: Properties of cube roots of unity.

Solution:

Step 1: Use identity

$$1 + \omega + \omega^2 = 0 \Rightarrow \omega^2 = -1 - \omega$$

Step 2: First term

$$1 - \omega + \omega^2 = 1 - \omega - 1 - \omega = -2\omega$$

Step 3: Second term

$$1 + \omega - \omega^2 = 1 + \omega + 1 + \omega = 2(1 + \omega)$$

But:

$$1 + \omega = -\omega^2$$

So:

$$1 + \omega - \omega^2 = -2\omega^2$$

Step 4: Compute

$$(-2\omega)^5 + (-2\omega^2)^5 = (-32)(\omega^5 + \omega^{10})$$

Since $\omega^3 = 1$:

$$\omega^5 = \omega^2, \quad \omega^{10} = \omega$$

$$= -32(\omega^2 + \omega) = -32(-1) = 32$$

Final Answer: $\boxed{32}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: Locus in complex plane.

Solution:

Step 1: Given

$$|z-4| < |z-2|$$

Let $z = x + iy$

Step 2: Square both sides

$$(x-4)^2 + y^2 < (x-2)^2 + y^2$$

Step 3: Simplify

$$x^2 - 8x + 16 < x^2 - 4x + 4$$

$$-8x + 16 < -4x + 4$$

$$12 < 4x \Rightarrow x > 3$$

Final Answer: $\boxed{\text{Re}(z) > 3}$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept: Amplitude (argument) of a complex number.

Solution:

Step 1: Given complex number

$$z = \sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right)$$

Step 2: Write in standard form

$$z = x + iy$$

$$\tan\theta = \frac{y}{x} = \frac{1 - \cos\frac{\pi}{5}}{\sin\frac{\pi}{5}}$$

Step 3: Use identity

$$\tan\left(\frac{\theta}{2}\right) = \frac{1 - \cos\theta}{\sin\theta}$$

Here $\theta = \frac{\pi}{5}$, so:

$$\arg(z) = \frac{\pi}{10}$$

Final Answer: $\boxed{\frac{\pi}{10}}$

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept: Vieta's relations and transformation of roots.

Solution:

Step 1: Original roots

$$\alpha + \beta = p, \quad \alpha\beta = r$$

Step 2: New roots

$$\frac{\alpha}{2},\ 2\beta$$

For new equation:

$$\frac{\alpha}{2} + 2\beta = q$$

Step 3: Substitute

$$\frac{\alpha}{2} + 2\beta = q \Rightarrow \alpha + 4\beta = 2q$$

Step 4: Solve system

$$\alpha + \beta = p, \quad \alpha + 4\beta = 2q$$

Subtract:

$$3\beta = 2q - p \Rightarrow \beta = \frac{2q - p}{3}$$

$$\alpha = p - \beta = \frac{4p - 2q}{3}$$

Step 5: Compute product

$$r = \alpha\beta = \frac{(4p - 2q)(2q - p)}{9}$$

$$= \frac{2}{9}(2p - q)(2q - p)$$

Final Answer: $\boxed{\frac{2}{9}(2p - q)(2q - p)}$

Answer: (D)

Go Back to Question 27

Q28.

Solution

Concept: Range analysis of exponential hyperbolic function.

Solution:

Step 1: Given equation

$$e^{\sin x} - e^{-\sin x} = 4$$

Step 2: Let $t = \sin x$

$$e^{t} - e^{-t} = 4$$

$$2\sinh t = 4 \Rightarrow \sinh t = 2$$

Step 3: Check feasibility Since $\sinh t$ is continuous and strictly increasing on $[-1, 1]$:

$$\sinh(1) \approx 1.175 < 2$$

So equation has no solution.

Final Answer: $\boxed{0}$

Answer: (D)

Go Back to Question 28

Q29.

Solution

Concept: Quadratic with consecutive integer roots.

Solution:

Step 1: Let roots

$$\alpha = n, \quad \beta = n + 1$$

Step 2: Coefficients

$$b = \alpha + \beta = 2n + 1, \quad c = n(n + 1)$$

Step 3: Compute discriminant

$$b^2 - 4c = (2n + 1)^2 - 4n(n + 1)$$

$$= 4n^2 + 4n + 1 - 4n^2 - 4n = 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept: Properties of harmonic progression.

Solution:

Step 1: In H.P.

$$\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in A.P.}$$

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

Step 2: Required expression

$$\frac{b+a}{b-a} + \frac{b+c}{b-c}$$

Rewrite using symmetry and H.P. condition, expression simplifies to:

$$2$$

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: Sum of infinite series using standard summation formulas.

Solution:

Step 1: General term

$$1 + \frac{1+2}{2} + \frac{1+2+3}{4} + \cdots$$

The nth numerator is:

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

Denominator:

$$2^{n-1}$$

So,

$$S = \sum_{n=1}^{\infty} \frac{n(n+1)}{2^n}$$

Step 2: Split series

$$S = \sum_{n=1}^{\infty} \frac{n^2+n}{2^n} = \sum_{n=1}^{\infty} \frac{n^2}{2^n} + \sum_{n=1}^{\infty} \frac{n}{2^n}$$

Step 3: Use standard results

$$\sum_{n=1}^{\infty} \frac{n}{2^n} = 2$$

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n} = 6$$

Hence,

$$S = 6 + 2 = 8$$

Final Answer: $\boxed{8}$

Answer: (C)

Go Back to Question 31

Q32.

Solution

Concept: Arithmetic progression and AM-GM inequality.

Solution:

Step 1: Let numbers in A.P.

$$a = b - d, \quad b, \quad c = b + d$$

Step 2: Product condition

$$(b - d)b(b + d) = 8$$

$$b(b^2 - d^2) = 8$$

$$b^3 - bd^2 = 8$$

Step 3: Since $d^2 \geq 0$

$$b^3 \geq 8$$

$$b \geq 2$$

Equality occurs when:

$$d = 0$$

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: Binomial expansion.

Solution:

$$1 - x - x^2 + x^3 = (1 - x)^2(1 + x)$$

$$(1 - x - x^2 + x^3)^6 = (1 - x)^{12}(1 + x)^6$$

Coefficient of x^7:

$$\sum_{r=0}^{6} \binom{6}{r}(-1)^{7-r}\binom{12}{7-r} = -132$$

Final Answer: $\boxed{-132}$

Answer: (D)

Go Back to Question 33

Q34.

Solution

Concept: Greatest binomial coefficient.

Solution:

$$2^n = 4096 = 2^{12} \Rightarrow n = 12$$

Greatest coefficient:

$$\binom{12}{6} = 924$$

Final Answer: $\boxed{924}$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept: Stars and bars.

Solution:

$$x_1 + x_2 + x_3 = 10, \quad x_i \geq 1$$

Putting

$$y_i = x_i - 1$$

gives

$$y_1 + y_2 + y_3 = 7$$

Number of solutions:

$$\binom{9}{2} = 36$$

Final Answer: $\boxed{36}$

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept: Probability by counting.

Solution:

Total numbers:

$$^8P_5 = 6720$$

For divisibility by 5, last digit must be 5.

Favourable numbers:

$$^7P_4 = 840$$

Probability:

$$\frac{840}{6720} = \frac{1}{8}$$

Final Answer: $\boxed{\frac{1}{8}}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept: Bayes' theorem.

Solution:

$$P(B|A^c) = 1 - 0.2 = 0.8$$

$$P(B) = 0.3(0.4) + 0.8(0.6) = 0.6$$

$$P(A|B) = \frac{0.3 \times 0.4}{0.6} = \frac{1}{5}$$

Final Answer: $\boxed{\frac{1}{5}}$

Answer: (B)

Go Back to Question 37

Q38.

Solution

Concept: Circular permutation.

Solution:

Arrange boys:

$$(5-1)! = 4!$$

Arrange girls in gaps:

$$5!$$

Total ways:

$$4! \times 5!$$

Final Answer: $\boxed{4! \times 5!}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: Distance along a line.

Solution:

Line through $(1, 2)$ with slope 1:

$$y = x + 1$$

Intersecting with

$$x + y + 5 = 0$$

gives point:

$$(-3, -2)$$

Distance:

$$\sqrt{(-4)^2 + (-4)^2} = 4\sqrt{2}$$

Final Answer: $\boxed{4\sqrt{2}}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: Perpendicular lines.

Solution:

Slopes:

$$m_1 = -1, \quad m_2 = a$$

For perpendicular lines:

$$m_1 m_2 = -1$$

$$(-1)(a) = -1 \Rightarrow a = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept: Length of tangent from a point to a circle.

Solution:

Step 1: Given circle

$$x^2 + y^2 + 4x + 6y + 1 = 0$$

For point $(x_1, y_1) = (1, 1)$, tangent length is:

$$\sqrt{S_1}$$

Step 2: Compute S_1

$$S_1 = 1^2 + 1^2 + 4(1) + 6(1) + 1$$

$$= 1 + 1 + 4 + 6 + 1 = 13$$

Step 3: Tangent length

$$= \sqrt{13}$$

Final Answer: $\boxed{\sqrt{13}}$

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept: Condition for touching circles.

Solution:

Step 1: First circle

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

Center:

$$C_1 = (2, 3)$$

Radius:

$$r_1 = \sqrt{2^2 + 3^2 + 12} = \sqrt{25} = 5$$

Step 2: Second circle

$$x^2 + y^2 + 6x + 18y + 26 = 0$$

Center:

$$C_2 = (-3, -9)$$

Radius:

$$r_2 = \sqrt{(-3)^2 + (-9)^2 - 26} = \sqrt{64} = 8$$

Step 3: Distance between centers

$$d = \sqrt{(2+3)^2 + (3+9)^2}$$

$$= \sqrt{25 + 144} = \sqrt{169} = 13$$

Step 4: Compare

$$d = r_1 + r_2 = 5 + 8 = 13$$

Hence circles touch externally.

Final Answer: Externally

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: Tangent to parabola and intercept form.

Solution:

Step 1: Parabola

$$y^2 = 4ax$$

Point:

$$(a, 2a)$$

Step 2: Tangent equation

For parabola $y^2 = 4ax$, tangent at (x_1, y_1) is:

$$yy_1 = 2a(x + x_1)$$

Substitute:

$$y(2a) = 2a(x + a)$$

$$y = x + a$$

Step 3: Intercepts

x-intercept:

$$y = 0 \Rightarrow x = -a$$

y-intercept:

$$x = 0 \Rightarrow y = a$$

Step 4: Area of triangle

$$\text{Area} = \frac{1}{2} \times a \times a = \frac{a^2}{2}$$

Final Answer: $\boxed{\frac{a^2}{2}}$

Answer: (C)

Go Back to Question 43

Q44.

Solution

Concept: Relation between latus rectum and minor axis of ellipse.

Solution:

Step 1: Standard formulas

Length of latus rectum:

$$\frac{2b^2}{a}$$

Minor axis:

$$2b$$

Given:

$$\frac{2b^2}{a} = \frac{1}{2}(2b)$$

$$\frac{2b^2}{a} = b$$

$$2b = a$$

Step 2: Use relation

$$b^2 = a^2(1 - e^2)$$

Since:

$$b = \frac{a}{2}$$

$$\frac{a^2}{4} = a^2(1 - e^2)$$

$$1 - e^2 = \frac{1}{4}$$

$$e^2 = \frac{3}{4}$$

$$e = \frac{\sqrt{3}}{2}$$

Final Answer: $\boxed{\frac{\sqrt{3}}{2}}$

Answer: (B)

Go Back to Question 44

Q45.

Solution

Concept: Standard equation of hyperbola.

Solution:

Step 1: Given

$$e = \frac{5}{4}, \quad c = 5$$

For hyperbola:

$$e = \frac{c}{a}$$

$$\frac{5}{4} = \frac{5}{a} \Rightarrow a = 4$$

$$a^2 = 16$$

Step 2: Find b^2

For hyperbola:

$$c^2 = a^2 + b^2$$

$$25 = 16 + b^2$$

$$b^2 = 9$$

Step 3: Equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Final Answer: $\boxed{\frac{x^2}{16} - \frac{y^2}{9} = 1}$

Answer: (A)

Go Back to Question 45

Q46.

Solution

Concept: Pair of perpendicular tangents to a parabola.

Solution:

Step 1: Equation of tangent

For parabola:

$$y^2 = 4ax$$

Tangent with slope m:

$$y = mx + \frac{a}{m}$$

Step 2: Perpendicular tangents

If one tangent has slope m, the perpendicular tangent has slope:

$$-\frac{1}{m}$$

Its equation:

$$y = -\frac{x}{m} - am$$

Step 3: Find intersection point

Solve:

$$mx + \frac{a}{m} = -\frac{x}{m} - am$$

Multiplying by m:

$$m^2x + a = -x - am^2$$

$$x(m^2 + 1) = -a(m^2 + 1)$$

$$x = -a$$

Thus locus is:

$$x = -a$$

Final Answer: $\boxed{x = -a}$

Answer: (B)

Go Back to Question 46

Q47.

Solution

Concept: Dot product using vector identity.

Solution:

Step 1: Given

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$\vec{a} + \vec{b} = -\vec{c}$$

Step 2: Square both sides

$$|\vec{a} + \vec{b}|^2 = |\vec{c}|^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{c}|^2$$

$$3^2 + 5^2 + 2(3)(5)\cos\theta = 7^2$$

$$9 + 25 + 30\cos\theta = 49$$

$$30\cos\theta = 15$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

Final Answer: $\boxed{60^\circ}$

Answer: (C)

Go Back to Question 47

Q48.

Solution

Concept: Volume using scalar triple product.

Solution:

Step 1: Write vectors

$$\vec{a} = (2, -3, 0)$$

$$\vec{b} = (1, 1, -1)$$

$$\vec{c} = (3, 0, -1)$$

Step 2: Compute $\vec{b} \times \vec{c}$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(2) - \hat{k}(3)$$

$$= (-1, -2, -3)$$

Step 3: Scalar triple product

$$V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$

$$= |(2)(-1) + (-3)(-2) + (0)(-3)|$$

$$= |-2 + 6| = 4$$

Final Answer: $\boxed{4}$

Answer: (A)

Go Back to Question 48

Q49.

Solution

Concept: Distance of point from plane.

Solution:

Step 1: Formula Distance from point (x_1, y_1, z_1) to plane:

$$Ax + By + Cz + D = 0$$

is

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Step 2: Substitute values

Plane:

$$x + 2y - 2z + 5 = 0$$

Point:

$$(1, 2, 3)$$

$$d = \frac{|1 + 4 - 6 + 5|}{\sqrt{1 + 4 + 4}}$$

$$= \frac{4}{3}$$

Final Answer: $\boxed{\frac{4}{3} \text{ units}}$

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept: Angle between two lines using direction ratios.

Solution:

Step 1: Direction vectors

$$\vec{d}_1 = (1, 2, 1)$$

$$\vec{d}_2 = (-1, 1, -2)$$

Step 2: Use formula

$$\cos\theta = \frac{\vec{d}_1 \cdot \vec{d}_2}{|\vec{d}_1||\vec{d}_2|}$$

Step 3: Compute dot product

$$\vec{d}_1 \cdot \vec{d}_2 = (1)(-1) + (2)(1) + (1)(-2)$$

$$= -1 + 2 - 2 = -1$$

Step 4: Compute magnitudes

$$|\vec{d}_1| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$$

$$|\vec{d}_2| = \sqrt{(-1)^2 + 1^2 + (-2)^2} = \sqrt{6}$$

Step 5: Find angle

$$\cos\theta = \frac{-1}{\sqrt{6}\cdot\sqrt{6}} = -\frac{1}{6}$$

$$\theta = \cos^{-1}\left(-\frac{1}{6}\right)$$

Final Answer: $\boxed{\cos^{-1}\left(-\frac{1}{6}\right)}$

Answer: (D)

Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	D	4	A	5	B
6	A	7	A	8	B	9	B	10	B
11	A	12	B	13	A	14	D	15	B
16	A	17	A	18	A	19	A	20	A
21	A	22	B	23	A	24	A	25	A
26	B	27	D	28	D	29	B	30	A
31	C	32	A	33	D	34	A	35	A
36	A	37	B	38	A	39	A	40	A
41	A	42	A	43	C	44	B	45	A
46	B	47	C	48	A	49	A	50	D