MHT-CET Physics Sample Paper-20

Duration: 45 Minutes **Maximum Marks: 50**

Instructions

- This paper contains a total of **50** Multiple Choice Questions.
- Each correct answer carries **+1 marks**.
- No negative marking for incorrect questions.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.
- No marks will be deducted for questions that are left unattempted.

Q1. A composite rod is made by joining a copper rod of length L and a steel rod of length $2L$, both having the same area of cross-section A. If their thermal conductivities are K_1 and K_2 respectively, the equivalent thermal conductivity of the composite rod for steady state heat flow along its length is:

(A) $\frac{3K_1K_2}{2K_1+K_2}$
(B) $\frac{3K_1K_2}{K_2+2K_1}$
(C) $\frac{K_1+2K_2}{3}$
(D) $\frac{2K_1+K_2}{3}$

Q2. A particle of mass m is moving in a horizontal circle of radius r under a centripetal force $F = -k/r^2$. The total energy of the particle is:

(A) $-\frac{k}{2r}$
(B) $\frac{k}{2r}$
(C) $-\frac{k}{r}$
(D) $\frac{k}{r}$

Q3. In a Young's double slit experiment, the intensity at a point where the path difference is $\lambda/6$ (λ being the wavelength of light) is I. If I_0 denotes the maximum intensity, then I/I_0 is equal to:

(A) $1/2$

(B) $\sqrt{3}/2$

(C) $3/4$

(D) $1/4$

Q4. A solid sphere and a hollow sphere of the same mass and same external radius are released from the top of a rough inclined plane. If they roll without slipping, the ratio of their acceleration ($a_{solid} : a_{hollow}$) is:

(A) 25 : 21

(B) 21 : 25

(C) 5 : 3

(D) 14 : 15

Q5. A charge Q is distributed uniformly over a ring of radius R. The electric potential at a point on the axis of the ring at a distance $R\sqrt{3}$ from the center is:

(A) $\frac{Q}{4\pi\epsilon_0 R}$

(B) $\frac{Q}{8\pi\epsilon_0 R}$

(C) $\frac{Q}{2\pi\epsilon_0 R}$

(D) $\frac{Q}{4\pi\epsilon_0 (2R)}$

Q6. A satellite is revolving around the Earth in a circular orbit of radius r. If its gravitational potential energy is U, then its total energy is:

(A) $U/2$

(B) $-U/2$

(C) $2U$

(D) $-2U$

Q7. The magnetic flux linked with a coil satisfies the relation $\phi = (4t^2+5t+2)$ mWb. The induced e.m.f. in the coil at $t = 2$ s is:

(A) 21 mV

(B) 16 mV

(C) 18 mV

(D) 20 mV

Q8. An ideal gas heat engine operates in a Carnot cycle between 227°C and 127°C. It absorbs 6×10^4 cal of heat at the higher temperature. The amount of heat converted into work is:

(A) 1.2×10^4 cal

(B) 2.4×10^4 cal

(C) 3.6×10^4 cal

(D) 4.8×10^4 cal

Q9. The work function of a metal surface is 4.2 eV. The maximum wavelength of incident radiation that can produce photoelectric effect is:

(A) 2955 Å

(B) 2000 Å

(C) 3500 Å

(D) 4200 Å

Q10. Two capacitors of 2 μF and 3 μF are charged to 100 V and 200 V respectively. They are then connected in parallel with their positive terminals together. The common potential is:

(A) 150 V

(B) 160 V

(C) 140 V

(D) 180 V

Q11. A simple pendulum has a time period T_1 on the surface of Earth and T_2 when taken to a height R above the Earth's surface (R is the radius of Earth). The ratio T_2/T_1 is:

(A) 1

(B) 2

(C) 4

(D) $\sqrt{2}$

Q12. If the mass of a radioactive sample is doubled, its half-life period will:

(A) Be doubled

(B) Be halved

(C) Remain the same

(D) Become four times

Q13. The fundamental frequency of a string stretched by a weight of 4 kg is 256 Hz. The weight required to produce its octave is:

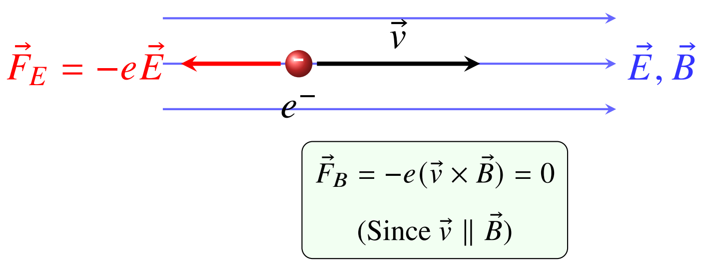
(A) 8 kg

(B) 12 kg

(C) 16 kg

(D) 24 kg

Q14. A uniform electric field and a uniform magnetic field are acting in the same direction. An electron is projected with a velocity in the same direction. Then:



(A) The electron will turn to its right

(B) The electron will turn to its left

(C) The electron velocity will increase

(D) The electron velocity will decrease

Q15. A light ray is incident on a glass slab of refractive index $\sqrt{3}$ at an angle such that the reflected and refracted rays are perpendicular to each other. The angle of incidence is:

(A) $30°$

(B) $45°$

(C) $60°$

(D) $90°$

Q16. A cylinder of mass M and radius R is rolling without slipping on a horizontal surface with angular velocity ω. It then encounters a smooth inclined plane of angle θ. The height reached by the cylinder on the incline is:

(A) $\frac{3R^2\omega^2}{4g}$

(B) $\frac{R^2\omega^2}{4g}$

(C) $\frac{R^2\omega^2}{2g}$

(D) $\frac{3R^2\omega^2}{2g}$

Q17. A liquid drop of radius R is broken into 1000 identical small droplets. If T is the surface tension of the liquid, the expenditure of energy is:

(A) $9\pi R^2 T$

(B) $36\pi R^2 T$

(C) $10\pi R^2 T$

(D) $40\pi R^2 T$

Q18. A particle is executing S.H.M. with amplitude A. At what distance from the mean position is its kinetic energy equal to its potential energy?

(A) $A/2$
(B) $A/\sqrt{2}$
(C) $A/\sqrt{3}$
(D) $A/4$

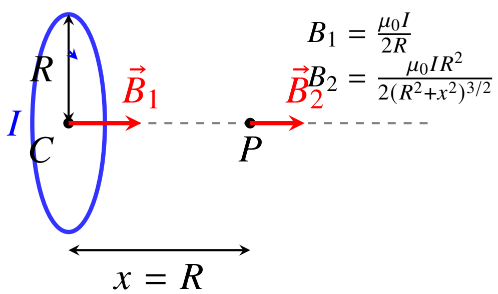
Q19. An electron is accelerated through a potential difference of $10,000$ V. Its de Broglie wavelength is approximately:

(A) 12.27 Å
(B) 1.227 Å
(C) 0.1227 Å
(D) 0.01227 Å

Q20. In an AC circuit, the instantaneous e.m.f. and current are given by $e = 100\sin(314t)$ volts and $i = 5\sin(314t + \pi/3)$ amperes. The power dissipated in the circuit is:

(A) 500 W
(B) 250 W
(C) 125 W
(D) 100 W

Q21. The magnetic field at the center of a circular current-carrying loop of radius R is B_1. The magnetic field at a point on its axis at a distance R from the center is B_2. The ratio B_1/B_2 is:



(A) $2\sqrt{2}$

(B) $1/\sqrt{2}$

(C) $\sqrt{2}$

(D) 2

Q22. The escape velocity for a body projected vertically upwards from the surface of Earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be:

(A) $11\sqrt{2}$ km/s

(B) $11/\sqrt{2}$ km/s

(C) 11 km/s

(D) 22 km/s

Q23. A transparent medium shows a critical angle of 30° when light passes from it to air. The speed of light in the medium is:

(A) 1.5×10^8 m/s

(B) 2×10^8 m/s

(C) 3×10^8 m/s

(D) 1.5×10^7 m/s

Q24. The self-inductance of a solenoid of length L, area of cross-section A and having N total turns is:

(A) $\mu_0 N^2 A/L$

(B) $\mu_0 NA/L$

(C) $\mu_0 N^2 AL$

(D) $\mu_0 NAL$

Q25. A transistor has $\beta = 100$. If the collector current changes by 10 mA, the change in emitter current is:

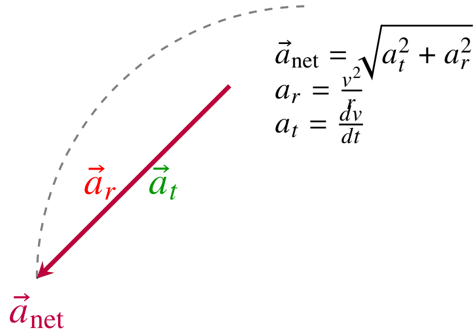
(A) 10.1 mA

(B) 9.9 mA

(C) 10 mA

(D) 11 mA

Q26. A car moves at a speed of 30 m/s on a circular track of radius 300 m. Its speed is increasing at the rate of 4 m/s^2. The net acceleration of the car is:



(A) 3 m/s^2

(B) 4 m/s^2

(C) 5 m/s^2

(D) 7 m/s^2

Q27. Two charges +1 μC and +5 μC are placed 0.1 m apart. What is the ratio of the force exerted by the first on the second to that exerted by the second on the first?

(A) 1 : 1

(B) 1 : 5

(C) 5 : 1

(D) 1 : 25

Q28. A tuning fork of frequency 480 Hz produces 10 beats per second when sounded together with a vibrating sonometer string. When the tension in the string is slightly increased, the beat frequency decreases to 5. The original frequency of the string was:

(A) 470 Hz

(B) 490 Hz

(C) 475 Hz

(D) 485 Hz

Q29. The de-Broglie wavelength of a neutron at thermal equilibrium with heavy water at temperature T (Kelvin) is:

(A) $\frac{h}{\sqrt{3mkT}}$

(B) $\frac{h}{\sqrt{mkT}}$

(C) $\frac{2h}{\sqrt{3mkT}}$

(D) $\frac{h}{2\sqrt{mkT}}$

Q30. In a common emitter amplifier, the audio signal voltage across the collector resistance of 2 kΩ is 2 V. If the current amplification factor of the transistor is 100 and the base resistance is 1 kΩ, then the input signal voltage is:

(A) 10 mV

(B) 20 mV

(C) 30 mV

(D) 15 mV

Q31. The molar specific heat at constant pressure of an ideal gas is $(7/2)R$. The ratio of specific heats γ is:

(A) 1.67

(B) 1.4

(C) 1.33

(D) 1.2

Q32. A ball is thrown vertically upwards with a velocity of 20 m/s from the top of a tower. It returns to the ground in 6 s. The height of the tower is (g = 10 m/s^2):

(A) 40 m

(B) 60 m

(C) 80 m

(D) 120 m

Q33. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV. If the stopping potential for radiation incident on this surface is 5 V, then the incident radiation lies in the:

(A) Ultraviolet region

(B) Infrared region

(C) Visible region

(D) X-ray region

Q34. A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance and a cell of e.m.f. 2 V and negligible internal resistance. A source of e.m.f. 10 mV is balanced against a length of 40 cm of the potentiometer wire. The value of external resistance is:

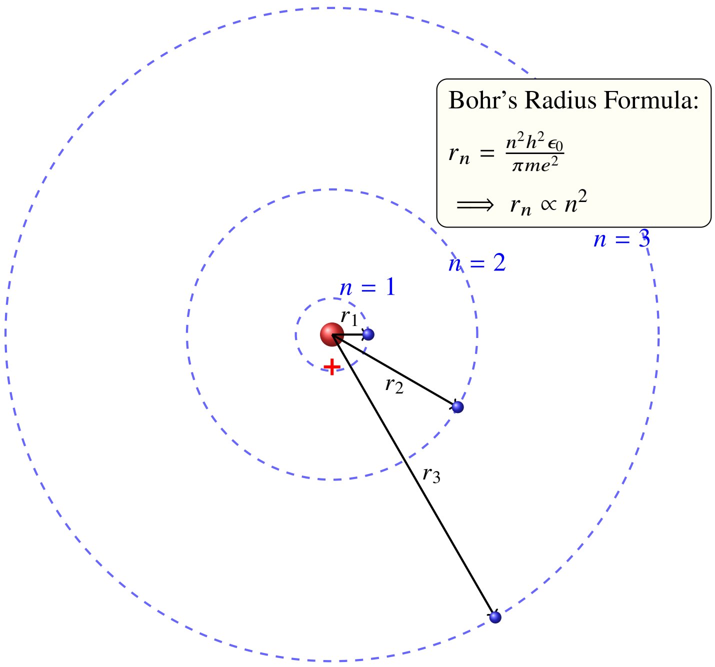
(A) 790 Ω

(B) 810 Ω

(C) 390 Ω

(D) 400 Ω

Q35. The ratio of the radii of the first three Bohr orbits of a hydrogen atom is:



(A) 1 : 2 : 3

(B) 1 : 4 : 9

(C) 1 : 8 : 27

(D) $1 : \sqrt{2} : \sqrt{3}$

Q36. An inductor of 20 mH, a capacitor of 100 μF and a resistor of 50 Ω are connected in series across a source of e.m.f. $V = 10\sin(314t)$. The power loss in the circuit is:

(A) 0.79 W

(B) 0.43 W

(C) 2.74 W

(D) 1.13 W

Q37. Two spheres of same material and radius R and $2R$ are heated to the same temperature and allowed to cool in the same surroundings. The ratio of their rates of cooling is:

(A) 1 : 2

(B) 2 : 1

(C) 1 : 4

(D) 4 : 1

Q38. A body of mass 2 kg is rotating in a vertical circle of radius 4 m. The difference in its kinetic energy at the top and bottom of the circle is:

(A) 80 J

(B) 160 J

(C) 40 J

(D) 240 J

Q39. A convex lens of focal length 20 cm is placed in contact with a concave lens of focal length 40 cm. The power of the combination is:

(A) +2.5 D

(B) −2.5 D

(C) +5 D

(D) −5 D

Q40. The binding energy per nucleon of $3Li^7$ and $2He^4$ are 5.60 MeV and 7.06 MeV respectively. In the nuclear reaction $3Li^7 + 1H^1 \rightarrow 2He^4 + 2He^4 + Q$, the value of Q is:

(A) 19.6 MeV

(B) 17.3 MeV

(C) 8.4 MeV

(D) 28.2 MeV

Q41. The wavelength of the first line of the Lyman series for hydrogen atom is L. The wavelength of the first line of the Balmer series for the same atom is:

(A) $27L/5$

(B) $5L/27$

(C) $32L/27$

(D) $27L/32$

Q42. A particle moves in a circle of radius 5 cm with constant speed and time period 0.2π s. The acceleration of the particle is:

(A) 5 m/s^2

(B) 25 m/s^2

(C) 36 m/s^2

(D) 15 m/s^2

Q43. The radiant energy from the sun incident normally at the surface of earth is 20 kcal/m^{2}min. What would have been the radiant energy incident normally on the earth if the sun had a temperature twice of the present one?

(A) 160 kcal/m^{2}min

(B) 40 kcal/m^{2}min

(C) 320 kcal/m^{2}min

(D) 80 kcal/m^{2}min

Q44. A mass m is attached to the end of a spring of force constant k. The properly adjusted spring and mass system is taken to the moon. Which of the following is true?

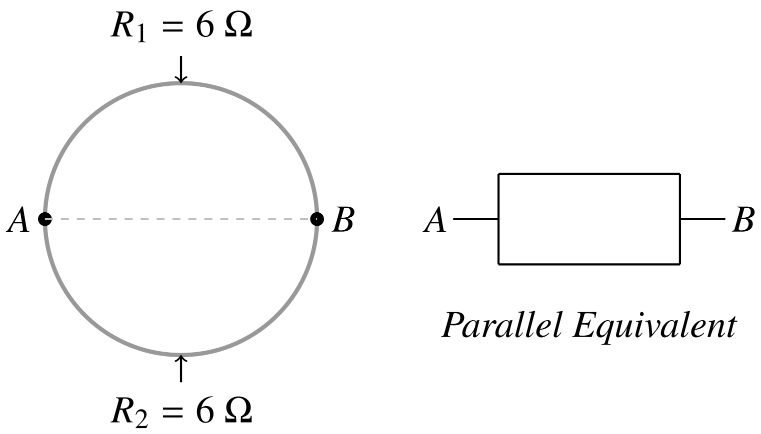
(A) The time period of oscillation increases

(B) The time period of oscillation decreases

(C) The time period of oscillation remains the same

(D) The frequency of oscillation increases

Q45. A wire of resistance 12 Ω is bent in the form of a circle. The effective resistance between the ends of any diameter is:



(A) $3\ \Omega$

(B) $6\ \Omega$

(C) $4\ \Omega$

(D) $24\ \Omega$

Q46. The magnetic field in a plane electromagnetic wave is given by $B = 2\times10^{-7}\sin(0.5\times 10^3 x + 1.5\times10^{11} t)$ T. The expression for the electric field is:

(A) $E = 60\sin(0.5\times10^3 x + 1.5\times10^{11} t)$ V/m

(B) $E = 30\sin(0.5\times10^3 x + 1.5\times10^{11} t)$ V/m

(C) $E = 20\sin(0.5\times10^3 x + 1.5\times10^{11} t)$ V/m

(D) $E = 6\sin(0.5\times10^3 x + 1.5\times10^{11} t)$ V/m

Q47. Two simple harmonic motions are represented by $y_1 = 10\sin(3\pi t + \pi/4)$ and $y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$. The ratio of their amplitudes is:

(A) 1 : 1

(B) 2 : 1

(C) 1 : 2

(D) 1 : 4

Q48. In a diffraction pattern by a single slit of width a, the first minimum is observed at an angle of 30° when light of wavelength 500 nm is incident on the slit. The first secondary maximum is observed at an angle of:

(A) $\sin^{-1}(1/4)$

(B) $\sin^{-1}(2/3)$

(C) $\sin^{-1}(3/4)$

(D) $\sin^{-1}(1/2)$

Q49. A body of mass M hits normally a rigid wall with velocity v and bounces back with the same velocity. The change in momentum is:

(A) Mv

(B) $2Mv$

(C) 0

(D) $Mv/2$

Q50. Which of the following gates is called a universal gate?

(A) OR gate

(B) AND gate

(C) NAND gate

(D) NOT gate

Detailed Solutions

Q1.

Solution

Concept:

When two rods are joined in series, the total thermal resistance is the sum of the individual resistances. Thermal resistance R_{th} is given by:

$$R_{th} = \frac{L}{KA}$$

For a composite rod of total length $L_{total} = L_1 + L_2$ and equivalent conductivity K_{eq}:

$$\frac{L_1 + L_2}{K_{eq}A} = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A}$$

Solution:

1. Here, $L_1 = L$ and $L_2 = 2L$. The total length is $L + 2L = 3L$. 2. The equivalent resistance equation becomes:

$$\frac{3L}{K_{eq}A} = \frac{L}{K_1 A} + \frac{2L}{K_2 A}$$

3. Canceling L/A from both sides:

$$\frac{3}{K_{eq}} = \frac{1}{K_1} + \frac{2}{K_2}$$

4. Simplify the right side:

$$\frac{3}{K_{eq}} = \frac{K_2 + 2K_1}{K_1 K_2}$$

5. Rearranging for K_{eq}:

$$K_{eq} = \frac{3K_1 K_2}{2K_1 + K_2}$$

Final Answer: The equivalent thermal conductivity is $\frac{3K_1K_2}{2K_1+K_2}$.

Answer: (A)

Q2.

Solution

Concept:
For a particle in circular motion, the centripetal force is provided by the given force F. Centripetal force is $F_c = \frac{mv^2}{r}$. Kinetic Energy (K.E.) is $\frac{1}{2}mv^2$. Potential Energy (P.E.) is obtained by integrating the force: $U = -\int F dr$. Total Energy $E = K.E. + P.E.$

Solution:
1. Magnitude of centripetal force: $\frac{mv^2}{r} = \frac{k}{r^2}$ 2. Thus, $mv^2 = \frac{k}{r}$. 3. Kinetic Energy: $K.E. = \frac{1}{2}mv^2 = \frac{k}{2r}$. 4. Potential Energy: $U = -\int \left(-\frac{k}{r^2}\right) dr = -\left[\frac{k}{r}\right] = -\frac{k}{r}$. 5. Total Energy: $E = K.E. + P.E. = \frac{k}{2r} - \frac{k}{r} = -\frac{k}{2r}$.

Final Answer: The total energy is $-\frac{k}{2r}$.

Answer: (A)

Q3.

Solution

Concept:
The resultant intensity I in Young's Double Slit Experiment is given by:

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

where ϕ is the phase difference. The relationship between phase difference (ϕ) and path difference (Δx) is:

$$\phi = \frac{2\pi}{\lambda}\Delta x$$

Solution:
1. Given path difference $\Delta x = \lambda/6$. 2. Calculate phase difference: $\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3} = 60^\circ$. 3. Substitute into the intensity formula:

$$I = I_0 \cos^2\left(\frac{60^\circ}{2}\right) = I_0 \cos^2(30^\circ)$$

4. Since $\cos(30^\circ) = \sqrt{3}/2$:

$$I = I_0 \left(\frac{\sqrt{3}}{2}\right)^2 = I_0 \left(\frac{3}{4}\right)$$

5. Therefore, $I/I_0 = 3/4$.

Final Answer: I/I_0 is equal to $3/4$.

Answer: (C)

Q4.

Solution

Concept:

The acceleration of an object rolling down an inclined plane without slipping is:

$$a = \frac{g \sin\theta}{1 + \frac{I}{MR^2}}$$

where I is the moment of inertia about the axis of rotation.

Solution:

1. For a solid sphere: $I_s = \frac{2}{5}MR^2$.

$$a_{solid} = \frac{g \sin\theta}{1 + \frac{2}{5}} = \frac{g \sin\theta}{7/5} = \frac{5}{7} g \sin\theta$$

2. For a hollow sphere: $I_h = \frac{2}{3}MR^2$.

$$a_{hollow} = \frac{g \sin\theta}{1 + \frac{2}{3}} = \frac{g \sin\theta}{5/3} = \frac{3}{5} g \sin\theta$$

3. Ratio $a_s : a_h = \frac{5}{7} : \frac{3}{5} = \frac{5\times5}{7\times3} = 25 : 21$.

Final Answer: The ratio of their acceleration is 25 : 21.

Answer: (A)

Q5.

Solution

Concept:

The electric potential V at a point on the axis of a uniform ring of charge Q and radius R at a distance x from the center is:

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{R^2 + x^2}}$$

Solution:

1. Given distance $x = R\sqrt{3}$. 2. Calculate the denominator: $\sqrt{R^2 + (R\sqrt{3})^2} = \sqrt{R^2 + 3R^2} = \sqrt{4R^2} = 2R$. 3. Substitute this into the potential formula:

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{2R}$$

4. This can be written as $V = \frac{Q}{8\pi\epsilon_0 R}$.

Final Answer: The electric potential is $\frac{Q}{8\pi\epsilon_0 R}$.

Answer: (B)

Q6.

Solution

Concept:

The Gravitational Potential Energy (U) of a satellite of mass m at a distance r from the center of Earth (mass M) is given by:

$$U = -\frac{GMm}{r}$$

The Kinetic Energy (K) is:

$$K = \frac{GMm}{2r}$$

The Total Energy (E) is the sum of K and U:

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

Solution:

1. From the equations above, we can see that $K = -U/2$. 2. Substituting K into the Total Energy expression:

$$E = -\frac{U}{2} + U \text{ (Incorrect approach, use direct ratio)}$$

3. Comparing $E = -\frac{GMm}{2r}$ and $U = -\frac{GMm}{r}$, we find:

$$E = \frac{1}{2}\left(-\frac{GMm}{r}\right) = \frac{U}{2}$$

Final Answer: The total energy is $U/2$.

Answer: (A)

Q7.

Solution

Concept:
According to Faraday's Law of Induction, the magnitude of the induced electromotive force (e.m.f.) is equal to the rate of change of magnetic flux (ϕ) through the circuit:

$$|e| = \frac{d\phi}{dt}$$

Solution:
1. Given $\phi = (4t^2 + 5t + 2)$ mWb. 2. Differentiate with respect to time t:

$$\frac{d\phi}{dt} = \frac{d}{dt}(4t^2 + 5t + 2) = 8t + 5$$

3. The induced e.m.f. at $t = 2$ s is:

$$e = 8(2) + 5 = 16 + 5 = 21$$

4. Since the flux was in mWb, the e.m.f. is in mV.
Final Answer: The induced e.m.f. is 21 mV.

Answer: (A)

Q8.

Solution

Concept:
The efficiency (η) of a Carnot engine is given by:

$$\eta = 1 - \frac{T_{sink}}{T_{source}} = \frac{W}{Q_{absorbed}}$$

where temperatures must be in Kelvin ($K = °C + 273$).
Solution:
1. Convert temperatures: $T_{source} = 227 + 273 = 500$ K $T_{sink} = 127 + 273 = 400$ K 2. Calculate efficiency:

$$\eta = 1 - \frac{400}{500} = 1 - 0.8 = 0.2$$

3. Work done $W = \eta \times Q_{absorbed}$:

$$W = 0.2 \times 6 \times 10^4 \text{ cal} = 1.2 \times 10^4 \text{ cal}$$

Final Answer: The heat converted into work is 1.2×10^4 cal.

Answer: (A)

Q9.

Solution

Concept:

The maximum wavelength (λ_{max}), also known as the threshold wavelength, is related to the work function (ϕ) of the metal by:

$$\phi = \frac{hc}{\lambda_{max}}$$

Using the approximation $hc \approx 12400$ eV · Å.

Solution:

1. Given $\phi = 4.2$ eV. 2. Rearrange formula: $\lambda_{max} = \frac{12400}{\phi}$ 3. Calculate:

$$\lambda_{max} = \frac{12400}{4.2} \approx 2952.38 \text{ Å}$$

4. The closest value in the options is 2955 Å.

Final Answer: The maximum wavelength is 2955 Å.

Answer: (A)

Q10.

Solution

Concept:

When two capacitors C_1 and C_2 charged to potentials V_1 and V_2 are connected in parallel (positive to positive), the common potential V is given by:

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Solution:

1. Given: $C_1 = 2\ \mu$F, $V_1 = 100$ V, $C_2 = 3\ \mu$F, $V_2 = 200$ V. 2. Total charge $Q = C_1V_1 + C_2V_2$:

$$Q = (2 \times 100) + (3 \times 200) = 200 + 600 = 800\ \mu\text{C}$$

3. Total capacitance $C_{eq} = C_1 + C_2 = 2 + 3 = 5\ \mu$F. 4. Common potential $V = \frac{800}{5} = 160$ V.

Final Answer: The common potential is 160 V.

Answer: (B)

Q11.

Solution

Concept:

The time period T of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where l is the length and g is the acceleration due to gravity. The value of g changes with altitude according to the relation $g_h = g\left(\frac{R}{R+h}\right)^2$.

Solution:

1. At the surface of the Earth:

$$T_1 = 2\pi\sqrt{\frac{l}{g}}$$

2. At height $h = R$:

$$g_h = g\left(\frac{R}{R+R}\right)^2 = g\left(\frac{R}{2R}\right)^2 = \frac{g}{4}$$

3. The new time period T_2 is:

$$T_2 = 2\pi\sqrt{\frac{l}{g/4}} = 2\pi\sqrt{\frac{4l}{g}} = 2\left(2\pi\sqrt{\frac{l}{g}}\right)$$

4. Therefore:

$$T_2 = 2T_1 \implies \frac{T_2}{T_1} = 2$$

Final Answer: The ratio T_2/T_1 is 2.

Answer: (B)

Q12.

Solution

Concept:

Radioactivity is a nuclear process that depends on the decay constant (λ) of the specific isotope. The half-life period ($T_{1/2}$) is related to the decay constant by:

$$T_{1/2} = \frac{0.693}{\lambda}$$

This value is an intrinsic property of the radioactive substance.

Solution:

1. The decay constant λ is independent of the mass or quantity of the sample. 2. Since $T_{1/2}$ depends only on λ, it does not change regardless of whether the mass is doubled, halved, or changed. 3. Therefore, the half-life period remains constant.

Final Answer: The half-life period will remain the same.

Answer: (C)

Q13.

Solution

Concept:
The fundamental frequency n of a stretched string is given by:

$$n = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

where T is the tension and μ is the mass per unit length. An "octave" refers to a frequency that is twice the fundamental frequency ($n' = 2n$).

Solution:
1. From the formula, $n \propto \sqrt{T}$. 2. Let the initial tension be $T_1 = 4$ kg-wt and initial frequency be n_1. 3. Let the required tension be T_2 for frequency $n_2 = 2n_1$. 4. Setting up the ratio:

$$\frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}} \implies \frac{2n_1}{n_1} = \sqrt{\frac{T_2}{4}}$$

5. Squaring both sides:

$$2^2 = \frac{T_2}{4} \implies 4 = \frac{T_2}{4}$$

6. $T_2 = 16$ kg-wt.
Final Answer: The weight required is 16 kg.

Answer: (C)

Q14.

Solution

Concept:
An electron moving in electric and magnetic fields experiences forces. The electric force is $\vec{F_e} = q\vec{E}$. The magnetic force is $\vec{F_m} = q(\vec{v} \times \vec{B})$.

Solution:
1. The magnetic field and velocity are in the same direction ($\theta = 0°$). 2. Magnetic force: $F_m = qvB\sin(0°) = 0$. So, the magnetic field does not affect the electron. 3. The electric field E is in the same direction as the velocity v. 4. For an electron (negative charge), the force $\vec{F_e} = -e\vec{E}$ is opposite to the direction of the electric field. 5. Since the force is opposite to the direction of motion, the electron will experience retardation. 6. Thus, the velocity will decrease.
Final Answer: The electron velocity will decrease.

Answer: (D)

Q15.

Solution

Concept:
According to Brewster's Law, when the reflected and refracted rays are perpendicular, the angle of incidence is the polarizing angle (i_p). It is related to the refractive index (μ) by:

$$\mu = \tan i_p$$

Solution:
1. Given $\mu = \sqrt{3}$. 2. Using Brewster's relation: $\sqrt{3} = \tan i_p$. 3. Since $\tan 60^\circ = \sqrt{3}$, the angle of incidence $i_p = 60^\circ$. 4. At this specific angle, the reflected and refracted rays are at 90° to each other.
Final Answer: The angle of incidence is 60°.

Answer: (C)

Q16.

Solution

Concept:
When a cylinder rolls without slipping, it possesses both translational kinetic energy ($K_t = \frac{1}{2}Mv^2$) and rotational kinetic energy ($K_r = \frac{1}{2}I\omega^2$). The total initial energy is the sum of these two. As it moves up a smooth inclined plane, this total kinetic energy is converted into gravitational potential energy ($U = Mgh$) at the highest point. For a solid cylinder, $I = \frac{1}{2}MR^2$ and $v = R\omega$.
Solution:
1. Total initial kinetic energy $K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. 2. Substituting $I = \frac{1}{2}MR^2$ and $v = R\omega$:

$$K_{total} = \frac{1}{2}M(R\omega)^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2$$

$$K_{total} = \frac{1}{2}MR^2\omega^2 + \frac{1}{4}MR^2\omega^2 = \frac{3}{4}MR^2\omega^2$$

3. At maximum height h, $K_{total} = Mgh$. 4. Equating the energies:

$$\frac{3}{4}MR^2\omega^2 = Mgh$$

5. Solving for h:

$$h = \frac{3R^2\omega^2}{4g}$$

Final Answer: The height reached by the cylinder is $\frac{3R^2\omega^2}{4g}$.

Answer: (A)

Q17.

Solution

Concept:
The surface energy of a liquid drop is given by $E = T \times A$, where T is surface tension and A is the surface area ($4\pi R^2$ for a sphere). When a drop is broken into n droplets, the total volume remains constant, but the total surface area increases. The work done (expenditure of energy) is $\Delta E = T(A_{final} - A_{initial})$.
Solution:
1. Volume of big drop = Volume of 1000 small droplets:

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3 \implies R = 10r \implies r = \frac{R}{10}$$

2. Initial surface area $A_i = 4\pi R^2$. 3. Final surface area $A_f = 1000 \times (4\pi r^2) = 1000 \times 4\pi \left(\frac{R}{10}\right)^2 = 1000 \times \frac{4\pi R^2}{100} = 10 \times 4\pi R^2$. 4. Increase in area $\Delta A = A_f - A_i = 40\pi R^2 - 4\pi R^2 = 36\pi R^2$. 5. Work done $W = T \cdot \Delta A = 36\pi R^2 T$.
Final Answer: The expenditure of energy is $36\pi R^2 T$.

Answer: (B)

Q18.

Solution

Concept:
In Simple Harmonic Motion (S.H.M.), the potential energy (P.E.) at displacement x is $\frac{1}{2}kx^2$ and the kinetic energy (K.E.) is $\frac{1}{2}k(A^2 - x^2)$, where A is the amplitude and k is the force constant.
Solution:
1. Given condition: $K.E. = P.E.$ 2. Substitute the expressions:

$$\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$$

3. Cancel $\frac{1}{2}k$ from both sides:

$$A^2 - x^2 = x^2$$

4. Rearrange:

$$A^2 = 2x^2 \implies x^2 = \frac{A^2}{2}$$

5. Taking the square root:

$$x = \frac{A}{\sqrt{2}}$$

Final Answer: The kinetic energy is equal to potential energy at $x = A/\sqrt{2}$.

Answer: (B)

Q19.

Solution

Concept:
The de Broglie wavelength (λ) of an electron accelerated through a potential difference V is given by the formula:

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$

This formula is derived from $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$.
Solution:
1. Given potential difference $V = 10,000$ V. 2. Substitute V into the formula:

$$\lambda = \frac{12.27}{\sqrt{10000}}$$

3. Since $\sqrt{10000} = 100$:

$$\lambda = \frac{12.27}{100} = 0.1227 \text{ Å}$$

Final Answer: The de Broglie wavelength is 0.1227 Å.

Answer: (C)

Q20.

Solution

Concept:
In an AC circuit, the average power dissipated is given by:

$$P = V_{rms} I_{rms} \cos\phi$$

where $V_{rms} = \frac{E_0}{\sqrt{2}}$, $I_{rms} = \frac{I_0}{\sqrt{2}}$, and ϕ is the phase difference between voltage and current.
Solution:
1. From the equations: $E_0 = 100$ V, $I_0 = 5$ A, and phase difference $\phi = \pi/3 = 60°$. 2. Calculate $V_{rms} \cdot I_{rms}$:

$$V_{rms} I_{rms} = \frac{100}{\sqrt{2}} \times \frac{5}{\sqrt{2}} = \frac{500}{2} = 250$$

3. The power factor is $\cos(60°) = 1/2$. 4. Power dissipated:

$$P = 250 \times \frac{1}{2} = 125 \text{ W}$$

Final Answer: The power dissipated in the circuit is 125 W.

Answer: (C)

Q21.

Solution

Concept:

The magnetic field (B) produced by a circular current-carrying loop of radius R at a distance x from its center along the axis is given by:

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

At the center of the loop, $x = 0$, the formula simplifies to $B_{center} = \frac{\mu_0 I}{2R}$.

Solution:

1. Magnetic field at the center (B_1):

$$B_1 = \frac{\mu_0 I}{2R}$$

2. Magnetic field at a point on the axis at distance $x = R$ (B_2):

$$B_2 = \frac{\mu_0 I R^2}{2(R^2 + R^2)^{3/2}} = \frac{\mu_0 I R^2}{2(2R^2)^{3/2}}$$

3. Simplify the denominator: $(2R^2)^{3/2} = 2^{3/2}(R^2)^{3/2} = 2\sqrt{2}R^3$. 4. Substitute back into B_2:

$$B_2 = \frac{\mu_0 I R^2}{2(2\sqrt{2}R^3)} = \frac{\mu_0 I}{4\sqrt{2}R}$$

5. Find the ratio B_1/B_2:

$$\frac{B_1}{B_2} = \frac{\frac{\mu_0 I}{2R}}{\frac{\mu_0 I}{4\sqrt{2}R}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

Final Answer: The ratio B_1/B_2 is $2\sqrt{2}$.

Answer: (A)

Q22.

Solution

Concept:

The escape velocity (v_e) from the surface of a planet is given by:

$$v_e = \sqrt{\frac{2GM}{R}}$$

where G is the gravitational constant, M is the mass of the planet, and R is its radius.

Solution:

1. The formula for escape velocity involves only the mass and radius of the planet and the gravitational constant. 2. It represents the minimum speed required for an object to break free from the planet's gravitational pull and reach infinity with zero total energy. 3. Crucially, the expression for v_e does not contain any term related to the angle of projection. 4. This means that as long as the object is projected from the surface and does not hit the ground immediately, the escape velocity is independent of the direction of projection. 5. Therefore, the escape velocity remains 11 km/s regardless of whether it is projected at 45°, 90°, or any other angle.

Final Answer: The escape velocity will be 11 km/s.

Answer: (C)

Q23.

Solution

Concept:
The critical angle (C) for a medium is the angle of incidence in the denser medium for which the angle of refraction in the rarer medium is $90°$. It is related to the refractive index (μ) of the medium by:

$$\mu = \frac{1}{\sin C}$$

The refractive index is also the ratio of the speed of light in vacuum (c) to the speed of light in the medium (v): $\mu = c/v$.

Solution:
1. Given critical angle $C = 30°$. 2. Calculate the refractive index μ:

$$\mu = \frac{1}{\sin 30°} = \frac{1}{0.5} = 2$$

3. Use the relation $\mu = c/v$ where $c = 3 \times 10^8$ m/s.

$$2 = \frac{3 \times 10^8}{v}$$

4. Solve for v:

$$v = \frac{3 \times 10^8}{2} = 1.5 \times 10^8 \text{ m/s}$$

Final Answer: The speed of light in the medium is 1.5×10^8 m/s.

Answer: (A)

Q24.

Solution

Concept:
The self-inductance (L) of a long solenoid is a measure of its ability to oppose changes in current. It is given by the formula:

$$L = \frac{\mu_0 N^2 A}{l}$$

where μ_0 is the permeability of free space, N is the total number of turns, A is the area of cross-section, and l is the length of the solenoid.

Solution:
1. The magnetic field inside a long solenoid is $B = \mu_0 n I$, where $n = N/l$ is the number of turns per unit length. 2. The total magnetic flux linked with the solenoid is $\Phi = NBA = N(\mu_0 \frac{N}{l} I)A$. 3. Simplifying the flux expression: $\Phi = \frac{\mu_0 N^2 A I}{l}$. 4. By definition, self-inductance is $L = \Phi/I$. 5. Therefore, $L = \frac{\mu_0 N^2 A}{l}$.

Final Answer: The self-inductance is $\mu_0 N^2 A/L$.

Answer: (A)

Q25.

Solution

Concept:

In a transistor, the relationship between the emitter current (I_E), base current (I_B), and collector current (I_C) is:

$$I_E = I_B + I_C$$

The current gain in common-emitter configuration is $\beta = \Delta I_C / \Delta I_B$.

Solution:

1. Given $\beta = 100$ and change in collector current $\Delta I_C = 10$ mA. 2. Calculate the change in base current ΔI_B:

$$\Delta I_B = \frac{\Delta I_C}{\beta} = \frac{10 \text{ mA}}{100} = 0.1 \text{ mA}$$

3. The change in emitter current is the sum of the changes in base and collector currents:

$$\Delta I_E = \Delta I_B + \Delta I_C$$

$$\Delta I_E = 0.1 \text{ mA} + 10 \text{ mA} = 10.1 \text{ mA}$$

Final Answer: The change in emitter current is 10.1 mA.

Answer: (A)

Q26.

Solution

Concept:

The net acceleration (a_{net}) of a particle moving in a circular path with changing speed is the vector sum of two perpendicular components: 1. **Centripetal acceleration (a_c):** Responsible for changing the direction of velocity. It is given by $a_c = v^2/r$. 2. **Tangential acceleration (a_t):** Responsible for changing the magnitude of velocity (speed). It is given by $a_t = dv/dt$. The net acceleration is calculated as:

$$a_{net} = \sqrt{a_c^2 + a_t^2}$$

Solution:

1. Given: speed $v = 30$ m/s, radius $r = 300$ m, and tangential acceleration $a_t = 4$ m/s^2. 2. Calculate the centripetal acceleration:

$$a_c = \frac{v^2}{r} = \frac{30^2}{300} = \frac{900}{300} = 3 \text{ m/s}^2$$

3. The tangential acceleration is already provided as $a_t = 4$ m/s^2. 4. Calculate the net acceleration:

$$a_{net} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25}$$

5. Thus, $a_{net} = 5$ m/s^2.

Final Answer: The net acceleration of the car is 5 m/s^2.

Answer: (C)

Q27.

Solution

Concept:

Newton's Third Law of Motion states that for every action, there is an equal and opposite reaction. In the context of electrostatics, Coulomb's Law gives the magnitude of the force between two point charges q_1 and q_2 separated by distance r:

$$F = k\frac{q_1 q_2}{r^2}$$

The force exerted by q_1 on q_2 is equal in magnitude and opposite in direction to the force exerted by q_2 on q_1.

Solution:

1. Let F_{12} be the force exerted by the first charge (+1 μC) on the second charge (+5 μC). 2. Let F_{21} be the force exerted by the second charge on the first charge. 3. According to Coulomb's law, the magnitude of both forces is:

$$|F_{12}| = |F_{21}| = k\frac{(1 \times 10^{-6})(5 \times 10^{-6})}{(0.1)^2}$$

4. Since the magnitudes are identical, the ratio of the forces is:

$$\frac{|F_{12}|}{|F_{21}|} = \frac{1}{1} = 1 : 1$$

5. This holds true regardless of the values of the charges or the distance between them.

Final Answer: The ratio of the forces is 1 : 1.

Answer: (A)

Q28.

Solution

Concept:
Beats are produced when two sound waves of slightly different frequencies (n_1 and n_2) interfere. The beat frequency is $|n_1 - n_2|$. The frequency of a sonometer string (n_s) is proportional to the square root of tension (T): $n_s \propto \sqrt{T}$. Therefore, increasing the tension increases the frequency of the string.

Solution:
1. Given tuning fork frequency $n_f = 480$ Hz and beat frequency = 10 Hz. 2. The original string frequency n_s could be $480 + 10 = 490$ Hz or $480 - 10 = 470$ Hz. 3. When tension is increased, n_s increases. 4. Case A: If $n_s = 490$ Hz, increasing tension makes $n_s > 490$. The difference $|n_s - 480|$ would increase (greater than 10). 5. Case B: If $n_s = 470$ Hz, increasing tension makes n_s move from 470 towards 480. The difference $|480 - n_s|$ would decrease. 6. Since the problem states the beat frequency decreased to 5, Case B must be correct. 7. Thus, the original frequency was 470 Hz.

Final Answer: The original frequency of the string was 470 Hz.

Answer: (A)

Q29.

Solution

Concept:
The de-Broglie wavelength (λ) is given by $\lambda = h/p$, where p is the momentum. For a particle in thermal equilibrium at temperature T, the average kinetic energy is:

$$K.E. = \frac{3}{2}kT$$

where k is the Boltzmann constant. The relationship between momentum and kinetic energy is $p = \sqrt{2m(K.E.)}$.

Solution:
1. Substitute the thermal kinetic energy into the momentum expression:

$$p = \sqrt{2m\left(\frac{3}{2}kT\right)} = \sqrt{3mkT}$$

2. Now, substitute this momentum into the de-Broglie equation:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}$$

3. This wavelength represents the wave-like property of a neutron moving with thermal energy.

Final Answer: The de-Broglie wavelength is $\frac{h}{\sqrt{3mkT}}$.

Answer: (A)

Q30.

Solution

Concept:

In a Common Emitter (CE) amplifier, the voltage gain (A_V) is the ratio of output signal voltage (V_{out}) to input signal voltage (V_{in}). It is also related to current gain (β), collector resistance (R_C), and base resistance (R_B):

$$A_V = \frac{V_{out}}{V_{in}} = \beta \frac{R_C}{R_B}$$

Solution:

1. Given: $V_{out} = 2$ V, $\beta = 100$, $R_C = 2$ k$\Omega = 2000\ \Omega$, and $R_B = 1$ k$\Omega = 1000\ \Omega$. 2. Calculate the voltage gain (A_V):

$$A_V = 100 \times \frac{2000}{1000} = 100 \times 2 = 200$$

3. Use the gain formula to find V_{in}:

$$V_{in} = \frac{V_{out}}{A_V} = \frac{2\text{ V}}{200}$$

4. Calculate:

$$V_{in} = 0.01\text{ V} = 10\text{ mV}$$

Final Answer: The input signal voltage is 10 mV.

Answer: (A)

Q31.

Solution

Concept:

The molar specific heat at constant pressure (C_P) and constant volume (C_V) for an ideal gas are related by Mayer's relation:

$$C_P - C_V = R$$

The ratio of specific heats is defined as $\gamma = C_P/C_V$. For an ideal gas, $C_P = \frac{f+2}{2}R$ and $C_V = \frac{f}{2}R$, where f is the number of degrees of freedom.

Solution:

1. Given $C_P = \frac{7}{2}R$. 2. Using Mayer's relation:

$$C_V = C_P - R = \frac{7}{2}R - R = \frac{5}{2}R$$

3. Calculate the ratio γ:

$$\gamma = \frac{C_P}{C_V} = \frac{(7/2)R}{(5/2)R} = \frac{7}{5}$$

4. Convert to decimal:

$$\gamma = 1.4$$

5. This value ($\gamma = 1.4$) typically corresponds to a diatomic gas.

Final Answer: The ratio of specific heats γ is 1.4.

Answer: (B)

Q32.

Solution

Concept:
The motion of an object thrown vertically upwards is governed by the kinematic equation:

$$S = ut + \frac{1}{2}at^2$$

When taking the point of projection as the origin, displacement (S) is the net change in position. For a ball returning to the ground from a tower of height H, the total displacement is $-H$.

Solution:
1. Let the top of the tower be the origin. 2. Initial velocity $u = +20$ m/s (upwards). 3. Acceleration $a = -g = -10$ m/s^2 (downwards). 4. Total time $t = 6$ s. 5. Displacement $S = -H$:

$$-H = (20)(6) + \frac{1}{2}(-10)(6^2)$$

6. Calculate:

$$-H = 120 - 5(36)$$

$$-H = 120 - 180$$

$$-H = -60 \implies H = 60 \text{ m}$$

Final Answer: The height of the tower is 60 m.

Answer: (B)

Q33.

Solution

Concept:
According to Einstein's photoelectric equation, the energy of an incident photon (E) is the sum of the work function (ϕ) and the maximum kinetic energy of the emitted electron (K_{max}):

$$E = \phi + K_{max}$$

The work function is related to threshold frequency (ν_0) by $\phi = h\nu_0$. The maximum kinetic energy is related to stopping potential (V_s) by $K_{max} = eV_s$.
Solution:
1. Given work function $\phi = 6.2$ eV (threshold energy). 2. Given stopping potential $V_s = 5$ V, so $K_{max} = 5$ eV. 3. Energy of incident photon:

$$E = 6.2 \text{ eV} + 5 \text{ eV} = 11.2 \text{ eV}$$

4. Wavelength of incident radiation:

$$\lambda = \frac{12400}{11.2} \approx 1107 \text{ Å}$$

5. Wavelengths between 100 Å and 4000 Å generally fall in the Ultraviolet (UV) region. Since 11.2 eV is significantly higher than the visible range (1.8 to 3.1 eV), it is UV.
Final Answer: The incident radiation lies in the Ultraviolet region.

Answer: (A)

Q34.

Solution

Concept:

A potentiometer works on the principle that the potential drop (V) across a wire is proportional to its length (l) when a constant current (I) flows through it: $V = kl$, where k is the potential gradient.

$$k = \frac{V_{wire}}{L_{total}} = \frac{I \cdot R_{wire}}{L_{total}}$$

The current I in the primary circuit is $I = \frac{E}{R_{ext}+R_{wire}}$.

Solution:

1. Balancing condition: $E_{cell} = k \cdot l_{balance} = \left(\frac{I \cdot R_{wire}}{L_{total}}\right) l_{balance}$. 2. Substitute I:

$$10 \times 10^{-3} = \left(\frac{2}{R_{ext} + 10} \times \frac{10}{100}\right) \times 40$$

3. Simplify:

$$0.01 = \frac{2}{R_{ext} + 10} \times 0.1 \times 40$$

$$0.01 = \frac{8}{R_{ext} + 10}$$

4. Solve for R_{ext}:

$$R_{ext} + 10 = \frac{8}{0.01} = 800$$

$$R_{ext} = 790\ \Omega$$

Final Answer: The value of external resistance is 790 Ω.

Answer: (A)

Q35.

Solution

Concept:

According to Bohr's model of the hydrogen atom, the radius (r_n) of the n^{th} orbit is given by:

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2}$$

This simplifies to $r_n \propto n^2$.

Solution:

1. The radius of the first orbit ($n = 1$): $r_1 \propto 1^2 = 1$. 2. The radius of the second orbit ($n = 2$): $r_2 \propto 2^2 = 4$. 3. The radius of the third orbit ($n = 3$): $r_3 \propto 3^2 = 9$. 4. Taking the ratio:

$$r_1 : r_2 : r_3 = 1^2 : 2^2 : 3^2$$

$$r_1 : r_2 : r_3 = 1 : 4 : 9$$

Final Answer: The ratio of the radii is 1 : 4 : 9.

Answer: (B)

Q36.

Solution

Concept:

The power dissipated in an AC circuit containing resistance (R), inductance (L), and capacitance (C) is given by:

$$P = I_{rms}^2 R$$

where $I_{rms} = \frac{V_{rms}}{Z}$. The impedance Z is calculated as $Z = \sqrt{R^2 + (X_L - X_C)^2}$, with $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.

Solution:

1. From $V = 10\sin(314t)$, we have $V_0 = 10$ V and $\omega = 314$ rad/s. 2. Calculate V_{rms}: $V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{10}{\sqrt{2}}$ V. 3. Calculate Reactances:

$$X_L = \omega L = 314 \times 20 \times 10^{-3} \approx 6.28\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 100 \times 10^{-6}} \approx \frac{1}{0.0314} \approx 31.85\ \Omega$$

4. Calculate Impedance (Z):

$$Z = \sqrt{50^2 + (6.28 - 31.85)^2} = \sqrt{2500 + (-25.57)^2} \approx \sqrt{2500 + 653.8} \approx \sqrt{3153.8} \approx 56.16\ \Omega$$

5. Calculate I_{rms}:

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{10}{\sqrt{2} \times 56.16}$$

6. Calculate Power $P = I_{rms}^2 R$:

$$P = \left(\frac{100}{2 \times 3153.8}\right) \times 50 = \frac{5000}{6307.6} \approx 0.79\ \text{W}$$

Final Answer: The power loss in the circuit is approximately 0.79 W.

Answer: (A)

Q37.

Solution

Concept:

According to Newton's Law of Cooling, the rate of loss of heat (dQ/dt) is proportional to the surface area (A) and the temperature difference with the surroundings. However, the **rate of cooling** (dT/dt) is given by:

$$\frac{dT}{dt} = \frac{\sigma Ae(T^4 - T_s^4)}{ms}$$

where m is mass, s is specific heat, and A is surface area. For spheres of the same material, $m = \rho \cdot \frac{4}{3}\pi R^3$ and $A = 4\pi R^2$.

Solution:

1. Substitute A and m into the rate of cooling formula:

$$\frac{dT}{dt} \propto \frac{Area}{Mass} \propto \frac{R^2}{R^3} \propto \frac{1}{R}$$

2. Let $R_1 = R$ and $R_2 = 2R$. 3. The ratio of rates of cooling is:

$$\frac{(dT/dt)_1}{(dT/dt)_2} = \frac{R_2}{R_1} = \frac{2R}{R} = \frac{2}{1}$$

4. Thus, the smaller sphere cools twice as fast as the larger one because it has a larger surface-area-to-mass ratio.

Final Answer: The ratio of their rates of cooling is 2 : 1.

Answer: (B)

Q38.

Solution

Concept:
When a body moves in a vertical circle, its total mechanical energy (Potential Energy + Kinetic Energy) remains conserved. Potential Energy (P.E.) at height h is Mgh. Let the bottom be $h = 0$ and the top be $h = 2R$.

Solution:
1. Conservation of energy between bottom (B) and top (T):

$$K.E._B + P.E._B = K.E._T + P.E._T$$

2. Rearranging to find the difference in Kinetic Energy:

$$K.E._B - K.E._T = P.E._T - P.E._B$$

3. Substitute the values:

$$\Delta K.E. = Mg(2R) - Mg(0) = 2MgR$$

4. Given $M = 2$ kg, $g = 10$ m/s^2, and $R = 4$ m:

$$\Delta K.E. = 2 \times 2 \times 10 \times 4 = 160 \text{ J}$$

Final Answer: The difference in kinetic energy is 160 J.

Answer: (B)

Q39.

Solution

Concept:
The power (P) of a lens is the reciprocal of its focal length in meters ($P = 1/f$). For a combination of thin lenses in contact, the total power is the algebraic sum of the individual powers:

$$P_{total} = P_1 + P_2 = \frac{1}{f_1} + \frac{1}{f_2}$$

Convex lenses have positive focal lengths, while concave lenses have negative focal lengths.

Solution:
1. Focal length of convex lens $f_1 = +20$ cm $= +0.2$ m. 2. Focal length of concave lens $f_2 = -40$ cm $= -0.4$ m. 3. Calculate individual powers:

$$P_1 = \frac{1}{0.2} = +5 \text{ D}$$

$$P_2 = \frac{1}{-0.4} = -2.5 \text{ D}$$

4. Total power:

$$P_{total} = 5 + (-2.5) = +2.5 \text{ D}$$

Final Answer: The power of the combination is +2.5 D.

Answer: (A)

Q40.

Solution

Concept:
In a nuclear reaction, the Q-value (energy released) is the difference between the total binding energy (B.E.) of the products and the total binding energy of the reactants:

$$Q = \sum B.E._{products} - \sum B.E._{reactants}$$

Total B.E. = (B.E. per nucleon) × (Number of nucleons).

Solution:
1. Reactants: $3Li^7$ and $1H^1$. - B.E. of $Li^7 = 7 \times 5.60 = 39.2$ MeV. - B.E. of $H^1 = 0$ MeV (single proton). - Total reactant B.E. = 39.2 MeV. 2. Products: Two $2He^4$ nuclei. - B.E. of one $He^4 = 4 \times 7.06 = 28.24$ MeV. - Total product B.E. $= 2 \times 28.24 = 56.48$ MeV. 3. Q-value:

$$Q = 56.48 - 39.2 = 17.28 \text{ MeV}$$

4. Rounding to the nearest option gives 17.3 MeV.

Final Answer: The value of Q is 17.3 MeV.

Answer: (B)

Q41.

Solution

Concept:
The wavelength (λ) of radiation emitted during an electronic transition in a hydrogen-like atom is given by the Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where R is the Rydberg constant. For the Lyman series, $n_1 = 1$, and for the first line, $n_2 = 2$. For the Balmer series, $n_1 = 2$, and for the first line, $n_2 = 3$.

Solution:

1. For the first line of the Lyman series ($2 \rightarrow 1$):

$$\frac{1}{L} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R\left(1 - \frac{1}{4}\right) = \frac{3R}{4} \implies R = \frac{4}{3L}$$

2. For the first line of the Balmer series ($3 \rightarrow 2$):

$$\frac{1}{\lambda_B} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right) = R\left(\frac{5}{36}\right)$$

3. Substitute the value of R:

$$\frac{1}{\lambda_B} = \left(\frac{4}{3L}\right)\left(\frac{5}{36}\right) = \frac{5}{27L}$$

4. Therefore, $\lambda_B = \frac{27L}{5}$.

Final Answer: The wavelength of the first line of the Balmer series is $27L/5$.

Answer: (A)

Q42.

Solution

Concept:
For a particle moving in a circle with constant speed, the acceleration is centripetal (a_c) and is directed towards the center. It is given by:

$$a_c = \omega^2 r$$

where ω is the angular velocity and r is the radius. The relationship between angular velocity and time period (T) is $\omega = 2\pi/T$.

Solution:
1. Given radius $r = 5$ cm $= 0.05$ m and time period $T = 0.2\pi$ s. 2. Calculate angular velocity:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2\pi} = 10 \text{ rad/s}$$

3. Calculate centripetal acceleration:

$$a_c = \omega^2 r = (10)^2 \times 0.05$$

$$a_c = 100 \times 0.05 = 5 \text{ m/s}^2$$

Final Answer: The acceleration of the particle is 5 m/s^2.

Answer: (A)

Q43.

Solution

Concept:
According to Stefan-Boltzmann Law, the total energy (E) radiated per unit area per unit time by a black body is directly proportional to the fourth power of its absolute temperature (T):

$$E = \sigma T^4$$

If the distance and size remain constant, the incident energy follows the same proportionality.
Solution:
1. Initial energy $E_1 = 20$ kcal/m^{2}min at temperature T. 2. New temperature $T_2 = 2T$. 3. Using the proportionality $E \propto T^4$:

$$\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{2T}{T}\right)^4 = 2^4 = 16$$

4. Calculate E_2:

$$E_2 = 16 \times E_1 = 16 \times 20 = 320 \text{ kcal/m}^2\text{min}$$

Final Answer: The radiant energy would have been 320 kcal/m^{2}min.

Answer: (C)

Q44.

Solution

Concept:
The time period (T) of a mass-spring system executing simple harmonic motion is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where m is the mass and k is the spring constant.
Solution:
1. The formula for the time period of a spring-mass system depends only on the mass and the stiffness of the spring. 2. Unlike a simple pendulum ($T = 2\pi\sqrt{l/g}$), it does not contain the term g (acceleration due to gravity). 3. When the system is taken to the moon, the mass m and the spring constant k remain unchanged. 4. Therefore, the time period of oscillation remains exactly the same as on Earth.
Final Answer: The time period of oscillation remains the same.

Answer: (C)

Q45.

Solution

Concept:
Resistance (R) of a wire is proportional to its length. When a wire is bent into a circle, a diameter divides it into two equal semicircular arcs. Each arc will have half the resistance of the original wire. These two arcs are connected in parallel between the two ends of the diameter.
Solution:
1. Original resistance = 12 Ω. 2. Resistance of each semicircular arc: $R_1 = R_2 = 12/2 = 6\ \Omega$. 3. Since they are in parallel across the diameter, the effective resistance R_{eff} is:

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

4. Therefore, $R_{eff} = 3\ \Omega$.
Final Answer: The effective resistance between the ends of any diameter is 3 Ω.

Answer: (A)

Q46.

Solution

Concept:
In a plane electromagnetic wave, the electric field (E) and magnetic field (B) are perpendicular to each other and to the direction of wave propagation. They are related by the equation:

$$E_0 = B_0 \cdot c$$

where c is the speed of light ($c = \omega/k$). The phase and frequency of the electric field are the same as those of the magnetic field.
Solution:
1. From the given equation $B = 2\times10^{-7}\sin(0.5\times10^3 x+1.5\times10^{11} t)$, we identify: - $B_0 = 2\times10^{-7}$ T - $k = 0.5 \times 10^3$ rad/m - $\omega = 1.5 \times 10^{11}$ rad/s 2. Calculate the speed of the wave:

$$c = \frac{\omega}{k} = \frac{1.5 \times 10^{11}}{0.5 \times 10^3} = 3 \times 10^8 \text{ m/s}$$

3. Calculate the amplitude of the electric field (E_0):

$$E_0 = B_0 \cdot c = (2 \times 10^{-7}) \times (3 \times 10^8) = 60 \text{ V/m}$$

4. Since the fields are in phase, the expression for E is:

$$E = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t) \text{ V/m}$$

Final Answer: The expression for the electric field is $E = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ V/m.

Answer: (A)

Q47.

Solution

Concept:
The standard form of S.H.M. is $y = A\sin(\omega t + \phi)$. For a combination of harmonic motions like $y = a\sin\omega t + b\cos\omega t$, the resultant amplitude R is given by:

$$R = \sqrt{a^2 + b^2}$$

Solution:
1. For the first motion $y_1 = 10\sin(3\pi t + \pi/4)$: - The amplitude $A_1 = 10$. 2. For the second motion $y_2 = 5\sin 3\pi t + 5\sqrt{3}\cos 3\pi t$: - Here $a = 5$ and $b = 5\sqrt{3}$. - The resultant amplitude $A_2 = \sqrt{5^2 + (5\sqrt{3})^2}$ - $A_2 = \sqrt{25 + 75} = \sqrt{100} = 10$. 3. The ratio of their amplitudes is:

$$A_1 : A_2 = 10 : 10 = 1 : 1$$

Final Answer: The ratio of their amplitudes is 1 : 1.

Answer: (A)

Q48.

Solution

Concept:
In single slit diffraction, the condition for minima is $a\sin\theta = n\lambda$ and the condition for secondary maxima is $a\sin\theta = (n + 1/2)\lambda$.

Solution:
1. For the first minimum ($n = 1$):

$$a\sin(30^\circ) = 1\cdot\lambda \implies a(1/2) = \lambda \implies a = 2\lambda$$

2. For the first secondary maximum ($n = 1$):

$$a\sin\theta' = (1 + 1/2)\lambda = \frac{3}{2}\lambda$$

3. Substitute $a = 2\lambda$:

$$(2\lambda)\sin\theta' = \frac{3}{2}\lambda$$

4. Solve for $\sin\theta'$:

$$\sin\theta' = \frac{3}{4} \implies \theta' = \sin^{-1}(3/4)$$

Final Answer: The first secondary maximum is observed at an angle of $\sin^{-1}(3/4)$.

Answer: (C)

Q49.

Solution

Concept:
Momentum ($\vec{p}$) is a vector quantity given by $m\vec{v}$. The change in momentum ($\Delta\vec{p}$) is the vector difference between final momentum and initial momentum:

$$\Delta\vec{p} = \vec{p}_{final} - \vec{p}_{initial}$$

Solution:
1. Let the direction towards the wall be positive. 2. Initial momentum: $p_i = M(+v) = Mv$. 3. Since the body bounces back with the same velocity, the final velocity is $-v$. 4. Final momentum: $p_f = M(-v) = -Mv$. 5. Change in momentum:

$$\Delta p = p_f - p_i = (-Mv) - (Mv) = -2Mv$$

6. The magnitude of the change in momentum is $2Mv$.
Final Answer: The change in momentum is $2Mv$.

Answer: (B)

Q50.

Solution

Concept:
A universal gate is a logic gate that can be used to implement any Boolean function without the need for any other type of gate. The NAND and NOR gates are the two primary universal gates.
Solution:
1. AND, OR, and NOT are considered basic gates. 2. By combining multiple NAND gates, one can create the functionality of NOT, AND, and OR gates. 3. For example, a NOT gate can be made by joining both inputs of a NAND gate. 4. Because all other logic gates can be constructed using only NAND gates (or only NOR gates), NAND is classified as a universal gate.
Final Answer: NAND gate is called a universal gate.

Answer: (C)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	A	3	C	4	A	5	B
6	A	7	A	8	A	9	A	10	B
11	B	12	C	13	C	14	D	15	C
16	A	17	B	18	B	19	C	20	C
21	A	22	C	23	A	24	A	25	A
26	C	27	A	28	A	29	A	30	A
31	B	32	B	33	A	34	A	35	B
36	A	37	B	38	B	39	A	40	B
41	A	42	A	43	C	44	C	45	A
46	A	47	A	48	C	49	B	50	C