MH-BOARD-CLASS-10-MATHEMATICS-GEOMETRY-71-N-926-2023 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**80	**Total questions :**96

General Instructions

Important instructions:

1. Each activity has to be answered in complete sentence/s. One word answers will **not** be given complete credit. Just the correct activity number written in case of options will **not** be given credit.
2. Web diagrams, flow charts, tables etc. are to be presented exactly as they are with answers.
3. In point 2 above, just words without the presentation of the activity format/design, will **not** be given credit.
4. Use of colour pencils/pens etc. is **not** allowed. (Only blue/black pens are allowed.)
5. Multiple answers to the same activity will be treated as wrong and will **not** be given any credit.
6. Maintain the sequence of the Sections/ Question Nos./ Activities throughout the activity sheet.

1. If a, b, c are sides of a triangle and $a^2 + b^2 = c^2$, name the type of triangle:

(A) Obtuse angled triangle

(B) Acute angled triangle

(C) Right angled triangle

(D) Equilateral triangle

Correct Answer: (C) Right angled triangle

Solution:

Step 1: Recall the Pythagoras Theorem.

According to the Pythagoras theorem, for a right-angled triangle having sides a, b (legs), and c (hypotenuse), the relation is given by:

$$a^2 + b^2 = c^2$$

Step 2: Compare with the given condition.

The question states that $a^2 + b^2 = c^2$. Hence, this satisfies the condition of the Pythagoras theorem.

Step 3: Conclusion.

Therefore, the triangle is a **Right-angled triangle**.

Quick Tip

If the sum of squares of two sides equals the square of the third side, the triangle is right-angled.

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2. Chords AB and CD of a circle intersect inside the circle at point E. If AE = 4, EB = 10, and CE = 8, then find ED:

(A) 7

(B) 5

(C) 8

(D) 9

Correct Answer: (D) 9

Solution:

Step 1: Apply the property of intersecting chords.

When two chords intersect inside a circle, the product of the segments of one chord equals the product of the segments of the other chord. That is:

$$AE \times EB = CE \times ED$$

Step 2: Substitute the given values.

$$4 \times 10 = 8 \times ED$$

Step 3: Simplify to find ED.

$$40 = 8 \times ED$$

$$ED = \frac{40}{8} = 5$$

Step 4: Verify.

Thus, $ED = 5$.

Correct Answer: (B) 5

Quick Tip

For intersecting chords in a circle, remember that the products of the two segments of each chord are equal.

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3. Co-ordinates of origin are:

(A) (0, 0)

(B) (0, 1)

(C) (1, 0)

(D) (1, 1)

Correct Answer: (A) (0, 0)

Solution:

Step 1: Understanding the coordinate system.

In the Cartesian coordinate system, every point is represented as an ordered pair (x, y), where x is the horizontal distance and y is the vertical distance from the origin.

Step 2: Definition of the origin.

The origin is the point where the X-axis and Y-axis intersect. At this point, both coordinates are zero. Hence, the coordinates of the origin are $(0, 0)$.

Step 3: Conclusion.

Therefore, the coordinates of the origin are **(0, 0)**.

Quick Tip

Always remember — the origin in the coordinate plane is denoted by $(0, 0)$ where both axes meet.

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4. If the radius of the base of a cone is 7 cm and the height is 24 cm, find its slant height:

(A) 23 cm

(B) 26 cm

(C) 31 cm

(D) 25 cm

Correct Answer: (B) 25 cm

Solution:

Step 1: Recall the formula for the slant height of a cone.

For a cone with radius r and height h, the slant height l is given by the Pythagoras theorem as:

$$l = \sqrt{r^2 + h^2}$$

Step 2: Substitute the given values.

Given $r = 7$ cm and $h = 24$ cm, substitute into the formula:

$$l = \sqrt{7^2 + 24^2}$$

$$l = \sqrt{49 + 576} = \sqrt{625}$$

Step 3: Simplify.

$$l = 25\,\text{cm}$$

Step 4: Conclusion.

The slant height of the cone is **25 cm**.

> **Quick Tip**
>
> In right circular cones, always use the Pythagoras theorem: $l = \sqrt{r^2 + h^2}$ to find the slant height.

(B) Solve the following sub-questions: 4

(1) If $\triangle ABC \sim \triangle PQR$ and $\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{16}{25}$, then find $AB : PQ$.

Solution:

Step 1: Recall the property of similar triangles.

If two triangles are similar, then the ratio of their areas is equal to the square of the ratio of their corresponding sides. That is,

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2$$

Step 2: Substitute the given values.

$$\frac{16}{25} = \left(\frac{AB}{PQ}\right)^2$$

Step 3: Take the square root on both sides.

$$\frac{AB}{PQ} = \frac{4}{5}$$

Step 4: Conclusion.

Hence, the ratio of the corresponding sides is:

$$AB : PQ = 4 : 5$$

Correct Answer: $AB : PQ = 4 : 5$

Quick Tip

For similar triangles, remember that the ratio of their areas equals the square of the ratio of their corresponding sides.

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(2) In $\triangle RST$, $\angle S = 90°$, $\angle T = 30°$, and $RT = 12$ cm, then find RS.

Solution:

Step 1: Understanding the triangle.

In $\triangle RST$, $\angle S = 90°$ and $\angle T = 30°$. Therefore, it is a 30°–60°–90° right triangle.

Step 2: Property of 30°–60°–90° triangle.

In a 30°–60°–90° triangle, the ratio of sides is given by:

$$1 : \sqrt{3} : 2$$

where the side opposite 30° is the smallest, and the hypotenuse is twice that side.

Step 3: Identify the sides.

Given that $RT = 12$ cm (hypotenuse), so the side opposite the 30° angle (RS) is half the hypotenuse.

$$RS = \frac{1}{2} \times RT = \frac{1}{2} \times 12 = 6\,\text{cm}$$

Step 4: Conclusion.

Hence, $RS = 6$ cm.

Correct Answer: $RS = 6$ cm

Quick Tip

In a 30°–60°–90° triangle, the hypotenuse is twice the side opposite the 30° angle.

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(3) If the radius of a circle is 5 cm, then find the length of the longest chord of the circle.

Solution:

Step 1: Recall the property of chords.

The longest chord of a circle passes through its center and is equal to the diameter of the circle.

Step 2: Use the relationship between radius and diameter.

$$\text{Diameter} = 2 \times \text{radius}$$

$$\text{Diameter} = 2 \times 5 = 10\,\text{cm}$$

Step 3: Conclusion.

Hence, the length of the longest chord = 10 cm.

Correct Answer: 10 cm

Quick Tip

The diameter is always the longest chord in a circle.

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(4) Find the distance between the points $O(0, 0)$ and $P(3, 4)$.

Solution:

Step 1: Recall the distance formula.

The distance between two points (x_1, y_1) and (x_2, y_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Step 2: Substitute the given values.

Here, $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (3, 4)$

$$d = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{9+16} = \sqrt{25}$$

Step 3: Simplify.

$$d = 5$$

Step 4: Conclusion.

Hence, the distance between the two points is 5 units.

Correct Answer: 5 units

Quick Tip

Always use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ for coordinate geometry problems.

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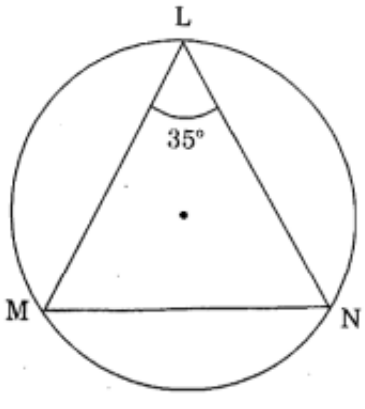
2. (A) Complete the following activities (any two): 4

(1) In the figure below, $\angle L = 35°$. Find:

(i) $m(\text{arc } MN)$

(ii) $m(\text{arc } MLN)$



Solution:

(i)

$$\angle L = \frac{1}{2}\, m(\text{arc } MN) \quad \text{(By the Inscribed Angle Theorem)}$$

$$35^\circ = \frac{1}{2}\, m(\text{arc } MN)$$

$$2 \times 35^\circ = m(\text{arc } MN)$$

$$\boxed{m(\text{arc } MN) = 70^\circ}$$

(ii)

$$m(\text{arc } MLN) = 360^\circ - m(\text{arc } MN) \quad \text{(By definition of measure of arc)}$$

$$m(\text{arc } MLN) = 360^\circ - 70^\circ$$

$$\boxed{m(\text{arc } MLN) = 290^\circ}$$

Final Answers:

(i) $m(\text{arc } MN) = 70^\circ$

(ii) $m(\text{arc } MLN) = 290^\circ$

Quick Tip

In a circle, the measure of an inscribed angle is half the measure of its intercepted arc.

The sum of the measures of the major and minor arcs is always 360°.

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(2) Show that, $\cot\theta + \tan\theta = \csc\theta \times \sec\theta$

Solution:

L.H.S. $= \cot\theta + \tan\theta$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta \times \cos\theta}$$

$$= \frac{1}{\sin\theta \times \cos\theta} \quad (\text{Since } \sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$$

$$= \csc\theta \times \sec\theta$$

Hence,

$$\text{L.H.S.} = \text{R.H.S.}$$

$$\boxed{\therefore \cot\theta + \tan\theta = \csc\theta \times \sec\theta}$$

Quick Tip

Always use the fundamental identity $\sin^2\theta + \cos^2\theta = 1$ to simplify trigonometric expressions.

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(3) Find the surface area of a sphere of radius 7 cm.

Solution:

Step 1: Formula for surface area of a sphere.

The formula for the surface area of a sphere is:

$$\text{Surface area} = 4\pi r^2$$

Step 2: Substitute the given values.

Given that $r = 7$ cm and $\pi = \frac{22}{7}$, we have:

$$\text{Surface area} = 4 \times \frac{22}{7} \times (7)^2$$

Step 3: Simplify the expression.

$$\text{Surface area} = 4 \times \frac{22}{7} \times 49$$

$$\text{Surface area} = 4 \times 22 \times 7$$

$$\text{Surface area} = 616 \text{ sq. cm.}$$

Step 4: Conclusion.

Therefore, the surface area of the sphere is:

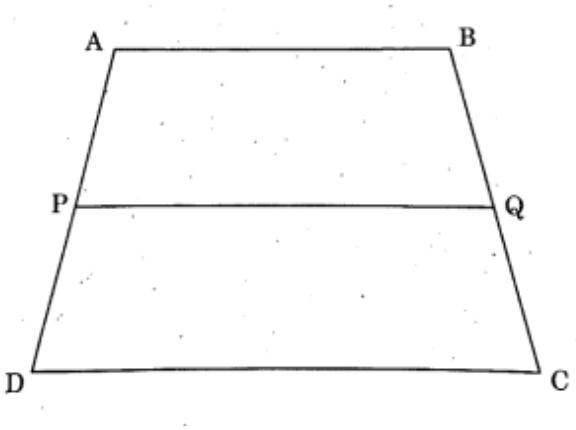
$$\boxed{616 \text{ sq. cm.}}$$

Quick Tip

The surface area of a sphere is always given by $4\pi r^2$. Don't forget to square the radius and multiply by 4.

(B) Solve the following sub-questions (Any four): 8

(1) In trapezium $ABCD$, side $AB \parallel PQ \parallel DC$. If $AP = 3$, $PD = 12$, and $QC = 14$, find BQ.



Solution:

Step 1: Understanding the given figure.

In trapezium $ABCD$, the sides AB, PQ, and DC are parallel. The lines AD and BC are transversals. Hence, the ratios of the corresponding segments on these transversals are equal.

$$\frac{AP}{PD} = \frac{BQ}{QC}$$

Step 2: Substitute the given values.

$$\frac{3}{12} = \frac{BQ}{14}$$

Step 3: Simplify the equation.

$$\frac{1}{4} = \frac{BQ}{14}$$

$$BQ = \frac{14}{4} = 3.5$$

Step 4: Conclusion.

Hence, $BQ = 3.5$ cm.

Correct Answer: $BQ = 3.5$ cm

Quick Tip

In trapeziums with parallel sides, the intercept theorem (or basic proportionality theorem) can be used to find missing segment lengths.

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(2) Find the length of the diagonal of a rectangle whose length is 35 cm and breadth is 12 cm.

Solution:

Step 1: Recall the formula for the diagonal of a rectangle.

The diagonal d of a rectangle is given by:

$$d = \sqrt{l^2 + b^2}$$

where l = length and b = breadth.

Step 2: Substitute the given values.

$$d = \sqrt{35^2 + 12^2}$$

$$d = \sqrt{1225 + 144}$$

$$d = \sqrt{1369}$$

$$d = 37\,\text{cm}$$

Step 3: Conclusion.

Hence, the length of the diagonal is **37 cm**.

Correct Answer: 37 cm

Quick Tip

The diagonal of a rectangle can always be found using the Pythagoras theorem.

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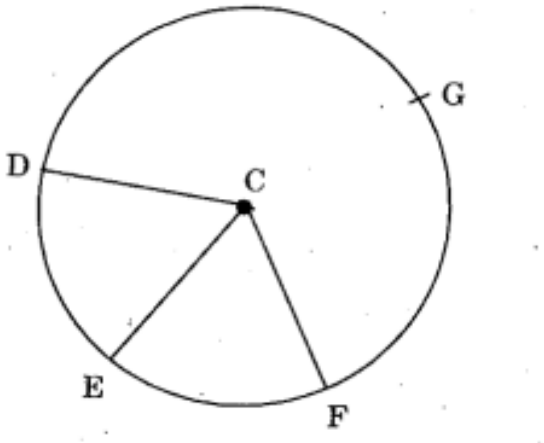
(3) In the given figure, points G, D, E, F are points on a circle with centre C. If $\angle ECF = 70°$ and $m(\text{arc } DGF) = 200°$, find:

(i) $m(\text{arc } DE)$

(ii) $m(\text{arc } DEF)$



Solution:

Step 1: Recall the relationship between central angle and its intercepted arc.

The measure of an arc is equal to the measure of its corresponding central angle.

Step 2: For arc DE.

Given that $\angle ECF = 70°$,

$$m(\text{arc } EF) = 70°$$

Also, the total measure of the circle is $360°$.

Given $m(\text{arc } DGF) = 200°$, so the remaining part of the circle (arc DEF) is:

$$m(\text{arc } DEF) = 360° - 200° = 160°$$

Now,

$$m(\text{arc } DEF) = m(\text{arc } DE) + m(\text{arc } EF)$$

$$160° = m(\text{arc } DE) + 70°$$

$$m(\text{arc } DE) = 90°$$

Step 3: Final values.

(i) $m(\text{arc } DE) = 90°$

(ii) $m(\text{arc } DEF) = 160°$

Correct Answers:

(i) $m(\text{arc } DE) = 90°$

(ii) $m(\text{arc } DEF) = 160°$

Quick Tip

In a circle, the measure of a major arc and minor arc always add up to $360°$. Use this property to find unknown arcs.

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(4) Show that points A$(-1, -1)$, B$(0, 1)$, and C$(1, 3)$ are collinear.

Solution:

Step 1: Recall the concept of collinearity.

Three points are collinear if the slopes of any two pairs of points are equal. That is,

$$\text{Slope of } AB = \text{Slope of } BC$$

Step 2: Find the slope of AB.

Using the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

For points A$(-1, -1)$ and B$(0, 1)$:

$$m_{AB} = \frac{1 - (-1)}{0 - (-1)} = \frac{2}{1} = 2$$

Step 3: Find the slope of BC.

For points B$(0, 1)$ and C$(1, 3)$:

$$m_{BC} = \frac{3 - 1}{1 - 0} = \frac{2}{1} = 2$$

Step 4: Compare the slopes.

$$m_{AB} = m_{BC} = 2$$

Since the slopes are equal, the points A, B, and C are collinear.

Correct Answer: Points A, B, and C are collinear.

Quick Tip

If the slopes of any two pairs of points are equal, the points are collinear.

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(5) A person is standing at a distance of 50 m from a temple looking at its top. The angle of elevation is $45°$. Find the height of the temple.

Solution:

Step 1: Understand the situation.

Let the height of the temple be h m and the distance from the person to the temple be 50 m.

The angle of elevation is $45°$.

Step 2: Apply the tangent trigonometric ratio.

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\tan 45^\circ = \frac{h}{50}$$

Step 3: Simplify.

$$1 = \frac{h}{50} \Rightarrow h = 50$$

Step 4: Conclusion.

Therefore, the height of the temple is **50 m**.

Correct Answer: 50 m

Quick Tip

When the angle of elevation is 45°, the height and base are equal in right-angled triangle problems.

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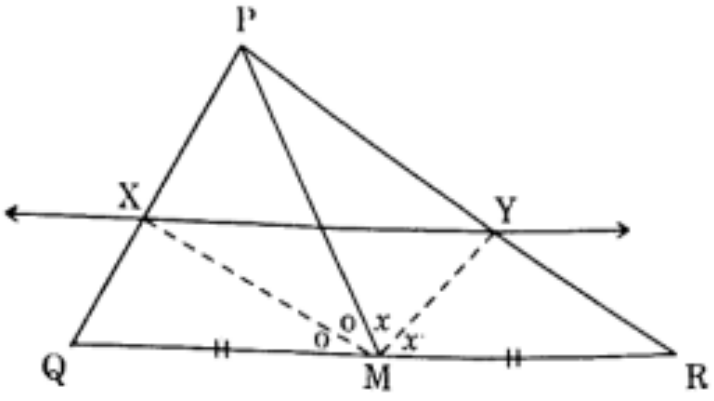
3. (A) Complete the following activities (any one): 3

(1) In $\triangle PQR$, seg PM is a median. Angle bisectors of $\angle PMQ$ and $\angle PMR$ intersect sides PQ and PR in points X and Y respectively. Prove that $XY \parallel QR$.



Solution:

Step 1: In $\triangle PMQ$,

Ray MX is the bisector of $\angle PMQ$.

By the **Angle Bisector Theorem**,

$$\frac{MP}{MQ} = \frac{PX}{XQ} \quad \text{..... (I)}$$

Step 2: In $\triangle PMR$,

Ray MY is the bisector of $\angle PMR$.

By the **Angle Bisector Theorem**,

$$\frac{MP}{MR} = \frac{PY}{YR} \quad \text{..... (II)}$$

Step 3: Since M is the midpoint of QR,

$$MQ = MR$$

Hence,

$$\frac{MP}{MQ} = \frac{MP}{MR} \quad \text{..... (III)}$$

Step 4: From (I), (II), and (III),

We get,

$$\frac{PX}{XQ} = \frac{PY}{YR}$$

Step 5: By the Converse of the Basic Proportionality Theorem (Thales Theorem),

$$XY \parallel QR$$

Hence, it is proved that $XY \parallel QR$.

Quick Tip

When medians and angle bisectors are involved in triangles, use the Angle Bisector Theorem and the Converse of the Basic Proportionality Theorem to prove parallel lines.

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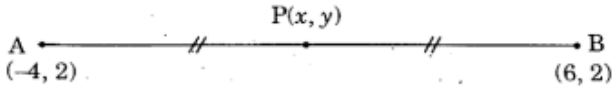
(2) Find the co-ordinates of point P where P is the midpoint of a line segment AB with A(-4, 2) and B(6, 2).



Solution:

Let $A(-4,2) = (x_1, y_1)$ and $B(6,2) = (x_2, y_2)$, and let $P(x,y)$ be the midpoint of AB.

Step 1: According to the midpoint theorem,

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}$$

Step 2: Substitute the given values.

$$x = \frac{-4+6}{2} = \frac{2}{2} = 1$$

$$y = \frac{2+2}{2} = \frac{4}{2} = 2$$

Step 3: Write the coordinates of midpoint P.

$$P(x,y) = (1,2)$$

Step 4: Conclusion.

Therefore, the co-ordinates of midpoint P are **(1, 2)**.

Correct Answer: $P(1,2)$

Quick Tip

The midpoint of a line segment joining (x_1, y_1) and (x_2, y_2) is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

(B) Solve the following sub-questions (any two): 6

(1) In $\triangle ABC$, seg AP is a median. If $BC = 18$, and $AB^2 + AC^2 = 260$, find AP.

Solution:

Step 1: Recall the formula for the length of a median.

If AP is a median to side BC in $\triangle ABC$, then

$$AB^2 + AC^2 = 2(AP^2 + \tfrac{1}{4}BC^2)$$

Step 2: Substitute the given values.

$$260 = 2(AP^2 + \tfrac{1}{4} \times 18^2)$$

$$260 = 2(AP^2 + \tfrac{1}{4} \times 324)$$

$$260 = 2(AP^2 + 81)$$

Step 3: Simplify the equation.

$$260 = 2AP^2 + 162$$

$$260 - 162 = 2AP^2$$

$$98 = 2AP^2$$

$$AP^2 = 49$$

Step 4: Find AP.

$$AP = \sqrt{49} = 7$$

Step 5: Conclusion.

Therefore, the length of median AP is **7 cm**.

Correct Answer: $AP = 7$ cm

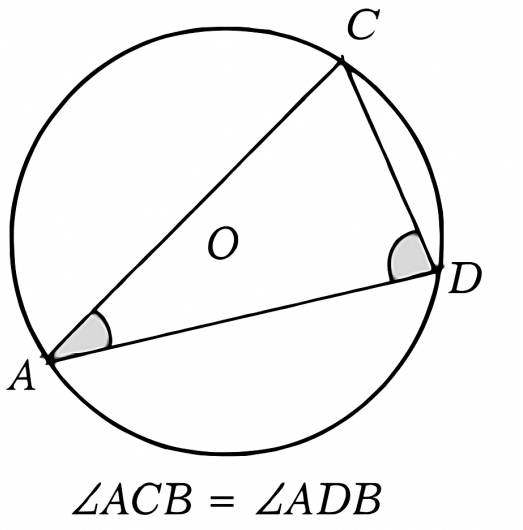
Quick Tip

For any triangle, the median length can be found using the formula $AB^2 + AC^2 = 2(AP^2 + \frac{1}{4}BC^2)$.

(2) Prove that, "Angles inscribed in the same arc are congruent."

Solution:

Given: A circle with center O, and points A, B, and C lying on the circle such that $\angle ACB$ and $\angle ADB$ are inscribed in the same arc AB.



To Prove: $\angle ACB = \angle ADB$

Proof:

Step 1: Join O to A, B, C, and D. Then, $\triangle OAC$, $\triangle OBC$, $\triangle OAD$, and $\triangle OBD$ are formed.

Step 2: $\angle AOB$ is the central angle subtending arc AB, and $\angle ACB$, $\angle ADB$ are inscribed angles subtending the same arc AB.

Step 3: By the property of a circle, the measure of an inscribed angle is half the measure of the central angle subtending the same arc.

$$\angle ACB = \frac{1}{2}\angle AOB \quad \text{and} \quad \angle ADB = \frac{1}{2}\angle AOB$$

Step 4: Therefore,

$$\angle ACB = \angle ADB$$

Hence proved.

Result: Angles inscribed in the same arc are congruent.

Quick Tip

All angles subtending the same arc (or equal arcs) in a circle are equal in measure.

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(3) Draw a circle of radius 3.3 cm. Draw a chord PQ of length 6.6 cm. Draw tangents to the circle at points P and Q.

Construction Steps:

1. Draw a circle with center O and radius 3.3 cm.
2. Draw a chord PQ of length 6.6 cm.
3. Draw the radius OP and OQ.
4. At points P and Q, draw lines perpendicular to OP and OQ respectively.
5. These perpendicular lines are the required tangents to the circle at points P and Q.

Result: The two tangents drawn at points P and Q are the required tangents.

Quick Tip

The tangent to a circle is always perpendicular to the radius at the point of contact.

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(4) The radii of the circular ends of a frustum are 14 cm and 6 cm respectively, and its height is 6 cm. Find its curved surface area. ($\pi = 3.14$)

Solution:

Step 1: Formula for curved surface area (C.S.A) of a frustum.

$$\text{C.S.A.} = \pi(r_1 + r_2)l$$

where $r_1 = 14$ cm, $r_2 = 6$ cm, and l = slant height.

Step 2: Find slant height using Pythagoras theorem.

$$l = \sqrt{(r_1 - r_2)^2 + h^2}$$

$$l = \sqrt{(14-6)^2 + 6^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10\,\text{cm}$$

Step 3: Substitute in the formula.

$$\text{C.S.A.} = 3.14 \times (14 + 6) \times 10$$

$$\text{C.S.A.} = 3.14 \times 20 \times 10 = 628\,\text{sq. cm}$$

Step 4: Conclusion.

Hence, the curved surface area of the frustum is **628 sq. cm**.

Correct Answer: 628 sq. cm

Quick Tip

For a frustum, always find the slant height first using $l = \sqrt{(r_1 - r_2)^2 + h^2}$ before calculating surface area.

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4. Solve the following sub-questions (any two): 8

(1) In $\triangle ABC$, seg $DE \parallel BC$. If $2A(\triangle ADE) = A(\triangle DBCE)$, find $AB : AD$ and show that $BC = \sqrt{3}\,DE$.

Solution:

Step 1: Given information.

In $\triangle ABC$, $DE \parallel BC$. Therefore, $\triangle ADE \sim \triangle ABC$ by **Basic Proportionality Theorem (BPT)**.

Step 2: Ratio of areas of similar triangles.

For two similar triangles, the ratio of their areas is equal to the square of the ratio of their corresponding sides.

$$\frac{A(\triangle ADE)}{A(\triangle ABC)} = \left(\frac{AD}{AB}\right)^2$$

Step 3: Given condition.

It is given that $2A(\triangle ADE) = A(\triangle DBCE)$. Now,

$$A(\triangle ABC) = A(\triangle ADE) + A(\triangle DBCE)$$

Substitute $A(\triangle DBCE) = 2A(\triangle ADE)$:

$$A(\triangle ABC) = A(\triangle ADE) + 2A(\triangle ADE) = 3A(\triangle ADE)$$

Step 4: Substitute in the area ratio.

$$\frac{A(\triangle ADE)}{A(\triangle ABC)} = \frac{1}{3}$$

Hence,

$$\left(\frac{AD}{AB}\right)^2 = \frac{1}{3}$$

$$\frac{AD}{AB} = \frac{1}{\sqrt{3}}$$

$$AB : AD = \sqrt{3} : 1$$

Step 5: Relation between BC and DE.

Since $\triangle ADE \sim \triangle ABC$,

$$\frac{BC}{DE} = \frac{AB}{AD} = \sqrt{3}$$

$$\therefore BC = \sqrt{3}\, DE$$

Step 6: Conclusion.

Hence, $AB : AD = \sqrt{3} : 1$ and $BC = \sqrt{3}\, DE$.

Correct Answer: $AB : AD = \sqrt{3} : 1$ and $BC = \sqrt{3}\, DE$

Quick Tip

In similar triangles, the ratio of areas is equal to the square of the ratio of their corresponding sides. Always use this relation to compare medians, sides, or heights.

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(2) $\triangle SHR \sim \triangle SVU$. In $\triangle SHR$, $SH = 4.5$ cm, $HR = 5.2$ cm, and $SR = 5.8$ cm. If $\frac{SH}{SV} = \frac{3}{5}$, construct $\triangle SVU$.

Solution (Construction Steps):

Step 1: Draw a line segment $HR = 5.2$ cm.

Step 2: With center H and radius 4.5 cm, draw an arc.

Step 3: With center R and radius 5.8 cm, draw another arc intersecting the first arc at point S. Join S to H and S to R. Thus, $\triangle SHR$ is obtained.

Step 4: Extend HS beyond S.

Step 5: From H, draw a ray HX below HS. On HX, mark 5 equal divisions (since denominator of ratio is 5).

Step 6: Join the 3rd division point (since numerator of ratio is 3) to S. Draw a line through this point parallel to SR to meet the extended HS at point V.

Step 7: Through V, draw a line parallel to HR meeting the extended R line at U.

Step 8: $\triangle SVU$ is the required triangle similar to $\triangle SHR$.

Result: The required triangle $\triangle SVU$ is constructed such that $\triangle SHR \sim \triangle SVU$.

Quick Tip

When constructing similar triangles, use the ratio of corresponding sides and draw parallel lines to maintain proportionality.

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(3) An ice-cream pot has a right circular cylindrical shape. The radius of the base is 12 cm and the height is 7 cm. This pot is completely filled with ice-cream. The entire ice-cream is given to students in the form of right circular cones having diameter 4 cm and height 3.5 cm. If each student is given one cone, how many students can be served?

Solution:

Step 1: Find the volume of the cylindrical pot.

$$V_1 = \pi r_1^2 h_1$$

Given $r_1 = 12$ cm and $h_1 = 7$ cm,

$$V_1 = 3.14 \times 12^2 \times 7 = 3.14 \times 144 \times 7 = 3165.12\,\text{cm}^3$$

Step 2: Find the volume of one ice-cream cone.

Diameter of cone = 4 cm, so radius $r_2 = 2$ cm, and height $h_2 = 3.5$ cm.

$$V_2 = \frac{1}{3}\pi r_2^2 h_2$$

$$V_2 = \frac{1}{3} \times 3.14 \times 2^2 \times 3.5 = \frac{1}{3} \times 3.14 \times 4 \times 3.5 = 14.66\,\text{cm}^3$$

Step 3: Find the number of students.

$$\text{Number of students} = \frac{V_1}{V_2} = \frac{3165.12}{14.66} \approx 216$$

Step 4: Conclusion.

Hence, **216 students** can be served one cone each.

Correct Answer: 216 students

Quick Tip

When solids are melted and recast, their volumes remain equal. Always use Volume of solid 1 = Volume of solid 2 for such problems.

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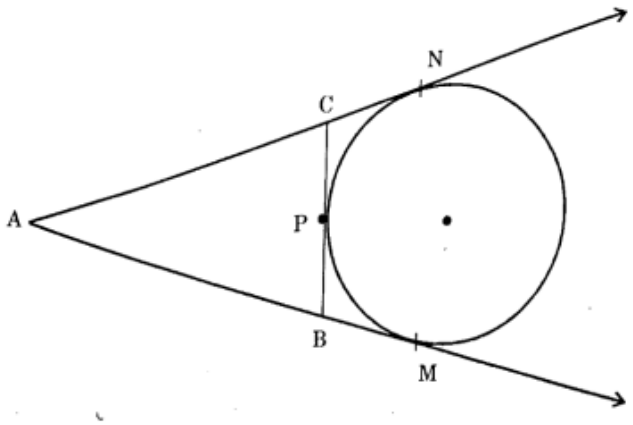
5. Solve the following sub-questions (Any one): 3

(1) A circle touches side BC at point P of $\triangle ABC$, from outside of the triangle. Further extended lines AC and AB are tangents to the circle at N and M respectively. Prove that:

$$AM = \frac{1}{2}(\text{Perimeter of } \triangle ABC)$$



Solution:

Step 1: Given.

A circle touches the sides of $\triangle ABC$ externally at points P, M, and N such that the tangents from a single external point to a circle are equal in length.

Step 2: Tangent length properties.

Let the tangents drawn from each vertex be as follows:

$$\text{From } A: \ AM = AN = x,$$

$$\text{From } B: \ BP = BM = y,$$

$$\text{From } C: \ CP = CN = z.$$

Step 3: Express the sides of the triangle.

$$AB = AM + MB = x + y,$$

$$BC = BP + PC = y + z,$$

$$CA = CN + NA = z + x.$$

Step 4: Find the perimeter of the triangle.

$$\text{Perimeter of } \triangle ABC = AB + BC + CA = (x + y) + (y + z) + (z + x)$$

$$\text{Perimeter} = 2(x + y + z)$$

Step 5: Relation of AM.

From the figure, $AM = x$.

Hence,

$$x + y + z = \frac{1}{2}(\text{Perimeter of } \triangle ABC)$$

Therefore,

$$AM = \frac{1}{2}(\text{Perimeter of } \triangle ABC)$$

Hence proved.

Result: $AM = \frac{1}{2}(\text{Perimeter of } \triangle ABC)$

Quick Tip

The tangents drawn from an external point to a circle are equal in length. Use this property to relate the perimeter of the triangle with the tangent segment lengths.

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(2) Eliminate θ if $x = r\cos\theta$ and $y = r\sin\theta$.

Solution:

Step 1: Given equations.

We have

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

Step 2: Square both equations and add them.

$$x^2 = r^2\cos^2\theta \quad \text{and} \quad y^2 = r^2\sin^2\theta$$

$$x^2 + y^2 = r^2(\cos^2\theta + \sin^2\theta)$$

Step 3: Simplify using the trigonometric identity.

Since $\cos^2\theta + \sin^2\theta = 1$, we get

$$x^2 + y^2 = r^2$$

Step 4: Conclusion.

Thus, the required relation after eliminating θ is

$$\boxed{x^2 + y^2 = r^2}$$

Correct Answer: $x^2 + y^2 = r^2$

Quick Tip

To eliminate θ in polar equations, square both $x = r\cos\theta$ and $y = r\sin\theta$ and add them to use $\sin^2\theta + \cos^2\theta = 1$.
