MPBSE Class 12th Higher Math - 2023 Question Paper

Time Allowed :3 Hour	Maximum Marks :80	Total Questions :23

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Attempt all questions.
2. Read the instructions carefully.
3. Marks alloted to each question are indicated against it.

1. i. In the function $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = 5x$:

(1) f is one-one onto
(2) f is many-one onto
(3) f is one-one but not onto
(4) f is neither one-one nor onto

—

1. ii. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ **is equal to:**

(1) π
(2) $\frac{\pi}{3}$
(3) $\frac{\pi}{3}$
(4) $\frac{2\pi}{3}$

—

1. iii. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ **and** $A + A' = I$**, then the value of** α **is:**

(1) $\frac{\pi}{6}$
(2) $\frac{\pi}{3}$
(3) π
(4) $\frac{3\pi}{2}$

—

1. iv. The oxidation number of Fe in $K_2[Fe(CN)_6]$ is:

(1) +6
(2) +4
(3) +3
(4) -4

---

2. (i) If $y = \sqrt{e^x}$, $x > 0$, then $\frac{dy}{dx}$ = _______

—

2. (ii) Rate of change of area of circle per second with respect to its radius r when $r = 5$ cm will be _______

—

2. (iii) $\int \left(\sin^{-1} x + \cos^{-1} x\right) dx =$ _______

—

2. (iv) The number of arbitrary constants in the particular solution of a differential equation of third order are _______

—

2. (v) The vector sum of the three sides of a triangle taken in order is _______

—

2. (vi) The direction cosines of x, y, and z-axis are _______ respectively.

—

2. (vii) $\int e^x[f(x) + f'(x)]dx =$ _______

---

3. Match the pairs correctly:

(i)	$\int \tan x \, dx$	$\log \lvert \sin x \rvert + c$
(ii)	$\int \cot x \, dx$	$\log \lvert \csc x \rvert - \cot x + c$
(iii)	$\int \sec x \, dx$	$\log \lvert \sec x + \tan x \rvert + c$
(iv)	$\int \csc x \, dx$	$-\log \lvert \csc x + \cot x \rvert + c$
(v)	$\int \frac{\cos x}{\sin x} \, dx$	$\log \lvert \sin x \rvert + c$
(vi)	Derivative of $\sin 2x$ with respect to x	$2 \cos 2x$

---

4. (i) What is trivial relation?

—

4. (ii) Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

—

4. (iii) What is column matrix?

—

4. (iv) Find magnitude of the vector $\hat{i} + \hat{j} + \hat{k}$

—

4. (v) If A and B are two independent events with $P(A) = 0.3$ and $P(B) = 0.4$, then find $P(B|A)$

---

5. (i) Order of differential equation $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$ is 2.

---

5. (ii) Angle between vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ is 60°.

---

5. (iii) Integrating factor of differential equation $\frac{dy}{dx} - y = \cos x$ is e^{-x}.

---

5. (iv) $f: x \to y$ **is an onto function then range of** $f = y$**.**

---

5. (v) $\tan^{-1} 1/2 + \tan^{-1} 1/3 = \tan^{-1} 1/5$**.**

---

5. (vi) If $A = [a_{ij}]_{m\times n}$ **and** $B = [b_{ij}]_{j\times p}$**, then order of** AB **will be** $m \times p$**.**

---

6. If $A = \{1, 2, 3\}$**,** $B = \{4, 5, 6, 7\}$**, and** $f = \{(1, 4), (2, 5), (3, 6)\}$ **is a function from A to B, then show that** f **is one-one.**

—
OR

6. Examine that the relation R **in the set** $\{1, 2, 3, 4\}$ **given by** $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3,$
is reflexive and transitive but not symmetric.

---

7. Prove that $\sin^{-1}(-x) = -\sin^{-1}(x)$**, where** $x \in [-1, 1]$

—
OR

7. Prove that $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$**, where** $x \in [-1, 1]$

---

8. If $A = \begin{pmatrix} -2 & 3 \\ 1 & 2 \end{pmatrix}$ **and** $B = \begin{pmatrix} -1 & 0 \\ 1 & 2 \end{pmatrix}$**, then find** $(A + B)$**.**

—
OR

8. If $A = \begin{pmatrix} 1 & -4 \\ 3 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ 1 & 2 \end{pmatrix}$, then find AB.

---

9. (i) Differentiate X^X with respect to X.

—

OR

9. (ii) If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then find $\frac{dy}{dx}$.

---

10. (i) Show that the function given by $f(x) = 12x - 3$ is increasing on R.

OR

10. (ii) Show that the function given by $f(x) = e^{3x}$ is increasing on R.

---

11. Evaluate

$$\int_{-1}^{1} \sin^5 x \cos^4 x \, dx$$

OR

11. Evaluate

$$\int_{-1}^{2} \left(x^3 - |x|\right) dx.$$

---

12. Find the projection of vector $\mathbf{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ on the vector $\mathbf{b} = 2\hat{i} + \hat{k}$.

OR

12. Find the area of a parallelogram whose adjacent sides are given by the vectors $\mathbf{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\mathbf{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

---

13. Show that for any two vectors a and b, always $|\mathbf{a} \cdot \mathbf{b}| \leq \|\mathbf{a}\|\|\mathbf{b}\|$.

OR

13. Find the value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

---

14. If direction ratios of a line are -18, 12, -4, then find its direction cosines.

OR

14. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$.

---

15. Find local maximum and local minimum values of the function given by

$$f(x) = 3x^4 + 4x^3 - 12x^2 + 12$$

OR

15. Find the interval in which the function given by

$$f(x) = x^2 - 4x + 6$$

is decreasing.

---

16. Find the area of the region bounded by the curve

$$y = x^2 \quad \text{and the line} \quad y = 4.$$

OR

16. Find the area of the region bounded by the curve

$$y^2 = 4x \quad \text{and the line} \quad x = 3.$$

17. Find the general solution of the differential equation

$$x\frac{dy}{dx} + 2y = x^2 \quad \text{where} \quad (x \neq 0).$$

OR

17. Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}.$$

18. Minimize $Z = 3x + 5y$ subject to the constraints:

$$x + 3y \geq 3, \quad x + y \geq 2, \quad x \geq 0, \quad y \geq 0.$$

OR

18. Maximize $Z = 3x + 9y$ subject to the constraints:

$$x + 3y \leq 60, \quad x + y \geq 10, \quad x \leq y, \quad x \geq 0, \quad y \geq 0.$$

19. Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?

OR

19. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. Suppose that the probability of drawing each ball is the same. What is the probability that both drawn balls are black?

---

20. Solve the equation:

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

where $a \neq 0$.

OR

20. Prove that:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

---

21. (i) Prove that if function f is differentiable at a point a, then it is also continuous at that point.

OR

21. (ii) Differentiate $\sin(x^2)$ with respect to x^2.

---

22. (i) Evaluate the integral:

$$\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$$

OR

22. (ii) Evaluate the integral:

$$\int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x + \cos^5 x}\, dx$$

---

23. Find the shortest distance between the lines:

$$\vec{r_1} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k} \quad \text{and} \quad \vec{r_2} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$$

OR

23. Find the shortest distance between parallel lines:

$$\vec{r_1} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}) \quad \text{and} \quad \vec{r_2} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$$
