

# Manipur Board Class 10 2026 Mathematics Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :80	Total questions :32
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## General Instructions

Read the following instructions very carefully and strictly follow them:

1. The paper is divided into Section A and Section B.
2. Section A includes objective-type questions.
3. All questions in Section A are compulsory.
4. Section B includes short answer, and long answer type questions.
5. Answers must be written legibly within the word limit.
6. Use of unfair means or electronic devices is prohibited.
7. Follow the correct format and instructions for each section.

## Section - A

**1. The number of days within which the stock exchange is supposed to resolve dispute at their end is**

- (A) 10
- (B) 15
- (C) 20
- (D) 30

**Correct Answer:** (B) 15

**Solution:**

**Step 1:** Understanding the dispute resolution timeline.

In the stock market grievance redressal mechanism, stock exchanges are required to handle and resolve investor disputes or complaints within a prescribed time limit. This timeline is fixed so that investor grievances are addressed quickly and efficiently without unnecessary delay.

**Step 2: Identifying the correct number of days.**

As per the standard dispute resolution framework followed by stock exchanges, the dispute is supposed to be resolved at their end within **15 days**. This makes the process time-bound and helps in protecting the interests of investors.

**Step 3: Comparison with other options.**

- **(A) 10:** Incorrect. This is shorter than the prescribed dispute resolution period.
- **(B) 15:** Correct. This is the prescribed number of days within which the stock exchange is expected to resolve the dispute at its level.
- **(C) 20:** Incorrect. This is more than the usual prescribed time limit.
- **(D) 30:** Incorrect. This is too long for the stock exchange level dispute resolution period in this context.

**Step 4: Conclusion.**

Therefore, the correct answer is **15 days**, since stock exchanges are supposed to resolve the dispute within this period at their end.

**Final Answer:** 15 days.

**Quick Tip**

For stock exchange and investor grievance questions, always remember that the dispute resolution process is generally time-bound to ensure speedy redressal. Here, the important number is **15 days**.

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**2. If  $x + 1$  is a factor of  $p(x) = 4x^2 + 3x + k$ , then the value of  $k$  is**

(A) 8

(B) -8

(C) 1

(D) -1

**Correct Answer:** (C) 1

**Solution:**

**Step 1:** Use the factor theorem.

If  $x + 1$  is a factor of the polynomial  $p(x)$ , then by the Factor Theorem, we must have:

$$p(-1) = 0$$

This means that when we substitute  $x = -1$  into the polynomial, the result must be zero.

**Step 2:** Substitute  $x = -1$  into the polynomial.

Given:

$$p(x) = 4x^2 + 3x + k$$

Now put  $x = -1$ :

$$p(-1) = 4(-1)^2 + 3(-1) + k$$

$$p(-1) = 4(1) - 3 + k$$

$$p(-1) = 1 + k$$

Since  $x + 1$  is a factor, we know:

$$1 + k = 0$$

**Step 3:** Solve for  $k$ .

From

$$1 + k = 0$$

we get:

$$k = -1$$

So, the required value of  $k$  is:

$$-1$$

**Step 4:** Compare with the given options.

- (A) 8: Incorrect. This does not make  $p(-1) = 0$ .
- (B) -8: Incorrect. This also does not satisfy the factor condition.
- (C) 1: Incorrect. This would give  $p(-1) = 2$ , not 0.
- (D) -1: Correct. This makes  $p(-1) = 0$ , so  $x + 1$  becomes a factor.

Hence, the correct answer is  $-1$ .

**Final Answer:**  $-1$ .

#### Quick Tip

Whenever a linear expression like  $x - a$  or  $x + b$  is given as a factor, directly apply the Factor Theorem: substitute the root value into the polynomial and equate it to zero.

**3. If one root of the equation  $2x^2 - 3x + k = 0$  be reciprocal of the other, then the value of  $k$  is**

- (A)  $\frac{3}{2}$
- (B)  $-\frac{3}{2}$
- (C) 3
- (D) 2

**Correct Answer:** (D) 2

**Solution:**

**Step 1: Understand the condition on the roots.**

Let the two roots of the quadratic equation be  $\alpha$  and  $\beta$ . The question says that one root is the reciprocal of the other. This means:

$$\beta = \frac{1}{\alpha}$$

Therefore, the product of the roots becomes:

$$\alpha\beta = \alpha \cdot \frac{1}{\alpha} = 1$$

So, for this quadratic equation, the product of the roots must be equal to 1.

**Step 2: Use the product of roots formula.**

For a quadratic equation of the form:

$$ax^2 + bx + c = 0$$

the product of the roots is:

$$\alpha\beta = \frac{c}{a}$$

Here,

$$a = 2, \quad c = k$$

So,

$$\alpha\beta = \frac{k}{2}$$

But from the given condition, we already know that:

$$\alpha\beta = 1$$

Hence,

$$\frac{k}{2} = 1$$

**Step 3: Find the value of  $k$ .**

Multiplying both sides by 2, we get:

$$k = 2$$

Thus, the required value of  $k$  is:

$$2$$

**Step 4: Compare with the options.**

- (A)  $\frac{3}{2}$ : Incorrect. This does not make the product of roots equal to 1.
- (B)  $-\frac{3}{2}$ : Incorrect. This also does not satisfy the reciprocal condition.
- (C) 3: Incorrect. This would give product of roots  $\frac{3}{2}$ .
- (D) 2: Correct. This gives product of roots  $\frac{2}{2} = 1$ .

Therefore, the correct value of  $k$  is 2.

**Final Answer:** 2.

### Quick Tip

If one root of a quadratic equation is the reciprocal of the other, then the product of the roots is always 1. Use  $\alpha\beta = \frac{c}{a}$  immediately.

**4. The sum of the first  $n$  terms of the AP whose first term is 1 and common difference is 2 is :**

- (A)  $3n$
- (B)  $2n - 1$
- (C)  $n^2$
- (D)  $n(n + 1)$

**Correct Answer:** (C)  $n^2$

**Solution:**

**Step 1: Identify the given AP.**

The arithmetic progression has first term  $a = 1$  and common difference  $d = 2$ . So the AP is:

$$1, 3, 5, 7, \dots$$

This is the sequence of the first  $n$  odd numbers.

**Step 2: Use the formula for the sum of  $n$  terms of an AP.**

The formula for the sum of the first  $n$  terms of an AP is:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

Substituting  $a = 1$  and  $d = 2$ , we get:

$$S_n = \frac{n}{2} [2(1) + (n - 1)(2)]$$

$$S_n = \frac{n}{2} [2 + 2n - 2]$$

$$S_n = \frac{n}{2} (2n)$$

$$S_n = n^2$$

**Step 3:** Compare with the given options.

- (A)  $3n$ : Incorrect. This is not the sum obtained from the AP formula.
- (B)  $2n - 1$ : Incorrect. This gives the  $n$ th term of the AP, not the sum.
- (C)  $n^2$ : Correct. The sum of the first  $n$  odd numbers is always  $n^2$ .
- (D)  $n(n + 1)$ : Incorrect. This is not the required sum for this AP.

**Step 4:** Conclusion.

Therefore, the sum of the first  $n$  terms of the AP with first term 1 and common difference 2 is:

$$n^2$$

**Final Answer:**  $n^2$ .

#### Quick Tip

The AP  $1, 3, 5, 7, \dots$  consists of odd numbers, and the sum of the first  $n$  odd numbers is always  $n^2$ .

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**5. Two concentric circles are of radii 6 cm and 10 cm. The length of a chord of the larger circle which touches the smaller circle is**

- (A) 8 cm
- (B) 12 cm
- (C) 16 cm
- (D) 18 cm

**Correct Answer:** (C) 16 cm

**Solution:**

**Step 1:** Understand the geometry of the figure.

There are two concentric circles, which means both circles have the same center. The radius of the smaller circle is 6 cm and the radius of the larger circle is 10 cm. A chord of the larger circle touches the smaller circle, so this chord is tangent to the smaller circle.

This means that the perpendicular distance from the common center to the chord is equal to the radius of the smaller circle, that is:

$$d = 6 \text{ cm}$$

**Step 2: Use the chord-length formula.**

For a circle of radius  $R$ , if the perpendicular distance of a chord from the center is  $d$ , then the length of the chord is:

$$2\sqrt{R^2 - d^2}$$

Here,

$$R = 10, \quad d = 6$$

So, the chord length is:

$$2\sqrt{10^2 - 6^2}$$

**Step 3: Simplify the expression.**

$$2\sqrt{100 - 36}$$

$$2\sqrt{64}$$

$$2 \times 8 = 16$$

Thus, the required length of the chord is:

$$16 \text{ cm}$$

**Step 4: Compare with the given options.**

- (A) 8 cm: Incorrect.
- (B) 12 cm: Incorrect.
- (C) 16 cm: Correct.
- (D) 18 cm: Incorrect.

Therefore, the correct answer is (C) 16 cm.

**Final Answer:** 16 cm.

#### Quick Tip

If a chord of a larger circle touches a smaller concentric circle, then the distance of the chord from the common center is equal to the radius of the smaller circle.

**6. Length of an arc of a sector of a circle with radius  $r$  and sectorial angle  $\theta$  (measured in degrees) is**

- (A)  $\frac{\pi r \theta}{360}$
- (B)  $\frac{\pi r \theta}{180}$
- (C)  $\frac{\pi r^2 \theta}{360}$
- (D)  $\frac{\pi r^2 \theta}{180}$

**Correct Answer:** (B)  $\frac{\pi r \theta}{180}$

**Solution:**

**Step 1:** Recall the formula for the circumference of a circle.

The total circumference of a circle of radius  $r$  is:

$$2\pi r$$

This corresponds to a central angle of  $360^\circ$ .

**Step 2:** Find the arc length for angle  $\theta$ .

If the full circle of  $360^\circ$  has arc length  $2\pi r$ , then a sector with angle  $\theta$  will have arc length:

$$\frac{\theta}{360} \times 2\pi r$$

**Step 3:** Simplify the formula.

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r$$

$$\begin{aligned} &= \frac{2\pi r\theta}{360} \\ &= \frac{\pi r\theta}{180} \end{aligned}$$

So, the required length of the arc is:

$$\frac{\pi r\theta}{180}$$

**Step 4: Compare with the given options.**

- (A)  $\frac{\pi r\theta}{360}$ : Incorrect. This is half the required value.
- (B)  $\frac{\pi r\theta}{180}$ : Correct.
- (C)  $\frac{\pi r^2\theta}{360}$ : Incorrect. This resembles an area-related expression, not arc length.
- (D)  $\frac{\pi r^2\theta}{180}$ : Incorrect. Arc length depends on  $r$ , not  $r^2$ .

Therefore, the correct option is (B).

**Final Answer:**  $\frac{\pi r\theta}{180}$ .

#### Quick Tip

For arc length with angle in degrees, always use

$$\text{Arc length} = \frac{\theta}{360} \times 2\pi r = \frac{\pi r\theta}{180}$$

Do not confuse it with the area of a sector, which uses  $r^2$ .

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**7. If  $p(x)$  is a polynomial of degree  $\geq 1$  and  $a$  is any real number, then the remainder when  $p(x)$  is divided by  $x - a$  is**

- (A)  $p(a)$
- (B)  $p(-a)$
- (C)  $-p(a)$
- (D)  $-p(-a)$

**Correct Answer:** (A)  $p(a)$

**Solution:**

**Step 1: Recall the Remainder Theorem.**

The Remainder Theorem states that if a polynomial  $p(x)$  is divided by a linear factor of the form  $x - a$ , then the remainder is obtained by substituting  $x = a$  into the polynomial. Thus, the remainder is:

$$p(a)$$

This is a standard result in polynomial theory.

**Step 2: Apply the theorem to the given expression.**

Here, the divisor is:

$$x - a$$

So, according to the Remainder Theorem, the remainder when  $p(x)$  is divided by  $x - a$  is:

$$p(a)$$

Hence, the required remainder is  $p(a)$ .

**Step 3: Compare with the given options.**

- **(A)  $p(a)$ :** Correct. This follows directly from the Remainder Theorem.
- **(B)  $p(-a)$ :** Incorrect. This would be the remainder if the divisor were  $x + a$ .
- **(C)  $-p(a)$ :** Incorrect. The remainder is not negative of  $p(a)$ .
- **(D)  $-p(-a)$ :** Incorrect. This does not match the theorem.

So, the correct option is **(A)  $p(a)$ .**

**Step 4: Conclusion.**

Therefore, whenever a polynomial  $p(x)$  is divided by  $x - a$ , the remainder is always equal to:

$$p(a)$$

Hence, the correct answer is  $p(a)$ .

**Final Answer:**  $p(a)$ .

#### Quick Tip

Remember this shortcut: for divisor  $x - a$ , put  $x = a$  in the polynomial. For divisor  $x + a$ , put  $x = -a$  in the polynomial.

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8. A point P is at a distance of 13 cm from the centre of a circle. If the radius of the circle is 5 cm, the length of the tangent from P to the circle is

- (A) 12 cm
- (B) 13 cm
- (C) 15 cm
- (D) 18 cm

**Correct Answer:** (A) 12 cm

**Solution:**

**Step 1: Understand the geometry of the figure.**

Let  $O$  be the centre of the circle and let  $PT$  be the tangent from external point  $P$  touching the circle at point  $T$ . The radius  $OT$  is perpendicular to the tangent  $PT$ . Therefore, triangle  $OPT$  is a right-angled triangle.

**Step 2: Write the given values.**

The distance from the centre to the external point is:

$$OP = 13 \text{ cm}$$

The radius of the circle is:

$$OT = 5 \text{ cm}$$

We need to find the tangent length:

$$PT = ?$$

**Step 3: Apply the Pythagoras theorem.**

Since triangle  $OPT$  is right-angled at  $T$ , we use:

$$OP^2 = OT^2 + PT^2$$

Substituting the values:

$$13^2 = 5^2 + PT^2$$

$$169 = 25 + PT^2$$

$$PT^2 = 169 - 25$$

$$PT^2 = 144$$

$$PT = 12 \text{ cm}$$

**Step 4:** Compare with the options and conclude.

- **(A) 12 cm:** Correct. This is the value obtained by calculation.
- **(B) 13 cm:** Incorrect. This is the distance from the centre to point  $P$ , not the tangent length.
- **(C) 15 cm:** Incorrect. This does not satisfy the right triangle relation.
- **(D) 18 cm:** Incorrect. This is greater than the hypotenuse, so it is not possible.

Hence, the length of the tangent from  $P$  to the circle is 12 cm.

**Final Answer:** 12 cm.

#### Quick Tip

For a tangent drawn from an external point, the radius at the point of contact is always perpendicular to the tangent. So, use the Pythagoras theorem directly.

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**9. The volume of the hemisphere of radius  $r$  is**

- (A)  $\frac{4}{3}\pi r^3$
- (B)  $\frac{1}{3}\pi r^3$
- (C)  $\frac{2}{3}\pi r^3$
- (D)  $4\pi r^3$

**Correct Answer:** (C)  $\frac{2}{3}\pi r^3$

**Solution:**

**Step 1: Recall the volume of a sphere.**

The volume of a sphere of radius  $r$  is given by the formula:

$$V = \frac{4}{3}\pi r^3$$

This is the standard formula for the volume enclosed by a complete sphere.

**Step 2: Understand what a hemisphere is.**

A hemisphere is exactly half of a sphere. Therefore, the volume of a hemisphere will be one-half of the volume of the full sphere.

So,

$$V_{\text{hemisphere}} = \frac{1}{2} \times \frac{4}{3}\pi r^3$$

**Step 3: Simplify the expression.**

Now simplify:

$$V_{\text{hemisphere}} = \frac{4}{6}\pi r^3$$

$$V_{\text{hemisphere}} = \frac{2}{3}\pi r^3$$

So, the volume of the hemisphere is:

$$\frac{2}{3}\pi r^3$$

**Step 4: Compare with the options.**

Among the given options:

- (A)  $\frac{4}{3}\pi r^3$ : This is the volume of a full sphere, not a hemisphere.
- (B)  $\frac{1}{3}\pi r^3$ : Incorrect value.
- (C)  $\frac{2}{3}\pi r^3$ : Correct. This is half the volume of a sphere.
- (D)  $4\pi r^3$ : Incorrect formula.

Therefore, the correct option is (C).

**Final Answer:**  $\frac{2}{3}\pi r^3$ .

### Quick Tip

Always remember: a hemisphere is half of a sphere. So, to find its volume, first write the sphere formula  $\frac{4}{3}\pi r^3$  and then divide it by 2.

**10. The remainder when  $4x^3 + 4x^2 + x - 4$  is divided by  $2x - 1$  is**

- (A) 2
- (B) -2
- (C) 4
- (D) -4

**Correct Answer:** (C) 4

**Solution:**

**Step 1: Use the Remainder Theorem for a linear divisor.**

When a polynomial  $f(x)$  is divided by a linear expression of the form  $ax - b$ , the remainder can be found by substituting  $x = \frac{b}{a}$  into the polynomial.

Here, the divisor is  $2x - 1$ . So, we put:

$$2x - 1 = 0$$

$$x = \frac{1}{2}$$

Therefore, the remainder is:

$$f\left(\frac{1}{2}\right)$$

**Step 2: Substitute  $x = \frac{1}{2}$  into the polynomial.**

Given polynomial:

$$f(x) = 4x^3 + 4x^2 + x - 4$$

Now substitute  $x = \frac{1}{2}$ :

$$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^2 + \frac{1}{2} - 4$$

**Step 3: Simplify each term carefully.**

First term:

$$4 \left(\frac{1}{2}\right)^3 = 4 \cdot \frac{1}{8} = \frac{1}{2}$$

Second term:

$$4 \left(\frac{1}{2}\right)^2 = 4 \cdot \frac{1}{4} = 1$$

Third term:

$$\frac{1}{2}$$

So,

$$f\left(\frac{1}{2}\right) = \frac{1}{2} + 1 + \frac{1}{2} - 4$$

Now combine:

$$\frac{1}{2} + \frac{1}{2} = 1$$

$$1 + 1 = 2$$

$$2 - 4 = -2$$

**Step 4: Match with the options.**

Thus, the remainder is:

$$-2$$

So, the correct option is:

$$(B) -2$$

**Final Answer:**  $-2$ .

#### Quick Tip

For division by a linear polynomial like  $ax - b$ , always find the remainder by putting  $x = \frac{b}{a}$  in the polynomial. Here, for  $2x - 1$ , use  $x = \frac{1}{2}$ .

### Section - B

**11. Solve the quadratic equation  $ax^2 + bx + c = 0$ , ( $a \neq 0$ ) by the method of completing square.**

**Solution:**

**Step 1: Write the given quadratic equation.**

We are given the quadratic equation:

$$ax^2 + bx + c = 0, \quad a \neq 0$$

**Step 2: Divide the whole equation by  $a$ .**

To make the coefficient of  $x^2$  equal to 1, divide every term by  $a$ :

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

**Step 3: Shift the constant term to the right side.**

Subtract  $\frac{c}{a}$  from both sides:

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

**Step 4: Complete the square.**

Take half of the coefficient of  $x$ , which is  $\frac{b}{2a}$ , and square it:

$$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

Now add  $\frac{b^2}{4a^2}$  to both sides:

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

**Step 5: Write the left side as a perfect square.**

The left-hand side becomes:

$$\left(x + \frac{b}{2a}\right)^2$$

So, we get:

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

**Step 6: Take square root on both sides.**

Taking square roots, we get:

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

**Step 7: Find the value of  $x$ .**

Subtract  $\frac{b}{2a}$  from both sides:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Step 8: State the final result.**

Hence, the solution of the quadratic equation  $ax^2 + bx + c = 0$  is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### Quick Tip

In completing the square, always make the coefficient of  $x^2$  equal to 1 first. Then add the square of half the coefficient of  $x$ .

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## 12. Solve graphically:

$$2x + 3y = 5$$

$$5x - 4y + 22 = 0$$

**Solution:**

**Step 1: Write both equations clearly.**

The given pair of linear equations is:

$$2x + 3y = 5$$

$$5x - 4y + 22 = 0$$

The second equation can be written as:

$$5x - 4y = -22$$

**Step 2: Understand graphical solution.**

To solve graphically, we draw the lines represented by both equations on the same graph.

The coordinates of their point of intersection give the required solution.

**Step 3: Find points for the first line.**

For the equation  $2x + 3y = 5$ :

If  $x = 1$ , then

$$2(1) + 3y = 5$$

$$2 + 3y = 5$$

$$3y = 3$$

$$y = 1$$

So one point is  $(1, 1)$ .

If  $x = 4$ , then

$$2(4) + 3y = 5$$

$$8 + 3y = 5$$

$$3y = -3$$

$$y = -1$$

So another point is  $(4, -1)$ .

**Step 4: Find points for the second line.**

For the equation  $5x - 4y = -22$ :

If  $x = -2$ , then

$$5(-2) - 4y = -22$$

$$-10 - 4y = -22$$

$$-4y = -12$$

$$y = 3$$

So one point is  $(-2, 3)$ .

If  $x = 2$ , then

$$5(2) - 4y = -22$$

$$10 - 4y = -22$$

$$-4y = -32$$

$$y = 8$$

So another point is  $(2, 8)$ .

**Step 5: Find the intersection point algebraically for confirmation.**

From the first equation:

$$2x + 3y = 5$$

From the second equation:

$$5x - 4y = -22$$

Multiply the first equation by 4:

$$8x + 12y = 20$$

Multiply the second equation by 3:

$$15x - 12y = -66$$

Now add both equations:

$$23x = -46$$

$$x = -2$$

Substitute  $x = -2$  into  $2x + 3y = 5$ :

$$2(-2) + 3y = 5$$

$$-4 + 3y = 5$$

$$3y = 9$$

$$y = 3$$

**Step 6: State the graphical solution.**

Thus, the two lines intersect at the point:

$$(-2, 3)$$

Hence, the graphical solution is:

$$x = -2, y = 3$$

#### Quick Tip

In graphical method, the solution of a pair of linear equations is the point where the two lines intersect.

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### 13. Write the statement of Euclid's Division Lemma.

**Solution:**

**Step 1: State the lemma in words.**

Euclid's Division Lemma states that for any two positive integers, there exist unique integers  $q$  and  $r$  such that a given integer can be expressed in the form of divisor multiplied by quotient plus remainder.

**Step 2: Write the mathematical form.**

If  $a$  and  $b$  are two positive integers, then there exist unique whole numbers  $q$  and  $r$  such that

$$a = bq + r$$

where  $0 \leq r < b$ .

**Step 3:** Explain the meaning of symbols.

Here,  $a$  is the dividend,  $b$  is the divisor,  $q$  is the quotient, and  $r$  is the remainder.

**Step 4:** Mention the condition on remainder.

The remainder must always be non-negative and smaller than the divisor. This condition makes the statement complete and correct.

**Step 5:** Final statement.

Therefore, the statement of Euclid's Division Lemma is: For any two positive integers  $a$  and  $b$ , there exist unique integers  $q$  and  $r$  satisfying

$$a = bq + r, \quad 0 \leq r < b$$

#### Quick Tip

Remember: Euclid's Division Lemma is based on the idea that Dividend = Divisor  $\times$  Quotient + Remainder, where the remainder is always less than the divisor.

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**14. Write down the quadratic equation whose roots are 2 and -3.**

**Solution:**

**Step 1:** Write the given roots.

The given roots of the quadratic equation are 2 and  $-3$ .

**Step 2:** Use the standard form of equation from roots.

If  $\alpha$  and  $\beta$  are the roots of a quadratic equation, then the equation is given by

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

**Step 3:** Find the sum and product of roots.

Here,

$$\alpha = 2, \quad \beta = -3$$

So, the sum of the roots is

$$\alpha + \beta = 2 + (-3) = -1$$

and the product of the roots is

$$\alpha\beta = 2 \times (-3) = -6$$

**Step 4: Substitute in the formula.**

Putting these values in the standard equation, we get

$$x^2 - (-1)x + (-6) = 0$$

**Step 5: Simplify the equation.**

On simplification, the required quadratic equation becomes

$$x^2 + x - 6 = 0$$

**Quick Tip**

To form a quadratic equation from given roots, use the formula  $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$ .

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**15. In a right triangle  $ABC$ , right angled at  $B$ , prove that  $\sin A = \cos(90^\circ - A)$  and  $\cos A = \sin(90^\circ - A)$ .**

**Solution:**

**Step 1: Write the given condition of the triangle.**

In  $\triangle ABC$ , it is given that  $\angle B = 90^\circ$ . Since the sum of all angles of a triangle is  $180^\circ$ , we get

$$\angle A + \angle C + 90^\circ = 180^\circ$$

So,

$$\angle A + \angle C = 90^\circ$$

Hence,

$$\angle C = 90^\circ - \angle A$$

**Step 2: Write  $\sin A$  in terms of sides.**

In a right triangle,

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

**Step 3: Write  $\cos(90^\circ - A)$  using angle  $C$ .**

Since

$$C = 90^\circ - A$$

we have

$$\cos(90^\circ - A) = \cos C$$

Now,

$$\cos C = \frac{\text{Adjacent side to } C}{\text{Hypotenuse}} = \frac{BC}{AC}$$

**Step 4: Compare both values.**

From above,

$$\sin A = \frac{BC}{AC}$$

and

$$\cos(90^\circ - A) = \frac{BC}{AC}$$

Therefore,

$$\sin A = \cos(90^\circ - A)$$

**Step 5: Prove the second identity.**

Now,

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

Also, since  $C = 90^\circ - A$ ,

$$\sin(90^\circ - A) = \sin C$$

and

$$\sin C = \frac{\text{Perpendicular to } C}{\text{Hypotenuse}} = \frac{AB}{AC}$$

Thus,

$$\cos A = \sin(90^\circ - A)$$

**Step 6: Write the conclusion.**

Hence proved,

$$\boxed{\sin A = \cos(90^\circ - A)}$$

and

$$\boxed{\cos A = \sin(90^\circ - A)}$$

### Quick Tip

Remember: In a right triangle, the two acute angles are complementary. So, sine of one acute angle is equal to cosine of the other, and cosine of one acute angle is equal to sine of the other.

**16. Prove that the area of a sector of sectorial angle  $\theta$  and radius  $r$  is  $\frac{\pi r^2 \theta}{360}$ .**

**Solution:**

**Step 1: Write the area of a complete circle.**

We know that the area of a circle of radius  $r$  is

$$\pi r^2$$

**Step 2: Understand the meaning of a sector.**

A sector is a part of a circle enclosed by two radii and the corresponding arc. If the angle of the sector is  $\theta^\circ$ , then it is only a fraction of the whole circle.

**Step 3: Find the fraction of the whole circle.**

Since the angle of the complete circle is  $360^\circ$ , the sector with angle  $\theta^\circ$  represents

$$\frac{\theta}{360}$$

part of the whole circle.

**Step 4: Multiply this fraction by the total area.**

Therefore, the area of the sector is equal to  $\frac{\theta}{360}$  times the total area of the circle. So,

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

**Step 5: Simplify the expression.**

Thus,

$$\text{Area of sector} = \frac{\pi r^2 \theta}{360}$$

**Step 6: Write the final result.**

Hence, the area of a sector of angle  $\theta$  and radius  $r$  is

$$\boxed{\frac{\pi r^2 \theta}{360}}$$

### Quick Tip

Remember: A sector is a fraction of a full circle, so its area is found by multiplying the total area  $\pi r^2$  by  $\frac{\theta}{360}$ .

---

## 17. Write the full form of SCORES.

### Solution:

#### Step 1: Identify the term SCORES.

SCORES is a term used in the field of consumer grievance redressal related to the securities market.

#### Step 2: Write the full form.

The full form of SCORES is

SEBI Complaints Redress System

#### Step 3: Explain its purpose.

It is an online platform provided by SEBI for investors to lodge and track complaints against listed companies, intermediaries, and other market-related entities.

#### Step 4: State the importance.

This system helps investors by offering a simple and transparent method for grievance redressal in the securities market.

#### Step 5: Final answer.

Therefore, the full form of SCORES is **SEBI Complaints Redress System**.

### Quick Tip

Remember: SCORES is linked with SEBI and is used for handling investor complaints in the securities market.

---

## 18. What is the first term of the quotient when $2x^3 + x^2 - 3x + 5$ is divided by $1 - 3x + x^2$ ?

**Solution:**

**Step 1: Rewrite the divisor in standard form.**

The divisor is given as  $1 - 3x + x^2$ . Writing it in descending powers of  $x$ , we get:

$$x^2 - 3x + 1$$

**Step 2: Identify the leading terms.**

The leading term of the dividend  $2x^3 + x^2 - 3x + 5$  is  $2x^3$ .

The leading term of the divisor  $x^2 - 3x + 1$  is  $x^2$ .

**Step 3: Divide the leading terms.**

To find the first term of the quotient, divide the leading term of the dividend by the leading term of the divisor:

$$\frac{2x^3}{x^2} = 2x$$

**Step 4: State the first term of the quotient.**

Hence, the first term of the quotient is:

$$2x$$

#### Quick Tip

In polynomial division, the first term of the quotient is always found by dividing the highest degree term of the dividend by the highest degree term of the divisor.

---

**19. How many tangents can be drawn to a circle through a point lying outside the circle?**

**Solution:**

**Step 1: Understand the position of the point.**

A point lying outside a circle is called an external point. From such a point, tangents can be drawn to touch the circle without cutting it.

**Step 2: Recall the tangent property.**

A tangent touches the circle at exactly one point. From an external point, it is always possible to draw two distinct tangents to the circle.

**Step 3: State the result.**

Therefore, the number of tangents that can be drawn to a circle from a point lying outside the circle is:

2

**Quick Tip**

Remember: From a point outside a circle, exactly two tangents can be drawn. From a point on the circle, only one tangent can be drawn.

---

**20. Write down the formula to find the area of a triangle whose vertices are  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$ .**

**Solution:**

**Step 1: Recall the coordinate geometry formula.**

The area of a triangle whose vertices are given in coordinate form can be found using the determinant formula.

**Step 2: Write the standard formula.**

If the vertices of the triangle are  $(x_1, y_1)$ ,  $(x_2, y_2)$ , and  $(x_3, y_3)$ , then its area is:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

**Step 3: Mention the absolute value.**

Absolute value is taken because area is always positive, even if the expression inside the bracket becomes negative.

**Step 4: State the final formula.**

Hence, the required formula is:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

### Quick Tip

For area of a triangle using coordinates, always use modulus signs because area cannot be negative.

## 21. State and prove Basic Proportionality Theorem.

### Solution:

#### Step 1: State the Basic Proportionality Theorem.

The Basic Proportionality Theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

#### Step 2: Write the given figure and assumption.

In  $\triangle ABC$ , let a line  $DE$  be drawn parallel to side  $BC$ , such that  $D$  lies on  $AB$  and  $E$  lies on  $AC$ . Then we have to prove that

$$\frac{AD}{DB} = \frac{AE}{EC}$$

#### Step 3: Compare the areas of triangles with the same altitude.

Since triangles  $\triangle BDE$  and  $\triangle CDE$  lie on the same base  $DE$  and between the same parallels  $DE \parallel BC$ , their areas are equal. Therefore,

$$\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$$

Also, triangles  $\triangle ADE$  and  $\triangle BDE$  have the same altitude from  $E$  on line  $AB$ . Hence,

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{AD}{DB}$$

Similarly, triangles  $\triangle ADE$  and  $\triangle CDE$  have the same altitude from  $D$  on line  $AC$ . Hence,

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)} = \frac{AE}{EC}$$

#### Step 4: Use the equality of areas.

Since

$$\text{ar}(\triangle BDE) = \text{ar}(\triangle CDE)$$

therefore,

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle CDE)}$$

So, we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

**Step 5: Write the conclusion.**

Hence, it is proved that if a line is drawn parallel to one side of a triangle, then it divides the other two sides in the same ratio. Therefore,

$$\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$$

#### Quick Tip

Remember: In Basic Proportionality Theorem, a line parallel to one side of a triangle divides the other two sides proportionally.

**OR,**

**Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.**

**Solution:**

**Step 1: Write the given condition of similarity.**

Let  $\triangle ABC \sim \triangle DEF$ . Since the triangles are similar, their corresponding angles are equal and their corresponding sides are proportional. Therefore,

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

**Step 2: Write the formula for area of a triangle.**

We know that the area of a triangle can be written as

$$\text{ar}(\triangle) = \frac{1}{2} \times \text{base} \times \text{height}$$

Let the altitudes corresponding to sides  $AB$  and  $DE$  be  $h_1$  and  $h_2$  respectively. Then,

$$\text{ar}(\triangle ABC) = \frac{1}{2} \cdot AB \cdot h_1$$

and

$$\text{ar}(\triangle DEF) = \frac{1}{2} \cdot DE \cdot h_2$$

**Step 3: Take the ratio of the areas.**

So,

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{\frac{1}{2} \cdot AB \cdot h_1}{\frac{1}{2} \cdot DE \cdot h_2} = \frac{AB}{DE} \cdot \frac{h_1}{h_2}$$

**Step 4: Use the property of similar triangles.**

In similar triangles, the ratio of corresponding altitudes is equal to the ratio of corresponding sides. Hence,

$$\frac{h_1}{h_2} = \frac{AB}{DE}$$

Substituting this in the above expression, we get

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AB}{DE} \cdot \frac{AB}{DE}$$

**Step 5: Simplify the result.**

Therefore,

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \left(\frac{AB}{DE}\right)^2$$

Similarly, we may also write

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \left(\frac{BC}{EF}\right)^2 = \left(\frac{CA}{FD}\right)^2$$

Thus, the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

#### Quick Tip

Remember: For similar triangles, not only the corresponding sides but also the altitudes are in the same ratio. Hence, areas are proportional to the squares of corresponding sides.

**22. Find the area of a circle whose radius is 7 cm.**

**Solution:**

**Step 1: Write the formula for area of a circle.**

The formula for the area of a circle is

$$A = \pi r^2$$

**Step 2: Identify the given radius.**

Here, the radius of the circle is

$$r = 7 \text{ cm}$$

**Step 3: Substitute the value in the formula.**

Putting  $r = 7 \text{ cm}$  in the formula, we get

$$A = \pi \times 7^2$$

**Step 4: Simplify the expression.**

Since

$$7^2 = 49$$

therefore,

$$A = \pi \times 49$$

**Step 5: Find the final answer.**

Using  $\pi = \frac{22}{7}$ , we get

$$A = \frac{22}{7} \times 49 = 154 \text{ cm}^2$$

So, the area of the circle is

$$\boxed{154 \text{ cm}^2}$$

#### Quick Tip

Remember: Area of a circle is found by using  $A = \pi r^2$ . First square the radius, then multiply by  $\pi$ .

---

**23. Define mode of a frequency distribution.**

**Solution:**

**Step 1: Understand the meaning of mode.**

Mode is a measure of central tendency. It tells us the value that appears most frequently in a set of observations or a frequency distribution.

**Step 2: Define it for frequency distribution.**

In a frequency distribution, the mode is the value or observation which has the highest frequency.

**Step 3: Explain its importance.**

It helps us identify the most common item or the most repeated value in the given data.

**Step 4: State the formal definition.**

Therefore, the mode of a frequency distribution is the observation corresponding to the maximum frequency.

**Step 5: Final answer.**

Hence, mode is defined as the value which occurs most frequently in the frequency distribution.

**Quick Tip**

Remember: Mode means the most repeated value. In a frequency table, just look for the value with the highest frequency.

---

**24. Show that**  $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c) \{(a - b)^2 + (b - c)^2 + (c - a)^2\}$ .

**Solution:**

**Step 1: Start with the right-hand side.**

We take the expression:

$$\frac{1}{2}(a + b + c) \{(a - b)^2 + (b - c)^2 + (c - a)^2\}$$

**Step 2: Expand the terms inside the bracket.**

Now,

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(b - c)^2 = b^2 - 2bc + c^2$$

$$(c - a)^2 = c^2 - 2ca + a^2$$

Adding these, we get:

$$\begin{aligned} & (a - b)^2 + (b - c)^2 + (c - a)^2 \\ &= a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2 \\ &= 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca \\ &= 2(a^2 + b^2 + c^2 - ab - bc - ca) \end{aligned}$$

**Step 3: Substitute this back into the expression.**

Substituting in the right-hand side, we get:

$$\begin{aligned} & \frac{1}{2}(a + b + c) \cdot 2(a^2 + b^2 + c^2 - ab - bc - ca) \\ &= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) \end{aligned}$$

**Step 4: Use the standard identity.**

We know the algebraic identity:

$$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$$

**Step 5: Conclude the proof.**

Therefore,

$$\frac{1}{2}(a + b + c) \{(a - b)^2 + (b - c)^2 + (c - a)^2\} = a^3 + b^3 + c^3 - 3abc$$

Hence, proved that:

$$\boxed{a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c) \{(a - b)^2 + (b - c)^2 + (c - a)^2\}}$$

### Quick Tip

Remember the identity:  $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ .  
Then use  $(a - b)^2 + (b - c)^2 + (c - a)^2 = 2(a^2 + b^2 + c^2 - ab - bc - ca)$ .

**25. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them saves Rs. 5000 per month, find their monthly incomes.**

**Solution:**

**Step 1: Assume the incomes and expenditures.**

Let the monthly incomes of the two persons be  $9x$  and  $7x$ , since their income ratio is 9 : 7.

Let their monthly expenditures be  $4y$  and  $3y$ , since their expenditure ratio is 4 : 3.

**Step 2: Use the condition of savings.**

It is given that each person saves Rs. 5000 per month.

So, for the first person:

$$9x - 4y = 5000$$

For the second person:

$$7x - 3y = 5000$$

**Step 3: Solve the pair of equations.**

We have:

$$9x - 4y = 5000 \quad \dots(1)$$

$$7x - 3y = 5000 \quad \dots(2)$$

Multiply equation (1) by 3:

$$27x - 12y = 15000$$

Multiply equation (2) by 4:

$$28x - 12y = 20000$$

Now subtract the first from the second:

$$(28x - 12y) - (27x - 12y) = 20000 - 15000$$

$$x = 5000$$

**Step 4: Find the monthly incomes.**

Now substitute  $x = 5000$ :

First person's income:

$$9x = 9 \times 5000 = 45000$$

Second person's income:

$$7x = 7 \times 5000 = 35000$$

**Step 5: Verify the result.**

From equation (2):

$$7(5000) - 3y = 5000$$

$$35000 - 3y = 5000$$

$$3y = 30000$$

$$y = 10000$$

So expenditures are  $4y = 40000$  and  $3y = 30000$ .

Savings are:

$$45000 - 40000 = 5000$$

$$35000 - 30000 = 5000$$

Hence, the answer is correct.

**Step 6: State the final answer.**

Therefore, the monthly incomes of the two persons are:

Rs. 45000 and Rs. 35000

### Quick Tip

In ratio problems involving income, expenditure, and savings, first assume the quantities in terms of variables according to the given ratios, then form equations using  $\text{Income} - \text{Expenditure} = \text{Savings}$ .

---

**26. If  $x, y, z$  are real numbers,  $x \neq 0$ , and  $xy = xz$ , prove that  $y = z$ .**

**Solution:**

**Step 1: Write the given equation.**

We are given that:

$$xy = xz$$

**Step 2: Bring all terms to one side.**

Subtract  $xz$  from both sides of the equation:

$$xy - xz = 0$$

**Step 3: Take out the common factor.**

Since  $x$  is common in both terms, factor it out:

$$x(y - z) = 0$$

**Step 4: Use the given condition  $x \neq 0$ .**

It is given that  $x \neq 0$ . Therefore, the product  $x(y - z) = 0$  can be zero only if:

$$y - z = 0$$

**Step 5: Conclude the result.**

From  $y - z = 0$ , we get:

$$y = z$$

Hence, it is proved that:

$$\boxed{y = z}$$

### Quick Tip

If  $ab = ac$  and  $a \neq 0$ , then by cancelling the non-zero common factor  $a$ , we get  $b = c$ .

---

**27. A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height  $h$ . At a point on the plane, the angle of elevation of the bottom of the flagstaff is  $\alpha$  and that of the top of the flagstaff is  $\beta$ . Prove that the height of the tower is**

$$\frac{h \tan \alpha}{\tan \beta - \tan \alpha}.$$

**Solution:**

**Step 1:** Assume the height of the tower and horizontal distance.

Let the height of the tower be  $H$  and the horizontal distance of the observation point from the foot of the tower be  $x$ .

The height of the flagstaff is already given as  $h$ .

Therefore, the total height up to the top of the flagstaff is:

$$H + h$$

**Step 2:** Form the trigonometric equation for the bottom of the flagstaff.

The bottom of the flagstaff is the top of the tower.

Since the angle of elevation of this point is  $\alpha$ , we have:

$$\begin{aligned} \tan \alpha &= \frac{\text{height of tower}}{\text{horizontal distance}} \\ \tan \alpha &= \frac{H}{x} \end{aligned}$$

So,

$$H = x \tan \alpha$$

**Step 3:** Form the trigonometric equation for the top of the flagstaff.

The angle of elevation of the top of the flagstaff is  $\beta$ .

Hence,

$$\tan \beta = \frac{H + h}{x}$$

**Step 4:** Substitute the value of  $H$  from Step 2.

Substituting  $H = x \tan \alpha$  into the second equation, we get:

$$\begin{aligned}\tan \beta &= \frac{x \tan \alpha + h}{x} \\ \tan \beta &= \tan \alpha + \frac{h}{x}\end{aligned}$$

**Step 5:** Find the value of  $x$ .

Rearranging the above equation:

$$\begin{aligned}\tan \beta - \tan \alpha &= \frac{h}{x} \\ x &= \frac{h}{\tan \beta - \tan \alpha}\end{aligned}$$

**Step 6:** Find the height of the tower.

We know that:

$$H = x \tan \alpha$$

Substituting the value of  $x$ :

$$\begin{aligned}H &= \frac{h}{\tan \beta - \tan \alpha} \cdot \tan \alpha \\ H &= \frac{h \tan \alpha}{\tan \beta - \tan \alpha}\end{aligned}$$

**Step 7:** State the required result.

Hence, the height of the tower is:

$$\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$$

### Quick Tip

In height and distance problems, first assign variables carefully, then form separate  $\tan \theta$  equations for each angle of elevation and eliminate the common horizontal distance.

**28. A metallic sphere of radius 9 cm is melted and recast to form a cylinder of radius 3 cm. Find the curved surface area of the cylinder.**

**Solution:**

**Step 1: Use the concept of conservation of volume.**

Since the metallic sphere is melted and recast into a cylinder, the volume of the sphere will be equal to the volume of the cylinder.

**Step 2: Write the volume of the sphere.**

Radius of the sphere is 9 cm.

So, volume of the sphere is:

$$\begin{aligned}V_{\text{sphere}} &= \frac{4}{3}\pi r^3 \\V_{\text{sphere}} &= \frac{4}{3}\pi(9)^3 \\V_{\text{sphere}} &= \frac{4}{3}\pi \cdot 729 \\V_{\text{sphere}} &= 972\pi\end{aligned}$$

**Step 3: Write the volume of the cylinder.**

Let the height of the cylinder be  $h$ .

Radius of the cylinder is 3 cm.

So, volume of the cylinder is:

$$V_{\text{cylinder}} = \pi r^2 h$$

$$V_{\text{cylinder}} = \pi(3)^2h$$

$$V_{\text{cylinder}} = 9\pi h$$

**Step 4:** Equate the two volumes to find the height.

Since both volumes are equal:

$$972\pi = 9\pi h$$

Cancel  $\pi$  from both sides:

$$972 = 9h$$

$$h = 108$$

**Step 5:** Find the curved surface area of the cylinder.

Curved surface area of a cylinder is given by:

$$\text{CSA} = 2\pi rh$$

Substituting  $r = 3$  cm and  $h = 108$  cm:

$$\text{CSA} = 2\pi(3)(108)$$

$$\text{CSA} = 648\pi$$

**Step 6:** State the final answer.

Hence, the curved surface area of the cylinder is:

$$\boxed{648\pi \text{ cm}^2}$$

If expressed numerically,

$$648\pi \approx 2036.57 \text{ cm}^2$$

#### Quick Tip

When one solid is melted and recast into another, their volumes remain equal. First find the missing dimension using volume, then calculate the required surface area.

---

**29. Prove that  $x^n - y^n$  is divisible by  $x + y$  only when  $n$  is even.**

**Solution:**

**Step 1: Use the divisibility condition.**

If a polynomial is divisible by  $x + y$ , then on putting  $x = -y$ , the polynomial must become zero.

So, consider:

$$x^n - y^n$$

**Step 2: Substitute  $x = -y$ .**

Replacing  $x$  by  $-y$ , we get:

$$(-y)^n - y^n$$

**Step 3: Analyze according to whether  $n$  is even or odd.**

Now,  $(-y)^n$  depends on the parity of  $n$ :

If  $n$  is even, then:

$$(-y)^n = y^n$$

So,

$$(-y)^n - y^n = y^n - y^n = 0$$

Hence,  $x + y$  divides  $x^n - y^n$ .

If  $n$  is odd, then:

$$(-y)^n = -y^n$$

So,

$$(-y)^n - y^n = -y^n - y^n = -2y^n \neq 0$$

Hence,  $x + y$  does not divide  $x^n - y^n$ .

**Step 4: State the conclusion.**

Therefore,  $x^n - y^n$  is divisible by  $x + y$  only when  $n$  is even.

**Step 5: Final statement.**

Thus, we have proved that:

$$x^n - y^n \text{ is divisible by } x + y \text{ only if } n \text{ is even}$$

#### Quick Tip

To check whether a polynomial is divisible by  $x + a$ , substitute  $x = -a$ . If the result is zero, then the polynomial is divisible by that factor.

---

**30. If  $\cos A = \frac{3}{5}$ , calculate  $\sin A$  and  $\tan A$ .**

**Solution:**

**Step 1: Use the identity between sine and cosine.**

We know that

$$\sin^2 A + \cos^2 A = 1$$

Given that

$$\cos A = \frac{3}{5}$$

So,

$$\cos^2 A = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

**Step 2: Find the value of  $\sin^2 A$ .**

Substituting in the identity, we get

$$\sin^2 A = 1 - \frac{9}{25} = \frac{25}{25} - \frac{9}{25} = \frac{16}{25}$$

**Step 3: Find the value of  $\sin A$ .**

Taking the positive square root for an acute angle,

$$\sin A = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

**Step 4:** Find the value of  $\tan A$ .

Now,

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$$

**Step 5:** Write the final answer.

Therefore,

$$\sin A = \frac{4}{5}, \quad \tan A = \frac{4}{3}$$

#### Quick Tip

Remember: If one trigonometric ratio is given, use identities like  $\sin^2 A + \cos^2 A = 1$  to find the others.

---

**31. Show that the square of an odd integer is of the form  $8k + 1$ .**

**Solution:**

**Step 1:** Represent an odd integer in general form.

Any odd integer can be written as

$$2n + 1$$

where  $n$  is an integer.

**Step 2:** Square the odd integer.

Now, squaring  $2n + 1$ , we get

$$(2n + 1)^2 = 4n^2 + 4n + 1$$

**Step 3:** Factor the expression.

Taking  $4n$  common from the first two terms,

$$4n^2 + 4n + 1 = 4n(n + 1) + 1$$

**Step 4:** Use the property of consecutive integers.

Here,  $n$  and  $n + 1$  are two consecutive integers. Out of two consecutive integers, one must be even. Therefore, the product  $n(n + 1)$  is always even. So, we can write

$$n(n + 1) = 2k$$

for some integer  $k$ .

**Step 5: Substitute and obtain the required form.**

Substituting  $n(n + 1) = 2k$  into the expression, we get

$$(2n + 1)^2 = 4(2k) + 1 = 8k + 1$$

Hence, the square of an odd integer is always of the form

$$\boxed{8k + 1}$$

### Quick Tip

To prove statements about odd integers, first write the odd number in the form  $2n + 1$ , then simplify step by step.

**32. Find the area of the quadrilateral whose vertices are  $(1, 1)$ ,  $(3, 4)$ ,  $(5, -2)$  and  $(4, -7)$  taken in order.**

**Solution:**

**Step 1: Write the coordinates in tabular form for using the shoelace formula.**

The given vertices of the quadrilateral taken in order are

$$(1, 1), \quad (3, 4), \quad (5, -2), \quad (4, -7)$$

Now, we write them in order and repeat the first point at the end:

$x$	$y$
1	1
3	4
5	-2
4	-7
1	1

**Step 2: Find the sum of the products of downward diagonals.**

Using the shoelace rule, we calculate

$$S_1 = (1 \times 4) + (3 \times -2) + (5 \times -7) + (4 \times 1)$$

$$S_1 = 4 - 6 - 35 + 4 = -33$$

**Step 3:** Find the sum of the products of upward diagonals.

Now,

$$S_2 = (1 \times 3) + (4 \times 5) + (-2 \times 4) + (-7 \times 1)$$

$$S_2 = 3 + 20 - 8 - 7 = 8$$

**Step 4:** Apply the formula for area of a quadrilateral.

The area of the quadrilateral is given by

$$\text{Area} = \frac{1}{2} |S_1 - S_2|$$

Substituting the values, we get

$$\text{Area} = \frac{1}{2} |-33 - 8|$$

$$\text{Area} = \frac{1}{2} |-41| = \frac{41}{2}$$

**Step 5:** Write the final answer.

Therefore, the area of the quadrilateral is

$$\boxed{\frac{41}{2} \text{ square units}}$$

or

$$\boxed{20.5 \text{ square units}}$$

#### Quick Tip

For polygons whose vertices are given in order, the shoelace formula is the fastest method to find the area directly from coordinates.

**Or,**

**If three consecutive vertices of a parallelogram are  $A(1, -2)$ ,  $B(3, 6)$  and  $C(5, 10)$ , find its fourth vertex.**

**Solution:**

**Step 1:** Use the property of a parallelogram.

In a parallelogram, if  $A$ ,  $B$ , and  $C$  are three consecutive vertices, then the fourth vertex  $D$  is obtained by using the vector relation

$$\vec{D} = \vec{A} + \vec{C} - \vec{B}$$

**Step 2:** Write the coordinates of the given vertices.

The given points are

$$A(1, -2), \quad B(3, 6), \quad C(5, 10)$$

**Step 3:** Apply the coordinate formula for the fourth vertex.

Using

$$D(x, y) = (x_A + x_C - x_B, y_A + y_C - y_B)$$

we get

$$D = (1 + 5 - 3, -2 + 10 - 6)$$

**Step 4:** Simplify the coordinates.

Now simplify each coordinate:

$$x = 1 + 5 - 3 = 3$$

$$y = -2 + 10 - 6 = 2$$

So,

$$D = (3, 2)$$

**Step 5:** Write the final answer.

Hence, the fourth vertex of the parallelogram is

$$\boxed{(3, 2)}$$

#### Quick Tip

If three consecutive vertices  $A$ ,  $B$ , and  $C$  of a parallelogram are known, then the fourth vertex is found by  $D = A + C - B$ .