Mizoram Board Class 12 Mathematics Question Paper with Solutions(Memory Based)

Time Allowed :3 Hour	**Maximum Marks :**60	**Total Questions :**24

General Instructions

Read the following instructions very carefully and strictly follow them:

- Answers to this Paper must be written on the paper provided separately.
- You will not be allowed to write during the first 15 minutes
- This time is to be spent in reading the question paper.
- The time given at the head of this Paper is the time allowed for writing the answers,
- The paper has four Sections.
- Section A is compulsory - All questions in Section A must be answered.
- You must attempt one question from each of the Sections B, C and D and one other question from any Section of your choice.

1. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by

$$R = \{(a, b) : |a - b| \textbf{ is even}\}$$

is an equivalence relation.

Solution:
Concept: A relation R on a set A is called an **equivalence relation** if it satisfies the following three properties:

- Reflexive: $(a, a) \in R$ for all $a \in A$
- Symmetric: If $(a, b) \in R$, then $(b, a) \in R$
- Transitive: If $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$

Here, the relation is defined by $|a - b|$ being even.
Step 1: Reflexive property.
For every element $a \in A$,

$$|a - a| = 0$$

Since 0 is even, $(a, a) \in R$ for all $a \in A$. Hence, the relation R is **reflexive**.
Step 2: Symmetric property.
If $(a, b) \in R$, then

$$|a - b| \text{ is even.}$$

But

$$|b - a| = |a - b|$$

Thus $|b-a|$ is also even, implying $(b,a) \in R$. Hence, the relation R is **symmetric**.
Step 3: Transitive property.
Suppose $(a,b) \in R$ and $(b,c) \in R$. Then

$$|a-b| \text{ is even} \quad \text{and} \quad |b-c| \text{ is even.}$$

This implies

$$a-b=2m, \qquad b-c=2n$$

for some integers m, n.
Adding the equations,

$$a-c=2m+2n=2(m+n)$$

Thus $a-c$ is even, which means

$$|a-c| \text{ is even}$$

Therefore $(a,c) \in R$. Hence, R is **transitive**.
Step 4: Conclusion.
Since the relation R satisfies **reflexive, symmetric, and transitive properties**, it is an **equivalence relation** on the set A.

Quick Tip

For relations defined using $|\mathbf{a}-\mathbf{b}|$, check parity (odd/even). Numbers with the **same parity (both even or both odd)** have an even difference. Thus, the equivalence classes here are:

$$\{1,3,5\}, \quad \{2,4\}.$$

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2. Find the principal value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$**.**

Solution:
Concept: To evaluate expressions involving inverse trigonometric functions, we use their **principal value ranges**:

- $\tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- $\sec^{-1} x \in [0,\pi], \ x \neq (-1,1)$

We use standard trigonometric values and identities to determine the angles.
Step 1: Evaluate $\tan^{-1}(\sqrt{3})$.

We know that

$$\tan\frac{\pi}{3} = \sqrt{3}$$

Since $\frac{\pi}{3}$ lies within the principal value range of $\tan^{-1} x$,

$$\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

Step 2: Evaluate $\sec^{-1}(-2)$.

Let

$$\sec^{-1}(-2) = \theta$$

Then

$$\sec\theta = -2$$

$$\cos\theta = -\frac{1}{2}$$

In the principal value range $[0, \pi]$, the angle whose cosine is $-\frac{1}{2}$ is

$$\theta = \frac{2\pi}{3}$$

Thus,

$$\sec^{-1}(-2) = \frac{2\pi}{3}$$

Step 3: Compute the required expression.

$$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$$

$$= \frac{\pi}{3} - \frac{2\pi}{3}$$

$$= -\frac{\pi}{3}$$

Step 4: Final Answer.

$$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = -\frac{\pi}{3}$$

Quick Tip

Remember the principal value ranges: $\tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\sec^{-1} x \in [0, \pi]$. Also recall standard values such as $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$ and $\sec^{-1}(-2) = \frac{2\pi}{3}$.

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3. If

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} \quad \textbf{and} \quad B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix},$$

verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Solution:
Concept: For any two non-singular matrices A and B,

$$(AB)^{-1} = B^{-1}A^{-1}$$

To verify this property, we compute AB, then find $(AB)^{-1}$, and separately compute $B^{-1}A^{-1}$. If both results are equal, the identity is verified.
Step 1: Find the product AB.

$$AB = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2(1) + 3(-1) & 2(-2) + 3(3) \\ 1(1) + (-4)(-1) & 1(-2) + (-4)(3) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$$

Step 2: Find $(AB)^{-1}$.
For a 2×2 matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Determinant of AB:

$$(-1)(-14) - (5)(5) = 14 - 25 = -11$$

Thus,

$$(AB)^{-1} = \frac{1}{-11} \begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{14}{11} & \frac{5}{11} \\ \frac{5}{11} & \frac{1}{11} \end{bmatrix}$$

Step 3: Find A^{-1} and B^{-1}.
For A:

$$|A| = (2)(-4) - (3)(1) = -8 - 3 = -11$$

$$A^{-1} = \frac{1}{-11} \begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$$

For B:

$$|B| = (1)(3) - (-2)(-1) = 3 - 2 = 1$$

$$B^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

Step 4: Compute $B^{-1}A^{-1}$.

$$B^{-1}A^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \left(\frac{1}{-11} \begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix} \right)$$

$$= \frac{1}{-11} \begin{bmatrix} 3(-4) + 2(-1) & 3(-3) + 2(2) \\ 1(-4) + 1(-1) & 1(-3) + 1(2) \end{bmatrix}$$

$$= \frac{1}{-11} \begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{14}{11} & \frac{5}{11} \\ \frac{5}{11} & \frac{1}{11} \end{bmatrix}$$

Step 5: Verification.

$$(AB)^{-1} = \begin{bmatrix} \frac{14}{11} & \frac{5}{11} \\ \frac{5}{11} & \frac{1}{11} \end{bmatrix} = B^{-1}A^{-1}$$

Hence,

$$(AB)^{-1} = B^{-1}A^{-1}$$

is verified.

Quick Tip

For invertible matrices, remember the important identity:

$$(AB)^{-1} = B^{-1}A^{-1}$$

The **order reverses** when taking the inverse of a product of matrices.

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4. Solve the following system of equations using the matrix method:

$$2x + 3y + 3z = 5, \qquad x - 2y + z = -4, \qquad 3x - y - 2z = 3.$$

Solution:
Concept: A system of linear equations can be written in matrix form as

$$AX = B$$

where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix. If A is non-singular, the solution is given by

$$X = A^{-1}B$$

Step 1: Write the system in matrix form.

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

Thus,

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

Step 2: Find the determinant of A.

$$|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$$

$$= 2\begin{vmatrix} -2 & 1 \\ -1 & -2 \end{vmatrix} - 3\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} + 3\begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix}$$

$$= 2(4+1) - 3(-2-3) + 3(-1+6)$$

$$= 10 + 15 + 15 = 40$$

Since $|A| \neq 0$, the matrix A is invertible.
Step 3: Find A^{-1}.
Using $A^{-1} = \frac{1}{|A|}\text{Adj}(A)$, we obtain

$$A^{-1} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

Step 4: Compute $X = A^{-1}B$.

$$X = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Step 5: Final solution.

$$x = 1, \qquad y = 2, \qquad z = -1$$

Quick Tip

To solve linear equations using matrices:

$$AX = B \Rightarrow X = A^{-1}B$$

First check that $|A| \neq 0$. Only then does the inverse exist and the system has a unique solution.

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5. Find the derivative of $(\sin x)^x + \sin^{-1}(\sqrt{x})$ with respect to x.

Solution:

Concept: To differentiate $(\sin x)^x$, we use **logarithmic differentiation** since both the base and exponent are functions of x. For $\sin^{-1}(\sqrt{x})$, we apply the **chain rule** along with the derivative formula of inverse trigonometric functions.

$$\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

Step 1: Differentiate $(\sin x)^x$ using logarithmic differentiation.

Let

$$y = (\sin x)^x$$

Taking logarithm on both sides,

$$\ln y = x \ln(\sin x)$$

Differentiating with respect to x:

$$\frac{1}{y}\frac{dy}{dx} = \ln(\sin x) + x\frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = (\sin x)^x \left[\ln(\sin x) + x \cot x\right]$$

Step 2: Differentiate $\sin^{-1}(\sqrt{x})$.

Let $u = \sqrt{x}$.

$$\frac{d}{dx}\sin^{-1}(\sqrt{x}) = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

Step 3: Combine the derivatives.

$$\frac{d}{dx}\left[(\sin x)^x + \sin^{-1}(\sqrt{x})\right]$$

$$= (\sin x)^x \left[\ln(\sin x) + x\cot x\right] + \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

Step 4: Final Answer.

$$\boxed{\frac{d}{dx}\left[(\sin x)^x + \sin^{-1}(\sqrt{x})\right] = (\sin x)^x \left[\ln(\sin x) + x\cot x\right] + \frac{1}{2\sqrt{x}\sqrt{1-x}}}$$

Quick Tip

For expressions of the form $f(x)^{g(x)}$, use **logarithmic differentiation**. Also remember:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

and apply the **chain rule** when the argument is a function like $\sqrt{x}$.

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6. Evaluate the integral

$$\int \frac{x\sin^{-1} x}{\sqrt{1-x^2}}\,dx.$$

Solution:
Concept: To evaluate this integral, we use the **substitution method**. Notice that the derivative of $\sin^{-1} x$ is

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

which appears in the denominator of the integrand.
Step 1: Use substitution.
Let

$$t = \sin^{-1} x$$

Then

$$\frac{dt}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$dt = \frac{dx}{\sqrt{1-x^2}}$$

Thus the integral becomes

$$\int x(\sin^{-1} x)\frac{dx}{\sqrt{1-x^2}} = \int x\,t\,dt$$

Step 2: Express x in terms of t.
Since $t = \sin^{-1} x$,

$$x = \sin t$$

Hence the integral becomes

$$\int t \sin t \, dt$$

Step 3: Apply integration by parts.
Let

$$u = t, \qquad dv = \sin t \, dt$$

Then

$$du = dt, \qquad v = -\cos t$$

Using the formula

$$\int u \, dv = uv - \int v \, du$$

$$\int t \sin t \, dt = -t \cos t + \int \cos t \, dt$$

$$= -t \cos t + \sin t + C$$

Step 4: Substitute back $t = \sin^{-1} x$.

$$= -(\sin^{-1} x) \cos(\sin^{-1} x) + \sin(\sin^{-1} x) + C$$

Now,

$$\sin(\sin^{-1} x) = x$$

and

$$\cos(\sin^{-1} x) = \sqrt{1 - x^2}$$

Thus,

$$\int \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} \, dx = x - (\sin^{-1} x)\sqrt{1 - x^2} + C$$

Step 5: Final Answer.

$$\boxed{\int \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} \, dx = x - (\sin^{-1} x)\sqrt{1 - x^2} + C}$$

Quick Tip

Whenever an integral contains $\sin^{-1} x$ along with $\frac{1}{\sqrt{1-x^2}}$, try the substitution

$$t = \sin^{-1} x$$

since its derivative naturally appears in the integrand.

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7. Find the general solution of the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = x^2.$$

Solution:
Concept: The given equation is a **linear differential equation** of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where

$$P(x) = \frac{1}{x}, \qquad Q(x) = x^2$$

The solution is obtained using the **integrating factor (I.F.)**:

$$\text{I.F.} = e^{\int P(x)\,dx}$$

Step 1: Find the integrating factor.

$$\text{I.F.} = e^{\int \frac{1}{x} dx}$$

$$= e^{\ln x} = x$$

Step 2: Multiply the equation by the integrating factor.

$$x\frac{dy}{dx} + y = x^3$$

The left-hand side becomes the derivative of a product:

$$\frac{d}{dx}(xy) = x^3$$

Step 3: Integrate both sides.

$$xy = \int x^3\,dx$$

$$xy = \frac{x^4}{4} + C$$

Step 4: Express the solution for y.

$$y = \frac{x^4}{4x} + \frac{C}{x}$$

$$y = \frac{x^3}{4} + \frac{C}{x}$$

Step 5: General solution.

$$\boxed{y = \frac{x^3}{4} + \frac{C}{x}}$$

Quick Tip

For a linear differential equation

$$\frac{dy}{dx} + P(x)y = Q(x),$$

first compute the integrating factor

$$\text{I.F.} = e^{\int P(x)\,dx}.$$

Multiplying the equation by the I.F. converts the left side into $\frac{d}{dx}(y \cdot \text{I.F.})$, making the equation easy to integrate.
