The Collegedunia NCERT Notes

The Ultimate NCERT Revision Notes for Class 12 Physics

Chapter 2: Electrostatic Potential and Capacitance

1 Introduction to Electrostatic Potential

1.1 Why Potential Concept is Needed

Before introducing electric potential, we must understand why the force concept alone is not sufficient in electrostatics.

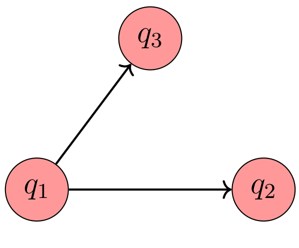
Limitation of Force Approach

In electrostatics, force between two point charges is given by:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Force is a vector quantity and requires vector addition when multiple charges are involved.

- Vector addition becomes complicated.
- Force does not directly give stored energy.
- For complex systems, energy method is easier.



Vector Force Addition

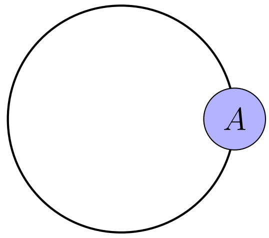
Because of this complexity, we introduce electric potential (a scalar quantity).

Conservative Nature of Electrostatic Force

Electrostatic force is conservative:

$$\oint \vec{F} \cdot d\vec{r} = 0$$

This means work depends only on initial and final positions.



Work in Closed Loop = 0

Work Done in Moving a Charge

If a charge q_0 is moved in electric field:

$$dW = \vec{F} \cdot d\vec{r}$$

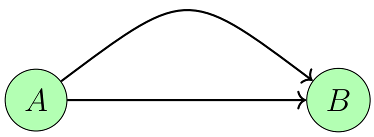
Since:

$$\vec{F} = q_0\vec{E}$$

$$W = \int_A^B q_0\vec{E} \cdot d\vec{r}$$

Path Independence of Work

$W_{A \to B}$ is independent of path



Same Work for All Paths

1.2 Conservative Field

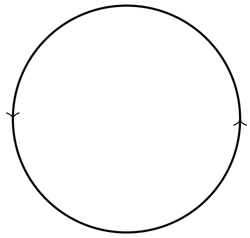
Definition

A field is conservative if:

$$\oint \vec{F} \cdot d\vec{r} = 0$$

Work Done in Closed Loop

$$W_{\text{closed loop}} = 0$$



Net Work = 0

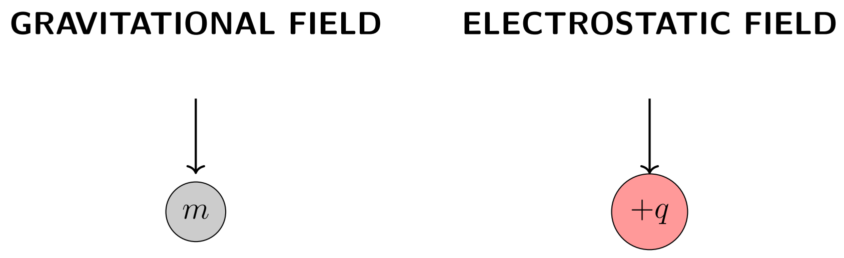
Electrostatic Field as Conservative Field

Electrostatic field due to stationary charges is conservative because:

- Path independent work
- Closed loop work zero
- Scalar potential exists

$$\vec{E} = -\nabla V$$

Comparison with Gravitational Field



Similarities:

- Both are conservative.
- Work is path independent.
- Potential energy can be defined.

Difference:

- Gravity is always attractive.
- Electrostatic force can be attractive or repulsive.

Conceptual Summary

- Electrostatic force is conservative.
- Work done is path independent.
- Closed loop work = 0.
- Conservative nature allows definition of potential.

2 Electrostatic Potential

2.1 Electric Potential (Definition)

Electric potential at a point is defined as the work done per unit positive test charge in bringing it from infinity to that point without acceleration.

$$V = \frac{W}{q}$$

where W = work done by external agent
q = magnitude of test charge

Nature of Electric Potential

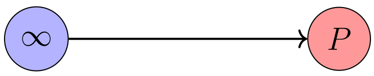
- Scalar quantity
- Has magnitude only (no direction)
- Can be positive or negative

SI Unit of Potential

SI unit of electric potential is **Volt (V)**.

$$1 \text{ Volt} = 1 \frac{\text{Joule}}{\text{Coulomb}}$$

Definition of 1 Volt:
If 1 Joule of work is required to move 1 Coulomb of charge, the potential difference is 1 Volt.



Work per Unit Charge

2.2 Electric Potential Due to a Point Charge

Consider a point charge q.

Work Done in Bringing Charge from Infinity

Force on test charge q_0:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$

Work done in bringing it from infinity to distance r:

$$W = \int_{\infty}^{r} F\,dr$$

After integration:

$$W = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$

Dividing by q_0:

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

Reference Point

Potential at infinity is taken as zero:

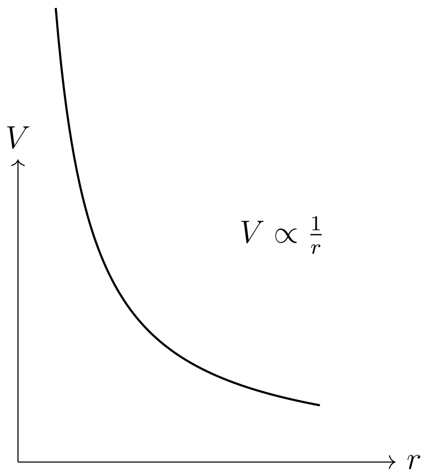
$$V(\infty) = 0$$

Sign of Potential

- For $+q$, potential is positive.
- For $-q$, potential is negative.

Graph of V vs r

$$V \propto \frac{1}{r}$$



2.3 Electric Potential Due to System of Charges

Superposition Principle

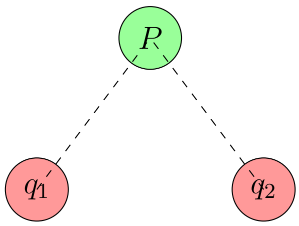
Total potential at a point due to many charges is the algebraic sum of individual potentials.

Since potential is scalar, we add algebraically.

$$V = V_1 + V_2 + V_3 + \dots$$

For multiple discrete charges:

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \sum \frac{q_i}{r_i}}$$



2.4 Potential Due to Electric Dipole

Consider a dipole with charges $+q$ and $-q$ separated by distance $2a$.
Dipole moment:

$$\vec{p} = q(2a)$$

(a) On Axial Line

Exact expression:

$$V = \frac{1}{4\pi\varepsilon_0} \left(\frac{q}{r-a} - \frac{q}{r+a} \right)$$

For large distance $r \gg a$:

$$\boxed{V \approx \frac{1}{4\pi\varepsilon_0} \frac{p}{r^2}}$$

(b) On Equatorial Line

Exact derivation:

$$V = \frac{1}{4\pi\varepsilon_0} \left(\frac{q}{\sqrt{r^2+a^2}} - \frac{q}{\sqrt{r^2+a^2}} \right)$$

$$\boxed{V = 0}$$

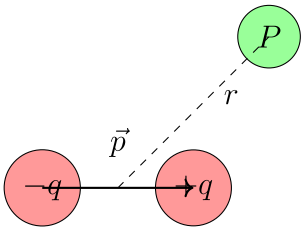
Direction: No net potential on equatorial line (for ideal dipole).

(c) At a General Point

For large distance:

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}}$$

where $\hat{r}$ is unit vector along position vector.



Comparison: Field vs Potential Dependence

- For point charge:

$$E \propto \frac{1}{r^2}, \quad V \propto \frac{1}{r}$$

- For dipole:

$$E \propto \frac{1}{r^3}, \quad V \propto \frac{1}{r^2}$$

Important Summary

- $V = \frac{W}{q}$
- $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$
- Add potentials algebraically.
- Dipole potential decreases faster than point charge.

3 Equipotential Surfaces

3.1 Definition

An equipotential surface is a surface on which electric potential has the same value at every point.

$$V = \text{constant}$$

This means that if a test charge moves from one point to another on the same surface, the potential difference is zero.

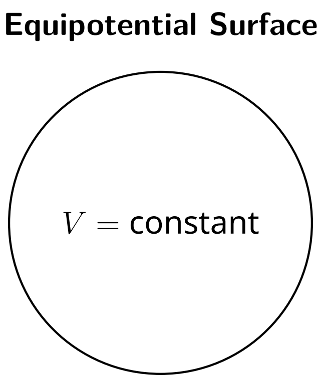
$$\Delta V = 0$$

Since work done is:

$$W = q\Delta V$$

$$W = 0$$

Thus, no work is required to move a charge along an equipotential surface.



3.2 Properties of Equipotential Surfaces

1. No Work Done Along Equipotential Surface

If a charge moves between two points A and B on the same surface:

$$V_A = V_B$$

$$W = q(V_B - V_A) = 0$$

Hence, motion along equipotential surface does not require energy.
This shows that electric force has no component along the surface.

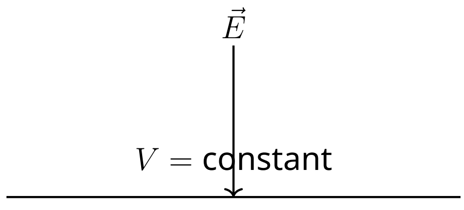
2. Electric Field is Perpendicular to Equipotential Surface

Electric field is related to potential as:

$$\vec{E} = -\nabla V$$

The electric field points in the direction of maximum decrease of potential.

If electric field had a component along the surface, then work would be done while moving along the surface. Since work is zero, electric field must be perpendicular.



3. Equipotential Surfaces Never Intersect

If two equipotential surfaces intersected:

- The point of intersection would have two different potentials.
- A point cannot have two different potential values.

Hence, equipotential surfaces never intersect.

4. Closer Surfaces Indicate Stronger Electric Field

Electric field magnitude:

$$E = \left| \frac{dV}{dr} \right|$$

If equipotential surfaces are very close:

- Potential changes rapidly.
- Electric field is strong.

If surfaces are far apart:

- Potential changes slowly.
- Electric field is weak.

3.3 Equipotential Surfaces for Different Charge Configurations

1. Single Point Charge

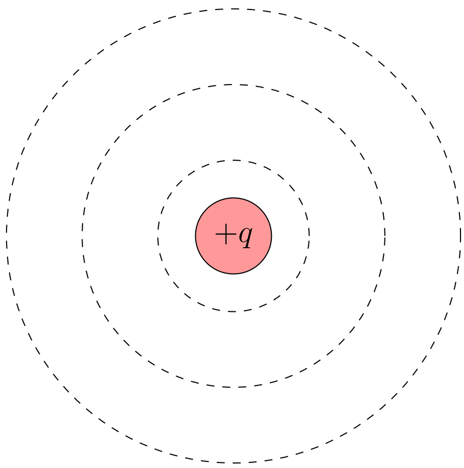
Potential due to a point charge:

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

Since potential depends only on distance r, all points at same distance form a sphere.

Therefore, equipotential surfaces are concentric spheres centered at the charge.

Concentric Spherical Surfaces



Field lines are radial and perpendicular to spherical surfaces.

2. Uniform Electric Field

In a uniform electric field:

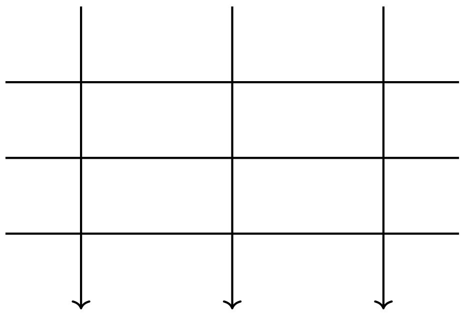
$$E = \text{constant}$$

Potential changes linearly with distance:

$$V = -Ex$$

Equipotential surfaces are parallel planes perpendicular to electric field lines.

Parallel Equipotential Planes



Spacing between planes is uniform because electric field is constant.

3. Electric Dipole

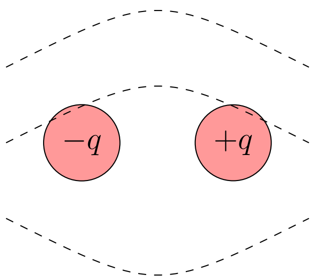
For a dipole consisting of charges $+q$ and $-q$ separated by distance $2a$:
Equipotential surfaces are more complex.

- Symmetric about dipole axis.
- Not spherical.
- Zero potential surface lies on equatorial plane.

Potential at a general point:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

Dipole Equipotential Pattern



Complete Concept Summary

- Equipotential surface $\Rightarrow$ constant potential.
- No work done along surface.
- Electric field is perpendicular to surface.
- Equipotential surfaces never intersect.
- Closer surfaces $\Rightarrow$ stronger electric field.
- Shape depends on charge distribution.

4 Relation Between Electric Field and Potential

4.1 Derivation of the Relation

Consider a small displacement dr of a test charge q in an electric field $\vec{E}$.

Work done by electric force:

$$dW = \vec{F} \cdot d\vec{r}$$

Since,

$$\vec{F} = q\vec{E}$$

$$dW = q\vec{E} \cdot d\vec{r}$$

By definition of potential difference:

$$dV = -\frac{dW}{q}$$

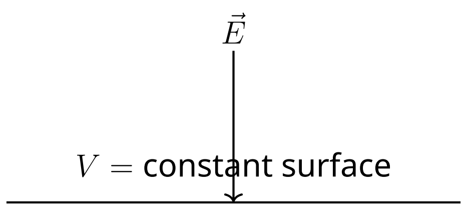
Substituting:

$$dV = -\vec{E} \cdot d\vec{r}$$

For motion along radial direction:

$$\boxed{E = -\frac{dV}{dr}}$$

This is the fundamental relation between electric field and potential.



4.2 Physical Meaning of Negative Sign

The negative sign indicates:

- Electric field points in direction of decreasing potential.
- Potential decreases in the direction of electric field.

If potential decreases rapidly, electric field is strong.
Example:
If potential decreases as we move to the right, electric field points toward right.

$$\vec{E} \rightarrow \text{from higher } V \text{ to lower } V$$

4.3 Potential Gradient

Potential gradient is defined as rate of change of potential with respect to distance.

$$\text{Potential Gradient} = \frac{dV}{dr}$$

Electric field is negative of potential gradient:

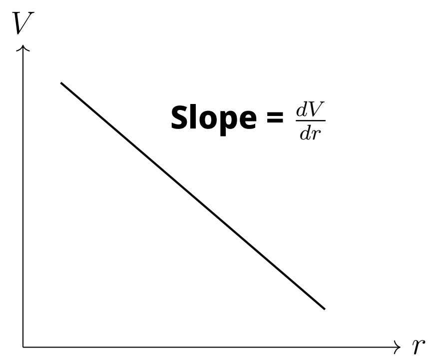
$$E = -\frac{dV}{dr}$$

Large gradient $\Rightarrow$ strong electric field.
Small gradient $\Rightarrow$ weak electric field.

4.4 Electric Field from Potential Graph

If graph of V vs r is given:

$$E = -\text{slope of } V\text{-}r \text{ graph}$$



Important Cases:

- Steep slope $\Rightarrow$ strong electric field.
- Gentle slope $\Rightarrow$ weak electric field.

- Zero slope $\Rightarrow$ zero electric field.

If graph is horizontal:

$$\frac{dV}{dr} = 0$$

$$E = 0$$

This means constant potential region has no electric field.

4.5 Integral Form of Relation

For finite displacement from point A to B:

$$dV = -\vec{E} \cdot d\vec{l}$$

Integrating from A to B:

$$\boxed{V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l}}$$

This is the general relation between potential difference and electric field.

- Valid for any path in electrostatic field.
- For conservative field, integral depends only on endpoints.

Complete Summary

- $E = -\frac{dV}{dr}$
- Electric field is negative gradient of potential.
- Field points toward decreasing potential.
- Slope of V–r graph gives electric field.
- $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{l}$

5 PART 2: Potential Energy

5.1 Potential Energy of System of Charges

Electrostatic potential energy of a system is defined as the total work done by an external agent in assembling the charges from infinity to their given positions without acceleration.

Since electrostatic force is conservative:

$$\text{Work done by external agent} = \text{Increase in Potential Energy}$$

5.2 Two Charge System

Consider two point charges q_1 and q_2 separated by distance r.

Step 1: Bringing First Charge

When q_1 is brought from infinity, no work is required because no other charge is present.

$$U_1 = 0$$

Step 2: Bringing Second Charge

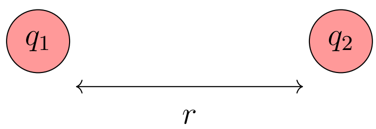
Now bring q_2 to distance r from q_1.
Potential due to q_1 at that point:

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r}$$

Work done in bringing q_2:

$$U = q_2 V$$

$$\boxed{U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}}$$



Important Observations:

- Like charges $\Rightarrow U > 0$
- Unlike charges $\Rightarrow U < 0$
- Depends only on separation r

5.3 Three Charge System

Consider three charges q_1, q_2, q_3 placed at distances r_{12}, r_{23} and r_{31}.

Work Done in Assembling

Step 1: Bring $q_1 \Rightarrow$ No work.
Step 2: Bring $q_2 \Rightarrow$ Work done:

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

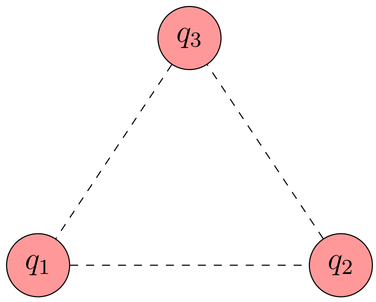
Step 3: Bring $q_3 \Rightarrow$ It interacts with both q_1 and q_2.

$$U_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}}$$

$$U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

Total energy:

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right)}$$



Each pair contributes independently to total potential energy.

5.4 Multiple Charge System

For a system of n charges:

$$\boxed{U = \sum_{i<j} \frac{1}{4\pi\varepsilon_0}\frac{q_i q_j}{r_{ij}}}$$

where:

- q_i, q_j are charges
- r_{ij} is distance between them

Number of Interaction Terms:

$$\frac{n(n-1)}{2}$$

Key Properties:

- Energy is scalar.

- Add algebraically.
- Depends only on relative positions.

Complete Summary

- Two charges: $U = \frac{kq_1q_2}{r}$
- Three charges: Sum of all pair energies
- Multiple charges: $U = \sum \frac{kq_iq_j}{r_{ij}}$
- Electrostatic potential energy equals work done in assembling charges.

6 Potential Energy in External Field

6.1 Single Charge in External Field

Consider a charge q placed in an external electric field where electric potential at that point is V.

By definition of potential:

$$V = \frac{W}{q}$$

Hence, potential energy of charge:

$$\boxed{U = qV}$$

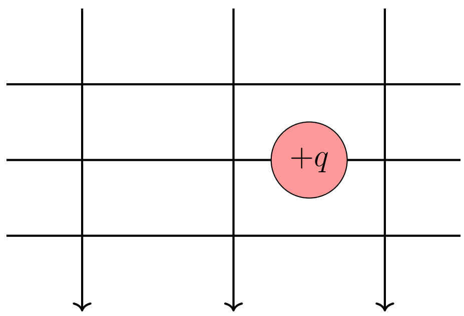
Physical Meaning

- If $V > 0$ and $q > 0$, then $U > 0$
- If $V > 0$ and $q < 0$, then $U < 0$

If a charge moves from point A to B:

$$\Delta U = q(V_B - V_A)$$

Charge in Uniform Electric Field



If the charge moves in direction of electric field:

- Potential decreases
- Potential energy decreases

6.2 Dipole in Uniform Electric Field

Consider a dipole of charges $+q$ and $-q$ separated by distance $2a$.
Dipole moment:

$$\vec{p} = q(2a)$$

Let dipole make angle θ with electric field $\vec{E}$.

(a) Torque on Dipole

Each charge experiences force:

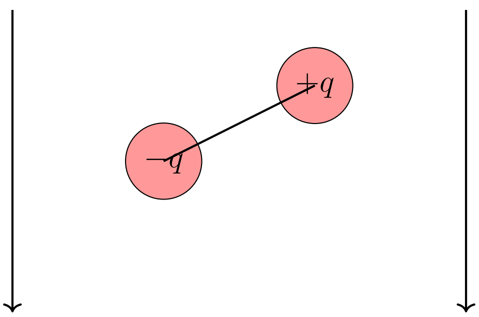
$$F = qE$$

Forces are equal and opposite but act at different points, producing torque.

$$\boxed{\tau = pE\sin\theta}$$

Torque tries to align dipole along electric field.

Dipole in Uniform Field



(b) Work Done in Rotation

If dipole rotates by small angle $d\theta$:

$$dW = \tau\, d\theta$$

$$dW = pE\sin\theta\, d\theta$$

Integrating from θ_1 to θ_2:

$$W = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$$

$$W = pE(\cos\theta_1 - \cos\theta_2)$$

(c) Potential Energy of Dipole

Potential energy is negative of work done by electric field.

$$\boxed{U = -\vec{p} \cdot \vec{E}}$$

$$\boxed{U = -pE \cos\theta}$$

This shows:

- Energy depends on orientation.
- Minimum when dipole aligns with field.

(d) Stable and Unstable Equilibrium

Case 1: $\theta = 0^\circ$

$$U_{\text{min}} = -pE$$

- Dipole parallel to field
- Stable equilibrium

Case 2: $\theta = 180^\circ$

$$U_{\text{max}} = +pE$$

- Dipole anti-parallel to field
- Unstable equilibrium

Case 3: $\theta = 90^\circ$

$$U = 0$$

Intermediate condition.

Complete Summary

- Single charge: $U = qV$
- Torque on dipole: $\tau = pE \sin\theta$
- Potential energy of dipole: $U = -\vec{p} \cdot \vec{E}$
- Stable equilibrium $\Rightarrow$ minimum energy
- Unstable equilibrium $\Rightarrow$ maximum energy

7 Electrostatics of Conductors

7.1 Conductors in Electrostatic Equilibrium

A conductor is a material that contains free charge carriers (usually electrons) that can move throughout the material.

When a conductor is placed in an electric field, charges rearrange themselves until electrostatic equilibrium is reached.

7.2 Free Charges in Conductor

Drift of Electrons

In a conductor:

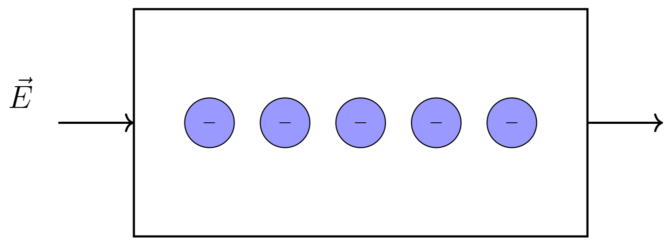
- Large number of free electrons are present.
- These electrons move randomly in absence of field.

When external electric field is applied:

- Electrons experience force $F = -eE$
- They drift opposite to direction of electric field.

This redistribution continues until internal electric field cancels the applied field.

Drift of Free Electrons



7.3 Conditions of Electrostatic Equilibrium

When conductor reaches electrostatic equilibrium, the following conditions are satisfied:

1. Electric Field Inside Conductor = 0

If electric field existed inside:

- Charges would continue to move.
- Equilibrium would not be achieved.

Hence,

$$\boxed{E_{\text{inside}} = 0}$$

2. Net Charge Resides on Surface

Since electric field inside is zero:

- Excess charges move to outer surface.
- No excess charge remains inside.

$$\text{Charge exists only on surface}$$

3. Electric Field Just Outside Conductor

Using Gauss's law:

$$E = \frac{\sigma}{\varepsilon_0}$$

where σ = surface charge density.
Field is normal (perpendicular) to surface.

4. Surface of Conductor is Equipotential

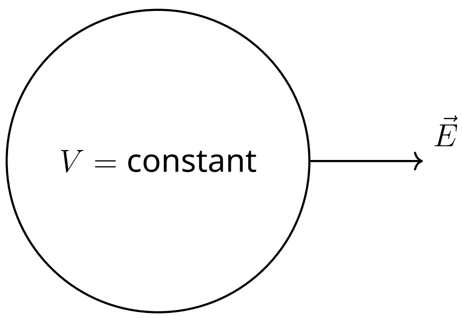
Since electric field inside is zero:

$$\frac{dV}{dr} = 0$$

Potential is constant throughout conductor.
Hence:

$$\text{Conductor surface is equipotential}$$

Field Perpendicular to Surface



7.4 Proof Using Gauss's Law

Apply Gauss's law inside conductor.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

Since $E_{\text{inside}} = 0$:

$$\oint \vec{E} \cdot d\vec{A} = 0$$

Therefore:

$$Q_{\text{enclosed}} = 0$$

This proves:

- No excess charge inside conductor.
- All charge resides on surface.

7.5 Electrostatic Shielding

Electrostatic shielding is the phenomenon in which the electric field inside a conductor is zero, even if external electric field is applied.

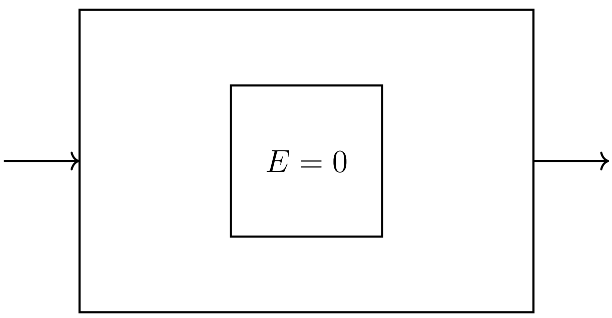
Faraday Cage Principle

If a hollow conductor is placed in an external electric field:

- Charges rearrange on outer surface.
- Field inside cavity becomes zero.

$$E_{\text{inside cavity}} = 0$$

Faraday Cage



Applications

- Protection of sensitive electronic instruments.
- Lightning protection systems.
- Shielding rooms for experiments.
- Microwave ovens (metal enclosure).

Complete Summary

- Free electrons drift under applied field.
- At equilibrium: $E_{\text{inside}} = 0$.
- Excess charge resides on surface.
- Surface is equipotential.
- Electric field just outside = σ/ε_0.
- Faraday cage provides electrostatic shielding.

8 PADielectrics and Polarisation

8.1 Dielectrics

A dielectric is an insulating material that does not allow free movement of charges but can be polarized when placed in an electric field.

Examples: Glass, mica, plastic, rubber, water.

8.2 Types of Molecules

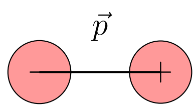
1. Polar Molecules

In polar molecules:

- Centres of positive and negative charges do not coincide.
- Molecule has permanent dipole moment.

Example: H_2O, NH_3

Polar Molecule



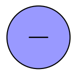
2. Non-Polar Molecules

In non-polar molecules:

- Centres of positive and negative charges coincide.
- No permanent dipole moment.

Example: O_2, N_2, CO_2

Non-Polar Molecule



8.3 Polarisation

Polarisation is the process of inducing dipole moment in dielectric material when placed in an electric field.

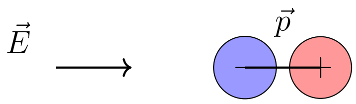
Induced Dipole Moment

In non-polar molecule:

- External electric field slightly shifts positive and negative charges.
- Induced dipole moment is created.

$$\vec{p}_{\text{induced}} \propto \vec{E}$$

Induced Dipole



Alignment of Dipoles

In polar molecules:

- Dipoles are randomly oriented without field.
- In external field, dipoles align partially along field.

8.4 Bound Charges

Polarisation causes appearance of bound charges.

Surface Bound Charge

Due to alignment of dipoles:

- Negative bound charge appears on one surface.
- Positive bound charge appears on opposite surface.

$$\sigma_b = \vec{P} \cdot \hat{n}$$

Volume Bound Charge (Conceptual)

If polarisation varies inside material:

$$\rho_b = -\nabla \cdot \vec{P}$$

8.5 Electric Field Inside Dielectric

When dielectric is placed in electric field:

- Induced field opposes external field.

Net field:

$$E = \frac{E_0}{K}$$

where K = dielectric constant.
Field inside dielectric is reduced.

8.6 Dielectric Constant (Relative Permittivity)

Dielectric constant is defined as:

$$\boxed{K = \frac{\varepsilon}{\varepsilon_0}}$$

where:

- ε = permittivity of medium
- ε_0 = permittivity of free space

Also,

$$K = \frac{E_0}{E}$$

8.7 Effect of Dielectric

1. Capacitance

$$C = KC_0$$

Capacitance increases by factor K.

2. Electric Field

$$E = \frac{E_0}{K}$$

Field decreases.

3. Potential

$$V = \frac{V_0}{K}$$

Potential decreases (if charge constant).

4. Energy

If charge constant:

$$U = \frac{U_0}{K}$$

If voltage constant:

$$U = KU_0$$

8.8 Dielectric Strength

Dielectric strength is the maximum electric field a dielectric can withstand without breakdown.

$$E_{\text{max}} = \text{Dielectric Strength}$$

Beyond this value:

- Insulator becomes conductor.
- Breakdown occurs.

Example:

- Air □ 3×10^6 V/m
- Mica □ very high dielectric strength

Complete Summary

- Dielectrics get polarized in electric field.
- Polar molecules have permanent dipole.
- Non-polar molecules form induced dipoles.
- Dielectric reduces electric field.
- $K = \varepsilon/\varepsilon_0$.
- Capacitance increases with dielectric.
- Dielectric strength determines breakdown limit.

9 Capacitors and Capacitance

9.1 Capacitance

A capacitor is a device used to store electric charge and electrical energy.
Capacitance is the ability of a conductor or system of conductors to store charge.

9.2 Definition

Capacitance is defined as the ratio of charge stored to the potential developed.

$$\boxed{C = \frac{Q}{V}}$$

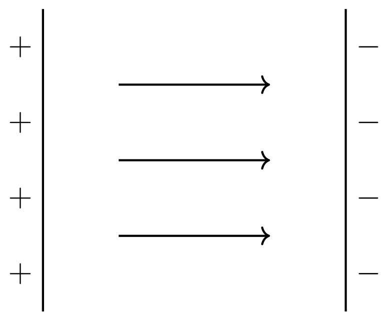
where:

- Q = charge stored
- V = potential difference

Important:

- Capacitance depends only on geometry and medium.
- It does not depend on Q or V separately.

Parallel Plate Capacitor



9.3 SI Unit of Capacitance

SI unit of capacitance is **Farad (F)**.
From definition:

$$C = \frac{Q}{V}$$

$$1 \text{ Farad} = 1 \frac{\text{Coulomb}}{\text{Volt}}$$

Practical capacitors are usually in:

- Microfarad (μF)
- Nanofarad (nF)
- Picofarad (pF)

9.4 Dimensions of Capacitance

From:

$$C = \frac{Q}{V}$$

Charge dimension:

$$[Q] = [IT]$$

Potential dimension:

$$[V] = \frac{[W]}{[Q]} = \frac{[ML^2T^{-2}]}{[IT]}$$

Therefore:

$$[C] = \frac{[IT]}{[ML^2T^{-2}I^{-1}]}$$

$$\boxed{[C] = M^{-1}L^{-2}T^4I^2}$$

9.5 Capacitance of an Isolated Spherical Conductor

Consider an isolated conducting sphere of radius R.
Potential of sphere:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

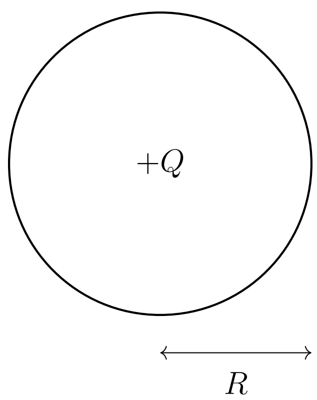
Using:

$$C = \frac{Q}{V}$$

$$C = \frac{Q}{\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}}$$

$$\boxed{C = 4\pi\varepsilon_0 R}$$

Isolated Conducting Sphere



Important Observations:

- Capacitance depends only on radius.
- Larger sphere $\Rightarrow$ larger capacitance.
- Independent of charge stored.

Complete Summary

- $C = Q/V$
- SI unit = Farad
- Dimension = $M^{-1}L^{-2}T^4I^2$
- Sphere: $C = 4\pi\varepsilon_0 R$
- Capacitance depends on geometry and medium only.

10 Parallel Plate Capacitor

10.1 Derivation Without Dielectric

Consider two large parallel conducting plates of area A separated by distance d in vacuum.

Surface charge density:

$$\sigma = \frac{Q}{A}$$

Electric field between plates:

$$E = \frac{\sigma}{\varepsilon_0}$$

$$E = \frac{Q}{\varepsilon_0 A}$$

Potential difference between plates:

$$V = Ed$$

$$V = \frac{Q}{\varepsilon_0 A}d$$

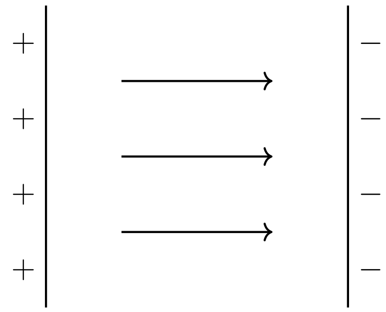
Using definition:

$$C = \frac{Q}{V}$$

$$C = \frac{Q}{\frac{Qd}{\varepsilon_0 A}}$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

Parallel Plate Capacitor (Vacuum)



10.2 With Dielectric Slab (Fully Filled)

If dielectric of permittivity ε is inserted fully:
Electric field reduces:

$$E = \frac{E_0}{K}$$

New capacitance:

$$\boxed{C = \frac{\varepsilon A}{d}}$$

where:

$$\varepsilon = K\varepsilon_0$$

Capacitance increases by factor K.

$$C = KC_0$$

10.3 Partially Filled Dielectric

Case 1: Dielectric fills part of area
System behaves like capacitors in parallel.

$$C = C_1 + C_2$$

Case 2: Dielectric fills part of thickness
System behaves like capacitors in series.

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

If dielectric slab thickness = t:

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

10.4 Multiple Dielectric Slabs

If multiple slabs of thickness d_1, d_2, d_3 and permittivities $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are inserted: They behave like capacitors in series.

$$\frac{1}{C} = \frac{d_1}{\varepsilon_1 A} + \frac{d_2}{\varepsilon_2 A} + \frac{d_3}{\varepsilon_3 A}$$

General form:

$$\boxed{\frac{1}{C} = \sum \frac{d_i}{\varepsilon_i A}}$$

10.5 Effect of Dielectric

Consider two cases:

Case 1: Battery Connected

Voltage remains constant.

$$V = \text{constant}$$

Capacitance increases:

$$C = KC_0$$

Charge becomes:

$$Q = CV$$

$$Q = KQ_0$$

Energy:

$$U = \frac{1}{2}CV^2$$

$$U = KU_0$$

Electric field decreases:

$$E = \frac{E_0}{K}$$

Case 2: Battery Disconnected

Charge remains constant.

$$Q = \text{constant}$$

Capacitance increases:

$$C = KC_0$$

Potential:

$$V = \frac{Q}{C}$$

$$V = \frac{V_0}{K}$$

Energy:

$$U = \frac{Q^2}{2C}$$

$$U = \frac{U_0}{K}$$

Electric field:

$$E = \frac{E_0}{K}$$

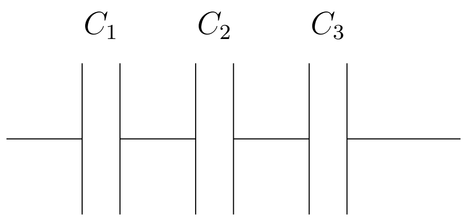
Complete Comparison Table

- Battery Connected:
  - V constant
  - Q increases
  - U increases
  - E decreases
- Battery Disconnected:
  - Q constant
  - V decreases
  - U decreases
  - E decreases

11 Combination of Capacitors

11.1 Series Combination

Consider capacitors C_1, C_2, C_3 connected in series.



Same Charge Concept

In series:

- Same charge Q flows through each capacitor.
- Charge on each capacitor is equal.

$$Q_1 = Q_2 = Q_3 = Q$$

Total potential difference:

$$V = V_1 + V_2 + V_3$$

Using $V = \frac{Q}{C}$:

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

Let equivalent capacitance be C_{eq}:

$$V = \frac{Q}{C_{\text{eq}}}$$

Comparing:

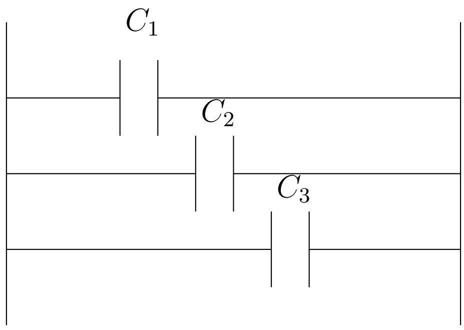
$$\boxed{\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Important Points

- Equivalent capacitance is less than smallest capacitor.
- Voltage divides across capacitors.

11.2 Parallel Combination

Consider capacitors C_1, C_2, C_3 connected in parallel.



Same Potential Concept

In parallel:

- Potential difference across each capacitor is same.

$$V_1 = V_2 = V_3 = V$$

Total charge:

$$Q = Q_1 + Q_2 + Q_3$$

Using $Q = CV$:

$$Q = C_1V + C_2V + C_3V$$

$$Q = V(C_1 + C_2 + C_3)$$

Let equivalent capacitance be C_{eq}:

$$Q = C_{\text{eq}}V$$

$$\boxed{C_{\text{eq}} = C_1 + C_2 + C_3}$$

Important Points

- Equivalent capacitance is greater than largest capacitor.
- Charge divides among capacitors.

Series vs Parallel Summary

- Series:
$$\frac{1}{C_{\text{eq}}} = \sum \frac{1}{C_i}$$
Same charge, different voltage.
- Parallel:
$$C_{\text{eq}} = \sum C_i$$
Same voltage, different charge.

12 Energy Stored in Capacitor

12.1 Work Done in Charging a Capacitor

Consider a capacitor being charged gradually from 0 to final charge Q.
At any instant, let charge on capacitor be q.
Potential at that instant:

$$V = \frac{q}{C}$$

Small work done in bringing small charge dq:

$$dW = V\,dq$$

$$dW = \frac{q}{C}dq$$

Total work done in charging from 0 to Q:

$$W = \int_0^Q \frac{q}{C}dq$$

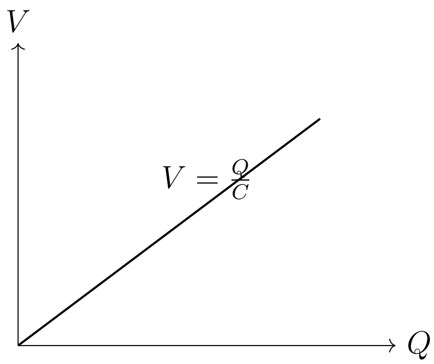
$$W = \frac{1}{C}\int_0^Q q\,dq$$

$$W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$W = \frac{Q^2}{2C}$$

This work is stored as electrostatic energy in the capacitor.
Using $Q = CV$:

$$\boxed{U = \frac{1}{2}CV^2}$$



Energy equals area under V–Q graph.

12.2 Alternative Forms of Energy

Using different relations:

1. Using $U = \frac{1}{2}CV^2$ **and** $Q = CV$

$$\boxed{U = \frac{1}{2}QV}$$

2. Using $V = \frac{Q}{C}$

$$\boxed{U = \frac{Q^2}{2C}}$$

Thus, energy stored in capacitor can be written in three equivalent forms:

$$U = \frac{1}{2}CV^2$$

$$U = \frac{1}{2}QV$$

$$U = \frac{Q^2}{2C}$$

12.3 Energy Density of Electric Field

Energy density is energy stored per unit volume.

$$u = \frac{U}{\text{Volume}}$$

For parallel plate capacitor:

$$U = \frac{1}{2}CV^2$$

Using:

$$C = \frac{\varepsilon_0 A}{d}$$

and

$$V = Ed$$

Substitute:

$$U = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(E^2 d^2)$$

$$U = \frac{1}{2}\varepsilon_0 A d E^2$$

Volume between plates:

$$\text{Volume} = Ad$$

Therefore,

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

For dielectric medium:

$$u = \frac{1}{2}\varepsilon E^2$$

12.4 Energy Stored in Electric Field

Important conclusion:

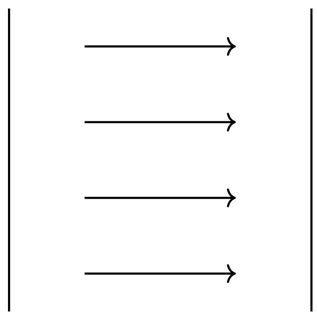
- Energy is not stored on plates.
- Energy is stored in electric field between plates.

Energy density exists wherever electric field exists.

$$U = \int \frac{1}{2}\varepsilon_0 E^2 \, dV$$

Thus, total electrostatic energy is energy of electric field in space.

Energy Stored in Electric Field



Complete Summary

- $U = \frac{1}{2}CV^2$
- $U = \frac{1}{2}QV$
- $U = \frac{Q^2}{2C}$
- Energy density: $u = \frac{1}{2}\varepsilon_0 E^2$
- Energy is stored in electric field.

13 Van de Graaff Generator

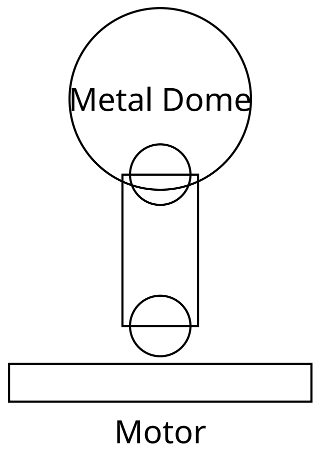
13.1 Construction

Van de Graaff generator is an electrostatic machine used to generate very high potential (millions of volts).

Main parts:

- Large hollow metallic spherical dome
- Insulating belt (rubber or silk)
- Two rollers (lower and upper)
- Motor to move the belt
- Comb-shaped metal brushes (pointed electrodes)

Van de Graaff Generator



13.2 Working Principle

The working principle is based on:

- Electrostatic induction
- Action of sharp points (charge concentration)
- Transfer of charge by moving belt

Sharp combs create strong electric field which helps transfer charge to the belt. The moving belt carries charge to the upper dome.

13.3 Charge Accumulation Mechanism

Step 1: Charging the Belt

Lower comb sprays charge onto moving insulating belt due to high electric field at sharp points.

Step 2: Charge Transport

Belt moves upward carrying charge.

Step 3: Transfer to Dome

Upper comb collects charge from belt and transfers it to metallic dome.
Since dome is hollow conductor:

- Charge spreads over outer surface.
- Electric field inside dome is zero.

Step 4: Continuous Accumulation

As belt keeps moving:

- More charge accumulates.
- Potential of dome increases.

Potential of spherical dome:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

Large radius R allows storage of large charge before breakdown.
Eventually, air breaks down and spark is produced when:

$$E_{\text{air}} \approx 3 \times 10^6 \text{ V/m}$$

13.4 Applications

- Generation of very high voltages
- Particle acceleration in nuclear physics
- X-ray production
- Demonstration of electrostatic principles
- Research in atomic and nuclear experiments

Complete Summary

- Works on electrostatic induction and charge transfer.
- Moving belt transports charge to dome.
- Charge accumulates on outer surface.
- Generates extremely high potential.
- Used in particle accelerators and research labs.