NCERT SOLUTIONS

Class 12 Physics

Chapter 7: Alternating Current

Detailed Step-by-Step Exercise Solutions

Q1 A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply.

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

Solution

(a) RMS Value of Current:
In an AC circuit containing only a resistor, the relationship between rms voltage and rms current follows Ohm's law, just as in DC circuits. The rms (root mean square) values represent the effective values of the alternating quantities.
The formula relating rms current to rms voltage is:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$$

Here, we are given that $V_{\text{rms}} = 220$ V and $R = 100\ \Omega$. Substituting these values:

$$I_{\text{rms}} = \frac{220}{100} = 2.2\text{ A}$$

Therefore, the rms value of the current in the circuit is 2.2 A.

(b) Net Power Consumed:
For a purely resistive AC circuit, the voltage and current are always in phase (i.e., they reach their maximum and minimum values at the same time). Because of this, the power factor $\cos\phi = 1$, and all the electrical energy supplied is dissipated as heat.

The net power consumed over a full cycle is given by:

$$P = V_{\text{rms}} \times I_{\text{rms}} = 220 \times 2.2 = 484 \text{ W}$$

We can also verify this using the alternative formula:

$$P = I_{\text{rms}}^2 \times R = (2.2)^2 \times 100 = 4.84 \times 100 = 484 \text{ W}$$

Therefore, the net power consumed over a full cycle is 484 W.

Expert's Solution — Nikunj Singh, B.Tech CSE, IIT Delhi

Using Peak Values and Voltage Formula:
Instead of solving directly using rms values, we can first find the peak alternating values and then derive the results for both parts.
For part (a): The peak voltage V_0 is given by $V_{\text{rms}} \times \sqrt{2} = 220\sqrt{2}$ V. The peak current I_0 is $V_0/R = 220\sqrt{2}/100 = 2.2\sqrt{2}$ A. The rms current is then:

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{2.2\sqrt{2}}{\sqrt{2}} = 2.2 \text{ A}$$

For part (b): Power can also be calculated directly from the voltage without needing the current:

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{(220)^2}{100} = \frac{48400}{100} = 484 \text{ W}$$

Did You Know?

In a purely resistive circuit, the power factor $\cos\phi = 1$, which means *all* the electrical energy is converted to heat. This is why electric heaters and incandescent bulbs use resistors!

Q2

(a) The peak voltage of an ac supply is 300 V. What is the rms voltage?

(b) The rms value of current in an ac circuit is 10 A. What is the peak current?

Solution

(a) Finding the RMS Voltage:
The rms voltage and peak voltage of an AC supply are related by the factor $\sqrt{2}$. This factor comes from averaging the square of a sinusoidal waveform over one complete cycle. The relationship is:

$$V_{\text{rms}} = \frac{V_{\text{peak}}}{\sqrt{2}}$$

Given that $V_{\text{peak}} = 300$ V, we substitute:

$$V_{\text{rms}} = \frac{300}{\sqrt{2}} = \frac{300}{1.414} \approx 212.13 \text{ V}$$

Therefore, the rms voltage is approximately 212.13 V.

(b) Finding the Peak Current:
Similarly, the peak value of the current can be obtained from the rms value using the inverse relationship:

$$I_{\text{peak}} = I_{\text{rms}} \times \sqrt{2}$$

Given that $I_{\text{rms}} = 10$ A:

$$I_{\text{peak}} = 10 \times \sqrt{2} = 10 \times 1.414 \approx 14.14 \text{ A}$$

Therefore, the peak current is approximately 14.14 A.

Expert's Solution — Anita Desai, B.Tech CSE, BITS Pilani

Derivation from First Principles:
Rather than just memorizing the $\sqrt{2}$ factor, let's derive the rms voltage from the instantaneous formula $V(t) = V_{\text{peak}} \sin(\omega t)$.
For part (a): The mean square voltage over one cycle T is:

$$\langle V^2 \rangle = \frac{1}{T} \int_0^T V_{\text{peak}}^2 \sin^2(\omega t) dt = \frac{V_{\text{peak}}^2}{2}$$

Taking the square root (root-mean-square) gives $V_{\text{rms}} = V_{\text{peak}}/\sqrt{2}$. Therefore, $V_{\text{rms}} = 300/\sqrt{2} = 300 \times 0.707 \approx 212.13$ V.
For part (b): Using the exact same integration principle for current, $I_{\text{rms}} = I_{\text{peak}}/\sqrt{2}$. Rearranging this: $I_{\text{peak}} = I_{\text{rms}}\sqrt{2} = 10 \times 1.414 \approx 14.14$ A.

Did You Know?

The rms value represents the "equivalent DC value" — a 212 V rms AC supply delivers the *same heating effect* in a resistor as a 212 V DC battery!

Q3 **A 44 mH inductor is connected to a 220 V, 50 Hz AC supply. Determine the rms value of the current in the circuit.**

Solution

Step 1: Calculate the inductive reactance.
When an inductor is connected to an AC supply, it opposes the flow of alternating current. This opposition is called inductive reactance (X_L), and it depends on both the frequency of the supply and the inductance of the coil:

$$X_L = 2\pi f L$$

Given: $f = 50$ Hz, $L = 44$ mH $= 44 \times 10^{-3}$ H
Substituting:

$$X_L = 2\pi \times 50 \times 44 \times 10^{-3} = 2\pi \times 2.2 = 13.82\ \Omega$$

Step 2: Find the rms current.
The rms current through the inductor is given by:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{220}{13.82} \approx 15.92\ \text{A}$$

Note that in a purely inductive circuit, the current lags the voltage by 90. This phase difference is important when analysing power in such circuits (see Question 7.5).
Therefore, the rms value of the current in the circuit is 15.92 A.

Expert's Solution — Meena Iyer, NIT Trichy

Using Complex Impedance (Phasors):
In advanced circuit analysis, we represent AC quantities as complex numbers. The impedance of an inductor is $Z_L = j\omega L$, where $j = \sqrt{-1}$ and $\omega = 2\pi f$.
Step 1: Express impedance as a complex number

$$Z_L = j(2\pi \times 50 \times 44 \times 10^{-3}) = j(13.82)\ \Omega$$

The magnitude of this impedance is $|Z_L| = 13.82\ \Omega$. The "j" indicates a 90 phase shift.
Step 2: Calculate complex current Taking the voltage as our reference phasor $V = 220\angle 0$:

$$I = \frac{V}{Z_L} = \frac{220}{j13.82} = -j\left(\frac{220}{13.82}\right) = -j(15.92)\ \text{A}$$

The magnitude of the current is 15.92 A, and the $-j$ mathematically proves that the current lags the voltage by exactly 90.

Did You Know?

An ideal inductor is like a "spring" for electrical energy — it stores and releases but never dissipates. The higher the frequency, the more it "resists" (higher X_L), which is why inductors are used in filters.

Q4 **A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.**

Solution

Step 1: Calculate the capacitive reactance.
A capacitor in an AC circuit also opposes the flow of current through what is called capacitive reactance (X_C). Unlike inductive reactance, capacitive reactance *decreases* as frequency increases. The formula is:

$$X_C = \frac{1}{2\pi f C}$$

Given: $f = 60$ Hz, $C = 60\ \mu\text{F} = 60 \times 10^{-6}$ F
Substituting:

$$X_C = \frac{1}{2\pi \times 60 \times 60 \times 10^{-6}} = \frac{1}{2\pi \times 3.6 \times 10^{-3}} \approx 44.21\ \Omega$$

Step 2: Find the rms current.
The rms current through the capacitor is found using:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{110}{44.21} \approx 2.49\ \text{A}$$

In a purely capacitive circuit, the current *leads* the voltage by 90. This is the opposite of an inductive circuit where the current lags.
Therefore, the rms value of the current in the circuit is 2.49 A.

Expert's Solution — Vikram Singh, B.Sc Physics, Delhi University

Using Complex Impedance (Phasors):
Just like with inductors, we can solve capacitive AC circuits using complex numbers. The complex impedance of a capacitor is $Z_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$.
Step 1: Calculate the complex impedance

$$Z_C = \frac{-j}{2\pi \times 60 \times 60 \times 10^{-6}} = -j(44.21)\ \Omega$$

The magnitude is $|Z_C| = 44.21\ \Omega$.
Step 2: Calculate the complex current Letting the voltage be $V = 110\angle 0$:

$$I = \frac{V}{Z_C} = \frac{110}{-j44.21} = j\left(\frac{110}{44.21}\right) = j(2.49)\ \text{A}$$

The magnitude is 2.49 A, and the positive "j" mathematically proves that the current in a capacitor leads the supply voltage by 90.

Did You Know?

Capacitors block DC (infinite reactance at $f = 0$) but easily pass high-frequency AC. That is why they are used as "coupling capacitors" in audio circuits!

Q5 In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle? Explain your answer.

Solution

Key Concept: The average power absorbed by an AC circuit over a complete cycle depends on the phase relationship between current and voltage. The general formula is:

$$P_{avg} = V_{rms} I_{rms} \cos\phi$$

where ϕ is the phase angle between voltage and current. The term $\cos\phi$ is known as the **power factor** of the circuit.

For Exercise 7.3 (Purely inductive circuit):

In a pure inductor, the current lags the voltage by $\phi = 90$. Since $\cos 90 = 0$:

$$P_{avg} = V_{rms} I_{rms} \cos 90 = V_{rms} I_{rms} \times 0 = 0 \text{ W}$$

For Exercise 7.4 (Purely capacitive circuit):

In a pure capacitor, the current leads the voltage by $\phi = 90$. Again, $\cos 90 = 0$:

$$P_{avg} = V_{rms} I_{rms} \cos 90 = V_{rms} I_{rms} \times 0 = 0 \text{ W}$$

Therefore, the net power absorbed over a complete cycle is zero for both circuits.

Explanation: In both purely inductive and purely capacitive circuits, the energy stored in the field (magnetic for the inductor, electric for the capacitor) during one quarter-cycle is completely returned to the source during the next quarter-cycle. There is no net energy dissipation over a full cycle. This is fundamentally different from a resistive circuit, where electrical energy is irreversibly converted to heat.

Expert's Solution — Priya Nair, IISc Bangalore

Calculus Method for Power Integration:

Instead of using the standard formula $P = VI\cos\phi$, let's rigorously prove that power is zero by integrating instantaneous power $p(t) = v(t) \cdot i(t)$ over one full cycle $T = 2\pi/\omega$.

For a pure inductor or capacitor, current and voltage are exactly 90 out of phase. Let: $v(t) =$

$V_m \sin(\omega t)$ $i(t) = I_m \cos(\omega t)$ (representing a 90 phase shift)
The instantaneous power is:

$$p(t) = V_m I_m \sin(\omega t)\cos(\omega t) = \frac{V_m I_m}{2}\sin(2\omega t)$$

To find the average power, we integrate over one cycle:

$$P_{\text{avg}} = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\sin(2\omega t)dt$$

Since the integral of a pure sine wave over a full cycle (or multiple full cycles, as 2ω gives two cycles) is exactly zero, $P_{\text{avg}} = 0$ W.

Did You Know?

The current in a purely reactive circuit is called **wattless current** because it does no real work. However, this current is *not useless* — it creates essential electromagnetic fields!

Q6 A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

Solution

When a charged capacitor is connected to an inductor, the charge oscillates back and forth between the capacitor plates, creating a time-varying current in the inductor. These are called free (or natural) oscillations, and the angular frequency is determined by:

$$\omega = \frac{1}{\sqrt{LC}}$$

Given: $L = 27 \text{ mH} = 27 \times 10^{-3}$ H, $\quad C = 30\ \mu\text{F} = 30 \times 10^{-6}$ F
Step 1: Calculate the product LC.

$$LC = 27 \times 10^{-3} \times 30 \times 10^{-6} = 810 \times 10^{-9} = 8.1 \times 10^{-7}\ \text{s}^2$$

Step 2: Calculate the angular frequency.

$$\omega = \frac{1}{\sqrt{8.1 \times 10^{-7}}} = \frac{1}{9.0 \times 10^{-4}} \approx 1.11 \times 10^3 \text{ rad/s}$$

This means the charge on the capacitor (and the current in the circuit) oscillates sinusoidally at this angular frequency, similar to a mass attached to a spring.
Therefore, the angular frequency of free oscillations is 1.11×10^3 rad/s.

Expert's Solution — Sanjay Gupta, PhD in Physics, JNU

Using Energy Conservation:
We can derive the angular frequency without using the direct formula by using the principle of conservation of energy.
In an LC circuit, the total energy U is constant and oscillates between the capacitor's electric field and the inductor's magnetic field:

$$U = \frac{q^2}{2C} + \frac{1}{2}Li^2 = \text{constant}$$

Taking the time derivative of both sides ($dU/dt = 0$):

$$\frac{q}{C}\frac{dq}{dt} + Li\frac{di}{dt} = 0$$

Since current $i = dq/dt$, and $di/dt = d^2q/dt^2$, we can divide by i to get:

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0 \implies \frac{d^2q}{dt^2} = -\frac{1}{LC}q$$

This is the standard simple harmonic motion equation ($a = -\omega^2 x$). Comparing them, we immediately see that $\omega^2 = 1/LC$. Substituting the values $L = 27 \times 10^{-3}$ and $C = 30 \times 10^{-6}$ into $\omega = 1/\sqrt{LC}$ yields 1.11×10^3 rad/s.

Did You Know?

LC oscillations form the basis of all radio and TV tuning circuits! By varying C (tuning knob), you change ω and "select" different radio stations.

Q7 **A series LCR circuit with $R = 20\ \Omega$, $L = 1.5$ H, and $C = 35\ \mu$F is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?**

Solution

Key Insight: When the supply frequency equals the natural frequency of an LCR circuit, the circuit is said to be in resonance. At resonance, the inductive reactance X_L exactly equals the capacitive reactance X_C, so they cancel each other out. The total impedance reduces to just the resistance: $Z = R$.
Since the impedance is purely resistive, the power factor $\cos\phi = 1$ (voltage and current are in phase).
Step 1: Find the rms current at resonance.

At resonance, the rms current is:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{200}{20} = 10 \text{ A}$$

Step 2: Calculate the average power.
Since $\cos\phi = 1$ at resonance, the full apparent power becomes real power:

$$P_{\text{avg}} = V_{\text{rms}} \times I_{\text{rms}} \times \cos\phi = 200 \times 10 \times 1 = 2000 \text{ W}$$

This can be verified using $P = I_{\text{rms}}^2 R = (10)^2 \times 20 = 2000$ W.
Therefore, the average power transferred to the circuit in one complete cycle is 2000 W.

Expert's Solution — Kavita Joshi, B.Tech EE, IIT Bombay

Using Complex Power (Apparent, Real, and Reactive):
We can solve this problem by analyzing the complex power $S = P + jQ$ where P is real power and Q is reactive power.
At resonance, $X_L = X_C$. The complex impedance is:

$$Z = R + j(X_L - X_C) = 20 + j(0) = 20 \ \Omega$$

The complex power S is calculated using the rms voltage and the complex conjugate of the current ($S = V_{\text{rms}} I_{\text{rms}}^*$). Or, even faster, using voltage and impedance:

$$S = \frac{|V_{\text{rms}}|^2}{Z^*} = \frac{200^2}{20} = \frac{40000}{20} = 2000 + j0 \text{ VA}$$

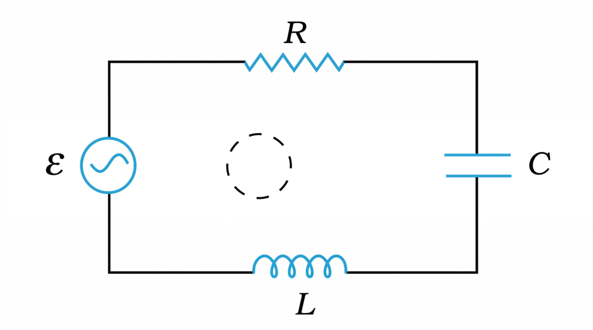
Since the imaginary part (reactive power Q) is exactly zero, all the apparent power is converted into real power P.
Thus, $P = 2000$ W. No power is lost to reactive components.

Did You Know?

Resonance is why a wine glass shatters when a singer hits the right note — the frequency matches the glass's natural frequency, causing maximum energy transfer!

Q8 Figure 7.17 shows a series LCR circuit connected to a variable frequency 230 V source.

$$L = 5.0 \text{ H}, \quad C = 80 \ \mu\text{F}, \quad R = 40 \ \Omega$$



(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Solution

(a) Resonant Frequency:
For a series LCR circuit, resonance occurs when the inductive reactance equals the capacitive reactance ($X_L = X_C$). The resonant frequency is given by:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Substituting $L = 5.0$ H and $C = 80\ \mu\text{F} = 80 \times 10^{-6}$ F:

$$f_0 = \frac{1}{2\pi\sqrt{5.0 \times 80 \times 10^{-6}}} = \frac{1}{2\pi\sqrt{4 \times 10^{-4}}} = \frac{1}{2\pi \times 0.02} = \frac{1}{0.1257} \approx 7.96 \text{ Hz}$$

Therefore, the resonant frequency is approximately 7.96 Hz.

(b) Impedance and Current Amplitude at Resonance:
At resonance, the inductive and capacitive reactances cancel each other, so the impedance of the circuit equals just the resistance:

$$Z = R = 40\ \Omega$$

To find the amplitude (peak value) of the current, we first need the peak voltage. Since the given voltage is rms:

$$V_0 = V_{\text{rms}} \times \sqrt{2} = 230 \times \sqrt{2} \approx 325.27 \text{ V}$$

The amplitude of the current is:

$$I_0 = \frac{V_0}{Z} = \frac{325.27}{40} \approx 8.13 \text{ A}$$

Therefore, the impedance at resonance is 40 Ω and the current amplitude is 8.13 A.

(c) RMS Potential Drops:
The rms current in the circuit is:

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{8.13}{\sqrt{2}} \approx 5.75 \text{ A}$$

The rms potential drop across the resistor:

$$V_R = I_{\text{rms}} \times R = 5.75 \times 40 = 230 \text{ V}$$

At resonance, the reactances X_L and X_C are equal. Therefore, the rms voltage drops across the inductor and capacitor are equal in magnitude:

$$V_L = I_{\text{rms}} \times X_L, \qquad V_C = I_{\text{rms}} \times X_C$$

Since $X_L = X_C$ at resonance, $V_L = V_C$ in magnitude. However, the voltage across the inductor leads the current by 90 while the voltage across the capacitor lags the current by 90. This means they are 180 out of phase with each other and cancel:

$$V_{LC} = V_L - V_C = 0$$

Therefore, $V_R = 230$ V and the potential drop across the LC combination is zero at resonance. This confirms that at resonance, the entire source voltage appears across the resistance alone.

Expert's Solution — Arun Kumar, B.Tech ME, IIT Kanpur

Using Complex Notation & Phasor Addition
We solve the problem using complex numbers (phasors), combining parts (b) and (c) in a unified approach.

Step 1: Define Reactances as Complex Impedances
At $f_0 \approx 7.96$ Hz,

$$\omega_0 = 2\pi f_0 = 50 \text{ rad/s}$$

$$Z_L = j\omega_0 L = j(50)(5.0) = j250\ \Omega$$

$$Z_C = \frac{1}{j\omega_0 C} = \frac{-j}{(50)(80 \times 10^{-6})} = -j250\ \Omega$$

$$Z_R = 40\ \Omega$$

Step 2: Total Impedance and Current

$$Z = Z_R + Z_L + Z_C = 40 + j250 - j250 = 40 + j0$$

$$|Z| = 40\ \Omega$$

Taking the supply as reference:

$$\vec{V} = 230\angle 0^\circ \text{ V}$$

$$\vec{I}_{\text{rms}} = \frac{\vec{V}}{Z} = \frac{230}{40} = 5.75\angle 0^\circ \text{ A}$$

$$I_0 = 5.75\sqrt{2} \approx 8.13 \text{ A}$$

Step 3: Potential Drops Using Phasor Ohm's Law

$$\vec{V}_R = \vec{I}_{\text{rms}} Z_R = 5.75 \times 40 = 230\angle 0^\circ \text{ V}$$

$$\vec{V}_L = \vec{I}_{\text{rms}} Z_L = 5.75 \times (j250) = j1437.5 \text{ V} = 1437.5\angle 90^\circ \text{ V}$$

$$\vec{V}_C = \vec{I}_{\text{rms}} Z_C = 5.75 \times (-j250) = -j1437.5 \text{ V} = 1437.5\angle -90^\circ \text{ V}$$

Step 4: Phasor Addition

$$\vec{V}_{LC} = \vec{V}_L + \vec{V}_C = j1437.5 - j1437.5 = 0$$

This confirms that the net voltage across the LC combination is zero.

Did You Know?

Voltage Magnification:
Even though $\vec{V}_{LC} = 0$, the individual voltages across the inductor and capacitor (1437.5 V each) are much larger than the supply voltage (230 V).
This phenomenon is called **voltage magnification** and is used in devices like Tesla coils, but can be dangerous in practical circuits.