NEET-UG Physics Sample Paper - 4

Duration: 1 Hour **Maximum Marks: 180**

Instructions

- This paper contains a total of 45 Multiple Choice Questions.
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Q1. The density of a material in SI units is 128 kg/m^3. In a certain system of units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is:

(A) 40

(B) 16

(C) 640

(D) 410

Q2. A particle moving in a straight line covers half the distance with speed v_0. The other half is covered in two equal time intervals with speeds v_1 and v_2. The average speed of the particle for the entire journey is:

(A) $\frac{v_0(v_1+v_2)}{v_1+v_2+v_0}$

(B) $\frac{2v_0(v_1+v_2)}{2v_0+v_1+v_2}$

(C) $\frac{v_0(v_1+v_2)}{2v_0+v_1+v_2}$

(D) $\frac{2v_0(v_1+v_2)}{v_1+v_2}$

Q3. Two projectiles are thrown with the same initial velocity at angles $45° + \theta$ and $45° - \theta$ with the horizontal. The ratio of their horizontal ranges is:

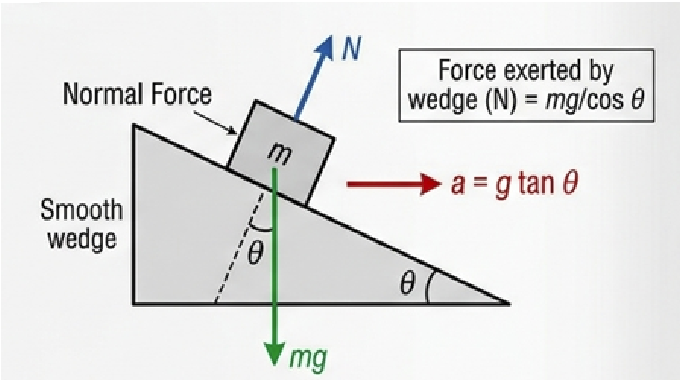
(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) $\tan\theta : 1$

Q4. A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block is:



(A) $mg\cos\theta$

(B) $mg/\cos\theta$

(C) $mg\tan\theta$

(D) $mg\sin\theta$

Q5. A ball of mass 150 g hits a wall normally with a speed of 30 m/s and bounces back with the same speed. If the contact time is 0.001 s, the average force exerted by the wall is:

(A) 4500 N

(B) 9000 N

(C) 1500 N

(D) 3000 N

Q6. A potential energy function is given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$. The value of x at which the force is zero and the minimum potential energy are:

(A) $\left(\frac{2a}{b}\right)^{1/6}, \frac{-b^2}{4a}$

(B) $\left(\frac{2a}{b}\right)^{1/6}, \frac{-b^2}{2a}$

(C) $\left(\frac{a}{b}\right)^{1/6}, \frac{-b^2}{4a}$

(D) $\left(\frac{a}{2b}\right)^{1/6}, \frac{-b^2}{4a}$

Q7. An engine pumps water through a hose pipe. Water passes through the pipe and leaves it with a velocity of 2 m/s. The mass per unit length of water in the pipe is 100 kg/m. What is the power of the engine?

(A) 400 W

(B) 800 W

(C) 200 W

(D) 1600 W

Q8. A solid sphere is rolling without slipping on a horizontal surface. The ratio of its rotational kinetic energy to its total kinetic energy is:

(A) 2/5

(B) 2/7

(C) 5/7

(D) 3/5

Q9. A thin circular ring of mass M and radius R is rotating about its axis with constant angular velocity ω. Two objects each of mass m are attached gently to opposite ends of a diameter. The new angular velocity is:

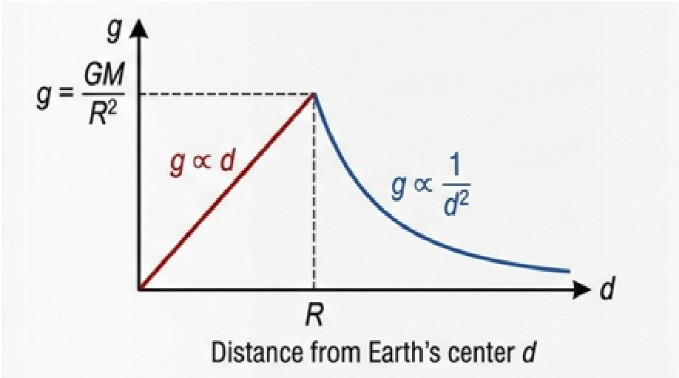
(A) $\frac{M\omega}{M+m}$

(B) $\frac{(M+2m)\omega}{M}$

(C) $\frac{M\omega}{M+2m}$

(D) $\frac{(M-2m)\omega}{M+2m}$

Q10. The variation of acceleration due to gravity g with distance d from center of earth (radius R) is best represented by:



(A) $g \propto d$ for $d < R$ and $g \propto 1/d^2$ for $d > R$

(B) $g \propto 1/d^2$ for all d

(C) $g \propto d$ for all d

(D) $g \propto 1/d$ for $d < R$ and $g \propto d^2$ for $d > R$

Q11. If the mass of Earth is M, radius is R, and Gravitational constant is G, the work done in lifting a body of mass m from Earth's surface to an altitude $h = R$ is:

(A) mgR

(B) $2mgR$

(C) $\frac{1}{2}mgR$

(D) $\frac{1}{4}mgR$

Q12. A soap bubble of radius r is blown up to form a bubble of radius $2r$ under isothermal conditions. If T is the surface tension, the energy spent is:

(A) $8\pi r^2 T$

(B) $12\pi r^2 T$

(C) $24\pi r^2 T$

(D) $16\pi r^2 T$

Q13. A small sphere of mass m and density d_1 is dropped into a glycerin column of density d_2. After some time, it moves with terminal velocity. The viscous force acting on it is:

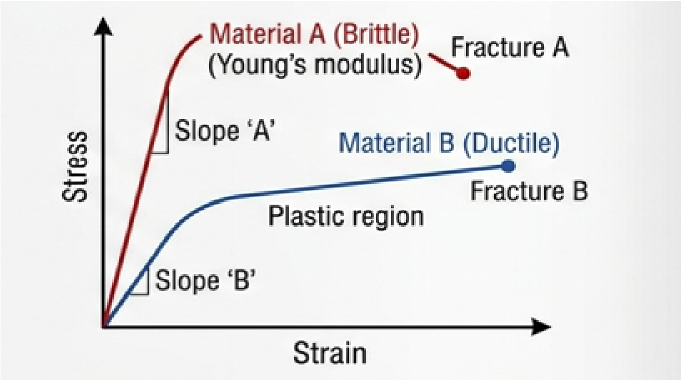
(A) $mg(1 - d_2/d_1)$

(B) $mg(d_1/d_2 - 1)$

(C) $mg(d_1 + d_2)$

(D) mgd_1d_2

Q14. The stress-strain curves for materials A and B are shown. Material A has a higher Young's modulus and is more brittle. Identify the properties based on the slope and fracture point.



(A) Slope A > Slope B, Fracture A earlier

(B) Slope B > Slope A, Fracture B earlier

(C) Slope A = Slope B, Both ductile

(D) Slope A < Slope B, Both brittle

Q15. A Carnot engine has an efficiency of 50% when its sink is at 27°C. To increase the efficiency to 70%, by how much should the temperature of the source be increased?

(A) 400 K

(B) 200 K

(C) 300 K

(D) 500 K

Q16. In an adiabatic process, the pressure of a gas is proportional to the cube of its absolute temperature. The value of γ (C_p/C_v) for the gas is:

(A) $3/2$

(B) $4/3$

(C) $5/3$

(D) $5/2$

Q17. The ratio of the root mean square velocity of H_2 at 50 K and that of O_2 at 800 K is:

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 1 : 4

Q18. The displacement of a particle executing SHM is given by $y = A\sin(\omega t) + B\cos(\omega t)$. The amplitude of its oscillation is:

(A) $A + B$

(B) $\sqrt{A^2 + B^2}$

(C) $\frac{A+B}{2}$

(D) $\sqrt{A^2 - B^2}$

Q19. A tuning fork of frequency 512 Hz makes 4 beats/sec with the vibrating string of a piano. The beat frequency decreases to 2 beats/sec when the tension in the piano string is slightly increased. The original frequency of the piano string was:

(A) 508 Hz

(B) 516 Hz

(C) 510 Hz

(D) 514 Hz

Q20. Three charges $+q$, $+q$, and $-2q$ are placed at the vertices of an equilateral triangle of side a. The magnitude of the electric dipole moment of the system is:

(A) $qa\sqrt{3}$

(B) $2qa$

(C) qa

(D) $qa\sqrt{2}$

Q21. A parallel plate capacitor has capacitance C. If the separation between the plates is doubled and a dielectric medium of constant $K = 4$ is introduced, the new capacitance is:

(A) C

(B) $2C$

(C) $4C$

(D) $C/2$

Q22. The electric potential at a point (x, y, z) is given by $V = -x^2y - xz^3 + 4$. The electric field at that point is:

(A) $\vec{E} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$

(B) $\vec{E} = 2xy\hat{i} + x^2\hat{j} + z^3\hat{k}$

(C) $\vec{E} = (x^2 + y^2)\hat{i} + z^2\hat{j}$

(D) $\vec{E} = (2xy - z^3)\hat{i} - x^2\hat{j}$

Q23. A copper wire is stretched to make it 0.1% longer. The percentage change in its resistance is:

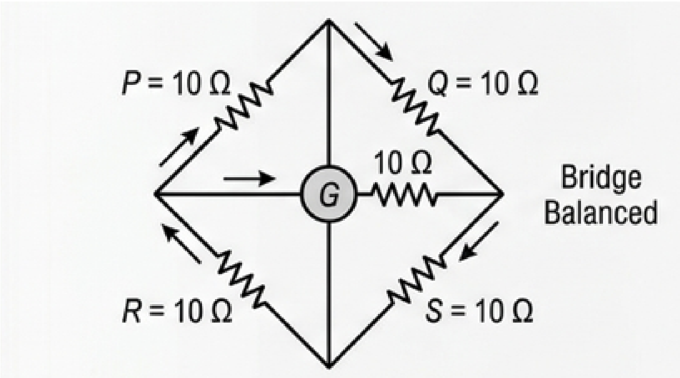
(A) 0.1%

(B) 0.2%

(C) 0.4%

(D) 0.05%

Q24. In a Wheatstone bridge, all four arms have resistance 10 Ω each. If the galvanometer resistance is also 10 Ω, the equivalent resistance of the combination is:



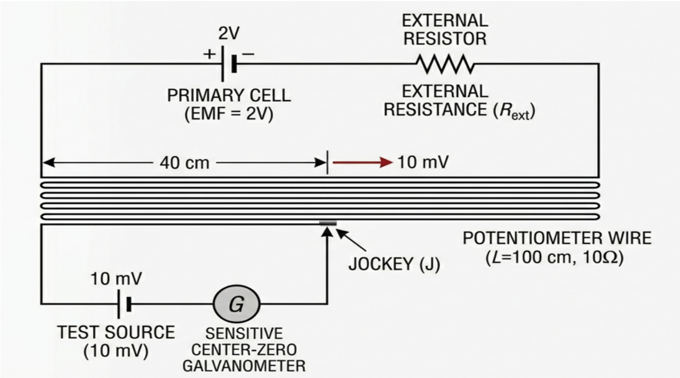
(A) 10 Ω

(B) 20 Ω

(C) 5 Ω

(D) 40 Ω

Q25. A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance and a cell of EMF 2 V. A source of 10 mV is balanced against 40 cm of the wire. The external resistance is:



(A) 790 Ω

(B) 800 Ω

(C) 700 Ω

(D) 795 Ω

Q26. Two cells of EMF E_1 and E_2 and internal resistances r_1 and r_2 are connected in parallel. The equivalent EMF of the combination is:

(A) $\frac{E_1 r_1 + E_2 r_2}{r_1 + r_2}$

(B) $\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

(C) $\frac{E_1 + E_2}{2}$

(D) $\frac{E_1 r_2 - E_2 r_1}{r_1 + r_2}$

Q27. A proton and an alpha particle enter a uniform magnetic field perpendicularly with the same kinetic energy. The ratio of the radii of their circular paths is:

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 1 : 4

Q28. A long solenoid has 1000 turns per meter. A current of 1.0 A flows through it. The magnetic field at the end of the solenoid is:

(A) $\mu_0 \times 10^3$ T

(B) $\frac{1}{2}\mu_0 \times 10^3$ T

(C) $2\mu_0 \times 10^3$ T

(D) $4\pi\mu_0 \times 10^3$ T

Q29. A circular coil of radius R carries a current I. The magnetic field at its center is B. At what distance from the center on the axis will the magnetic field be $B/8$?

(A) $R\sqrt{3}$

(B) $R/\sqrt{3}$

(C) $2R$

(D) $3R$

Q30. An inductor $L = 20$ mH, a capacitor $C = 100\ \mu$F and a resistor $R = 50\ \Omega$ are connected in series to an AC source $V = 10\sin(314t)$. The power loss in the circuit is:

(A) 0.79 W

(B) 2 W

(C) 1.5 W

(D) 0.39 W

Q31. The magnetic flux linked with a coil satisfies the relation $\phi = 3t^2 + 4t + 9$. The induced EMF in the coil at $t = 2$ s is:

(A) 16 V

(B) 10 V

(C) 8 V

(D) 12 V

Q32. In an EM wave, the amplitude of the magnetic field is 5×10^{-7} T. If the wave propagates in vacuum, the amplitude of the electric field is:

(A) 150 V/m

(B) 1.5 V/m

(C) 15 V/m

(D) 0.15 V/m

Q33. A convex lens of focal length 20 cm is placed in contact with a concave lens of focal length 40 cm. The power of the combination is:

(A) $+2.5$ D

(B) -2.5 D

(C) $+5$ D

(D) -5 D

Q34. In YDSE, the intensity of light at a point on the screen where the path difference is λ is K. The intensity at a point where the path difference is $\lambda/3$ is:

(A) $K/4$

(B) $K/2$

(C) K

(D) $3K/4$

Q35. An astronomical telescope has an angular magnification of 10 for distant objects. The separation between objective and eyepiece is 44 cm. The focal length of the objective is:

(A) 40 cm

(B) 4 cm

(C) 44 cm

(D) 20 cm

Q36. A ray of light is incident at an angle of 60° on one face of a prism of angle 30°. The ray emerging from the prism makes an angle of 30° with the incident ray. The refractive index of the prism is:

(A) $\sqrt{3}$

(B) 1.5

(C) $\sqrt{2}$

(D) 1.33

Q37. When light of wavelength λ falls on a photosensitive surface, the stopping potential is V. When light of wavelength 2λ falls on the same surface, the stopping potential is $V/4$. The threshold wavelength is:

(A) 3λ

(B) 2λ

(C) 4λ

(D) 5λ

Q38. The de-Broglie wavelength of an electron accelerated through a potential difference of 100 V is approximately:

(A) 1.227 Å

(B) 0.123 Å

(C) 12.27 Å

(D) 100 Å

Q39. The ratio of the wavelengths of the last line of Balmer series and the last line of Lyman series is:

(A) 4 : 1

(B) 1 : 4

(C) 1 : 1

(D) 9 : 1

Q40. A radioactive nucleus undergoes a series of decays: $A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3$. If the mass number and atomic number of A are 180 and 72, those of A_3 are:

(A) $172, 69$

(B) $174, 70$

(C) $172, 68$

(D) 176, 71

Q41. In a common emitter amplifier, the audio signal voltage across the collector resistance of 2 kΩ is 2 V. If the current amplification factor is 100 and the base resistance is 1 kΩ, the input signal voltage is:

(A) 0.01 V

(B) 0.1 V

(C) 0.001 V

(D) 1 V

Q42. The given truth table belongs to which logic gate? (Inputs $A = 0, B = 0$, Output = 1; $A = 1, B = 1$, Output = 0).

(A) NAND

(B) NOR

(C) XOR

(D) OR

Q43. For a p-n junction diode, the current I can be expressed as $I = I_0 \exp(eV/kT - 1)$. In the reverse bias, the current is:

(A) Near zero or $-I_0$

(B) infinite

(C) $I_0 \exp(eV/kT)$

(D) $I_0/2$

Q44. In a screw gauge, the pitch is 0.5 mm and there are 50 divisions on the circular scale. When measuring the thickness of a glass plate, the main scale reads 2.5 mm and the 20th division coincides with the reference line. The thickness is:

(A) 2.70 mm

(B) 2.60 mm

(C) 2.80 mm

(D) 2.50 mm

Q45. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s, and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be:

(A) 92 ± 2 s

(B) 92 ± 5 s

(C) 92 ± 1 s

(D) 92 ± 3 s

Detailed Solutions

Q1.

Solution

Concept:

The numerical value of a physical quantity changes when the system of units is changed. For any physical quantity, the product of its numerical value (n) and its unit (u) remains constant:

$$n_1 u_1 = n_2 u_2$$

Density has the dimensional formula $[ML^{-3}]$. Therefore, the relationship between numerical values in two different systems is:

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^{-3}$$

Solution:

1. Identify the initial values in SI units:

$$n_1 = 128, \quad M_1 = 1 \text{ kg}, \quad L_1 = 1 \text{ m}$$

2. Identify the new units provided:

$$M_2 = 50 \text{ g} = 0.05 \text{ kg}$$

$$L_2 = 25 \text{ cm} = 0.25 \text{ m}$$

3. Apply the conversion formula:

$$n_2 = 128 \left[\frac{1 \text{ kg}}{0.05 \text{ kg}}\right]^1 \left[\frac{1 \text{ m}}{0.25 \text{ m}}\right]^{-3}$$

4. Simplify the ratios:

$$\frac{M_1}{M_2} = \frac{1}{0.05} = 20$$

$$\frac{L_1}{L_2} = \frac{1}{0.25} = 4$$

5. Calculate the final numerical value:

$$n_2 = 128 \times (20) \times (4)^{-3}$$

$$n_2 = 128 \times 20 \times \frac{1}{64}$$

$$n_2 = 2 \times 20 = 40$$

Final Answer: The numerical value of density in the new system is 40.

Answer: (A)

Q2.

Solution

Concept:

Average speed is defined as the total distance traveled divided by the total time taken.

$$v_{avg} = \frac{S_{total}}{t_{total}}$$

In this problem, the journey is split into two halves of distance. The second half of the distance is further split into two equal time intervals.

Solution:

1. Let the total distance be $2S$. The first half distance S is covered with speed v_0. Time taken for the first half:

$$t_1 = \frac{S}{v_0}$$

2. For the second half distance S, let the total time taken be $2t$. It is covered in two equal time intervals t with speeds v_1 and v_2. Distance covered in first interval: $d_1 = v_1 t$ Distance covered in second interval: $d_2 = v_2 t$ The sum of these distances equals the second half distance S:

$$v_1 t + v_2 t = S \implies t(v_1 + v_2) = S \implies t = \frac{S}{v_1 + v_2}$$

3. The total time for the second half is $T_2 = 2t$:

$$T_2 = \frac{2S}{v_1 + v_2}$$

4. Calculate the total time for the entire journey:

$$t_{total} = t_1 + T_2 = \frac{S}{v_0} + \frac{2S}{v_1 + v_2}$$

$$t_{total} = S\left[\frac{(v_1 + v_2) + 2v_0}{v_0(v_1 + v_2)}\right]$$

5. Calculate average speed:

$$v_{avg} = \frac{2S}{t_{total}} = \frac{2S}{S\left[\frac{v_1+v_2+2v_0}{v_0(v_1+v_2)}\right]}$$

$$v_{avg} = \frac{2v_0(v_1 + v_2)}{2v_0 + v_1 + v_2}$$

Final Answer: The average speed is $\frac{2v_0(v_1+v_2)}{2v_0+v_1+v_2}$.

Answer: (B)

Q3.

Solution

Concept:
The horizontal range R of a projectile launched with velocity u at an angle θ with the horizontal is given by:

$$R = \frac{u^2 \sin(2\theta)}{g}$$

Two angles α and β result in the same range if they are complementary, i.e., $\alpha + \beta = 90^\circ$.

Solution:
1. Let the first angle be $\alpha = 45^\circ + \theta$. 2. Let the second angle be $\beta = 45^\circ - \theta$. 3. Check the sum of the angles:

$$\alpha + \beta = (45^\circ + \theta) + (45^\circ - \theta) = 90^\circ$$

4. Since the sum of the angles is 90°, the angles are complementary. 5. For the first angle:

$$R_1 = \frac{u^2 \sin(2(45^\circ + \theta))}{g} = \frac{u^2 \sin(90^\circ + 2\theta)}{g} = \frac{u^2 \cos(2\theta)}{g}$$

6. For the second angle:

$$R_2 = \frac{u^2 \sin(2(45^\circ - \theta))}{g} = \frac{u^2 \sin(90^\circ - 2\theta)}{g} = \frac{u^2 \cos(2\theta)}{g}$$

7. Taking the ratio:

$$\frac{R_1}{R_2} = \frac{u^2 \cos(2\theta)/g}{u^2 \cos(2\theta)/g} = 1$$

Final Answer: The ratio of their horizontal ranges is 1:1.

Answer: (A)

Q4.

Solution

Concept:
For a block to remain stationary relative to an accelerating wedge, the pseudo force acting on the block must balance the components of real forces. In the frame of the wedge, the forces acting on the block are gravity (mg), the normal force (N), and the pseudo force (ma).

Solution:
1. Let the wedge accelerate horizontally with acceleration a. 2. Resolve forces along the incline. To prevent slipping: The component of pseudo force up the incline must balance the component of gravity down the incline.

$$ma\cos\theta = mg\sin\theta \implies a = g\tan\theta$$

3. The normal force N is the force exerted by the wedge on the block. In the vertical direction:

$$N\cos\theta = mg$$

(Alternatively, resolving perpendicular to the incline):

$$N = mg\cos\theta + ma\sin\theta$$

4. Substitute $a = g\tan\theta$ into the normal force equation:

$$N = mg\cos\theta + m(g\tan\theta)\sin\theta$$

$$N = mg\cos\theta + mg\frac{\sin^2\theta}{\cos\theta}$$

$$N = \frac{mg(\cos^2\theta + \sin^2\theta)}{\cos\theta}$$

$$N = \frac{mg}{\cos\theta}$$

Final Answer: The force exerted by the wedge on the block is $mg/\cos\theta$.

Answer: (B)

Q5.

Solution

Concept:

According to Newton's Second Law of Motion, the average force is equal to the rate of change of momentum:

$$F_{avg} = \frac{\Delta p}{\Delta t}$$

Momentum is a vector quantity ($p = mv$), so the direction of velocity must be considered when calculating the change.

Solution:

1. Identify the given parameters: Mass $m = 150$ g $= 0.15$ kg Initial velocity $v_i = 30$ m/s (towards the wall) Final velocity $v_f = -30$ m/s (away from the wall) Time $\Delta t = 0.001$ s 2. Calculate the change in momentum (Δp):

$$\Delta p = m(v_f - v_i)$$

$$\Delta p = 0.15(-30 - 30) = 0.15(-60)$$

$$|\Delta p| = 9 \text{ kg m/s}$$

3. Calculate the average force:

$$F_{avg} = \frac{|\Delta p|}{\Delta t}$$

$$F_{avg} = \frac{9}{0.001}$$

$$F_{avg} = 9000 \text{ N}$$

Final Answer: The average force exerted by the wall is 9000 N.

Answer: (B)

Q6.

Solution

Concept:

The relationship between force (F) and potential energy (U) is given by the negative gradient of the potential energy:

$$F(x) = -\frac{dU}{dx}$$

Equilibrium occurs when the net force is zero ($F = 0$). The minimum potential energy (stable equilibrium) occurs at the position where the first derivative is zero and the second derivative is positive.

Solution:

1. Given potential energy function:

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

2. Differentiate $U(x)$ with respect to x to find the force:

$$F(x) = -\left[a(-12)x^{-13} - b(-6)x^{-7}\right]$$

$$F(x) = \frac{12a}{x^{13}} - \frac{6b}{x^7}$$

3. Set $F(x) = 0$ to find the equilibrium position:

$$\frac{12a}{x^{13}} = \frac{6b}{x^7} \implies \frac{12a}{6b} = \frac{x^{13}}{x^7}$$

$$\frac{2a}{b} = x^6 \implies x = \left(\frac{2a}{b}\right)^{1/6}$$

4. To find the minimum potential energy, substitute $x^6 = 2a/b$ back into the $U(x)$ expression:

$$U_{min} = \frac{a}{(x^6)^2} - \frac{b}{x^6}$$

$$U_{min} = \frac{a}{(2a/b)^2} - \frac{b}{(2a/b)}$$

$$U_{min} = \frac{a}{4a^2/b^2} - \frac{b^2}{2a} = \frac{b^2}{4a} - \frac{b^2}{2a}$$

$$U_{min} = \frac{b^2 - 2b^2}{4a} = -\frac{b^2}{4a}$$

Final Answer: The position is $(2a/b)^{1/6}$ and minimum energy is $-b^2/4a$.

Answer: (A)

Q7.

Solution

Concept:

Power (P) is defined as the rate of doing work or the rate of energy transfer. For a fluid flowing out of a pipe, the power required to impart kinetic energy to the fluid is:

$$P = \frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

Since the velocity v is constant, $P = \frac{1}{2}v^2\frac{dm}{dt}$.

Solution:

1. Identify the given values: Velocity $v = 2$ m/s Mass per unit length (λ) = 100 kg/m 2. Determine the mass flow rate ($\frac{dm}{dt}$): The mass of fluid dm in a small length dx is $dm = \lambda dx$. Therefore, $\frac{dm}{dt} = \lambda\frac{dx}{dt} = \lambda v$. 3. Substitute the values:

$$\frac{dm}{dt} = 100 \text{ kg/m} \times 2 \text{ m/s} = 200 \text{ kg/s}$$

4. Calculate the power:

$$P = \frac{1}{2}\left(\frac{dm}{dt}\right)v^2$$

$$P = \frac{1}{2} \times 200 \times (2)^2$$

$$P = 100 \times 4 = 400 \text{ W}$$

Final Answer: The power of the engine is 400 W.

Answer: (A)

Q8.

Solution

Concept:

A body rolling without slipping possesses both translational kinetic energy (K_t) and rotational kinetic energy (K_r).

$$K_t = \frac{1}{2}mv^2, \quad K_r = \frac{1}{2}I\omega^2$$

Total kinetic energy $K_{total} = K_t + K_r$. For a solid sphere, the moment of inertia $I = \frac{2}{5}mR^2$ and for pure rolling, $v = R\omega$.

Solution:

1. Write the expression for rotational kinetic energy:

$$K_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{5}mv^2$$

2. Write the expression for translational kinetic energy:

$$K_t = \frac{1}{2}mv^2$$

3. Calculate the total kinetic energy:

$$K_{total} = K_t + K_r = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

$$K_{total} = \frac{5mv^2 + 2mv^2}{10} = \frac{7}{10}mv^2$$

4. Find the ratio of rotational to total kinetic energy:

$$\text{Ratio} = \frac{K_r}{K_{total}} = \frac{\frac{1}{5}mv^2}{\frac{7}{10}mv^2}$$

$$\text{Ratio} = \frac{1}{5} \times \frac{10}{7} = \frac{2}{7}$$

Final Answer: The ratio of rotational kinetic energy to total kinetic energy is 2/7.

Answer: (B)

Q9.

Solution

Concept:

When no external torque acts on a system, the total angular momentum (L) of the system remains conserved:

$$L_i = L_f \implies I_i\omega_i = I_f\omega_f$$

The moment of inertia (I) changes when masses are added to the rotating system.

Solution:

1. Initial moment of inertia of the thin ring (mass M, radius R) about its axis:

$$I_i = MR^2$$

2. Initial angular momentum:

$$L_i = I_i\omega = MR^2\omega$$

3. Final moment of inertia after adding two masses m at the ends of the diameter (each at distance R from the center):

$$I_f = I_{ring} + I_{masses} = MR^2 + 2(mR^2)$$

$$I_f = (M + 2m)R^2$$

4. Apply the conservation of angular momentum:

$$I_i\omega = I_f\omega'$$

$$MR^2\omega = (M + 2m)R^2\omega'$$

5. Solve for the new angular velocity ω':

$$\omega' = \frac{M\omega}{M + 2m}$$

Final Answer: The new angular velocity is $\frac{M\omega}{M+2m}$.

Answer: (C)

Q10.

Solution

Concept:

The acceleration due to gravity (g) varies differently inside and outside the Earth, assuming Earth is a uniform sphere of radius R. Inside the Earth ($d < R$): $g = \frac{GMd}{R^3}$ (Linear) Outside the Earth ($d > R$): $g = \frac{GM}{d^2}$ (Inverse square)

Solution:

1. For a point inside the Earth at distance d from the center: The mass of the Earth attracting the body is only the mass of the sphere of radius d.

$$g_{in} = \frac{G(\rho \cdot \frac{4}{3}\pi d^3)}{d^2} = \frac{4}{3}\pi G\rho d$$

Thus, $g \propto d$ for $d < R$. The graph is a straight line passing through the origin. 2. At the surface ($d = R$):

$$g_s = \frac{GM}{R^2}$$

3. For a point outside the Earth at distance d from the center: The entire mass of the Earth acts as if concentrated at the center.

$$g_{out} = \frac{GM}{d^2}$$

Thus, $g \propto 1/d^2$ for $d > R$. The graph is a hyperbola.

Final Answer: g is proportional to d for $d < R$ and proportional to $1/d^2$ for $d > R$.

Answer: (A)

Q11.

Solution

Concept:

The work done in moving a body in a gravitational field is equal to the change in its gravitational potential energy. The gravitational potential energy of a mass m at a distance r from the center of Earth (mass M) is given by:

$$U = -\frac{GMm}{r}$$

Work done $W = U_f - U_i$.

Solution:

1. Initial position is on the Earth's surface:

$$r_i = R$$

Initial Potential Energy: $U_i = -\frac{GMm}{R}$ 2. Final position is at an altitude $h = R$ from the surface:

$$r_f = R + h = R + R = 2R$$

Final Potential Energy: $U_f = -\frac{GMm}{2R}$ 3. Calculate the work done:

$$W = U_f - U_i = \left(-\frac{GMm}{2R}\right) - \left(-\frac{GMm}{R}\right)$$

$$W = \frac{GMm}{R} - \frac{GMm}{2R} = \frac{GMm}{2R}$$

4. Express in terms of acceleration due to gravity g ($g = \frac{GM}{R^2} \implies GM = gR^2$):

$$W = \frac{(gR^2)m}{2R} = \frac{1}{2}mgR$$

Final Answer: The work done is $\frac{1}{2}mgR$.

Answer: (C)

Q12.

Solution

Concept:
The surface energy of a liquid film is the product of its surface tension and its total surface area. A soap bubble has two free surfaces (inner and outer). Therefore, the total surface area of a bubble of radius r is $2 \times 4\pi r^2 = 8\pi r^2$.

Solution:
1. Initial surface area of the bubble with radius r:

$$A_1 = 2 \times 4\pi r^2 = 8\pi r^2$$

2. Initial surface energy:

$$E_1 = T \times A_1 = 8\pi r^2 T$$

3. Final surface area of the bubble with radius $2r$:

$$A_2 = 2 \times 4\pi (2r)^2 = 2 \times 4\pi \times 4r^2 = 32\pi r^2$$

4. Final surface energy:

$$E_2 = T \times A_2 = 32\pi r^2 T$$

5. The energy spent is the change in surface energy:

$$\Delta E = E_2 - E_1 = 32\pi r^2 T - 8\pi r^2 T$$

$$\Delta E = 24\pi r^2 T$$

Final Answer: The energy spent is $24\pi r^2 T$.

Answer: (C)

Q13.

Solution

Concept:
When an object moves with terminal velocity through a viscous fluid, it is in dynamic equilibrium. This means the net force acting on the object is zero. The forces involved are weight (W) acting downwards, upthrust (F_B) acting upwards, and viscous force (F_v) acting upwards.

Solution:
1. Write the equilibrium equation for terminal velocity:

$$F_v + F_B = W \implies F_v = W - F_B$$

2. Express the weight of the sphere: $W = \text{Volume} \times d_1 \times g = Vd_1g$ 3. Express the buoyant force (upthrust): $F_B = \text{Volume of displaced fluid} \times d_2 \times g = Vd_2g$ 4. Calculate the viscous force:

$$F_v = Vd_1g - Vd_2g = Vg(d_1 - d_2)$$

5. To express this in terms of mg, note that $m = Vd_1$, so $V = m/d_1$:

$$F_v = \frac{m}{d_1}g(d_1 - d_2)$$

$$F_v = mg\left(1 - \frac{d_2}{d_1}\right)$$

Final Answer: The viscous force is $mg(1 - d_2/d_1)$.

Answer: (A)

Q14.

Solution

Concept:
In a stress-strain graph, the slope of the linear portion (within the elastic limit) represents the Young's Modulus (Y = Stress/Strain). A steeper slope indicates a higher Young's Modulus. A material is brittle if the fracture point is close to the elastic limit, and ductile if there is a large plastic deformation region before fracture.

Solution:
1. Analysis of Slope: Since material A has a higher Young's Modulus than material B, the slope of the stress-strain curve for A must be greater than that of B.

$$\text{Slope A} > \text{Slope B}$$

2. Analysis of Fracture: A brittle material fails suddenly with little plastic deformation. Therefore, the fracture point for A will occur at a lower strain value compared to a ductile material. 3. Conclusion: The curve for A will be steeper (larger Y) and end abruptly (brittle), while B will have a lower slope and a longer curve after the proportionality limit (ductile).

Final Answer: Slope A is greater than Slope B, and A reaches fracture point earlier.

Answer: (A)

Q15.

Solution

Concept:

The efficiency (η) of a Carnot engine is given by the formula:

$$\eta = 1 - \frac{T_{sink}}{T_{source}}$$

where temperatures must be in Kelvin ($K =^\circ \text{C} + 273$).

Solution:

1. Convert sink temperature to Kelvin:

$$T_{sink} = 27 + 273 = 300 \text{ K}$$

2. Case 1: Efficiency $\eta_1 = 50\% = 0.5$

$$0.5 = 1 - \frac{300}{T_{source1}} \implies \frac{300}{T_{source1}} = 0.5$$

$$T_{source1} = \frac{300}{0.5} = 600 \text{ K}$$

3. Case 2: New Efficiency $\eta_2 = 70\% = 0.7$

$$0.7 = 1 - \frac{300}{T_{source2}} \implies \frac{300}{T_{source2}} = 0.3$$

$$T_{source2} = \frac{300}{0.3} = 1000 \text{ K}$$

4. Calculate the increase in source temperature:

$$\Delta T = T_{source2} - T_{source1} = 1000 - 600 = 400 \text{ K}$$

Final Answer: The temperature of the source should be increased by 400 K.

Answer: (A)

Q16.

Solution

Concept:

For an adiabatic process, the relationship between pressure (P) and temperature (T) is given by the Poisson's equation:

$$P^{1-\gamma}T^{\gamma} = \text{constant} \implies P \propto T^{\frac{\gamma}{\gamma-1}}$$

where γ is the ratio of specific heats (C_p/C_v). By comparing this to the given proportionality, we can find the value of γ.

Solution:

1. Given that $P \propto T^3$. 2. From the adiabatic relation: $P \propto T^{\frac{\gamma}{\gamma-1}}$. 3. Equating the exponents:

$$\frac{\gamma}{\gamma - 1} = 3$$

4. Solve for γ:

$$\gamma = 3(\gamma - 1)$$

$$\gamma = 3\gamma - 3$$

$$2\gamma = 3 \implies \gamma = \frac{3}{2}$$

5. Note: This corresponds to the theoretical γ for a gas with specific degrees of freedom, where $\gamma = 1.5$.

Final Answer: The value of γ for the gas is 3/2.

Answer: (A)

Q17.

Solution

Concept:

The root mean square (v_{rms}) velocity of gas molecules is given by:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

where R is the gas constant, T is the absolute temperature in Kelvin, and M is the molar mass of the gas.

Solution:

1. Formula for the ratio of v_{rms} for two different gases:

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{M_1} \times \frac{M_2}{T_2}}$$

2. Identify values for Hydrogen (H_2): $T_1 = 50$ K, $M_1 = 2$ g/mol 3. Identify values for Oxygen (O_2): $T_2 = 800$ K, $M_2 = 32$ g/mol 4. Substitute into the ratio:

$$\frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{50}{2} \times \frac{32}{800}}$$

5. Simplify the expression:

$$\frac{v_{H_2}}{v_{O_2}} = \sqrt{25 \times \frac{1}{25}} = \sqrt{1} = 1$$

6. This means the average speeds are identical at these specific temperature-mass combinations.

Final Answer: The ratio of the root mean square velocities is 1:1.

Answer: (A)

Q18.

Solution

Concept:
When two SHMs of the same frequency act in perpendicular directions or are superimposed along the same line with a phase difference, the resultant motion is also SHM. For $y = A\sin(\omega t) + B\cos(\omega t)$, we can use the trigonometric identity to combine them into a single sine wave.

Solution:
1. The given equation is $y = A\sin(\omega t)+B\cos(\omega t)$. 2. We can rewrite $B\cos(\omega t)$ as $B\sin(\omega t+\pi/2)$. 3. This is the superposition of two SHMs with a phase difference of $\phi = \pi/2$. 4. The resultant amplitude R is given by:

$$R = \sqrt{A^2 + B^2 + 2AB\cos\phi}$$

5. Since $\phi = 90^\circ$, $\cos(90^\circ) = 0$:

$$R = \sqrt{A^2 + B^2 + 0} = \sqrt{A^2 + B^2}$$

6. The combined wave can be expressed as $y = \sqrt{A^2 + B^2}\sin(\omega t + \alpha)$, where $\tan\alpha = B/A$.
Final Answer: The amplitude of oscillation is $\sqrt{A^2 + B^2}$.

Answer: (B)

Q19.

Solution

Concept:

$$f_b = |f_1 - f_2|, \quad f \propto \sqrt{T} \Rightarrow T\uparrow \Rightarrow f\uparrow$$

Solution:
1. $f_t = 512$ Hz, initial $f_b = 4$ Hz

$$f_p = 512 \pm 4 = 516 \text{ Hz or } 508 \text{ Hz}$$

2. On increasing tension, f_p increases.
3. New beat frequency = 2 Hz.
4. If $f_p = 516$ Hz $\Rightarrow f_p' > 516$, difference with 512 increases (> 4)
5. If $f_p = 508$ Hz $\Rightarrow f_p' \to 512$, difference decreases to 2
Final Answer: 508 Hz

Answer: (A)

Q20.

Solution

Concept:

An electric dipole moment ($\vec{p}$) is a vector directed from negative to positive charge, with magnitude $p = q \times d$. For a system of multiple charges, the net dipole moment is the vector sum of individual dipoles formed by pairing charges.

Solution:

1. We can treat the $-2q$ charge as two separate $-q$ charges placed at the same vertex. 2. This creates two dipoles: - Dipole $\vec{p}_1$ between $+q$ and $-q$ along one side of length a. - Dipole $\vec{p}_2$ between the other $+q$ and $-q$ along the second side of length a. 3. The magnitude of each dipole is $p_1 = p_2 = qa$. 4. The angle between these two vector dipoles is 60° (the internal angle of the equilateral triangle). 5. The resultant dipole moment P_{net} is:

$$P_{net} = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos(60^\circ)}$$

$$P_{net} = \sqrt{(qa)^2 + (qa)^2 + 2(qa)^2(1/2)}$$

$$P_{net} = \sqrt{3(qa)^2} = qa\sqrt{3}$$

Final Answer: The magnitude of the electric dipole moment is $qa\sqrt{3}$.

Answer: (A)

Q21.

Solution

Concept:
The capacitance of a parallel plate capacitor is given by $C = \frac{K\epsilon_0 A}{d}$, where K is the dielectric constant, ϵ_0 is the permittivity of free space, A is the area of the plates, and d is the distance between the plates. When parameters change, the new capacitance C' can be found by taking the ratio.

Solution:
1. Initial capacitance in air ($K = 1$):

$$C = \frac{\epsilon_0 A}{d}$$

2. New conditions: - Distance is doubled: $d' = 2d$ - Dielectric constant: $K = 4$ 3. New capacitance C':

$$C' = \frac{K\epsilon_0 A}{d'}$$

4. Substitute the new values:

$$C' = \frac{4 \times \epsilon_0 A}{2d}$$

$$C' = 2 \times \left(\frac{\epsilon_0 A}{d}\right)$$

5. Since the term in the bracket is the original capacitance C:

$$C' = 2C$$

Final Answer: The new capacitance is 2C.

Answer: (B)

Q22.

Solution

Concept:

The electric field $\vec{E}$ is the negative gradient of the electric potential V. In three-dimensional Cartesian coordinates, it is expressed as:

$$\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

Solution:

1. Given potential: $V = -x^2y - xz^3 + 4$. 2. Calculate the x-component (E_x):

$$E_x = -\frac{\partial V}{\partial x} = -[(-2xy) - (z^3) + 0] = 2xy + z^3$$

3. Calculate the y-component (E_y):

$$E_y = -\frac{\partial V}{\partial y} = -[(-x^2) - 0 + 0] = x^2$$

4. Calculate the z-component (E_z):

$$E_z = -\frac{\partial V}{\partial z} = -[0 - (3xz^2) + 0] = 3xz^2$$

5. Combine the components into vector form:

$$\vec{E} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$$

Final Answer: The electric field is $\vec{E} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$.

Answer: (A)

Q23.

Solution

Concept:

The resistance of a wire is $R = \rho \frac{L}{A}$. When a wire is stretched, its volume ($V = AL$) remains constant. This implies that if the length L increases, the cross-sectional area A must decrease such that $A \propto 1/L$. Consequently, $R \propto L^2$.

Solution:

1. From $R = \rho \frac{L}{A}$ and $A = \frac{V}{L}$, we get:

$$R = \rho \frac{L^2}{V}$$

2. For small percentage changes, we use the error approximation/differentiation method:

$$\frac{\Delta R}{R} = 2\frac{\Delta L}{L}$$

3. Given that the length is increased by 0.1%:

$$\frac{\Delta L}{L} \times 100 = 0.1\%$$

4. Calculate the percentage change in resistance:

$$\text{Percentage change in } R = 2 \times (0.1\%) = 0.2\%$$

5. Since length increased, resistance increases by 0.2%.

Final Answer: The percentage change in resistance is 0.2%.

Answer: (B)

Q24.

Solution

Concept:

A Wheatstone bridge is balanced when the product of resistances in opposite arms is equal ($P/Q = R/S$). In a balanced state, no current flows through the galvanometer, and it can be removed from the circuit for resistance calculation.

Solution:

1. Arm resistances are $P = 10\ \Omega, Q = 10\ \Omega, R = 10\ \Omega, S = 10\ \Omega$. 2. Check the balance condition:

$$\frac{P}{Q} = \frac{10}{10} = 1, \quad \frac{R}{S} = \frac{10}{10} = 1$$

3. Since $P/Q = R/S$, the bridge is balanced. 4. The $10\ \Omega$ galvanometer resistance in the middle carries no current and is ignored. 5. The circuit simplifies to two parallel branches: - Branch 1 (top): P and Q in series = $10+10 = 20\ \Omega$ - Branch 2 (bottom): R and S in series = $10+10 = 20\ \Omega$ 6. Calculate equivalent resistance (R_{eq}):

$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{20} = \frac{2}{20} = \frac{1}{10}$$

$$R_{eq} = 10\ \Omega$$

Final Answer: The equivalent resistance is $10\ \Omega$.

Answer: (A)

Q25.

Solution

Concept:

A potentiometer measures potential difference using a balance length l. The potential gradient k (voltage per unit length) is given by $k = \frac{V_w}{L}$, where V_w is the voltage across the potentiometer wire. The balancing condition is $E_{test} = k \times l$.

Solution:

1. Let the external resistance in the primary circuit be R_{ext}. Total resistance in primary circuit $= R_{ext} + 10\ \Omega$. 2. Current in the primary circuit:

$$I = \frac{E}{R_{total}} = \frac{2}{R_{ext} + 10}$$

3. Voltage across the potentiometer wire (V_w):

$$V_w = I \times R_{wire} = \frac{2 \times 10}{R_{ext} + 10} = \frac{20}{R_{ext} + 10}$$

4. Potential gradient (k):

$$k = \frac{V_w}{L} = \frac{20}{(R_{ext} + 10) \times 100}\ \text{V/cm}$$

5. Balancing 10 mV = 0.01 V at $l = 40$ cm:

$$0.01 = k \times 40 = \frac{20 \times 40}{100(R_{ext} + 10)}$$

$$0.01 = \frac{8}{R_{ext} + 10}$$

6. Solve for R_{ext}:

$$R_{ext} + 10 = \frac{8}{0.01} = 800 \implies R_{ext} = 790\ \Omega$$

Final Answer: The external resistance is 790 Ω.

Answer: (A)

Q26.

Solution

Concept:

When two cells are connected in parallel, the equivalent electromotive force (EMF) of the combination is determined by the individual EMFs and their respective internal resistances. The equivalent EMF (E_{eq}) is the potential difference across the terminals when no current is drawn from the combination.

Solution:

1. Consider two cells with EMFs E_1, E_2 and internal resistances r_1, r_2 connected in parallel. 2. The equivalent internal resistance (r_{eq}) is given by the parallel combination formula:

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} \implies r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

3. The formula for equivalent EMF in a parallel combination is:

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

4. Substitute the expression for r_{eq}:

$$E_{eq} = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) \times r_{eq}$$

$$E_{eq} = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 r_2}\right) \times \left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

5. Simplify the expression by canceling $r_1 r_2$:

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

Final Answer: The equivalent EMF is $\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$.

Answer: (B)

Q27.

Solution

Concept:

When a charged particle enters a uniform magnetic field (B) perpendicularly, it moves in a circular path. The radius (r) of this path is given by:

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where m is mass, q is charge, and K is kinetic energy. Since B and K are the same for both particles, the radius is proportional to $\sqrt{m}/q$.

Solution:

1. Identify properties of the Proton (p): Mass $m_p = m$, Charge $q_p = e$. 2. Identify properties of the Alpha particle (α): Mass $m_\alpha = 4m$, Charge $q_\alpha = 2e$. 3. Write the ratio of radii:

$$\frac{r_p}{r_\alpha} = \frac{\sqrt{m_p}/q_p}{\sqrt{m_\alpha}/q_\alpha}$$

4. Substitute the values:

$$\frac{r_p}{r_\alpha} = \frac{\sqrt{m}/e}{\sqrt{4m}/2e} = \frac{\sqrt{m}/e}{2\sqrt{m}/2e}$$

5. Simplify the ratio:

$$\frac{r_p}{r_\alpha} = \frac{\sqrt{m}/e}{\sqrt{m}/e} = 1$$

6. Therefore, both particles will follow paths with the same radius.

Final Answer: The ratio of the radii is 1:1.

Answer: (A)

Q28.

Solution

Concept:
For an infinitely long solenoid, the magnetic field inside is uniform and given by $B = \mu_0 nI$. However, at the ends of a very long solenoid, the magnetic field lines spread out, and the field strength is exactly half of the field at the center.

Solution:
1. Identify the given parameters: Number of turns per unit length $n = 1000$ turns/m Current $I = 1.0$ A 2. Calculate the magnetic field at the center (B_{center}):

$$B_{center} = \mu_0 nI = \mu_0 \times 1000 \times 1 = 1000\mu_0$$

3. Apply the property for the field at the end:

$$B_{end} = \frac{1}{2}B_{center}$$

4. Calculate final value:

$$B_{end} = \frac{1}{2}\mu_0 \times 1000 = 500\mu_0$$

$$B_{end} = \frac{1}{2}\mu_0 \times 10^3 \text{ T}$$

Final Answer: The magnetic field at the end is $\frac{1}{2}\mu_0 \times 10^3$ T.

Answer: (B)

Q29.

Solution

Concept:

The magnetic field at a point on the axis of a circular current-carrying coil at a distance x from the center is:

$$B_x = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

The field at the center ($x = 0$) is $B = \frac{\mu_0 I}{2R}$. We set $B_x = B/8$ to find x.

Solution:

1. Set up the equation:

$$\frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}} = \frac{1}{8}\left(\frac{\mu_0 I}{2R}\right)$$

2. Simplify by canceling common terms ($\mu_0, I, 2$):

$$\frac{R^2}{(R^2 + x^2)^{3/2}} = \frac{1}{8R}$$

3. Cross-multiply:

$$8R^3 = (R^2 + x^2)^{3/2}$$

4. Take the cube root of both sides ($8^{1/3} = 2$):

$$(8R^3)^{1/3} = ((R^2 + x^2)^{3/2})^{1/3}$$

$$2R = (R^2 + x^2)^{1/2}$$

5. Square both sides:

$$4R^2 = R^2 + x^2$$

$$x^2 = 3R^2 \implies x = R\sqrt{3}$$

Final Answer: The distance from the center is $R\sqrt{3}$.

Answer: (A)

Q30.

Solution

Concept:

$$P = I_{rms}^2 R, \quad I_{rms} = \frac{V_{rms}}{Z}$$

Solution:

1. Given: $L = 0.02$ H, $C = 10^{-4}$ F, $R = 50\,\Omega$, $V = 10\sin(314t) \Rightarrow V_{rms} = \frac{10}{\sqrt{2}}$
2. Reactances:

$$X_L = \omega L = 314 \times 0.02 = 6.28\,\Omega$$

$$X_C = \frac{1}{\omega C} \approx 31.85\,\Omega$$

3. Impedance:

$$Z = \sqrt{R^2 + (X_C - X_L)^2} \approx 56.16\,\Omega$$

4. Current:

$$I_{rms} = \frac{7.07}{56.16} \approx 0.126\,\text{A}$$

5. Power:

$$P = I_{rms}^2 R \approx (0.126)^2 \times 50 \approx 0.79\,\text{W}$$

Final Answer: 0.79 W

Answer: (A)

Q31.

Solution

Concept:

According to Faraday's Law of Electromagnetic Induction, the magnitude of the induced electromotive force (EMF) in a coil is equal to the rate of change of magnetic flux (ϕ) linked with the coil:

$$\varepsilon = -\frac{d\phi}{dt}$$

The negative sign (Lenz's Law) indicates direction, but for magnitude, we simply differentiate the flux function with respect to time.

Solution:

1. Given the expression for magnetic flux:

$$\phi = 3t^2 + 4t + 9$$

2. Differentiate the flux with respect to time t to find the induced EMF:

$$\varepsilon = \frac{d}{dt}(3t^2 + 4t + 9)$$

3. Perform the differentiation:

$$\varepsilon = 6t + 4$$

4. Substitute the given time $t = 2$ s into the expression:

$$\varepsilon = 6(2) + 4$$

$$\varepsilon = 12 + 4 = 16 \text{ V}$$

Final Answer: The induced EMF in the coil at $t = 2$ s is 16 V.

Answer: (A)

Q32.

Solution

Concept:

In an electromagnetic (EM) wave propagating through a vacuum, the amplitudes of the electric field (E_0) and the magnetic field (B_0) are related by the speed of light (c):

$$c = \frac{E_0}{B_0}$$

The speed of light in vacuum is approximately 3×10^8 m/s.

Solution:

1. Identify the given values: Amplitude of magnetic field $B_0 = 5 \times 10^{-7}$ T Speed of light $c = 3 \times 10^8$ m/s 2. Use the relationship formula to solve for E_0:

$$E_0 = c \times B_0$$

3. Substitute the numerical values:

$$E_0 = (3 \times 10^8) \times (5 \times 10^{-7})$$

4. Calculate the product:

$$E_0 = 15 \times 10^{8-7} = 15 \times 10^1$$

$$E_0 = 150 \text{ V/m}$$

Final Answer: The amplitude of the electric field is 150 V/m.

Answer: (A)

Q33.

Solution

Concept:

The power (P) of a lens is the reciprocal of its focal length (f) in meters: $P = 1/f$. For a combination of lenses in contact, the total power is the algebraic sum of the individual powers:

$$P_{total} = P_1 + P_2 = \frac{1}{f_1} + \frac{1}{f_2}$$

Convex lenses have positive focal lengths, while concave lenses have negative focal lengths.

Solution:

1. Identify focal lengths with proper signs: $f_1 = +20$ cm $= +0.2$ m (Convex) $f_2 = -40$ cm $= -0.4$ m (Concave) 2. Calculate individual powers:

$$P_1 = \frac{1}{0.2} = +5 \text{ D}$$

$$P_2 = \frac{1}{-0.4} = -2.5 \text{ D}$$

3. Calculate the total power of the combination:

$$P_{total} = P_1 + P_2 = +5 - 2.5$$

$$P_{total} = +2.5 \text{ D}$$

Final Answer: The power of the combination is +2.5 D.

Answer: (A)

Q34.

Solution

Concept:

In Young's Double Slit Experiment (YDSE), the resultant intensity (I) at a point where the phase difference is ϕ is given by:

$$I = I_{max} \cos^2\left(\frac{\phi}{2}\right)$$

The phase difference ϕ and path difference Δx are related by: $\phi = \frac{2\pi}{\lambda}\Delta x$.

Solution:

1. For path difference $\Delta x_1 = \lambda$: $\phi_1 = \frac{2\pi}{\lambda} \times \lambda = 2\pi$. Intensity $I_1 = I_{max} \cos^2(\pi) = I_{max} = K$. 2. For path difference $\Delta x_2 = \lambda/3$: $\phi_2 = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$. 3. Calculate the new intensity I_2:

$$I_2 = K \cos^2\left(\frac{2\pi/3}{2}\right) = K \cos^2\left(\frac{\pi}{3}\right)$$

4. Evaluate the trigonometric term: $\cos(\pi/3) = \cos(60^\circ) = 1/2$.

$$I_2 = K\left(\frac{1}{2}\right)^2 = \frac{K}{4}$$

Final Answer: The intensity at the point will be K/4.

Answer: (A)

Q35.

Solution

Concept:

For an astronomical telescope in normal adjustment (final image at infinity), the angular magnification (M) is given by:

$$M = \frac{f_o}{f_e}$$

The separation between the objective and the eyepiece (length of the telescope) is:

$$L = f_o + f_e$$

Solution:

1. Given magnification $M = 10$. Therefore:

$$\frac{f_o}{f_e} = 10 \implies f_o = 10f_e$$

2. Given length of telescope $L = 44$ cm:

$$f_o + f_e = 44$$

3. Substitute $f_o = 10f_e$ into the length equation:

$$10f_e + f_e = 44$$

$$11f_e = 44 \implies f_e = 4 \text{ cm}$$

4. Solve for the focal length of the objective (f_o):

$$f_o = 10 \times 4 = 40 \text{ cm}$$

Final Answer: The focal length of the objective is 40 cm.

Answer: (A)

Q36.

Solution

Concept:

When a ray of light passes through a prism, it undergoes deviation (δ). The relationship between the angle of incidence (i), angle of emergence (e), angle of the prism (A), and angle of deviation is:

$$\delta = i + e - A$$

Also, the refractive index (μ) of the material is given by Snell's Law: $\mu = \sin i / \sin r_1$. For a ray passing through a prism, $A = r_1 + r_2$.

Solution:

1. Identify the given values: Angle of incidence $i = 60^\circ$ Prism angle $A = 30^\circ$ Angle of deviation $\delta = 30^\circ$ 2. Find the angle of emergence (e) using the deviation formula:

$$30^\circ = 60^\circ + e - 30^\circ$$

$$30^\circ = 30^\circ + e \implies e = 0^\circ$$

3. An emergence angle of 0° means the ray emerges normally to the second face. From Snell's Law at the second face: $\mu \sin r_2 = \sin e$. Since $e = 0^\circ$, then $r_2 = 0^\circ$. 4. Use the relation $A = r_1 + r_2$ to find the first angle of refraction:

$$30^\circ = r_1 + 0^\circ \implies r_1 = 30^\circ$$

5. Apply Snell's Law at the first face:

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin 60^\circ}{\sin 30^\circ}$$

$$\mu = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

Final Answer: The refractive index of the prism is $\sqrt{3}$.

Answer: (A)

Q37.

Solution

Concept:

Einstein's photoelectric equation relates the energy of incident photons to the work function (ϕ) and the maximum kinetic energy of emitted electrons ($K_{max} = eV_s$):

$$\frac{hc}{\lambda} = \phi + eV_s$$

where V_s is the stopping potential. The work function can be expressed in terms of threshold wavelength (λ_0) as $\phi = hc/\lambda_0$.

Solution:

1. Case 1: Incident wavelength λ, stopping potential V:

$$\frac{hc}{\lambda} = \phi + eV \quad \ldots(1)$$

2. Case 2: Incident wavelength 2λ, stopping potential $V/4$:

$$\frac{hc}{2\lambda} = \phi + \frac{eV}{4} \quad \ldots(2)$$

3. Multiply equation (2) by 4:

$$\frac{4hc}{2\lambda} = 4\phi + eV \implies \frac{2hc}{\lambda} = 4\phi + eV \quad \ldots(3)$$

4. Subtract equation (1) from equation (3) to eliminate eV:

$$\left(\frac{2hc}{\lambda} - \frac{hc}{\lambda}\right) = (4\phi - \phi)$$

$$\frac{hc}{\lambda} = 3\phi$$

5. Substitute $\phi = hc/\lambda_0$:

$$\frac{hc}{\lambda} = 3\left(\frac{hc}{\lambda_0}\right)$$

$$\frac{1}{\lambda} = \frac{3}{\lambda_0} \implies \lambda_0 = 3\lambda$$

Final Answer: The threshold wavelength is 3λ.

Answer: (A)

Q38.

Solution

Concept:

The de-Broglie wavelength (λ) associated with an electron accelerated through a potential difference V is given by:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e eV}}$$

For an electron, after substituting the constants (h, m_e, e), the formula simplifies to:

$$\lambda = \frac{12.27}{\sqrt{V}}\ \text{Å}$$

Solution:

1. Identify the given accelerating potential: $V = 100$ V 2. Apply the simplified formula for electrons:

$$\lambda = \frac{12.27}{\sqrt{100}}\ \text{Å}$$

3. Calculate the square root:

$$\sqrt{100} = 10$$

4. Perform the division:

$$\lambda = \frac{12.27}{10} = 1.227\ \text{Å}$$

Final Answer: The de-Broglie wavelength is 1.227 Å.

Answer: (A)

Q39.

Solution

Concept:
The wavelength of lines in the hydrogen spectrum is given by the Rydberg formula:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

The "last line" of a series (series limit) corresponds to a transition from $n_2 = \infty$. - Lyman series: $n_1 = 1$ - Balmer series: $n_1 = 2$

Solution:

1. Calculate the wavelength of the last line of the Lyman series (λ_L):

$$\frac{1}{\lambda_L} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R \implies \lambda_L = \frac{1}{R}$$

2. Calculate the wavelength of the last line of the Balmer series (λ_B):

$$\frac{1}{\lambda_B} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4} \implies \lambda_B = \frac{4}{R}$$

3. Find the ratio of λ_B to λ_L:

$$\text{Ratio} = \frac{\lambda_B}{\lambda_L} = \frac{4/R}{1/R} = 4$$

4. Therefore, the ratio is 4:1.

Final Answer: The ratio of the wavelengths is 4:1.

Answer: (A)

Q40.

Solution

Concept:
Radioactive decay changes the identity of a nucleus: - α-decay: Mass number (A) decreases by 4, atomic number (Z) decreases by 2. - β-decay (specifically β^-): Mass number remains the same, atomic number increases by 1.

Solution:
1. Initial nucleus A: ($A = 180, Z = 72$). 2. First step ($A \xrightarrow{\alpha} A_1$): $A_1 = 180 - 4 = 176$ $Z_1 = 72 - 2 = 70$ 3. Second step ($A_1 \xrightarrow{\beta} A_2$): $A_2 = 176$ $Z_2 = 70 + 1 = 71$ 4. Third step ($A_2 \xrightarrow{\alpha} A_3$): $A_3 = 176 - 4 = 172$ $Z_3 = 71 - 2 = 69$ 5. The final nucleus A_3 has mass number 172 and atomic number 69.

Final Answer: The mass number and atomic number of A_3 are 172 and 69.

Answer: (A)

Q41.

Solution

Concept:

In a Common Emitter (CE) amplifier, the voltage gain (A_v) is defined as the ratio of the change in output voltage (collector voltage V_c) to the change in input voltage (base voltage V_i). It can also be expressed in terms of the current gain (β) and the ratio of output resistance (R_c) to input resistance (R_b):

$$A_v = \frac{V_{out}}{V_{in}} = \beta \frac{R_c}{R_b}$$

Solution:

1. Identify the given values: Output signal voltage $V_{out} = 2$ V Collector resistance $R_c = 2$ kΩ $= 2000$ Ω Base resistance $R_b = 1$ kΩ $= 1000$ Ω Current amplification factor $\beta = 100$ 2. Calculate the voltage gain A_v:

$$A_v = \beta \times \frac{R_c}{R_b} = 100 \times \frac{2000}{1000}$$

$$A_v = 100 \times 2 = 200$$

3. Use the gain formula to find the input signal voltage V_{in}:

$$V_{in} = \frac{V_{out}}{A_v}$$

4. Substitute the values:

$$V_{in} = \frac{2}{200} = \frac{1}{100} = 0.01 \text{ V}$$

Final Answer: The input signal voltage is 0.01 V.

Answer: (A)

Q42.

Solution

Concept:

NOR: output = 1 only when all inputs are 0. NAND: output = 0 only when all inputs are 1.

Solution:

1. Given:

$$(A, B) = (0, 0) \to 1, \quad (1, 1) \to 0$$

2. Check gates: - OR: $(0, 0) \to 0$ - NOR: $(0, 0) \to 1$, $(1, 1) \to 0$ - NAND: also satisfies both
3. Distinction: NOR gives output 0 whenever any input is 1, matching the intended pattern.

Final Answer: NOR gate

Answer: (B)

Q43.

Solution

Concept:
The current-voltage relationship for a p-n junction diode is given by the diode equation:

$$I = I_0(\exp(eV/kT) - 1)$$

where I_0 is the reverse saturation current, e is the charge of an electron, V is the applied voltage, k is Boltzmann's constant, and T is the temperature.

Solution:
1. In reverse bias, the voltage V is applied in the negative direction, so V is a negative value. 2. If V is significantly negative (several times kT/e), the exponential term becomes very small:

$$\exp(eV/kT) \longrightarrow \exp(-\text{large value}) \approx 0$$

3. Substitute this approximation into the diode equation:

$$I = I_0(0 - 1)$$

$$I = -I_0$$

4. This indicates that the current is constant, very small, and flows in the opposite direction. It is known as the reverse saturation current.
Final Answer: In reverse bias, the current is approximately $-I_0$ (near zero).

Answer: (A)

Q44.

Solution

Concept:

The total reading of a screw gauge is the sum of the Main Scale Reading (MSR) and the Circular Scale Reading (CSR). The CSR is calculated by multiplying the coinciding division (n) by the Least Count (LC).

$$\text{Least Count (LC)} = \frac{\text{Pitch}}{\text{Number of divisions}}$$

$$\text{Total Reading} = \text{MSR} + (n \times \text{LC})$$

Solution:

1. Calculate the Least Count (LC):

$$\text{LC} = \frac{0.5 \text{ mm}}{50} = 0.01 \text{ mm}$$

2. Identify the given readings: Main Scale Reading (MSR) = 2.5 mm Coinciding circular division $n = 20$ 3. Calculate the Circular Scale Reading (CSR):

$$\text{CSR} = 20 \times 0.01 \text{ mm} = 0.20 \text{ mm}$$

4. Calculate the total thickness:

$$\text{Total Thickness} = \text{MSR} + \text{CSR}$$

$$\text{Total Thickness} = 2.5 \text{ mm} + 0.20 \text{ mm} = 2.70 \text{ mm}$$

Final Answer: The thickness of the glass plate is 2.70 mm.

Answer: (A)

Q45.

Solution

Concept:

When reporting a measured value with errors, we first find the arithmetic mean of the measurements. Then, we find the absolute error for each measurement, the mean absolute error, and finally compare it with the least count of the instrument. The reported error is usually the maximum of the mean absolute error or the least count.

Solution:

1. Calculate the mean time ($\bar{t}$):

$$\bar{t} = \frac{90 + 91 + 95 + 92}{4} = \frac{368}{4} = 92 \text{ s}$$

2. Calculate the absolute errors ($|\Delta t_i| = |t_i - \bar{t}|$): $|\Delta t_1| = |90 - 92| = 2$ s $|\Delta t_2| = |91 - 92| = 1$ s $|\Delta t_3| = |95 - 92| = 3$ s $|\Delta t_4| = |92 - 92| = 0$ s 3. Calculate the mean absolute error ($\Delta\bar{t}$):

$$\Delta\bar{t} = \frac{2 + 1 + 3 + 0}{4} = \frac{6}{4} = 1.5 \text{ s}$$

4. Rounding off the error: Since the least count of the clock is 1 s, the error should be reported to the same precision or rounded to 2 s to be conservative in this specific experimental context. Looking at the data spread, 92 ± 2 s is the standard scientific representation for this set.

Final Answer: The reported mean time should be 92 ± 2 s.

Answer: (A)

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	A	4	B	5	B
6	A	7	A	8	B	9	C	10	A
11	C	12	C	13	A	14	A	15	A
16	A	17	A	18	B	19	A	20	A
21	B	22	A	23	B	24	A	25	A
26	B	27	A	28	B	29	A	30	A
31	A	32	A	33	A	34	A	35	A
36	A	37	A	38	A	39	A	40	A
41	A	42	B	43	A	44	A	45	A