



General Instructions

- (i) The total marks for the examination is 240 marks to be attempted in 3 hours.
- (ii) The question paper consists of 80 questions, divided into 4 sections- Biology, Chemistry, Mathematics, and Physics each having 20 questions.
- (iii) Each correct answer carries 3 marks and an incorrect answer has a negative marking of 1.
- (iv) Please note: The sequence of questions and options were different in the examination.

Biology

1. Mendel's law of independent assortment implies that

- (A) alleles segregate independent of one another during gamete formation.
- (B) alleles segregate independent of their location on the chromosome during gamete formation.
- (C) traits are encoded by independent genes.
- (D) traits are always encoded by different chromosomes.

2. Bacteria have cell wall made of a polymer of N-acetylglucosamine and N-acetylmuramic acid connected by β -1,4 glycosidic bonds. Lysozyme, an enzyme, cleaves the glycosidic bond and disrupts the cell wall structure. In an experiment, Gram-positive and Gram-negative bacteria, which differ in terms of cell envelope and peptidoglycan thickness, are treated with lysozyme and subsequently placed in a hypotonic solution. The correct statement about the effect of the treatment is:

- (A) Gram-positive bacteria are lysed, but not Gram-negative bacteria.
 - (B) Both Gram-positive and Gram-negative bacteria are lysed.
 - (C) Only Gram-negative bacteria are lysed.
 - (D) Neither Gram-positive nor Gram-negative bacteria are lysed.
-

3. The fate of a cell in which pyruvate dehydrogenase is completely inhibited will be:

- (A) Glycolysis continues with NAD^+ regeneration via lactate fermentation, yielding 2 ATP per glucose molecule.
 - (B) Pyruvate accumulates and enters the Krebs cycle via an alternative pathway, yielding 32 ATP per glucose molecule.
 - (C) Glycolysis stops due to NAD^+ depletion, and no ATP is produced.
 - (D) Pyruvate is converted to acetyl CoA, yielding 8 ATP per glucose molecule via fermentation.
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4. Two spherical unicellular organisms, X and Y, have the same cytoplasmic composition, metabolic pathways, and live in identical nutrient-rich aerobic conditions. The diameters of X and Y are, respectively, 10 and 20 μm . When nutrients are not limiting, the ratio of total oxygen consumption per hour per cell of organisms Y to X is:

- (A) 8:1
 - (B) 1:2
 - (C) 2:1
 - (D) 4:1
-

5. In an experiment, bacteria were grown for 500 generations in a medium containing ^{14}N (light isotope), then transferred to a medium with ^{15}N (heavy isotope) for one generation, and finally transferred back to the ^{14}N medium for one more generation. Assuming all cells divide synchronously and replication is semi-conservative, the ratio of $^{14}\text{N}^{15}\text{N}$ to $^{14}\text{N}^{14}\text{N}$ double-stranded DNA at the end of the experiment is:

- (A) 1:1
 - (B) 1:2
 - (C) 1:3
 - (D) 1:4
-

6. In the roots of plants, casparian strip made of suberin forms a ring around the cells of the endodermis. It effectively blocks the entry of water and mineral ions into the stele via the apoplast. A defect in the casparian strip leads to

- (A) increased solute leakage and reduced root hydraulic conductivity.
 - (B) decreased solute leakage and increased root hydraulic conductivity.
 - (C) increased solute leakage and increased root hydraulic conductivity.
 - (D) decreased solute leakage and reduced root hydraulic conductivity.
-

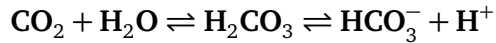
7. The ability of phloem to distribute sugar depends on the companion cell and sieve element working as a complex. Consider the following statements with regard to possible effects of reduction in companion cell plasmodesmata.

- (i) Sugar loading into sieve tubes reduces due to collapse in osmotic pressure gradient.**
- (ii) Water moves from companion cells into the xylem to maintain osmotic balance.**
- (iii) The sieve tubes collapse structurally due to loss of turgor pressure.**
- (iv) Reverse loading of sugar in phloem begins immediately due to pressure inversion.**

Under the given condition, the correct statements are:

- (A) (i) and (iii)
 - (B) (i) and (ii)
 - (C) (iii) and (iv)
 - (D) (ii) and (iv)
-

8. Carbonic anhydrase catalyzes the following reaction in red blood cells.



This reaction acts as a buffering system in blood. Consequence of failure of the enzyme to catalyze the reverse reaction is:

- (A) Blood becomes acidic and the affinity of haemoglobin for O_2 decreases.
 - (B) The pCO_2 increases and the affinity of haemoglobin for CO_2 increases.
 - (C) The pO_2 of the blood decreases and the affinity of haemoglobin for O_2 increases.
 - (D) Blood becomes acidic and the formation of carbamino-haemoglobin decreases.
-

9. Events occurring during signal propagation in a chemical synapse are given below.

P: Postsynaptic membrane potential changes.

Q: Synaptic vesicle fuses with the presynaptic membrane.

R: Action potential depolarizes the presynaptic terminal.

S: Neurotransmitter binds to receptors on the postsynaptic membrane.

T: Neurotransmitter is released into the synaptic cleft.

The correct sequence of occurrence of these events is:

- (A) R, Q, T, S, P
 - (B) Q, R, T, S, P
 - (C) R, T, Q, P, S
 - (D) Q, R, P, S, T
-

10. In a population of 1000 moths, 450 moths were brown-colored with homozygous genotype and 350 albino moths that were homozygous for the recessive allele. Assuming random mating within the population, the frequency of heterozygotes in the next generation would be:

- (A) 0.495
- (B) 0.200
- (C) 0.800

(D) 0.505

11. In a haplodiploid ant species, sons are haploid and produced from unfertilized eggs, while daughters are diploid and produced from fertilized eggs. A single queen establishes a colony after mating with three different unrelated males. The queen uses the sperm from all three males equally to produce her female offspring. The average genetic relatedness between two randomly selected female offspring in this colony will be:

(A) 0.417

(B) 0.500

(C) 0.750

(D) 0.333

12. In the axon of a giant squid neuron,

(i) The resting membrane potential is -70 mV.

(ii) At the equilibrium potential of Na^+ ($E_{\text{Na}^+} = +55 \text{ mV}$) there is no net movement of Na^+ ions across the membrane.

In an experiment, when the axon is stimulated, the voltage-gated Na^+ channels open. The membrane potential peaks at +30 mV, where the resistance of the axonal membrane for Na^+ flow is $1 \times 10^6 \Omega$. The net Na^+ current (I_{Na^+}) across the membrane and the direction of ionic movement at the peak is:

(A) $I_{\text{Na}^+} = 25 \text{ nA}$; direction = into the cell

(B) $I_{\text{Na}^+} = 100 \text{ nA}$; direction = out of the cell

(C) $I_{\text{Na}^+} = 25 \text{ nA}$; direction = out of the cell

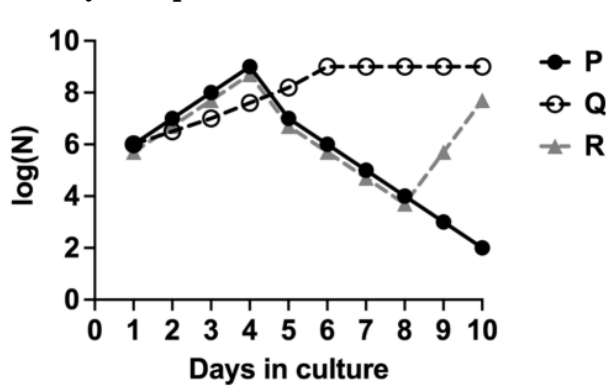
(D) $I_{\text{Na}^+} = 100 \text{ nA}$; direction = into the cell

13. Consider a mitochondrion in which the F_0 subunit of ATP synthase is mutated, such that it can still passively conduct protons but is mechanically uncoupled from the F_1 subunit. If such a mitochondrion is provided with succinate, ADP, and Pi in aerobic conditions, the correct

outcome based on chemiosmotic theory is:

- (A) Continuous oxygen consumption and no ATP synthesis.
- (B) No oxygen consumption and no ATP synthesis.
- (C) Rapid initial oxygen consumption that plateaus at a low rate and normal ATP synthesis.
- (D) Continuous oxygen consumption and normal ATP synthesis.

14. Three different bacterial strains (P, Q, and R) when grown independently in a culture medium showed typical growth curves. Equal number of cells from all three actively growing cultures are mixed and grown together for 10 days. The number of bacteria (N) are counted each day and plotted.



Assuming nutrients are not limiting, the correct explanation for the observed trend is:

- (A) Q is an antibiotic producer, P is antibiotic sensitive, and R is antibiotic sensitive but evolves resistance during the course of the experiment.
- (B) Q is an antibiotic producer, P is antibiotic sensitive, and R is antibiotic resistant.
- (C) R is an antibiotic producer, P is antibiotic sensitive, and Q is antibiotic resistant.
- (D) P is an antibiotic producer, Q is antibiotic resistant, and R is antibiotic sensitive but evolves resistance during the course of the experiment.

15. There is a difference between the predicted and observed global carbon budget. The possible reason is:

- (A) Higher CO₂ levels stimulate plant growth and productivity, leading to more carbon being se-

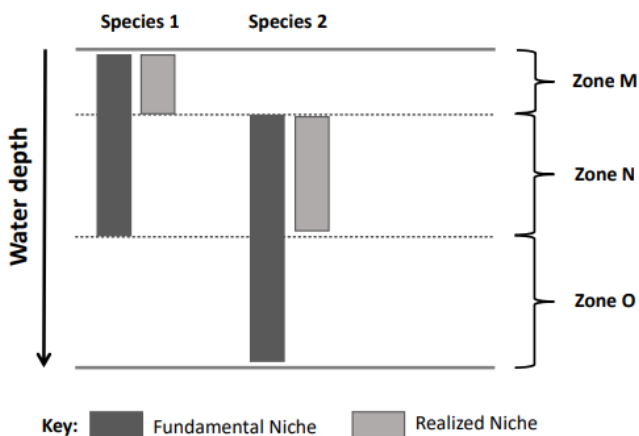
questered in plant tissues and root systems.

(B) Higher CO₂ levels lead to increased soil respiration rates, releasing more carbon into the atmosphere.

(C) Oceans are absorbing more carbon due to increased temperature, resulting in increased acidification.

(D) The thinning of the stratospheric ozone layer allows higher levels of ultraviolet radiation directly breaking down CO₂ molecules.

16. The fundamental niche of a species is the full range of environmental conditions and resources a species can potentially occupy in the absence of limitations. The realized niche is the actual, smaller subset of the fundamental niche that the species occupies due to biotic limitations. The distribution of populations of Species 1 and Species 2 over various levels towards the shore of an aquatic habitat is shown in the figure.



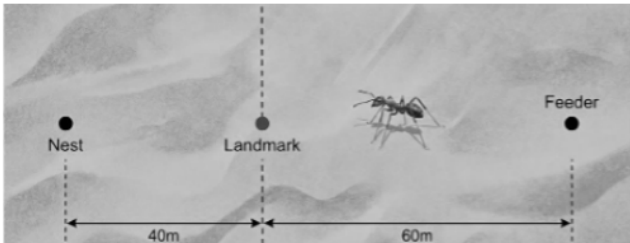
Consider the following statements based on the distribution.

- (i) Competition for resources between Species 1 and Species 2 is mainly in the zones M and N.
- (ii) Species 2 is a stronger competitor as compared to Species 1.
- (iii) In absence of Species 2, Species 1 could prevail in zone O.
- (iv) Species 2 probably has predators who thrive in zone O.

The correct statement(s) is(are):

- (A) (ii) and (iv)
- (B) (i) only
- (C) (ii) only
- (D) (i) and (iii)

17. Desert ants are known to use landmarks (reference points), pedometers (memorising step counts), and optic flow (rate of passing objects across its eye to estimate navigation speed) to navigate across deserts. A desert ant is trained to walk 100 m from its nest to a feeder in an artificial arena as depicted in the schematic. The arena had a single tall landmark placed at 40 m from the nest.



At the feeder, certain manipulations were performed before allowing the ant to return to its nest with the food.

Manipulation 1: Stilts were attached to its legs, which doubled the step length.

Manipulation 2: The optic flow of the arena was doubled.

While the ant uses the landmark exclusively for long-distance (>55 m) navigation, it switches to optic flow and pedometer simultaneously and exclusively for short-distance navigation. Due to the manipulations, the distance from the feeder where the ant will search its nest is:

- (A) 110 m
- (B) 100 m
- (C) 120 m
- (D) 160 m

18. The table shows the presence (+) / absence (-) of features 1 - 5 for animals P - U.

	P	Q	R	S	T	U
1	-	-	-	-	-	+
2	-	-	-	-	+	+
3	-	-	-	+	+	+
4	-	-	+	+	+	+
5	-	+	+	+	+	+

Based on the table, the correct statement is:

- (A) If Q and R represent lamprey and fish respectively, then feature 4 could be jaws.
- (B) Feature 5 is the most recently evolved trait.
- (C) If S, T and U are respectively amphibian, reptile and mammal, then feature 3 could be amniotic egg.
- (D) If S, T and U respectively are salamander, turtle and leopard, then features 1 and 2 could respectively represent hair and four legs.

19. The trophic relationships between species in a habitat can be represented in the form of a food web matrix where, rows represent consumers/ predators, and columns represent resources/prey. If the organism of a species in the row eats the organism of a particular species in the column, then a '1' is placed in the intersection. Otherwise a '0' is placed in the intersection. Such a matrix for species M - Q is given below.

		Resources/Prey				
		M	N	O	P	Q
Consumers/Predators	M	0	0	0	0	0
	N	1	0	0	0	0
	O	1	1	0	0	0
	P	0	1	1	0	0
	Q	1	0	0	0	0

Based on the matrix, the correct statement is:

- (A) Species P is both a secondary and tertiary consumer.
- (B) Scarcity of species M can directly affect primary consumers O and P.
- (C) There are two producer species in this ecosystem.
- (D) Species N is an omnivore.

20. A researcher wants to insert a linear DNA fragment into a plasmid DNA. The linear DNA fragment has two MfeI restriction sites at its ends. The plasmid DNA has only one EcoRI restriction site. The plasmid was digested with EcoRI and the DNA fragment was digested with MfeI. The recognition sequences of the restriction enzymes on one strand are given.

EcoRI site: 5' - G ↓ AATTC - 3'

MfeI site: 5' - C ↓ AATTG - 3'

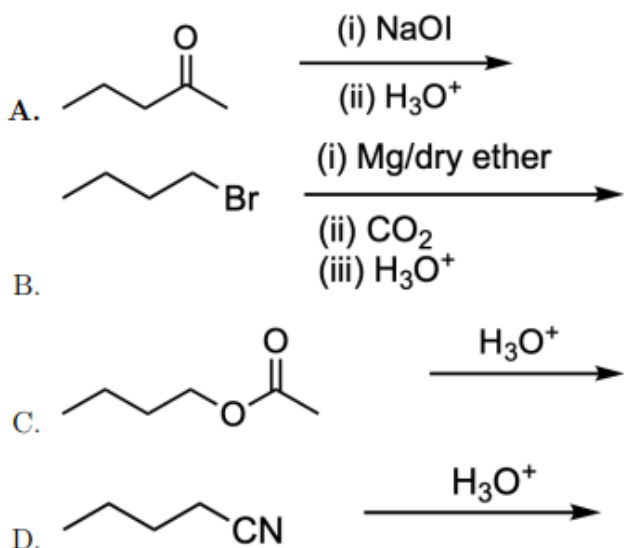
(arrow indicates enzyme cut sites)

Based on this information, the DNA fragments generated by EcoRI and MfeI

- (A) can ligate to each other, but the resultant product can neither be digested by EcoRI nor MfeI.
(B) cannot ligate to each other because the overhangs generated are from two different enzymes.
(C) can ligate to each other, and the resultant product can be digested by EcoRI but not MfeI.
(D) can ligate to each other, and the resultant product can be digested by MfeI but not EcoRI.

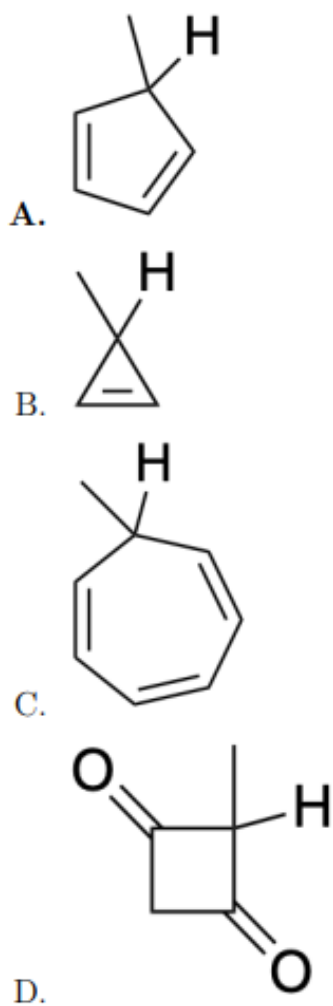
Chemistry

21. The reaction that produces butanoic acid as a product is:



- (A) A
(B) B
(C) C
(D) D

22. The compound that produces aromatic species upon deprotonation of the indicated hydrogen is:



- (A) 5-methylcyclopentadiene
 (B) 3-methylcyclopropene
 (C) 7-methylcycloheptatriene
 (D) cyclobutane-1,3-dione derivative

23. Two students (P and Q) independently made three measurements (denoted by I, II, and III) of the same property. The expected correct value is 9.0. The observed values by the students are provided in the given table.

Student	I	II	III	Average
P	8.9	9.1	9.0	9.0
Q	8.0	9.0	10.0	9.0

Based on the given data, the correct statement is:

- (A) Both P and Q are equally accurate but P is more precise than Q.
(B) P is more accurate but less precise than Q.
(C) P is less accurate but more precise than Q.
(D) Both P and Q are equally precise but P is more accurate than Q.
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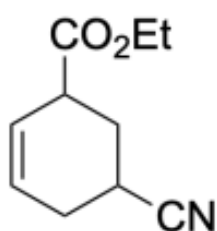
24. Species that are both isoelectronic and isostructural are:

- (A) NH_4^+ and BH_4^-
(B) O_3 and NO_2^+
(C) N_2O and NO_2
(D) NH_2^- and BH_4^-
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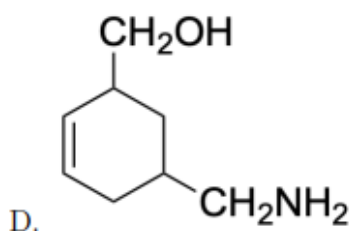
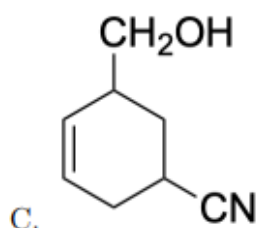
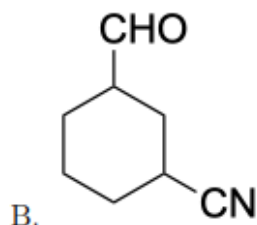
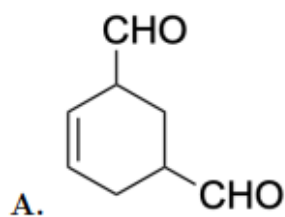
25. Two galvanic cells, Cell I and Cell II, operate at the same temperature and involve two different redox systems. It is observed that at a particular value of the reaction quotient $Q_I = Q_{II} = Q_0$, the ratio of EMF of the two cells (E_I/E_{II}) as well as the ratio of change of Gibbs free energy for the two cells ($\Delta G_I/\Delta G_{II}$) are both 1/3. Assuming standard cell reactions written in the spontaneous direction and reversible operation, the pair of cells can satisfy these conditions is:

- (A) Cell I: $\text{Zn(s)}|\text{Zn}^{2+}(\text{aq})||\text{Ag}^+(\text{aq})|\text{Ag(s)}$ and Cell II: $\text{Zn(s)}|\text{Zn}^{2+}(\text{aq})||\text{Cu}^{2+}(\text{aq})|\text{Cu(s)}$
(B) Cell I: $\text{Ag}^+(\text{aq})|\text{Ag(s)}||\text{Fe}^{2+}(\text{aq})|\text{Fe}^{3+}(\text{aq})$ and Cell II: $\text{Al(s)}|\text{Al}^{3+}(\text{aq})||\text{Ag}^+(\text{aq})|\text{Ag(s)}$
(C) Cell I: $\text{Zn(s)}|\text{Zn}^{2+}(\text{aq})||\text{Cu}^{2+}(\text{aq})|\text{Cu(s)}$ and Cell II: $\text{Al(s)}|\text{Al}^{3+}(\text{aq})||\text{Ag}^+(\text{aq})|\text{Ag(s)}$
(D) Cell I: $\text{Fe}^{2+}(\text{aq})|\text{Fe}^{3+}(\text{aq})||\text{Cu}^{2+}(\text{aq})|\text{Cu}^+(\text{aq})$ and Cell II: $\text{Cu(s)}|\text{Cu}^{2+}(\text{aq})||\text{Ag}^+(\text{aq})|\text{Ag(s)}$
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26. Consider the following compound.

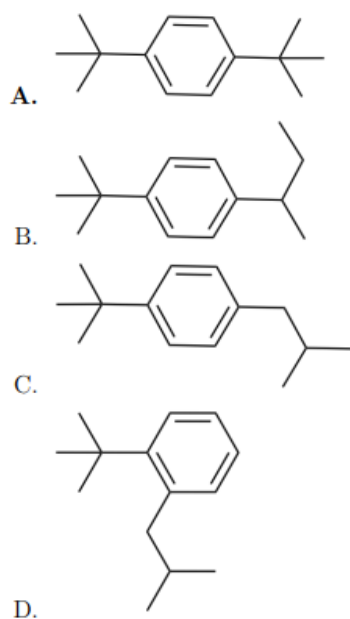


The compound undergoes reaction with two equivalents of diisobutyl aluminium hydride followed by acid hydrolysis to provide P as the major product. The structure of P is:



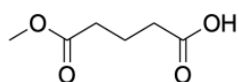
- (A) Structure with both functional groups reduced to $-\text{CHO}$.
(B) Structure with ester reduced to $-\text{CHO}$ and nitrile remaining intact as $-\text{CN}$.
(C) Structure with ester reduced to $-\text{CH}_2\text{OH}$ and nitrile remaining intact as $-\text{CN}$.
(D) Structure with ester reduced to $-\text{CH}_2\text{OH}$ and nitrile reduced to $-\text{CH}_2\text{NH}_2$.

27. *tert*-Butylbenzene reacts with 1-chloro-2-methylpropane in the presence of anhydrous AlCl_3 . The major product in this reaction is:

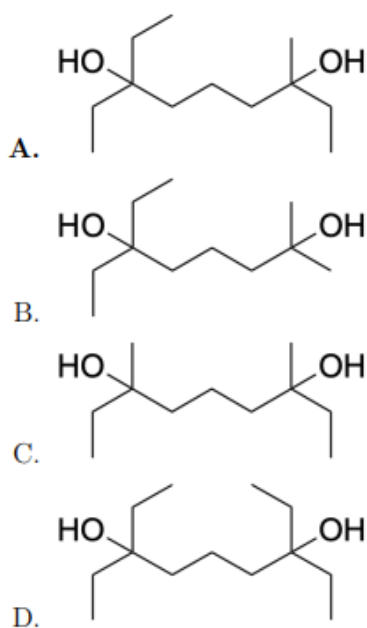


- (A) 1,4-di-tert-butylbenzene
 (B) 1-tert-butyl-4-isobutylbenzene
 (C) 1-tert-butyl-4-sec-butylbenzene
 (D) 1,2-di-tert-butylbenzene

28. Consider the following compound.

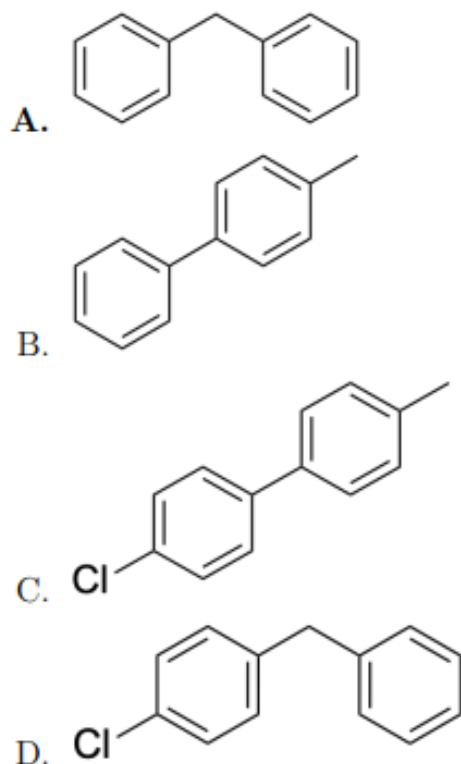


The compound reacts with SOCl_2 to produce P. P reacts with dimethyl cadmium to produce Q. Q upon reaction with ethyl magnesium bromide (3 equiv) followed by water workup produces R. The structure of R is:



- (A) Structure with one tertiary alcohol containing two ethyl groups and another containing one methyl and one ethyl group.
- (B) Structure with one secondary alcohol and one tertiary alcohol.
- (C) Structure with two tertiary alcohols containing only methyl groups.
- (D) Structure with two tertiary alcohols containing only ethyl groups.

29. Acetylene is passed through red hot iron tube at 873 K to produce P. P reacts with chlorine gas in the presence of anhydrous AlCl_3 to produce Q. Q reacts with benzyl chloride and sodium to produce R as the major product. The structure of R is:



- (A) diphenylmethane
 (B) biphenyl
 (C) 4-chlorobiphenyl
 (D) benzyl 4-chlorobenzoate

30. 1,2-dimethylbenzene on treatment with chromic oxide in acetic anhydride at 273-283 K followed by hydrolysis produces P. P on heating with concentrated NaOH followed by hydrolysis provides Q. The functional groups present in the product Q are:

- (A) One carboxylic acid and one alcohol
 (B) Two aldehydes
 (C) One aldehyde and one carboxylic acid
 (D) Two carboxylic acids

31. The increasing order of acidic strength for the aqua complexes $[V(H_2O)_6]^{2+}$, $[Fe(H_2O)_6]^{2+}$,

$[\text{Fe}(\text{H}_2\text{O})_6]^{3+}$, and $[\text{Co}(\text{H}_2\text{O})_6]^{3+}$ is:

- (A) $[\text{V}(\text{H}_2\text{O})_6]^{2+} < [\text{Fe}(\text{H}_2\text{O})_6]^{2+} < [\text{Fe}(\text{H}_2\text{O})_6]^{3+} < [\text{Co}(\text{H}_2\text{O})_6]^{3+}$
(B) $[\text{Fe}(\text{H}_2\text{O})_6]^{2+} < [\text{Fe}(\text{H}_2\text{O})_6]^{3+} < [\text{V}(\text{H}_2\text{O})_6]^{2+} < [\text{Co}(\text{H}_2\text{O})_6]^{3+}$
(C) $[\text{Co}(\text{H}_2\text{O})_6]^{3+} < [\text{Fe}(\text{H}_2\text{O})_6]^{3+} < [\text{Fe}(\text{H}_2\text{O})_6]^{2+} < [\text{V}(\text{H}_2\text{O})_6]^{2+}$
(D) $[\text{Fe}(\text{H}_2\text{O})_6]^{2+} < [\text{Fe}(\text{H}_2\text{O})_6]^{3+} < [\text{Co}(\text{H}_2\text{O})_6]^{3+} < [\text{V}(\text{H}_2\text{O})_6]^{2+}$

32. Consider the following table of three lanthanoid ions X, Y, and Z and their properties.

Lanthanoid Ions	Properties
X	Diamagnetism
Y	Reducing agent in solution
Z	Empty 4f orbitals

Atomic numbers of Ce, Eu, and Lu are 58, 63, and 71, respectively. Given these atomic numbers, the lanthanoid ions X, Y, and Z, respectively, are:

- (A) Lu^{3+} , Eu^{2+} , and Ce^{4+}
(B) Lu^{3+} , Ce^{4+} , and Eu^{2+}
(C) Eu^{2+} , Ce^{4+} , and Lu^{3+}
(D) Eu^{2+} , Lu^{3+} , and Ce^{4+}

33. Consider the following complex ions:

$[\text{Ni}(\text{CN})_4]^{2-}$, $[\text{Fe}(\text{CN})_6]^{3-}$, $[\text{Cu}(\text{H}_2\text{O})_6]^{2+}$, and $[\text{Co}(\text{CN})_6]^{3-}$.

The complex ions that are expected to show diamagnetic behavior at room temperature are:

- (A) $[\text{Ni}(\text{CN})_4]^{2-}$ and $[\text{Co}(\text{CN})_6]^{3-}$
(B) $[\text{Ni}(\text{CN})_4]^{2-}$ and $[\text{Cu}(\text{H}_2\text{O})_6]^{2+}$
(C) $[\text{Co}(\text{CN})_6]^{3-}$ and $[\text{Fe}(\text{CN})_6]^{3-}$
(D) $[\text{Cu}(\text{H}_2\text{O})_6]^{2+}$ and $[\text{Co}(\text{CN})_6]^{3-}$

34. Consider the following species:

- (i) ClO^-
- (ii) ClO_2^-
- (iii) ClO_3^-
- (iv) ClO_4^-

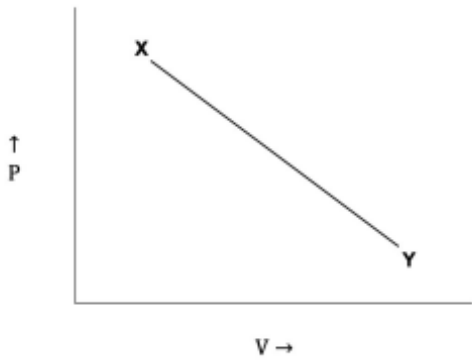
Among them, the species that undergoes (undergo) disproportionation reactions is (are):

- (A) (i), (ii), and (iii) only
- (B) (iv) only
- (C) (i) and (ii) only
- (D) (iii) and (iv) only

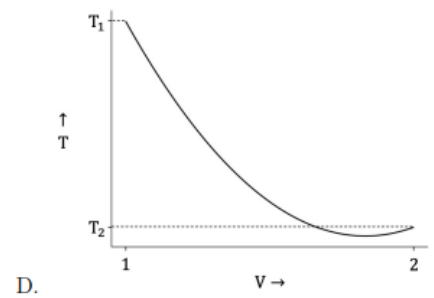
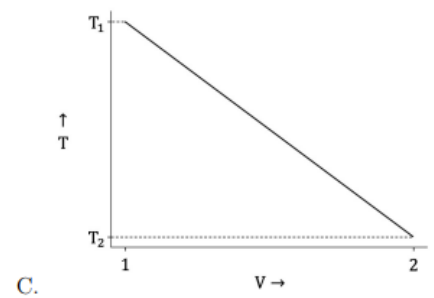
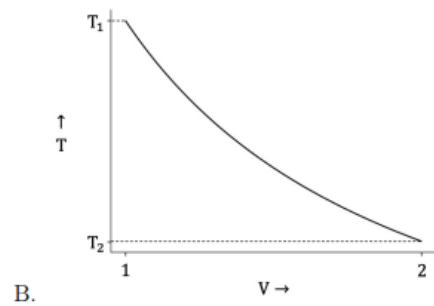
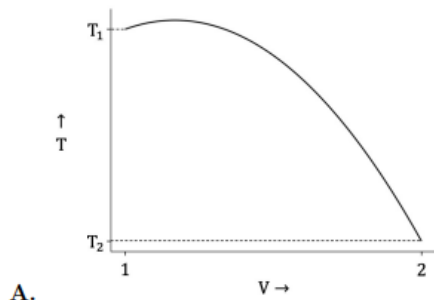
35. X, Y, and Z are three volatile liquids, which when mixed make an ideal solution. At a given temperature T_0 , the constituents X, Y, and Z have pure vapour pressures in the ratio 3:2:1. A mixture prepared with mole-fractions of X, Y, and Z in the ratio 1:2:3 starts to boil at temperature T_0 at 1.5 atm pressure. At temperature T_0 , the partial vapour pressure (in atm) of Y is:

- (A) $\frac{3}{5}$
- (B) $\frac{9}{10}$
- (C) $\frac{1}{6}$
- (D) $\frac{9}{20}$

36. Consider a reversible thermodynamic process involving one mole of ideal gas represented by a line on the $P - V$ diagram connecting two states X (4 bar, 1 L, T_1) and Y (1 bar, 2 L, T_2) as shown in the figure:



During the process the change in temperature (in K) as a function of volume (in L) is best represented as:

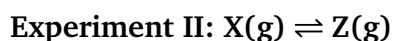
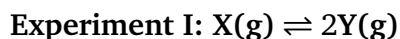


- (A) A parabolic curve opening downwards with a maximum temperature.
- (B) A hyperbolic curve decreasing monotonically.
- (C) A straight line with a negative slope.
- (D) A parabolic curve opening upwards with a minimum temperature.
-

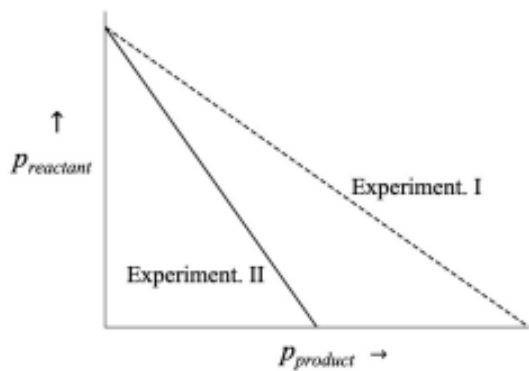
37. PQ_2 is a sparingly soluble salt with solubility product $K_{sp} = 4 \times 10^{-12}$ in aqueous medium at some given temperature. It is observed that upon addition of a highly soluble salt RQ at the same temperature, the solubility of PQ_2 drops by a factor of 100. The concentration (in millimoles per litre) of added RQ in the solution is closest to:

- (A) 2
- (B) 0.2
- (C) 20
- (D) 0.02
-

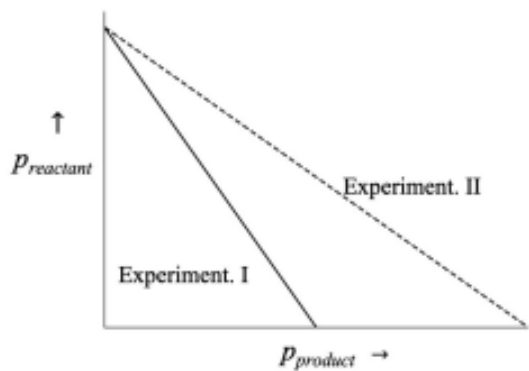
38. Consider the following chemical reactions performed at identical temperature and volume:



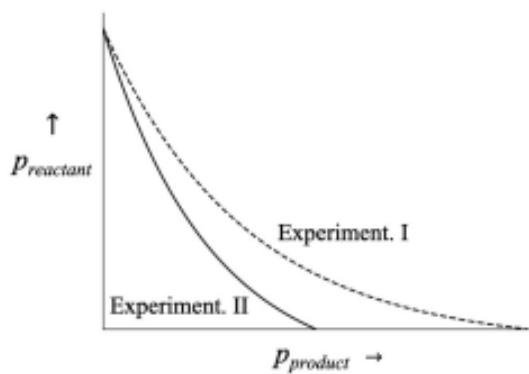
The partial pressure of the reactant and product are denoted, respectively, by p_{reactant} and p_{product} during the course of the reactions. Assuming ideal gas behaviour, the correct plot of p_{reactant} versus p_{product} for Experiments I and II is:



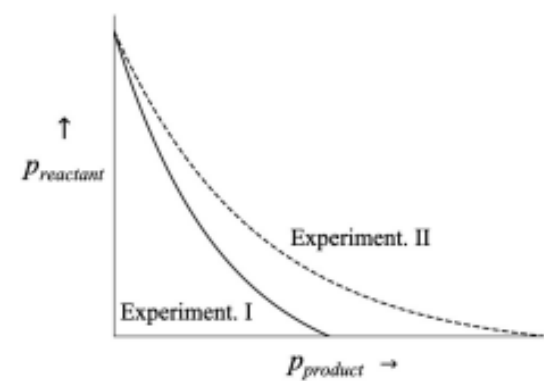
A.



B.



C.



D.

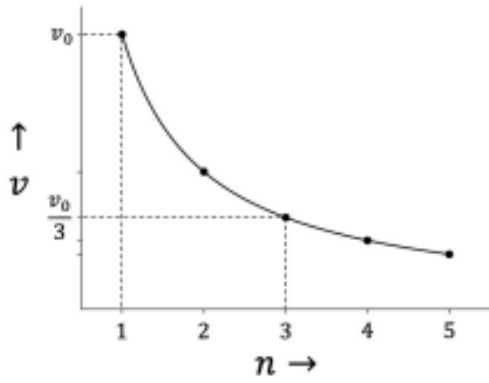
(A) A

(B) B

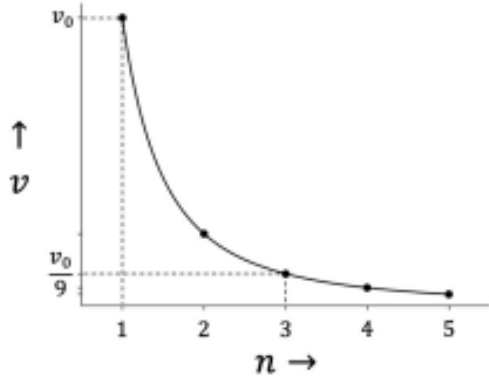
(C) C

(D) D

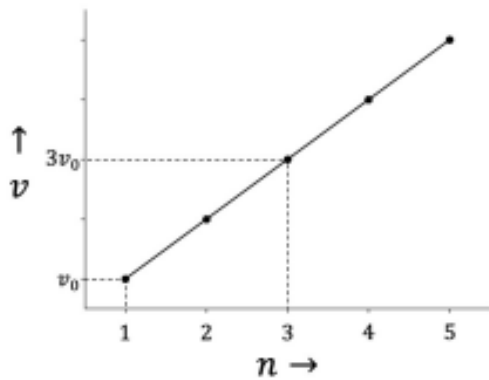
39. If velocity of an electron in the first Bohr orbit is denoted by v_0 , then the velocity (v) of the electron in other orbits (as a function of principle quantum number 'n') is represented as:



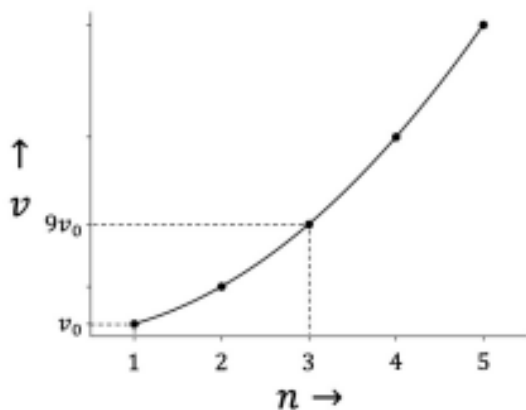
A.



B.



C.



D.

- (A) A
- (B) B
- (C) C
- (D) D

40. Consider two reactions.

I. A zero order reaction: $X \rightarrow P_1$.

II. A first order reaction: $Y \rightarrow P_2$.

Both reactions begin simultaneously with the same initial concentration 'C', that is $X_0 = Y_0 = C$.

It is observed that the concentrations of the reactants fall to one-third of their initial values at the same time, provided the rate constants satisfy a certain ratio. If k_0 and k_1 denote, respectively, the zero and first order rate constants then the ratio k_0/k_1 necessary for this condition to be satisfied is:

- (A) $\frac{2C}{3 \ln 3}$
- (B) $\frac{C}{\ln 3}$
- (C) $\frac{C}{\ln 2}$
- (D) $\frac{3C}{2 \ln 2}$

41. A possible solution of the system of equations

$$x^2 - 8xy + 16y^2 = 0$$

$$(\log_{10} x)^2 + 2(\log_{10} x)(\log_{10} y) + (\log_{10} y)^2 = 4$$

is

- (A) $x = \frac{1}{5}, y = \frac{1}{20}$
 - (B) $x = 100, y = 25$
 - (C) $x = 40, y = 10$
 - (D) $x = \frac{4}{25}, y = \frac{1}{16}$
-

42. The parabola $y = -x^2 + 16$ intersects the x -axis at points A and B. Further, the parabola intersects the line $y = 7$ at points M and N. Then the area of the quadrilateral with vertices A, B, M and N is

- (A) 49
 - (B) 42
 - (C) 56
 - (D) 35
-

43. The domain of the real function $f(x) = \sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right)$ is

- (A) $[-2, -1] \cup [1, 2]$
 - (B) $[-2, -1]$
 - (C) $[1, 2]$
 - (D) $[-2, 2]$
-

44. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a twice differentiable function such that $f''(x) = -f(x)$. Let $h : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that $h'(x) = f(x)^2 + (f'(x))^2$ for all $x \in \mathbb{R}$. If $h(0) = 1$ and

$h\left(\frac{1}{3}\right) = \frac{5}{3}$, then the value of $h(100)$ is

- (A) 201
 - (B) 101
 - (C) 100
 - (D) 202
-

45. Suppose $\lim_{x \rightarrow -2} \frac{bx^2 + 15x + 15 + b}{x^2 + x - 2} = L$, where b and L are real numbers. Then

- (A) $b = 3$ and $L = -1$
 - (B) $b = -3$ and $L = -1$
 - (C) $b = 3$ and $L = 1$
 - (D) $b = -3$ and $L = 1$
-

46. If $x \in (0, 1)$ and $\sin^{-1} x - \sin^{-1} \frac{1}{4} = \frac{\pi}{3}$, then x equals

- (A) $\frac{1}{8}(1 + 3\sqrt{5})$
 - (B) $\frac{1}{6}(1 + 2\sqrt{5})$
 - (C) $\frac{1}{6}(1 + 4\sqrt{3})$
 - (D) $\frac{1}{5}(2 + 3\sqrt{2})$
-

47. Let $S = \{(x, y) \mid x, y \in \mathbb{N}, 1 \leq x \leq 15, 1 \leq y \leq 20\}$ be a set. Let \mathcal{R} be the equivalence relation on S defined by $(x, y)\mathcal{R}(x', y')$ if and only if $x + y = x' + y'$. Then the number of equivalence classes of \mathcal{R} on S is

- (A) 34
- (B) 35
- (C) 15
- (D) 20

48. For a complex number $z = x + iy$, where $x, y \in \mathbb{R}$, denote $\hat{z} = y + ix$. The locus of z satisfying $|z + \hat{z}| = |z - \hat{z}|$ in the complex plane is

- (A) union of the real axis and the imaginary axis
- (B) the real axis
- (C) a circle
- (D) the straight line $y = x$

49. Let m and p be real numbers such that the polynomial $f(x) = x^2 + mx + p$ has two distinct negative rational roots. Then the polynomial $g(x) = x^2 - (m^2 - 2p)x + p^2$ has distinct

- (A) positive rational roots
- (B) positive irrational roots
- (C) negative rational roots
- (D) negative irrational roots

50. Amar, Akbar, and Anthony write a test independently. The probability that Amar passes the test and Akbar fails the test is $\frac{3}{20}$. The probability that Akbar passes the test and Anthony fails the test is $\frac{1}{4}$. The probability that Amar and Anthony both pass the test is $\frac{2}{5}$. Then the probability that at least one of Amar, Akbar and Anthony fails the test is

- (A) $\frac{7}{10}$
- (B) $\frac{3}{10}$
- (C) $\frac{3}{5}$
- (D) $\frac{4}{5}$

51. The probability that the sum of two integers m and n , where $m, n \in \{1, 2, \dots, 50\}$, chosen randomly and independently, being divisible by 3 is

- (A) 0.3336
 - (B) 0.3332
 - (C) 0.3333
 - (D) 0.3338
-

52. Let a variable line L meet x -axis and y -axis at points A and B, respectively. Suppose the distance of the line L from the origin is 3 units. Then the equation of the locus of the point C that divides the line segment AB internally in the ratio 2 : 1 is

- (A) $\frac{4}{x^2} + \frac{1}{y^2} = 1$
 - (B) $\frac{1}{x^2} + \frac{4}{y^2} = 1$
 - (C) $x^2 + 4y^2 = 9$
 - (D) $4x^2 + y^2 = 9$
-

53. In a building, an elevator starts from the ground floor (that is, 0th floor) with 10 passengers and stops at every floor until the 15th floor which is the topmost floor. No new passengers enter the elevator on any floor from the 1st through the 15th floor. If all the passengers got off the elevator then the probability that at most one passenger got off at each floor is

- (A) $\frac{15!}{5! \times 15^{10}}$
 - (B) $\frac{10!}{15^{10}}$
 - (C) $\frac{15!}{10^{15}}$
 - (D) $\frac{15!}{5! \times 10^{15}}$
-

54. Let S be the set of real values of k for which the system of equations

$$kx + y + z = k$$

$$x + ky + z = k$$

$$x + y + kz = k$$

has no solution. Then $|S|$ is

- (A) 1
 - (B) 2
 - (C) 0
 - (D) ∞
-

55. The definite integral

$$\int_{\pi/2}^{\pi} \frac{\sin x - x \cos x}{x(x + \sin x)} dx$$

equals

- (A) $\log\left(1 + \frac{2}{\pi}\right)$
 - (B) $\log\left(1 + \frac{\pi}{2}\right)$
 - (C) $1 + \frac{2}{\pi}$
 - (D) $1 + \frac{\pi}{2}$
-

56. The number of solutions of the equation $|\sin(\pi x)| = \frac{1}{50}(x^2 + 1)$ in \mathbb{R} is

- (A) 28
 - (B) 26
 - (C) 14
 - (D) 13
-

57. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = x^5 + x^3$ and let $g(x) = f^{-1}(x)$ be the inverse of f . If $g''(-2) = \frac{a}{b}$ where a and b are positive coprime integers, then the value of a is

- (A) 13
 - (B) 26
 - (C) 39
-

(D) 256

58. Let S_1 and S_2 be two circles drawn inside a unit square ABCD, touching each other externally. Further, the circle S_1 touches the sides AD and DC; and the circle S_2 touches the sides AB and BC. If the area of S_2 is twice the area of S_1 , then the radius of S_1 is

- (A) $3\sqrt{2} - 4$
 - (B) $3 - 2\sqrt{2}$
 - (C) $2 - \sqrt{2}$
 - (D) $\frac{3}{\sqrt{2}} - 2$
-

59. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} x \left(\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} \right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Then

- (A) there exists a constant C such that $|f(x)| \leq C$ for all $x \in \mathbb{R}$.
 - (B) f is monotonically increasing in the interval (-1, 1).
 - (C) f is differentiable at $x = 0$.
 - (D) f is not continuous at $x = 0$.
-

60. The limit

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \left[1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} \right]$$

- (A) equals 2
- (B) equals 1
- (C) equals 0

(D) does not exist

Physics

61. The angular speed of the Earth's rotation is $7.3 \times 10^{-5} \text{ rad} \cdot \text{s}^{-1}$. Take the radius of the Earth at the equator to be 6400 km. Then the ratio (a_c/g) of the magnitude of the centripetal acceleration a_c at a point on the equator to g , is of the order

- (A) 10^{-3}
 - (B) 10^{-5}
 - (C) 10^0
 - (D) 10^{-1}
-

62. An electromagnetic wave travels from vacuum into a non-magnetic dielectric medium with permittivity $\epsilon = 4\epsilon_0$. If the ratio (E_0/B_0) of the electric field amplitude E_0 to the magnetic field amplitude B_0 in vacuum is equal to the speed of light c , then the corresponding ratio in the given medium is

- (A) $c/2$
 - (B) c
 - (C) $c/\sqrt{2}$
 - (D) $c/4$
-

63. Among the following, the waves responsible for energy transfer in vacuum are

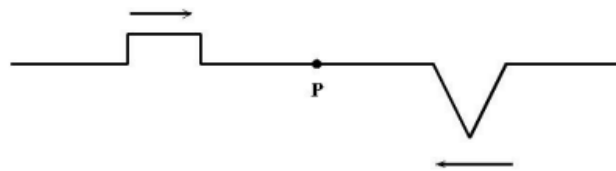
- (A) infrared waves.
- (B) sound waves.
- (C) SONAR waves.

(D) seismic waves.

64. Let A be the mass number, Z be the atomic number and N be the neutron number of a nucleus. Then the statement which is always true, is

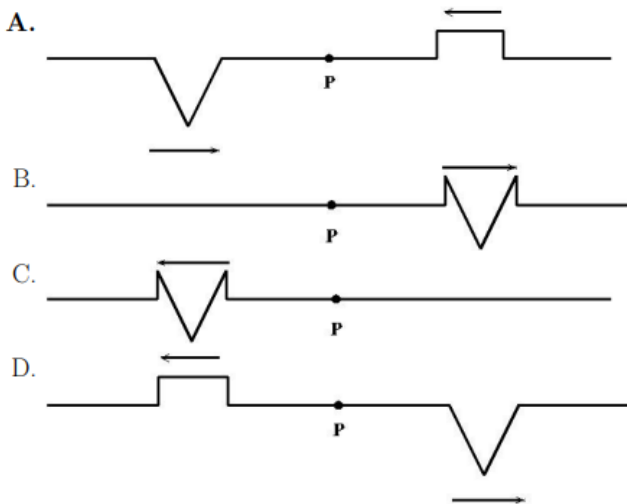
- (A) $A^2 \geq NZ$
- (B) $A \geq 2N$
- (C) $A \geq 2Z$
- (D) $AN \geq Z^2$

65. Two wave pulses, one rectangular and other triangular, approach each other as shown in



the figure below.

They overlap at the point P at time t . The diagram best representing the appearance of the wave pulses at a time $t' > t$ is



- (A) [A]
- (B) [B]
- (C) [C]
- (D) [D]

66. Ice on land mass melting into ocean, due to global warming, is estimated to be around 1.3×10^{15} kg per year. The density of sea water is $1025 \text{ kg} \cdot \text{m}^{-3}$. Assuming that the melting rate of ice remains constant and the effective surface area covered by the oceans is $3.6 \times 10^{14} \text{ m}^2$, the estimated average rise in sea level (in m) in the next 75 years will be closest to

- (A) 0.25
- (B) 0.45
- (C) 0.15
- (D) 0.50

67. A spherical glass bottle having negligible wall thickness is placed in air. When the bottle is completely filled with water, its focal length is f . If the water is replaced by another transparent liquid of higher refractive index, then the focal length changes to f' . Then the correct option is

- (A) $f' < f$
- (B) $f' > f$
- (C) $f' = f = \infty$
- (D) $f' = f$, but finite

68. Physicist Luis Alvarez and his collaborators hypothesised that the extinction of dinosaurs was due to the impact of an asteroid with the Earth. They estimated the mass and the radius of the asteroid to be about 2×10^{15} kg and 10 km respectively. Take the mass of the Earth to be 6×10^{24} kg. The gravitational acceleration (in SI units) of the Earth due to the asteroid just before the impact would be of the order

- (A) 10^{-9}
- (B) 10^1
- (C) 10^{-1}
- (D) 10^{-5}

69. An infinitely long straight wire with uniform line charge density λ lies at a perpendicular distance d from a point O. The total electric flux through the surface of a sphere of radius $R > d$ centred at O, is

- (A) $\frac{2\lambda}{\epsilon_0} \sqrt{R^2 - d^2}$
(B) $\frac{2\lambda}{\epsilon_0} \sqrt{Rd}$
(C) $\frac{2\lambda}{\epsilon_0} \sqrt{R^2 + d^2}$
(D) 0

70. A sphere made of a material with density $12 \text{ kg} \cdot \text{m}^{-3}$ and weighing 100 N in vacuum is immersed in a container of gas. Its weight in gas is 85 N. The density of the gas in $\text{kg} \cdot \text{m}^{-3}$ is closest to

- (A) 1.80
(B) 0.01
(C) 80.00
(D) 0.55

71. One mole of a monatomic ideal gas undergoes a transformation from an initial state with temperature 290 K and volume 30 litres to a final state with temperature 310 K and volume 16 litres. On the pressure–volume ($P - V$) diagram, this process is represented by a straight line path. The magnitude of the work done (in joules) during this process is close to

- (A) 1690
(B) 1939
(C) 877
(D) 1375

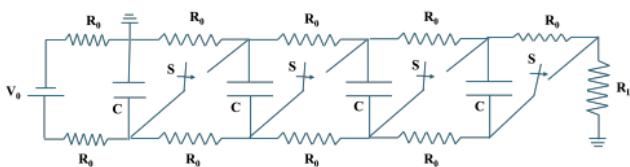
72. The electron in the hydrogen atom is in the third excited state. Its potential energy (in eV) is

- (A) -1.70
- (B) -1.51
- (C) -0.85
- (D) -3.02

73. A sphere is given an initial push so that it starts rolling (without slipping) up an inclined plane. During its climb

- (A) the direction of the force of friction on the sphere is up the incline.
- (B) the direction of the net force on the sphere is up the incline.
- (C) the net torque on the sphere is zero.
- (D) the work done by the force of friction on the sphere is negative.

74. All the capacitors in a given circuit are initially fully charged with all switches open. At a later time t , all the switches are simultaneously closed. The current flowing through the circuit at that instant is given by



- (A) $4V_0/R_L$
- (B) V_0/R_0
- (C) $3V_0/R_0$
- (D) V_0/R_L

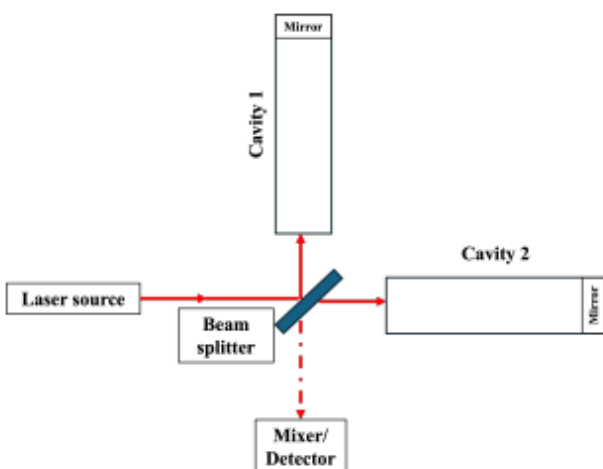
75. An incandescent lamp of wattage W is filled with argon gas and has a tungsten filament. The collision frequency of evaporated tungsten atoms with the argon atoms is found to be proportional to W^α . The most reasonable value of α is

- (A) $\frac{1}{8}$
- (B) $\frac{1}{4}$
- (C) $\frac{1}{2}$
- (D) 0

76. Two protons are separated by a distance of 1 nanometer. The magnitudes of gravitational, electromagnetic and strong nuclear forces between them are denoted by F_G , F_E and F_S respectively. Select the correct option.

- (A) $F_E > F_G > F_S$
- (B) $F_E > F_S > F_G$
- (C) $F_S > F_G > F_E$
- (D) $F_S > F_E > F_G$

77. A laser beam of wavelength $1 \mu\text{m}$ is split and sent into two vacuum cavities of equal length L as shown in the figure. A detector can register an interference signal only if the phase difference between the returning beams is at least 5×10^{-11} rad. A certain physical effect changes the length of cavity 2 by an amount ΔL such that $\Delta L/L \approx 10^{-21}$. The minimum cavity length (in km) needed for measuring this physical effect is approximately



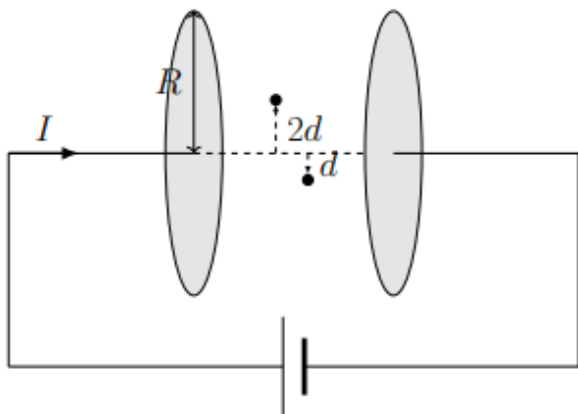
- (A) 4

- (B) 1
- (C) 2
- (D) 16

78. The critical electric field required to produce electron-positron pairs depends on the physical constants h , c , m_e and e . Use dimensional analysis and assume that the dimensionless coefficient is of order one. The magnitude of the critical electric field, in SI units, is of the order

- (A) 10^{18}
- (B) 10^{21}
- (C) 10^{24}
- (D) 10^{15}

79. A parallel plate capacitor with circular plates of radius R is being charged as shown in the figure. Let $B(r)$ be the induced magnetic field at a distance r from the central axis between the plates. Assuming $d \ll R$, the ratio $\frac{B(2d)}{B(d)}$, while charging, is



- (A) 2
- (B) $\frac{1}{2}$
- (C) $\frac{1}{4}$
- (D) 1

80. Two point bodies of masses m and $3m$ are connected by a massless spring of spring constant $k = m\omega_0^2$ and kept on a frictionless horizontal surface. The spring is extended by a small distance l over its natural length at time $t = 0$ and then released so that the masses execute simple harmonic motion. The maximum speed of the particle with mass m is given by

- (A) $\frac{\sqrt{3}\omega_0 l}{2}$
(B) $\frac{2\omega_0 l}{\sqrt{3}}$
(C) $\frac{\omega_0 l}{\sqrt{3}}$
(D) $\frac{3\omega_0 l}{4}$
-