NEST Mathematics Sample Paper – 8

Duration: 45 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **20 Multiple Choice Questions (single correct answer)**, modelled on the Mathematics section of **NEST 2026**.
- Each correct answer carries **+3 marks**. There is a deduction of -1 **mark** for each incorrect answer; **no marks** are deducted for an unattempted question.
- Every question has exactly **four options**, of which only **one** is correct. Choose carefully.
- Personal calculators, log tables, mobile phones, and other electronic gadgets are strictly prohibited in the examination hall.
- A simple on-screen (virtual) calculator is provided in the computer-based test interface and may be used; blank sheets for rough work are supplied at the exam centre.

Q1. On the set $A = \{1, 2, 3\}$, consider the relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$. Which one of the following statements is correct?

(A) R is reflexive and symmetric but not transitive

(B) R is an equivalence relation

(C) R is symmetric and transitive but not reflexive

(D) R is reflexive but neither symmetric nor transitive

Q2. The maximum value of $f(x) = 6 \sin x \cos x + 4$ is

(A) 10

(B) 7

(C) 6

(D) 13

Q3. If the quadratic equations $x^2-5x+6=0$ and $x^2-9x+k=0$ have exactly one common root, then the sum of all possible values of k is

(A) 32

(B) 14

(C) 18

(D) 26

Q4. Six books are to be arranged on a shelf. The number of arrangements in which two particular books are never placed next to each other is

(A) 240

(B) 600

(C) 720

(D) 480

Q5. The sum of the binomial coefficients $\binom{8}{0}+\binom{8}{1}+\binom{8}{2}+\cdots+\binom{8}{8}$ is

(A) 128

(B) 256

(C) 255

(D) 512

Q6. The sum of the series $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{9\cdot 10}$ is

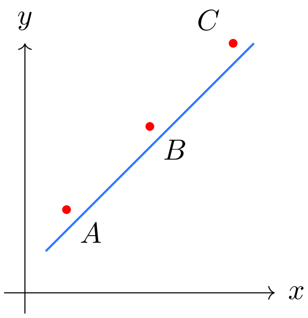
(A) $\frac{1}{10}$

(B) $\frac{10}{9}$

(C) $\frac{9}{10}$

(D) 1

Q7. The three points $A(1,2)$, $B(3,6)$ and $C(k,10)$ are collinear. The value of k is



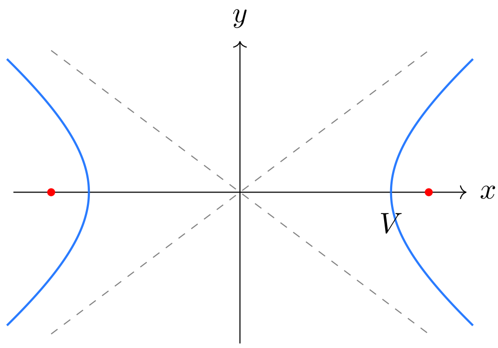
(A) 4

(B) 6

(C) 5

(D) 7

Q8. The equation of the hyperbola with vertices $(\pm4, 0)$ and foci $(\pm5, 0)$ is



(A) $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$

(B) $\dfrac{x^2}{16} - \dfrac{y^2}{25} = 1$

(C) $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$

(D) $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

Q9. For the function $f(x) = \begin{cases} x+2, & x<1 \\ 3x-1, & x \geq 1 \end{cases}$, the value of $\lim_{x\to1} f(x)$ is

(A) 3, since both one-sided limits equal 3

(B) 2, since the left-hand limit dominates

(C) the limit does not exist

(D) 4, the average of the two pieces

Q10. The mean of 10 observations is 14. If one more observation of value 25 is included, the mean of the resulting 11 observations is

(A) 15

(B) 14.5

(C) 16

(D) 13.5

Q11. The value of $\cos\left(\sin^{-1}\frac{3}{5}\right)$ is

(A) $\frac{3}{4}$

(B) $\frac{4}{5}$

(C) $\frac{3}{5}$

(D) $\frac{5}{4}$

Q12. If $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ and $AX = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$, then the column matrix X is

(A) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

(B) $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(D) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Q13. If $\begin{vmatrix} x & 2 \\ 3 & x \end{vmatrix} = 10$, then the positive value of x is

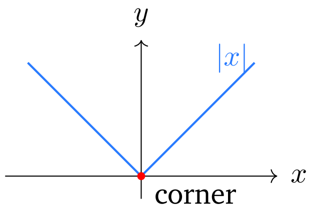
(A) 5

(B) $\sqrt{10}$

(C) 4

(D) 6

Q14. Using logarithmic differentiation, $\dfrac{dy}{dx}$ for $y = x^x$ (for $x > 0$) is



(A) $x^x(1 + \ln x)$

(B) $x \cdot x^{x-1}$

(C) $x^x \ln x$

(D) $x^x(1 - \ln x)$

Q15. The equation of the normal to the curve $y = x^2$ at the point $(1, 1)$ is

(A) $2x - y - 1 = 0$

(B) $x - 2y + 1 = 0$

(C) $2x + y - 3 = 0$

(D) $x + 2y - 3 = 0$

Q16. The value of $\displaystyle\int \frac{dx}{x^2 + 9}$ is (with C an arbitrary constant)

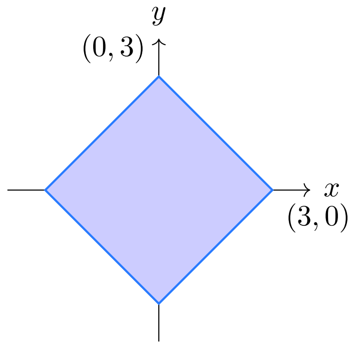
(A) $\dfrac{1}{3} \tan^{-1} \dfrac{x}{3} + C$

(B) $\tan^{-1} \dfrac{x}{3} + C$

(C) $3 \tan^{-1} \dfrac{x}{3} + C$

(D) $\dfrac{1}{9} \tan^{-1} \dfrac{x}{3} + C$

Q17. The area of the region described by $|x| + |y| \leq 3$ is



(A) 9

(B) 18

(C) 36

(D) 12

Q18. The number of arbitrary constants in the general solution of the differential equation $\dfrac{d^3y}{dx^3} + 2\dfrac{dy}{dx} = 0$ is

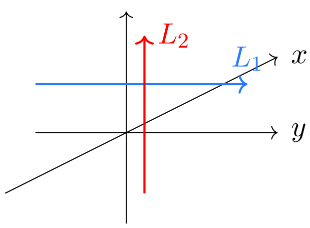
(A) 2

(B) 1

(C) 3

(D) 4

Q19. The shortest distance between the skew lines $\vec{r} = \hat{j} + s\,\hat{i}$ and $\vec{r} = \hat{k} + t\,\hat{j}$ is



(A) $\dfrac{1}{\sqrt{2}}$

(B) 1

(C) $\frac{1}{2}$

(D) $\sqrt{2}$

Q20. For two events A and B, $P(A) = 0.4$ and $P(B) = 0.5$. If A and B are *independent*, then $P(A \cup B)$ equals

(A) 0.9

(B) 0.2

(C) 0.6

(D) 0.7

Detailed Solutions

Q1.

Solution

Concept — Equivalence relation: A relation must be reflexive, symmetric and transitive simultaneously to be an equivalence relation.

Step 1 — Reflexive: $(1,1),(2,2),(3,3)$ are all present, so R is reflexive.

Step 2 — Symmetric: The only off-diagonal pairs are $(1,2)$ and $(2,1)$, which occur together, so R is symmetric.

Step 3 — Transitive: Check all chains. We have $(1,2)$ and $(2,1)$, requiring $(1,1)$ which is present; $(2,1)$ and $(1,2)$ require $(2,2)$ which is present. No chain forces a missing pair, so R is transitive. Hence R is an equivalence relation.

Why other options are wrong:

- (A) wrongly claims transitivity fails; all required pairs are present.
- (C) wrongly claims reflexivity fails, but all three diagonal pairs are present.
- (D) wrongly denies symmetry, but $(1,2)$ and $(2,1)$ both appear.

Final Answer: R is an equivalence relation $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q1

Q2.

Solution

Concept — Double-angle identity: $2\sin x\cos x = \sin 2x$, and $\sin 2x$ ranges over $[-1,1]$.

Step 1 — Rewrite: $f(x) = 6\sin x\cos x + 4 = 3(2\sin x\cos x) + 4 = 3\sin 2x + 4$.

Step 2 — Maximise: The maximum of $\sin 2x$ is 1, so $f_{\max} = 3(1) + 4 = 7$.

Why other options are wrong:

- (A) $10 = 6 + 4$ uses $\sin x\cos x = 1$, impossible since $\sin x\cos x \le \frac{1}{2}$.
- (C) 6 drops the constant or the coefficient.
- (D) 13 adds the amplitudes incorrectly.

Final Answer: maximum value $= 7 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept — Common root: A common root must satisfy both equations. Find the roots of the factorable equation, then test which makes the second equation consistent.

Step 1 — Roots of first equation: $x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0$, so $x = 2$ or $x = 3$.

Step 2 — Substitute each into the second: If $x = 2$: $4 - 18 + k = 0 \Rightarrow k = 14$. If $x = 3$: $9 - 27 + k = 0 \Rightarrow k = 18$.

Step 3 — Sum of values: The possible k values are 14 and 18, summing to 32.

Why other options are wrong:

- (B) 14 keeps only the root $x = 2$.
- (C) 18 keeps only the root $x = 3$.
- (D) 26 comes from an arithmetic slip in adding.

Final Answer: sum $= 14 + 18 = 32 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q3

Q4.

Solution

Concept — Never together: (Never together) = (total) − (always together), gluing the two books into one block for the latter.

Step 1 — Total arrangements: Six distinct books give $6! = 720$.

Step 2 — Two books together: Treat the pair as one block: 5 units arrange in $5! = 120$ ways, and the block internally in $2! = 2$ ways, giving $120 \times 2 = 240$.

Step 3 — Never together: $720 - 240 = 480$.

Why other options are wrong:

- (A) 240 is the "together" count, the opposite of what is asked.
- (B) 600 subtracts only 120 (forgets the internal 2!).
- (C) 720 is the unrestricted total.

Final Answer: $480 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Sum of all binomial coefficients: $\sum_{r=0}^{n}\binom{n}{r} = 2^n$, obtained by setting $x = 1$ in $(1+x)^n$.

Step 1 — Apply with $n = 8$: The sum equals 2^8.

Step 2 — Evaluate: $2^8 = 256$.

Why other options are wrong:

- (A) $128 = 2^7$ uses $n = 7$.
- (C) $255 = 2^8 - 1$ drops one coefficient.
- (D) $512 = 2^9$ uses $n = 9$.

Final Answer: $2^8 = 256 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q5

Q6.

Solution

Concept — Telescoping by partial fractions: $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$, so successive terms cancel.

Step 1 — Rewrite each term: The sum becomes

$$\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{9}-\frac{1}{10}\right).$$

Step 2 — Cancel: All interior terms cancel, leaving $1 - \frac{1}{10} = \frac{9}{10}$.

Why other options are wrong:

- (A) $\frac{1}{10}$ is the leftover negative tail, not the sum.
- (B) $\frac{10}{9}$ inverts the answer.
- (D) 1 forgets the final $-\frac{1}{10}$.

Final Answer: $\frac{9}{10} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q6

Q7.

Solution

Concept — Collinearity via slopes: Three points are collinear iff the slope between any two pairs is equal.

Step 1 — Slope of AB: With $A(1,2)$, $B(3,6)$: slope $= \frac{6-2}{3-1} = \frac{4}{2} = 2$.

Step 2 — Slope of BC equals 2: With $B(3,6)$, $C(k,10)$: $\frac{10-6}{k-3} = 2 \Rightarrow \frac{4}{k-3} = 2 \Rightarrow k-3 = 2 \Rightarrow k = 5$.

Why other options are wrong:

- (A) 4 solves $k-3=1$ (wrong slope).
- (B) 6 uses $\frac{4}{k-3} = \frac{4}{3}$.
- (D) 7 takes $k-3=4$ (slope 1).

Final Answer: $k = 5 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q7

Q8.

Solution

Concept — Standard hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$: Vertices $(\pm a, 0)$, foci $(\pm c, 0)$ with $c^2 = a^2 + b^2$.

Step 1 — Read a and c: Vertices $(\pm 4, 0) \Rightarrow a = 4$; foci $(\pm 5, 0) \Rightarrow c = 5$.

Step 2 — Find b^2: $b^2 = c^2 - a^2 = 25 - 16 = 9$.

Step 3 — Write equation: $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

Why other options are wrong:

- (B) uses $b^2 = 25$ (takes c^2 as b^2).
- (C) swaps a^2 and c^2.
- (D) swaps a^2 and b^2.

Final Answer: $\frac{x^2}{16} - \frac{y^2}{9} = 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q8

Q9.

Solution

Concept — Limit of a piecewise function: The limit exists iff the left-hand limit (LHL) and right-hand limit (RHL) are equal.

Step 1 — LHL ($x \to 1^-$): Use $x + 2$: $\lim_{x \to 1^-} (x + 2) = 1 + 2 = 3$.

Step 2 — RHL ($x \to 1^+$): Use $3x - 1$: $\lim_{x \to 1^+} (3x - 1) = 3 - 1 = 2$.

Step 3 — Compare: $\text{LHL} = 3 \neq 2 = \text{RHL}$, so the limit does not exist.

Why other options are wrong:

- (A) 3 uses only the LHL.
- (B) 2 uses only the RHL.
- (D) 4 averages the one-sided limits, which is not how limits work.

Final Answer: the limit does not exist $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q9

Q10.

Solution

Concept — Mean after adding an observation: New mean $= \dfrac{\text{old sum} + \text{new value}}{\text{new count}}$.

Step 1 — Old sum: Mean 14 over 10 observations gives sum $= 14 \times 10 = 140$.

Step 2 — New sum and count: New sum $= 140 + 25 = 165$ over 11 observations.

Step 3 — New mean: $\dfrac{165}{11} = 15$.

Why other options are wrong:

- (B) 14.5 averages 14 and 25 wrongly.
- (C) 16 divides by 10 instead of 11.
- (D) 13.5 treats 25 as below the mean.

Final Answer: new mean $= 15 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q10

Q11.

Solution

Concept — Composite of inverse trig: Let $\theta = \sin^{-1}\frac{3}{5}$, so $\sin\theta = \frac{3}{5}$ with $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, hence $\cos\theta \geq 0$.

Step 1 — Use the identity: $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$.

Step 2 — Conclude: $\cos\left(\sin^{-1}\frac{3}{5}\right) = \frac{4}{5}$ (positive, since θ is in the first quadrant).

Why other options are wrong:

- (A) $\frac{3}{4}$ is $\tan\theta$, not $\cos\theta$.
- (C) $\frac{3}{5}$ repeats $\sin\theta$.
- (D) $\frac{5}{4}$ is the reciprocal $\sec\theta$.

Final Answer: $\frac{4}{5} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q11

Q12.

Solution

Concept — Solving $AX = B$: Either invert A or solve the linear system directly. Here we solve directly.

Step 1 — Write equations: With $X = \binom{p}{q}$: $2p + q = 5$ and $p + q = 3$.

Step 2 — Eliminate: Subtract the second from the first: $(2p+q) - (p+q) = 5 - 3 \Rightarrow p = 2$. Then $q = 3 - p = 1$.

Step 3 — Verify: $2(2) + 1 = 5$ ✓and $2 + 1 = 3$ ✓, so $X = \binom{2}{1}$.

Why other options are wrong:

- (A) $\binom{1}{3}$ fails $2(1) + 3 = 5$ but $1 + 3 = 4 \neq 3$.
- (B) $\binom{3}{-1}$ fails the second equation $3 - 1 = 2 \neq 3$.
- (C) $\binom{1}{2}$ gives $2(1) + 2 = 4 \neq 5$.

Final Answer: $X = \binom{2}{1} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — 2×2 determinant: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Step 1 — Expand: $\begin{vmatrix} x & 2 \\ 3 & x \end{vmatrix} = x \cdot x - 2 \cdot 3 = x^2 - 6$.

Step 2 — Solve: $x^2 - 6 = 10 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$. The positive value is $x = 4$.

Why other options are wrong:

- (A) 5 solves $x^2 = 25$ (uses $+15$ instead of $+6$).
- (B) $\sqrt{10}$ ignores the -6 term entirely.
- (D) 6 sets $x^2 = 36$ from a wrong expansion.

Final Answer: $x = 4 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q13

Q14.

Solution

Concept — Logarithmic differentiation: For a variable base and exponent, take $\ln$ of both sides before differentiating. The figure illustrates a non-differentiable corner (of $|x|$) to contrast with the smooth x^x.

Step 1 — Take logarithm: $y = x^x \Rightarrow \ln y = x \ln x$.

Step 2 — Differentiate: $\frac{1}{y}\frac{dy}{dx} = \ln x + x \cdot \frac{1}{x} = \ln x + 1$.

Step 3 — Solve: $\frac{dy}{dx} = y(\ln x + 1) = x^x(1 + \ln x)$.

Why other options are wrong:

- (B) $x \cdot x^{x-1}$ treats the exponent as a constant (power rule misuse).
- (C) $x^x \ln x$ drops the $+1$ from differentiating $x \ln x$.
- (D) $x^x(1 - \ln x)$ has a wrong sign.

Final Answer: $x^x(1 + \ln x) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q14

Q15.

Solution

Concept — Normal to a curve: The normal slope is the negative reciprocal of the tangent slope $\frac{dy}{dx}$ at the point.

Step 1 — Tangent slope: $y = x^2 \Rightarrow \frac{dy}{dx} = 2x$. At $(1, 1)$, slope $= 2$.

Step 2 — Normal slope: Negative reciprocal $= -\frac{1}{2}$.

Step 3 — Equation: $y - 1 = -\frac{1}{2}(x - 1) \Rightarrow 2y - 2 = -(x - 1) \Rightarrow x + 2y - 3 = 0$.

Why other options are wrong:

- (A) $2x - y - 1 = 0$ is the tangent line (slope 2).
- (B) $x - 2y + 1 = 0$ has slope $+\frac{1}{2}$ (wrong sign).
- (C) $2x + y - 3 = 0$ has slope -2 (reciprocal not taken).

Final Answer: $x + 2y - 3 = 0 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q15

Q16.

Solution

Concept — Standard integral: $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$.

Step 1 — Identify a: Here $a^2 = 9$, so $a = 3$.

Step 2 — Apply formula: $\int \frac{dx}{x^2 + 9} = \frac{1}{3} \tan^{-1} \frac{x}{3} + C$.

Why other options are wrong:

- (B) omits the factor $\frac{1}{a} = \frac{1}{3}$.
- (C) multiplies by $a = 3$ instead of dividing.
- (D) uses $\frac{1}{a^2} = \frac{1}{9}$ instead of $\frac{1}{a}$.

Final Answer: $\frac{1}{3} \tan^{-1} \frac{x}{3} + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q16

Q17.

Solution

Concept — Region $|x| + |y| \le a$: This is a square (a "diamond") with diagonals along the axes, vertices at $(\pm a, 0)$ and $(0, \pm a)$.

Step 1 — Identify the diagonals: For $a = 3$, each diagonal has length $2a = 6$ (from -3 to 3).

Step 2 — Area of a rhombus: Area $= \frac{1}{2}d_1 d_2 = \frac{1}{2}(6)(6) = 18$. (Equivalently $2a^2 = 2 \cdot 9 = 18$.)

Why other options are wrong:

- (A) $9 = a^2$ takes only one of the four triangles times something wrong.
- (C) 36 uses the full $d_1 d_2$ without the $\frac{1}{2}$.
- (D) 12 is an arithmetic slip.

Final Answer: area $= 18 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q17

Q18.

Solution

Concept — Arbitrary constants: The number of arbitrary constants in the general solution of a differential equation equals its order (the highest derivative present).

Step 1 — Identify the order: The highest derivative is $\frac{d^3y}{dx^3}$, so the order is 3.

Step 2 — Conclude: A third-order equation has a general solution with exactly 3 arbitrary constants.

Why other options are wrong:

- (A) 2 would correspond to a second-order equation.
- (B) 1 corresponds to a first-order equation.
- (D) 4 overcounts the order.

Final Answer: 3 arbitrary constants $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q18

Q19.

Solution

Concept — Shortest distance between skew lines: For $\vec{r} = \vec{a}_1 + s\vec{b}_1$ and $\vec{r} = \vec{a}_2 + t\vec{b}_2$, the shortest distance is $d = \dfrac{|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$.

Step 1 — Identify vectors: Line L_1: $\vec{a}_1 = \hat{j}$, $\vec{b}_1 = \hat{i}$. Line L_2: $\vec{a}_2 = \hat{k}$, $\vec{b}_2 = \hat{j}$.

Step 2 — Cross product: $\vec{b}_1 \times \vec{b}_2 = \hat{i} \times \hat{j} = \hat{k}$, so $|\vec{b}_1 \times \vec{b}_2| = 1$.

Step 3 — Scalar triple part: $\vec{a}_2 - \vec{a}_1 = \hat{k} - \hat{j}$; its dot with $\hat{k}$ is 1. Hence $d = \dfrac{|1|}{1} = 1$.

Why other options are wrong:

- (A) $\frac{1}{\sqrt{2}}$ uses a wrong (non-unit) cross-product magnitude.
- (C) $\frac{1}{2}$ halves the numerator without reason.
- (D) $\sqrt{2}$ takes $|\hat{k} - \hat{j}|$ instead of its projection on $\hat{k}$.

Final Answer: shortest distance $= 1 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q19

Q20.

Solution

Concept — Independent events: If A and B are independent, $P(A \cap B) = P(A)\,P(B)$, and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Step 1 — Intersection: $P(A \cap B) = 0.4 \times 0.5 = 0.2$.

Step 2 — Union: $P(A \cup B) = 0.4 + 0.5 - 0.2 = 0.7$.

Why other options are wrong:

- (A) 0.9 treats A, B as mutually exclusive (subtracts nothing).
- (B) 0.2 is $P(A \cap B)$, not the union.
- (C) 0.6 uses a wrong intersection value.

Final Answer: $P(A \cup B) = 0.7 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	A	4	D	5	B
6	C	7	C	8	A	9	C	10	A
11	B	12	D	13	C	14	A	15	D
16	A	17	B	18	C	19	B	20	D