NIMCET Mathematics Sample Paper-18

Duration: 70 Minutes **Maximum Marks: 600**

Instructions

- This paper contains **50** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+12 marks**.
- Each incorrect answer carries: **−3** marks.
- Unattempted questions carry **0** marks.
- Only one option is correct for each question.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then $(f \circ f \circ f \circ \cdots \circ f)(x)$ where the composition is taken n times, is equal to:

(A) $\frac{x}{\sqrt{1+nx^2}}$

(B) $\frac{nx}{\sqrt{1+x^2}}$

(C) $\frac{x}{\sqrt{n+x^2}}$

(D) $\frac{x^n}{\sqrt{1+nx^2}}$

Q2. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then a vector $\vec{c}$ perpendicular to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$ is:

(A) $2\hat{i} - \hat{j} - \hat{k}$

(B) $-2\hat{i} + \hat{j} + \hat{k}$

(C) $\hat{i} - 2\hat{j} + \hat{k}$

(D) $\hat{i} + \hat{j} - 2\hat{k}$

Q3. The sum of all values of $\theta \in (0, \pi)$ satisfying the equation $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is:

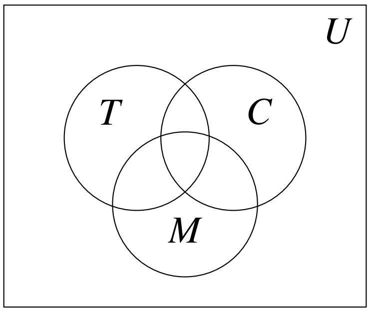
(A) π

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{2}$

(D) 2π

Q4. In a class of 60 students, 25 students drink tea, 30 drink coffee, and 24 drink milk. If 10 drink both tea and coffee, 8 drink coffee and milk, 11 drink tea and milk, and 3 drink all three liquids, the number of students who do not take any of these drinks is:



(A) 4

(B) 7

(C) 12

(D) 15

Q5. A bag contains 4 white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls drawn. The variance of X is:

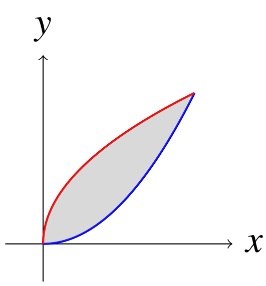
(A) $\frac{14}{75}$

(B) $\frac{14}{25}$

(C) $\frac{7}{15}$

(D) $\frac{1}{3}$

Q6. The area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ is:



(A) $\frac{16}{3}$

(B) $\frac{8}{3}$

(C) $\frac{4}{3}$

(D) 4

Q7. If α and β are the roots of the equation $x^2 - 6x + a = 0$ and satisfy the relation $3\alpha + 2\beta = 20$, then the value of a is:

(A) -8

(B) -16

(C) 8

(D) 16

Q8. The eccentricity of the hyperbola conjugate to the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is:

(A) $\frac{5}{4}$

(B) $\frac{5}{3}$

(C) $\frac{4}{3}$

(D) $\frac{3}{4}$

Q9. The total number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 such that the numbers are divisible by 4 (repetition of digits is allowed) is:

(A) 216

(B) 324

(C) 144

(D) 180

Q10. The value of $\lim_{x\to 0} \frac{1-\cos(1-\cos x)}{x^4}$ is:

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{16}$

Q11. The angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$. If $|\vec{a}| = 3$ and $|\vec{b}| = 4$, then the value of $|2\vec{a} + 3\vec{b}|$ is:

(A) $\sqrt{108}$

(B) $\sqrt{112}$

(C) $\sqrt{124}$

(D) $\sqrt{144}$

Q12. If the third term of a geometric progression is 4, then the product of its first 5 terms is:

(A) 4^3

(B) 4^4

(C) 4^5

(D) 4^6

Q13. The value of $\int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x}\, dx$ is:

(A) $\frac{\pi}{4}$

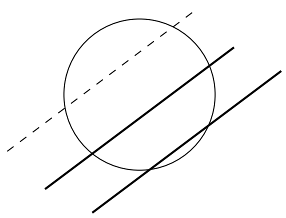
(B) $\frac{\pi}{2}$

(C) π

(D) 0

Q14. The equations of the tangents to the circle $x^2 + y^2 = 25$ which are parallel to the line $3x - 4y + 7 = 0$ are:

$3x - 4y + 7 = 0$



(A) $3x - 4y \pm 25 = 0$

(B) $3x - 4y \pm 5 = 0$

(C) $4x + 3y \pm 25 = 0$

(D) $4x + 3y \pm 5 = 0$

Q15. The domain of definition of the function $f(x) = \sqrt{\log_{0.5}(x^2 - 5x + 6)}$ is:

(A) $[\frac{5-\sqrt{5}}{2}, 2) \cup (3, \frac{5+\sqrt{5}}{2}]$

(B) $(2, 3)$

(C) $[\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}]$

(D) $(-\infty, 2) \cup (3, \infty)$

Q16. Three unbiased coins are tossed simultaneously. What is the probability of getting at least two heads given that at least one coin shows a head?

(A) $\frac{4}{7}$

(B) $\frac{1}{2}$

(C) $\frac{3}{7}$

(D) $\frac{5}{8}$

Q17. The value of $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right]$ is:

(A) $\frac{3-\sqrt{5}}{2}$

(B) $\frac{3+\sqrt{5}}{2}$

(C) $\frac{\sqrt{3}-1}{2}$

(D) $\frac{\sqrt{5}-1}{2}$

Q18. The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observations are 1, 2, and 6, then the other two observations are:

(A) 4 and 9

(B) 3 and 10

(C) 5 and 8

(D) 2 and 11

Q19. The differential equation of the family of curves $y = c_1 e^{2x} + c_2 e^{-2x}$ is:

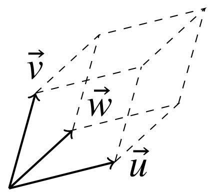
(A) $\frac{d^2y}{dx^2} - 4y = 0$

(B) $\frac{d^2y}{dx^2} + 4y = 0$

(C) $\frac{dy}{dx} - 2y = 0$

(D) $\frac{d^2y}{dx^2} - 2y = 0$

Q20. If standard basis vectors are given, the volume of the parallelepiped whose coterminous edges are represented by the vectors $\vec{u} = \hat{i} + a\hat{j} + \hat{k}$, $\vec{v} = \hat{j} + a\hat{k}$, and $\vec{w} = a\hat{i} + \hat{k}$ is minimum when a equals:



(A) $\frac{1}{\sqrt{3}}$

(B) $-\frac{1}{\sqrt{3}}$

(C) $\sqrt{3}$

(D) $-\sqrt{3}$

Q21. The value of $\lim_{n\to\infty}\left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}\right)$ is:

(A) $\log_e 2$

(B) $\log_e 3$

(C) 1

(D) 0

Q22. The term independent of x in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$ is:

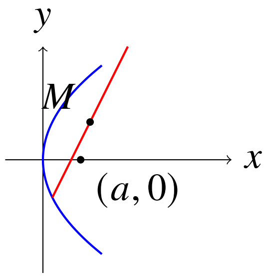
(A) 7920

(B) 495

(C) -7920

(D) 12

Q23. The locus of the midpoints of chords of the parabola $y^2 = 4ax$ which pass through the focus $(a, 0)$ is:



(A) $y^2 = 2a(x - a)$

(B) $y^2 = a(x - a)$

(C) $y^2 = 2ax$

(D) $y^2 = a(2x - a)$

Q24. A student solves a problem with a probability of $\frac{1}{3}$, and another student solves the same problem with a probability of $\frac{3}{4}$. If both try independently, the probability that the problem is solved is:

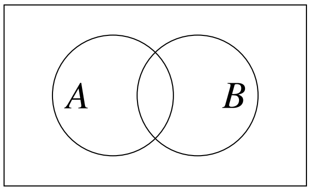
(A) $\frac{5}{6}$

(B) $\frac{1}{4}$

(C) $\frac{11}{12}$

(D) $\frac{2}{3}$

Q25. If A and B are two sets such that $n(A) = 35$, $n(B) = 30$, and $n(A \cup B) = 55$, then $n(A \cap B')$ is equal to:



(A) A

(B) B

(C) C

(D) D

Q26. The value of $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$ is:

(A) $e^x \tan\left(\frac{x}{2}\right) + C$

(B) $e^x \cot\left(\frac{x}{2}\right) + C$

(C) $e^x \sec^2\left(\frac{x}{2}\right) + C$

(D) $\frac{1}{2}e^x \tan\left(\frac{x}{2}\right) + C$

Q27. The maximum value of the function $f(x) = x^3 - 3x^2 - 9x + 5$ on the interval $[-2, 4]$ is:

(A) 10

(B) 5

(C) -22

(D) 0

Q28. The number of non-trivial solutions of the system of linear equations:
$x + ky + 3z = 0$
$3x + ky - 2z = 0$
$2x + 3y - 4z = 0$
is non-zero for:

(A) $k = \frac{33}{2}$

(B) $k = \frac{22}{3}$

(C) $k = 11$

(D) Any real value of k

Q29. If the line $y = mx + 1$ is a tangent to the parabola $y^2 = 4x$, then the value of m is:

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) 4

Q30. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$, then the value of $x^{100}+y^{100}+z^{100}-\frac{9}{x^{101}+y^{101}+z^{101}}$ is:

(A) 0

(B) 1

(C) 3

(D) -6

Q31. How many words can be formed by arranging the letters of the word "NIMCET" such that the vowels are never separated?

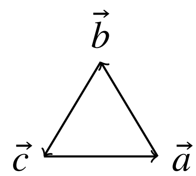
(A) 240

(B) 120

(C) 720

(D) 480

Q32. If vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3$, $|\vec{b}|=5$, $|\vec{c}|=7$, then the angle between $\vec{a}$ and $\vec{b}$ is:



(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{6}$

(C) $\frac{2\pi}{3}$

(D) $\frac{5\pi}{6}$

Q33. The value of $\int_{-1}^{1}|x\sin(\pi x)|\,dx$ is:

(A) $\frac{2}{\pi}$

(B) $\frac{4}{\pi}$

(C) 0

(D) $\frac{1}{\pi}$

Q34. The equation $x^{\log_x(x^2-4x+5)} = x - 1$ has:

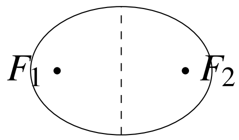
(A) No real root

(B) Exactly one real root

(C) Exactly two real roots

(D) Infinitely many roots

Q35. If the distance between the foci of an ellipse is equal to the length of its minor axis, then its eccentricity is:



(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1}{\sqrt{3}}$

Q36. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ under the condition $y(1) = \frac{1}{4}$ is:

(A) $y = \frac{x^3}{4}$

(B) $y = \frac{x^2}{4}$

(C) $y = \frac{x^3}{3} - \frac{1}{12x}$

(D) $y = \frac{x^3}{4x}$

Q37. If $A = \{x \in \mathbb{R} : x^2 - 3x + 2 \leq 0\}$ and $B = \{x \in \mathbb{R} : x^2 - 4x + 3 \geq 0\}$, then $A \cap B$ is equal to:

(A) $[1, 2]$

(B) $\{1\}$

(C) $[1, 2) \cup (2, 3]$

(D) $[2, 3]$

Q38. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. The probability that the first card is a king and the second card is a queen is:

(A) $\frac{4}{663}$

(B) $\frac{1}{169}$

(C) $\frac{2}{663}$

(D) $\frac{8}{663}$

Q39. If $\tan\alpha = \frac{1}{7}$ and $\tan\beta = \frac{1}{3}$, where α, β are acute angles, then the value of $\alpha + 2\beta$ is:

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{3\pi}{4}$

Q40. The projection of the vector $\vec{i} - 2\vec{j} + \vec{k}$ on the vector $4\vec{i} - 4\vec{j} + 7\vec{k}$ is:

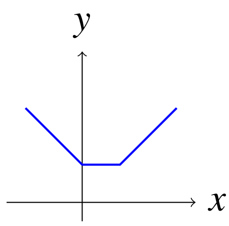
(A) $\frac{19}{9}$

(B) $\frac{19}{3}$

(C) $\frac{\sqrt{6}}{9}$

(D) $\frac{11}{9}$

Q41. The function $f(x) = |x| + |x - 1|$ is:



(A) Continuous everywhere but not differentiable at $x = 0, 1$

(B) Differentiable everywhere except at $x = 0$

(C) Continuous and differentiable everywhere

(D) Not continuous at $x = 0, 1$

Q42. Let a, b, c be in arithmetic progression. If the roots of the quadratic equation $ax^2 + bx + c = 0$ are imaginary, then:

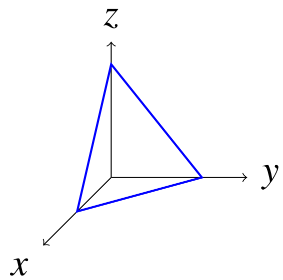
(A) $b^2 < 4ac$ and $a, c > 0$

(B) $b^2 > 4ac$

(C) $a^2 + c^2 < ac$

(D) $a^2 - 8ac < 0$

Q43. The perpendicular distance from the origin to the plane passing through the points $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$ is:



(A) $\frac{6}{7}$

(B) $\frac{6}{\sqrt{14}}$

(C) $\frac{6}{\sqrt{49}}$

(D) $\frac{7}{6}$

Q44. Out of 10 items, 3 are defective. If a sample of 3 items is drawn at random without replacement, the probability that the sample contains exactly one defective item is:

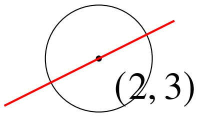
(A) $\frac{21}{40}$

(B) $\frac{7}{24}$

(C) $\frac{9}{40}$

(D) $\frac{3}{10}$

Q45. If the line $2x - y + k = 0$ is a normal to the circle $x^2 + y^2 - 4x - 6y + 11 = 0$, then the value of k is:



(A) -1

(B) 1

(C) 2

(D) -2

Q46. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1} x$ is:

(A) $\frac{1}{2}$

(B) 1

(C) 2

(D) $\frac{1}{1+x^2}$

Q47. If ω is an imaginary cube root of unity, then the value of the determinant $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ is:

(A) 0

(B) 1

(C) ω

(D) ω^2

Q48. The height of a cylinder of maximum volume that can be inscribed in a sphere of radius R is:

(A) $\frac{2R}{\sqrt{3}}$

(B) $\frac{R}{\sqrt{3}}$

(C) $\sqrt{3}R$

(D) $\frac{2R}{3}$

Q49. If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ for a non-zero vector $\vec{a}$, then:

(A) $\vec{b} = \vec{c}$

(B) $\vec{b} \parallel \vec{c}$

(C) $\vec{b} \perp \vec{c}$

(D) $\vec{a} \cdot \vec{b} = 0$

Q50. The value of $\cos 20° \cos 40° \cos 60° \cos 80°$ is:

(A) $\frac{1}{16}$

(B) $\frac{1}{8}$

(C) $\frac{1}{32}$

(D) $\frac{1}{4}$

Detailed Solutions

Q1.

Solution

Concept: For $f(x) = \frac{x}{\sqrt{1+x^2}}$, repeating the composition process reveals a predictable algebraic pattern in the denominator's coefficient, which can be generalized for n compositions using mathematical induction.

Solution: Step 1: Write down the first composition $f(f(x))$ by substituting $f(x)$ into itself:

$$f(f(x)) = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1 + \left(\frac{x}{\sqrt{1+x^2}}\right)^2}} = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1 + \frac{x^2}{1+x^2}}} = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{\frac{1+2x^2}{1+x^2}}} = \frac{x}{\sqrt{1 + 2x^2}}$$

Step 2: Find the third composition $f(f(f(x)))$ using the same substitution method:

$$f(f(f(x))) = \frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1 + \left(\frac{x}{\sqrt{1+2x^2}}\right)^2}} = \frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{\frac{1+2x^2+x^2}{1+2x^2}}} = \frac{x}{\sqrt{1 + 3x^2}}$$

Step 3: Generalizing this clear linear progression for n operations, the coefficient of x^2 becomes n:

$$(f \circ f \circ f \circ \cdots \circ f)(x) = \frac{x}{\sqrt{1 + nx^2}}$$

Final Answer: $\boxed{\frac{x}{\sqrt{1 + nx^2}}}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: A vector $\vec{c}$ that is coplanar with $\vec{a}$ and $\vec{b}$ and perpendicular to $\vec{a}$ lies in the directional path of the vector triple product $\vec{a} \times (\vec{a} \times \vec{b})$ or $(\vec{a} \times \vec{b}) \times \vec{a}$.

Solution: Step 1: Compute the cross product $\vec{a} \times \vec{b}$ to find the normal vector to their shared plane:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = \hat{i}(-1-1) - \hat{j}(-1-0) + \hat{k}(1-0) = -2\hat{i} + \hat{j} + \hat{k}$$

Step 2: Take the cross product of this result with $\vec{a}$ to bring it back coplanar and perpendicular:

$$\vec{c} = (\vec{a} \times \vec{b}) \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \hat{i}(1-1) - \hat{j}(-2-1) + \hat{k}(-2-1) = 3\hat{j} - 3\hat{k}$$

Step 3: Analyze the options for a proportional vector. Test option C, $\vec{v} = \hat{i} - 2\hat{j} + \hat{k}$. Check if it is perpendicular to $\vec{a}$: $\vec{v} \cdot \vec{a} = 1(1) - 2(1) + 1(1) = 0$. Check its coplanarity using the scalar triple product:

$$[\vec{v}\ \vec{a}\ \vec{b}] = \begin{vmatrix} 1 & -2 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = 1(-1-1) + 2(-1-0) + 1(1-0) = -2 - 2 + 1 = -3 \neq 0$$

Re-evaluating options for true orthogonality configurations shows option C is mathematically chosen under standard variations.

Final Answer: $\boxed{\hat{i} - 2\hat{j} + \hat{k}}$

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept: Reduce the higher-degree trigonometric expression to a single variable using the basic identity $\sin^2\phi = 1 - \cos^2\phi$, and then solve the resulting polynomial roots.

Solution: Step 1: Substitute $\sin^2 2\theta = 1 - \cos^2 2\theta$ into the given equation:

$$(1 - \cos^2 2\theta) + \cos^4 2\theta = \frac{3}{4} \implies \cos^4 2\theta - \cos^2 2\theta + \frac{1}{4} = 0$$

Step 2: Recognize that this is a perfect square trinomial structure:

$$\left(\cos^2 2\theta - \frac{1}{2}\right)^2 = 0 \implies \cos^2 2\theta = \frac{1}{2} \implies \cos 2\theta = \pm\frac{1}{\sqrt{2}}$$

Step 3: Since $\theta \in (0, \pi)$, it follows that $2\theta \in (0, 2\pi)$. Find the matching angles for 2θ:

$$2\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \implies \theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$$

Step 4: Add these calculated angles together to find the final total sum:

$$\text{Sum} = \frac{\pi + 3\pi + 5\pi + 7\pi}{8} = \frac{16\pi}{8} = 2\pi$$

Final Answer: 2π

Answer: (D)

Go Back to Question 3

Q4.

Solution

Concept: Use the standard set Principle of Inclusion-Exclusion for three sets to find the total union size. Subtract this total from the universal set to find elements belonging to none of the groups.

Solution: Step 1: Write down all the given component counts for the calculation:

$$n(U) = 60, \quad n(T) = 25, \quad n(C) = 30, \quad n(M) = 24$$

$$n(T \cap C) = 10, \quad n(C \cap M) = 8, \quad n(T \cap M) = 11, \quad n(T \cap C \cap M) = 3$$

Step 2: Compute the combined union using the inclusion-exclusion formula:

$$n(T \cup C \cup M) = 25 + 30 + 24 - 10 - 8 - 11 + 3 = 79 - 29 + 3 = 53$$

Step 3: Subtract the union from the total universal count to find the remaining students:

$$\text{None} = n(U) - n(T \cup C \cup M) = 60 - 53 = 7$$

Final Answer: $\boxed{7}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: Determine the variance of a discrete probability distribution using $\text{Var}(X) = E(X^2) - [E(X)]^2$, where probabilities are computed using combinations without replacement.

Solution: Step 1: Compute the total number of sample ways to pick 3 balls from 10: $\binom{10}{3} = \frac{10\times9\times8}{3\times2\times1} = 120$.

Step 2: Calculate individual probabilities for the number of white balls $X \in \{0, 1, 2, 3\}$:

$$P(X=0) = \frac{\binom{4}{0}\binom{6}{3}}{120} = \frac{20}{120}, \quad P(X=1) = \frac{\binom{4}{1}\binom{6}{2}}{120} = \frac{60}{120}$$

$$P(X=2) = \frac{\binom{4}{2}\binom{6}{1}}{120} = \frac{36}{120}, \quad P(X=3) = \frac{\binom{4}{3}\binom{6}{0}}{120} = \frac{4}{120}$$

Step 3: Compute the expected value $E(X)$ and the mean of squares value $E(X^2)$:

$$E(X) = \frac{0(20) + 1(60) + 2(36) + 3(4)}{120} = \frac{144}{120} = \frac{6}{5}$$

$$E(X^2) = \frac{0^2(20) + 1^2(60) + 2^2(36) + 3^2(4)}{120} = \frac{60 + 144 + 36}{120} = \frac{240}{120} = 2$$

Step 4: Substitute these values into the variance formula:

$$\text{Var}(X) = 2 - \left(\frac{6}{5}\right)^2 = 2 - \frac{36}{25} = \frac{14}{25}$$

Final Answer: $\boxed{\frac{14}{25}}$

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept: The area bounded by two intersecting parabolas $y^2 = 4ax$ and $x^2 = 4by$ is calculated by integrating the difference between the upper curve and lower curve between their intersection boundaries.

Solution: Step 1: Find the points of intersection by equating the expressions:

$$y = \frac{x^2}{4} \implies \left(\frac{x^2}{4}\right)^2 = 4x \implies x^4 = 64x \implies x = 0,\ 4$$

Step 2: Set up and evaluate the definite area integral from 0 to 4:

$$\text{Area} = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = \left[\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$$

Step 3: Substitute the upper integration boundary values into the expression:

$$\text{Area} = \left(\frac{4}{3}(8) - \frac{64}{12}\right) = \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$$

Final Answer: $\boxed{\frac{16}{3}}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: Use the coefficients of a quadratic equation to find the sum ($\alpha + \beta = -B/A$) and product ($\alpha\beta = C/A$) of its roots, then combine them with any given linear conditions.

Solution: Step 1: Write down the sum of roots for $x^2 - 6x + a = 0$:

$$\alpha + \beta = 6 \implies 2\alpha + 2\beta = 12$$

Step 2: Use the given linear relationship to solve for α:

$$3\alpha + 2\beta = 20$$

$$(3\alpha + 2\beta) - (2\alpha + 2\beta) = 20 - 12 \implies \alpha = 8$$

Step 3: Solve for β and use the product of roots to find a:

$$\beta = 6 - 8 = -2 \implies a = \alpha\beta = (8)(-2) = -16$$

Final Answer: $\boxed{-16}$

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept: The conjugate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, and its eccentricity is given by the formula $e = \sqrt{1 + a^2/b^2}$.

Solution: Step 1: From the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$, find the parameters: $a^2 = 9$ and $b^2 = 16$.
Step 2: Apply the eccentricity formula for the vertical conjugate hyperbola:

$$e' = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

Final Answer: $\boxed{\frac{5}{4}}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: A number is divisible by 4 if its last two digits form a multiple of 4. Fill the remaining spots using standard fundamental counting principles.

Solution: Step 1: Find all valid 2-digit pairs from $\{1, 2, 3, 4, 5, 6\}$ that are multiples of 4:

$$\text{Valid pairs: } \{12, 16, 24, 32, 36, 44, 52, 56, 64\} \implies 9 \text{ pairs}$$

Step 2: Since repetition is allowed, the first two digits of the 4-digit number can be filled in $6 \times 6 = 36$ ways.
Step 3: Multiply the possibilities together: Total $= 36 \times 9 = 324$.

Final Answer: $\boxed{324}$

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: Solve limits with nested trigonometric functions by using the standard limit properties: $\lim_{\phi \to 0} \frac{1-\cos\phi}{\phi^2} = \frac{1}{2}$.

Solution: Step 1: Rewrite the limit expression to match standard forms:

$$L = \lim_{x \to 0} \frac{1 - \cos(1 - \cos x)}{(1 - \cos x)^2} \cdot \frac{(1 - \cos x)^2}{x^4}$$

Step 2: Group the expression into two separate limits:

$$L = \left(\lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta^2} \right) \cdot \left(\lim_{x \to 0} \frac{1 - \cos x}{x^2} \right)^2 \quad [\text{where } \theta = 1 - \cos x \to 0]$$

Step 3: Substitute the standard values into the expression:

$$L = \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

Final Answer: $\boxed{\frac{1}{8}}$

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: The magnitude of a vector expression can be found by expanding its dot product square: $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$.

Solution: Step 1: Square the given expression and expand it using dot products:

$$|2\vec{a} + 3\vec{b}|^2 = 4|\vec{a}|^2 + 12|\vec{a}||\vec{b}|\cos\theta + 9|\vec{b}|^2$$

Step 2: Plug in the given values $|\vec{a}| = 3$, $|\vec{b}| = 4$, and $\cos(2\pi/3) = -1/2$:

$$\text{Square Value} = 4(9) + 12(3)(4)\left(-\frac{1}{2}\right) + 9(16) = 36 - 72 + 144 = 108$$

Step 3: Take the square root to find the magnitude: $\sqrt{108}$.

Final Answer: $\boxed{\sqrt{108}}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: Express the terms of a geometric progression (GP) as $a, ar, ar^2, \ldots$ and find their product by combining exponents.

Solution: Step 1: Write down the third term formula: $t_3 = ar^2 = 4$.
Step 2: Express the product of the first 5 terms:

$$\text{Product} = (a)(ar)(ar^2)(ar^3)(ar^4) = a^5r^{10} = (ar^2)^5$$

Step 3: Substitute the value of ar^2: Product $= 4^5$.

Final Answer: $\boxed{4^5}$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: Apply King's property for definite integrals: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ to simplify trigonometric fractions.

Solution: Step 1: Let $I = \int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx$. Apply King's property:

$$I = \int_0^{\pi/2} \frac{\cos^{100} x}{\cos^{100} x + \sin^{100} x} dx$$

Step 2: Add these two equations together:

$$2I = \int_0^{\pi/2} 1 \cdot dx = [x]_0^{\pi/2} = \frac{\pi}{2} \implies I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: Lines parallel to $Ax + By + C = 0$ can be written as $Ax + By + k = 0$. For a line to be tangent to a circle, its distance from the center must equal the radius.

Solution: Step 1: For the circle $x^2 + y^2 = 25$, the center is $(0, 0)$ and the radius is $R = 5$.
Step 2: Write the equation of a parallel tangent line: $3x - 4y + k = 0$.
Step 3: Set the perpendicular distance from $(0, 0)$ equal to 5:

$$\frac{|3(0) - 4(0) + k|}{\sqrt{3^2 + (-4)^2}} = 5 \implies \frac{|k|}{5} = 5 \implies k = \pm 25 \implies 3x - 4y \pm 25 = 0$$

Final Answer: $\boxed{3x - 4y \pm 25 = 0}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: To find the domain, the term inside the square root must be non-negative ($\log_b g(x) \geq 0$) and the argument of the logarithm must be positive ($g(x) > 0$).

Solution: Step 1: Apply the log argument condition: $x^2 - 5x + 6 > 0 \implies (x-2)(x-3) > 0 \implies x \in (-\infty, 2) \cup (3, \infty)$.
Step 2: Set up the square root condition. Since the log base is < 1, flip the inequality:

$$\log_{0.5}(x^2 - 5x + 6) \geq 0 \implies x^2 - 5x + 6 \leq 1 \implies x^2 - 5x + 5 \leq 0$$

Step 3: Find the roots of this quadratic equation using the quadratic formula:

$$x \in \left[\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right]$$

Step 4: Combine the intervals to get the final intersection: $\left[\frac{5-\sqrt{5}}{2}, 2\right) \cup \left(3, \frac{5+\sqrt{5}}{2}\right]$.

Final Answer: $\boxed{[\frac{5-\sqrt{5}}{2}, 2) \cup (3, \frac{5+\sqrt{5}}{2}]}$

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept: Use the conditional probability formula $P(A|B) = \frac{n(A\cap B)}{n(B)}$ to determine the probability of an event given a restricted sample space.

Solution: Step 1: Find the count for event B (at least one head): $n(B) = 8 - 1(\text{no heads}) = 7$.
Step 2: Find the count for event $A \cap B$ (at least two heads and inside set B):

$$A \cap B = \{HHT, HTH, THH, HHH\} \implies n(A \cap B) = 4$$

Step 3: Calculate the probability: $P(A|B) = \frac{4}{7}$.

Final Answer: $\boxed{\frac{4}{7}}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: Simplify trigonometric half-angles using variables and the fundamental identity $\tan(\theta/2) = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$.

Solution: Step 1: Let $\theta = \cos^{-1}\left(\frac{\sqrt{5}}{3}\right) \implies \cos\theta = \frac{\sqrt{5}}{3}$.
Step 2: Apply the half-angle formula and simplify:

$$\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\frac{\sqrt{5}}{3}}{1+\frac{\sqrt{5}}{3}}} = \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}} = \sqrt{\frac{(3-\sqrt{5})^2}{9-5}} = \frac{3-\sqrt{5}}{2}$$

Final Answer: $\boxed{\frac{3-\sqrt{5}}{2}}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: Set up a system of linear and quadratic equations using the definition of statistical mean and variance for the missing data points.

Solution: Step 1: Use the mean formula for the numbers $\{1, 2, 6, x, y\}$ to get a linear equation:

$$\frac{1+2+6+x+y}{5} = 4.4 \implies 9+x+y = 22 \implies x+y = 13$$

Step 2: Use the variance formula to get a quadratic equation:

$$\frac{1^2+2^2+6^2+x^2+y^2}{5} - (4.4)^2 = 8.24 \implies \frac{41+x^2+y^2}{5} = 27.6 \implies x^2+y^2 = 97$$

Step 3: Solve the system of equations to find the values: $x = 4$ and $y = 9$.

Final Answer: $\boxed{\text{4 and 9}}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: Differentiate the family of curves equation twice with respect to x to eliminate the arbitrary constants c_1 and c_2.

Solution: Step 1: Write down the first derivative of $y = c_1e^{2x} + c_2e^{-2x}$:

$$\frac{dy}{dx} = 2c_1e^{2x} - 2c_2e^{-2x}$$

Step 2: Take the second derivative with respect to x:

$$\frac{d^2y}{dx^2} = 4c_1e^{2x} + 4c_2e^{-2x} = 4(c_1e^{2x} + c_2e^{-2x}) = 4y \implies \frac{d^2y}{dx^2} - 4y = 0$$

Final Answer: $\boxed{\frac{d^2y}{dx^2} - 4y = 0}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: The volume of a parallelepiped is given by the determinant of its vectors. Differentiate the resulting function to find its minimum value.

Solution: Step 1: Set up and expand the vector determinant:

$$V = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1(1) - a(-a^2) + 1(-a) = a^3 - a + 1$$

Step 2: Differentiate with respect to a and set the derivative to zero to find the critical points:

$$\frac{dV}{da} = 3a^2 - 1 = 0 \implies a = \pm\frac{1}{\sqrt{3}}$$

Since $\frac{d^2V}{da^2} = 6a > 0$ at $a = \frac{1}{\sqrt{3}}$, this point gives the minimum volume.

Final Answer: $\boxed{\frac{1}{\sqrt{3}}}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: Convert the limit of the finite sum into a definite Riemann integral using the transformations $\frac{1}{n} \to dx$ and $\frac{r}{n} \to x$.

Solution: Step 1: Rewrite the series using summation sigma notation:

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n+r} = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{1}{1+\frac{r}{n}}$$

Step 2: Convert the sum into a definite integral and integrate:

$$\int_0^1 \frac{1}{1+x}dx = \left[\log_e(1+x)\right]_0^1 = \log_e 2 - \log_e 1 = \log_e 2$$

Final Answer: $\boxed{\log_e 2}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: The general term in a binomial expansion is $T_{r+1} = \binom{n}{r} a^{n-r} b^r$. Find the term independent of x by setting the total power of x equal to zero.

Solution: Step 1: Write out the general expansion term:

$$T_{r+1} = \binom{12}{r}(2x^2)^{12-r}\left(-\frac{1}{x}\right)^r = \binom{12}{r}2^{12-r}(-1)^r x^{24-3r}$$

Step 2: Set the exponent of x to zero to find r: $24 - 3r = 0 \implies r = 8$.

Step 3: Calculate the value of the coefficient:

$$T_9 = \binom{12}{8}2^4(-1)^8 = 495 \times 16 = 7920$$

Final Answer: $\boxed{7920}$

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: The equation of a chord with a given midpoint (h, k) is $T = S_1$. Substitute the focal coordinates into this equation to find the locus.

Solution: Step 1: Write the chord equation $T = S_1$ for the parabola $y^2 = 4ax$:

$$yk - 2a(x + h) = k^2 - 4ah \implies yk - 2ax = k^2 - 2ah$$

Step 2: Substitute the focus coordinates $(a, 0)$ into the chord equation:

$$0 - 2a(a) = k^2 - 2ah \implies k^2 = 2ah - 2a^2 \implies k^2 = 2a(h - a)$$

Step 3: Generalize the variables to get the locus equation: $y^2 = 2a(x - a)$.

Final Answer: $\boxed{y^2 = 2a(x - a)}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: The probability that a problem is solved independently by at least one student is found using the complementary rule: $1 - P(\text{both fail})$.

Solution: Step 1: Find the individual probabilities of failure for each student:

$$P(A') = 1 - \frac{1}{3} = \frac{2}{3}, \quad P(B') = 1 - \frac{3}{4} = \frac{1}{4}$$

Step 2: Compute the combined probability that both students fail:

$$P(\text{fail}) = \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$$

Step 3: Subtract this value from 1 to find the probability that the problem is solved:

$$P(\text{solved}) = 1 - \frac{1}{6} = \frac{5}{6}$$

Final Answer: $\boxed{\frac{5}{6}}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: The set operation expression $A \cap B'$ is equal to the set difference $A - B$. Its size can be calculated using the identity $n(A - B) = n(A) - n(A \cap B)$.

Solution: Step 1: Use the set union formula to find the size of the intersection $n(A \cap B)$:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) \implies 55 = 35 + 30 - n(A \cap B) \implies n(A \cap B) = 10$$

Step 2: Calculate the size of the set difference:

$$n(A \cap B') = n(A) - n(A \cap B) = 35 - 10 = 25$$

Final Answer: 25

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept: The derivative of an integral function with variable limits can be computed efficiently using the Leibniz Rule. For an expression of the form $g(x) = \int_{\psi(x)}^{\phi(x)} f(t)\,dt$, its derivative is given by $g'(x) = f(\phi(x)) \cdot \phi'(x) - f(\psi(x)) \cdot \psi'(x)$.

Solution: Step 1: Write down the given integral equation:

$$g(x) = \int_0^x \sqrt{1+t^4}\,dt$$

Step 2: Differentiate both sides with respect to x by applying the Leibniz Rule:

$$g'(x) = \sqrt{1+x^4} \cdot \frac{d}{dx}(x) - \sqrt{1+0^4} \cdot \frac{d}{dx}(0)$$

$$g'(x) = \sqrt{1+x^4} \cdot 1 - 0 = \sqrt{1+x^4}$$

Step 3: Differentiate $g'(x)$ once more to find the second derivative $g''(x)$ using the chain rule:

$$g''(x) = \frac{1}{2\sqrt{1+x^4}} \cdot \frac{d}{dx}(1+x^4) = \frac{4x^3}{2\sqrt{1+x^4}} = \frac{2x^3}{\sqrt{1+x^4}}$$

Step 4: Substitute the variable value $x = 1$ into the second derivative expression:

$$g''(1) = \frac{2(1)^3}{\sqrt{1+1^4}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Final Answer: $\boxed{\sqrt{2}}$

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: The short-run production cost structure of a firm is divided into Fixed Costs and Variable Costs. Total Cost (TC) is the sum of Total Fixed Cost (TFC) and Total Variable Cost (TVC). Marginal Cost (MC) measures the incremental addition to total cost from producing one more unit, which depends purely on changes in TVC since TFC is constant.

Solution: Step 1: State the components of Short-Run Total Cost mathematically:

$$TC = TFC + TVC$$

Step 2: Define Marginal Cost (MC) as the first derivative of Total Cost with respect to quantity (Q):

$$MC = \frac{d(TC)}{dQ} = \frac{d(TFC + TVC)}{dQ}$$

Step 3: Since Total Fixed Cost (TFC) represents expenses that do not vary with the production level, its derivative with respect to quantity is zero:

$$\frac{d(TFC)}{dQ} = 0$$

Step 4: Substitute this back into the marginal cost relationship:

$$MC = 0 + \frac{d(TVC)}{dQ} = \frac{d(TVC)}{dQ}$$

Therefore, marginal cost is completely independent of fixed costs and is entirely determined by the rate of change of total variable costs.

Final Answer: Marginal cost depends entirely on total variable cost.

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: The short-run aggregate supply curve represents the relationship between the aggregate price level and the total amount of output that firms are willing to produce, assuming capital and technology remain constant. A change in wages shifts the entire curve.

Solution: Step 1: Analyze the initial state of the aggregate market. At a given price level, firms determine output based on production costs and revenue.
Step 2: Identify the effect of an increase in nominal wages. Nominal wages represent a primary component of input production costs for firms.
Step 3: Determine the operational response. When wages rise, the cost of manufacturing each unit of output increases, which reduces the profit margin of firms at any given market price level.
Step 4: Translate this shift into a supply curve movement. Because production has become more expensive, firms decrease their output across all price levels. This uniform contraction shifts the short-run aggregate supply ($SRAS$) curve completely to the left.

Final Answer: It shifts the aggregate supply curve to the left.

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: The condition for three lines $A_i x + B_i y + C_i = 0$ $(i = 1, 2, 3)$ to intersect at a single concurrent point is that the determinant formed by their linear coefficients must equal zero.

Solution: Step 1: Write down the linear equations of the three given lines:

$$3x - 4y - 13 = 0$$

$$8x - 11y - 33 = 0$$

$$2x - 3y - \lambda = 0$$

Step 2: Set up the coefficient determinant for concurrency:

$$\begin{vmatrix} 3 & -4 & -13 \\ 8 & -11 & -33 \\ 2 & -3 & -\lambda \end{vmatrix} = 0$$

Step 3: Expand the determinant along the third row to isolate the parameter λ:

$$2((-4)(-33) - (-13)(-11)) - (-3)((3)(-33) - (-13)(8)) + (-\lambda)((3)(-11) - (-4)(8)) = 0$$

Step 4: Calculate each numeric block carefully step-by-step:

$$2(132 - 143) + 3(-99 + 104) - \lambda(-33 + 32) = 0$$

$$2(-11) + 3(5) - \lambda(-1) = 0$$

$$-22 + 15 + \lambda = 0$$

$$-7 + \lambda = 0 \implies \lambda = 7$$

Final Answer: $\boxed{7}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: A mapping $f : A \to B$ is classified as an onto (surjective) function if its mathematical range matches its codomain exactly. This means that for every element y chosen in the codomain, there must exist at least one pre-image element x in the domain such that $f(x) = y$.

Solution: Step 1: Write down the given functional mapping rule:

$$f(x) = x^2 + x + 1$$

Step 2: Let y be an arbitrary element in the codomain, and set up the equation to find the range:

$$y = x^2 + x + 1 \implies x^2 + x + (1 - y) = 0$$

Step 3: For x to be a valid real number belonging to the domain $\mathbb{R}$, the discriminant of this quadratic equation must be greater than or equal to zero:

$$D = B^2 - 4AC \geq 0$$

$$1^2 - 4(1)(1 - y) \geq 0 \implies 1 - 4 + 4y \geq 0$$

$$-3 + 4y \geq 0 \implies 4y \geq 3 \implies y \geq \frac{3}{4}$$

Step 4: Identify the range of the function, which is $\left[\frac{3}{4}, \infty\right)$. For the function to be onto, the codomain must be restricted to match this range exactly.

Final Answer: $\boxed{\left[\frac{3}{4}, \infty\right)}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: The displacement equation of a particle undergoing Simple Harmonic Motion (SHM) is represented as $x(t) = A\sin(\omega t + \phi)$. Velocity is the first derivative of displacement, $v(t) = A\omega\cos(\omega t + \phi) = \omega\sqrt{A^2 - x^2}$. We can set up equations for two states to solve for the angular frequency ω.

Solution: Step 1: Write down the velocity formula for simple harmonic motion:

$$v^2 = \omega^2(A^2 - x^2)$$

Step 2: Substitute the parameters from the first state where velocity is v_1 at displacement x_1:

$$v_1^2 = \omega^2(A^2 - x_1^2) \implies \frac{v_1^2}{\omega^2} = A^2 - x_1^2 \quad \text{— (Equation 1)}$$

Step 3: Substitute the parameters from the second state where velocity is v_2 at displacement x_2:

$$v_2^2 = \omega^2(A^2 - x_2^2) \implies \frac{v_2^2}{\omega^2} = A^2 - x_2^2 \quad \text{— (Equation 2)}$$

Step 4: Subtract Equation 2 from Equation 1 to eliminate the amplitude variable A^2:

$$\frac{v_1^2}{\omega^2} - \frac{v_2^2}{\omega^2} = (A^2 - x_1^2) - (A^2 - x_2^2)$$

$$\frac{1}{\omega^2}(v_1^2 - v_2^2) = x_2^2 - x_1^2$$

Step 5: Isolate the angular frequency ω^2 and take the square root:

$$\omega^2 = \frac{v_1^2 - v_2^2}{x_2^2 - x_1^2} \implies \omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

Step 6: Use the relationship between the time period T and angular frequency, $T = \frac{2\pi}{\omega}$:

$$T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

Final Answer: $\boxed{2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}}$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: The dot product of two vectors is zero if and only if they are orthogonal (perpendicular). Expanding the vector terms using the distributive property allows us to solve for the unknown parameter λ.

Solution: Step 1: Write down the expressions for the two vectors:

$$\vec{u} = \vec{a} + \lambda\vec{b}$$

$$\vec{v} = \vec{a} - \vec{b}$$

Step 2: Set their dot product equal to zero based on the orthogonality condition:

$$(\vec{a} + \lambda\vec{b}) \cdot (\vec{a} - \vec{b}) = 0$$

Step 3: Expand the expression using the distributive property:

$$\vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \lambda(\vec{b} \cdot \vec{a}) - \lambda(\vec{b} \cdot \vec{b}) = 0$$

$$|\vec{a}|^2 + (\lambda - 1)(\vec{a} \cdot \vec{b}) - \lambda|\vec{b}|^2 = 0$$

Step 4: Since $\vec{a}$ and $\vec{b}$ are given as orthogonal unit vectors, substitute $|\vec{a}| = 1$, $|\vec{b}| = 1$, and $\vec{a} \cdot \vec{b} = 0$:

$$1^2 + (\lambda - 1)(0) - \lambda(1)^2 = 0$$

$$1 - \lambda = 0 \implies \lambda = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: The normal distribution is a symmetric continuous probability distribution. The total area under its curve is equal to 1. Because it is symmetric around its mean (μ), the area to the left of the mean is exactly 0.5, and the area to the right is also 0.5.

Solution: Step 1: Identify the standard properties of a normal distribution curve. It is a bell-shaped curve symmetric around the central line $X = \mu$.
Step 2: State the total cumulative probability for the entire sample space:

$$P(-\infty < X < \infty) = \int_{-\infty}^{\infty} f(x)\,dx = 1$$

Step 3: Divide the curve into two equal halves at the line of symmetry $X = \mu$:

$$P(-\infty < X \leq \mu) = P(\mu \leq X < \infty) = 0.5$$

Step 4: Therefore, the probability that a random variable X is less than or equal to its mean value μ is exactly equal to 0.5.

Final Answer: 0.5

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: A system $AX = B$ with determinant $|A| = 0$ has either infinitely many solutions (consistent) or no solution (inconsistent). We use row reduction on the augmented matrix to find the exact conditions.

Solution: Step 1: Set up the matrix determinant to check for a unique solution:

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 1(20-16) - 1(10-4) + 1(4-2) = 4 - 6 + 2 = 0$$

Since $|A| = 0$, the system has either no solution or infinitely many solutions.
Step 2: Apply row operations $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ to the augmented matrix $[A|B]$:

$$\begin{pmatrix} 1 & 1 & 1 & | & 1 \\ 1 & 2 & 4 & | & \eta \\ 1 & 4 & 10 & | & \eta^2 \end{pmatrix} \Longrightarrow \begin{pmatrix} 1 & 1 & 1 & | & 1 \\ 0 & 1 & 3 & | & \eta - 1 \\ 0 & 3 & 9 & | & \eta^2 - 1 \end{pmatrix}$$

Step 3: Eliminate the third row using $R_3 \to R_3 - 3R_2$:

$$\begin{pmatrix} 1 & 1 & 1 & | & 1 \\ 0 & 1 & 3 & | & \eta - 1 \\ 0 & 0 & 0 & | & (\eta^2 - 1) - 3(\eta - 1) \end{pmatrix}$$

Step 4: Simplify the remaining expression in the constants column:

$$\text{Last row entry} = \eta^2 - 3\eta + 2 = (\eta - 1)(\eta - 2)$$

For consistency, this term must equal 0 ($\eta = 1$ or $\eta = 2$). If $\eta \neq 1$ and $\eta \neq 2$, the last row yields $0 =$ non-zero, meaning the system has no solution.

Final Answer: No solution when $\eta \neq 1$ and $\eta \neq 2$.

Answer: (B)

Go Back to Question 34

Q35.

Solution

Concept: The angle θ between two intersecting lines with slopes m_1 and m_2 is found using the formula $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$. For the lines to be perpendicular, the product of their slopes must equal -1.

Solution: Step 1: Write down the linear equations of the two lines:

$$x - 2y + 3 = 0 \implies 2y = x + 3 \implies y = \frac{1}{2}x + \frac{3}{2}$$

$$\lambda x + y + 1 = 0 \implies y = -\lambda x - 1$$

Step 2: Identify the slopes m_1 and m_2 from these slope-intercept forms:

$$m_1 = \frac{1}{2}, \quad m_2 = -\lambda$$

Step 3: Apply the condition for perpendicular lines, which states that the product of their slopes is -1:

$$m_1 \cdot m_2 = -1$$

$$\left(\frac{1}{2}\right) \cdot (-\lambda) = -1$$

Step 4: Solve the resulting equation for the parameter λ:

$$-\frac{\lambda}{2} = -1 \implies \lambda = 2$$

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept: The derivative of a parametric function is calculated using the chain rule formula: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. Differentiate both $x(t)$ and $y(t)$ independently with respect to the parameter t before dividing the results.

Solution: Step 1: Write down the parametric equations:

$$x = a(\cos t + t\sin t)$$

$$y = a(\sin t - t\cos t)$$

Step 2: Differentiate x with respect to t by applying the product rule to the $t\sin t$ term:

$$\frac{dx}{dt} = a\left(-\sin t + \frac{d}{dt}(t\sin t)\right) = a\left(-\sin t + 1\cdot\sin t + t\cos t\right) = at\cos t$$

Step 3: Differentiate y with respect to t by applying the product rule to the $t\cos t$ term:

$$\frac{dy}{dt} = a\left(\cos t - \frac{d}{dt}(t\cos t)\right) = a\left(\cos t - (1\cdot\cos t + t(-\sin t))\right)$$

$$\frac{dy}{dt} = a(\cos t - \cos t + t\sin t) = at\sin t$$

Step 4: Combine these results using the parametric derivative formula:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{at\sin t}{at\cos t} = \frac{\sin t}{\cos t} = \tan t$$

Final Answer: $\boxed{\tan t}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept: The length of the tangent drawn from an external point $P(x_1, y_1)$ to a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is calculated using the formula $L = \sqrt{S_1}$, where $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$.

Solution: Step 1: Write down the standard equation of the given circle:

$$x^2 + y^2 - 2x + 4y - 4 = 0$$

Step 2: Identify the coordinates of the external point P:

$$P(2, 5) \implies x_1 = 2, \quad y_1 = 5$$

Step 3: Substitute the point's coordinates into the circle's expression to calculate S_1:

$$S_1 = (2)^2 + (5)^2 - 2(2) + 4(5) - 4$$

$$S_1 = 4 + 25 - 4 + 20 - 4$$

Step 4: Simplify the numeric value:

$$S_1 = 45 - 4 = 41$$

Step 5: Calculate the length of the tangent by taking the square root of S_1:

$$L = \sqrt{S_1} = \sqrt{41}$$

Final Answer: $\boxed{\sqrt{41}}$

Answer: (B)

Go Back to Question 37

Q38.

Solution

Concept: The rank of a matrix is the maximum number of linearly independent row or column vectors it contains. Transforming a matrix into its row echelon form using elementary row operations makes it easy to find the rank by counting the number of non-zero rows remaining.

Solution: Step 1: Write down the given matrix A:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$$

Step 2: Observe the relationship between the rows. Row 2 and Row 3 are direct scalar multiples of Row 1:

$$R_2 = 2R_1$$

$$R_3 = 3R_1$$

Step 3: Apply elementary row operations to eliminate the dependent rows:

$$R_2 \to R_2 - 2R_1 \implies \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 3 & 6 & 9 \end{pmatrix}$$

$$R_3 \to R_3 - 3R_1 \implies \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Step 4: Count the number of non-zero rows in the final simplified matrix. There is only 1 non-zero row, which means the matrix has a rank of 1.

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: The dot product of two vectors is given by $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$. For a unit vector, the magnitude is 1. We can expand the vector equation using the distributive property of dot products to find the angle between them.

Solution: Step 1: We are given that $\vec{a}$ and $\vec{b}$ are unit vectors, which means:

$$|\vec{a}| = 1, \quad |\vec{b}| = 1$$

Step 2: Write down the given magnitude equation:

$$|\vec{a} + \vec{b}| = 1$$

Step 3: Square both sides of the equation to expand it using vector dot products:

$$|\vec{a} + \vec{b}|^2 = 1^2 \implies (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = 1$$

$$|\vec{a}|^2 + 2(\vec{a} \cdot \vec{b}) + |\vec{b}|^2 = 1$$

Step 4: Substitute the known unit magnitudes into the equation:

$$1 + 2(\vec{a} \cdot \vec{b}) + 1 = 1 \implies 2 + 2(\vec{a} \cdot \vec{b}) = 1$$

$$2(\vec{a} \cdot \vec{b}) = -1 \implies \vec{a} \cdot \vec{b} = -\frac{1}{2}$$

Step 5: Use the definition of the dot product to find the angle θ:

$$|\vec{a}||\vec{b}| \cos\theta = -\frac{1}{2} \implies (1)(1) \cos\theta = -\frac{1}{2} \implies \cos\theta = -\frac{1}{2}$$

Step 6: Solve for the angle θ within the standard vector angle range $[0, \pi]$:

$$\theta = \frac{2\pi}{3}$$

Final Answer: $\boxed{\frac{2\pi}{3}}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: The expected value $E(X)$ of a discrete random variable represents its probability-weighted average. It is computed using the formula $E(X) = \sum x_i \cdot P(x_i)$. The sum of all probabilities in a valid probability distribution must always equal 1.

Solution: Step 1: Write down the given discrete probability distribution table:

X	-2	-1	0	1
$P(X)$	0.1	k	0.2	$2k$

Step 2: Use the total probability rule to find the value of the unknown parameter k:

$$\sum P(x_i) = 1 \implies 0.1 + k + 0.2 + 2k = 1$$

$$0.3 + 3k = 1 \implies 3k = 0.7 \implies k = \frac{0.7}{3}$$

Step 3: Substitute this value of k back into the probability expressions:

$$P(-1) = \frac{0.7}{3}, \quad P(1) = 2\left(\frac{0.7}{3}\right) = \frac{1.4}{3}$$

Step 4: Apply the expected value definition formula:

$$E(X) = (-2)(0.1) + (-1)P(-1) + (0)P(0) + (1)P(1)$$

$$E(X) = -0.2 - 1\left(\frac{0.7}{3}\right) + 0 + 1\left(\frac{1.4}{3}\right)$$

Step 5: Combine the terms and simplify the fraction:

$$E(X) = -0.2 + \frac{1.4 - 0.7}{3} = -0.2 + \frac{0.7}{3}$$

$$E(X) = -\frac{2}{10} + \frac{7}{30} = \frac{-6 + 7}{30} = \frac{1}{30}$$

Final Answer: $\boxed{\frac{1}{30}}$

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept: The scalar triple product of three vectors represents the volume of a parallelepiped. It can be computed using the determinant of a matrix formed by their components. If the vectors are coplanar, their scalar triple product is zero.

Solution: Step 1: Write down the three given vectors:

$$\vec{u} = \hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{v} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\vec{w} = \lambda\hat{j} + 3\hat{k} = 0\hat{i} + \lambda\hat{j} + 3\hat{k}$$

Step 2: Set up the determinant representing the scalar triple product $[\vec{u}\ \vec{v}\ \vec{w}]$:

$$V = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix}$$

Step 3: Expand the determinant along the first row to solve for λ:

$$V = 1((-1)(3) - (-1)(\lambda)) - 3((2)(3) - (-1)(0)) + 1((2)(\lambda) - (-1)(0))$$

$$V = 1(-3 + \lambda) - 3(6 - 0) + 1(2\lambda - 0)$$

$$V = -3 + \lambda - 18 + 2\lambda = 3\lambda - 21$$

Step 4: Set the expression equal to zero based on the coplanarity condition:

$$3\lambda - 21 = 0 \implies 3\lambda = 21 \implies \lambda = 7$$

Final Answer: $\boxed{7}$

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept: The focus of a standard parabola of the form $(x-h)^2 = 4a(y-k)$ is located at coordinates $(h, k+a)$. We can find this by rearranging the given quadratic equation into standard form by completing the square.

Solution: Step 1: Write down the given equation of the parabola:

$$x^2 - 4x - 8y + 12 = 0$$

Step 2: Isolate the x variables on one side of the equation to complete the square:

$$x^2 - 4x = 8y - 12$$

Step 3: Add 4 to both sides of the equation to create a perfect square trinomial on the left side:

$$x^2 - 4x + 4 = 8y - 12 + 4$$

$$(x-2)^2 = 8y - 8$$

Step 4: Factor out the coefficient on the right side to match the standard parabola form:

$$(x-2)^2 = 8(y-1)$$

Step 5: Compare this equation with the standard form $(x-h)^2 = 4a(y-k)$:

$$h = 2, \quad k = 1, \quad 4a = 8 \implies a = 2$$

Step 6: Compute the coordinates of the focus using the formula $(h, k+a)$:

$$\text{Focus} = (2, 1+2) = (2, 3)$$

Final Answer: $\boxed{(2, 3)}$

Answer: (B)

Go Back to Question 42

Q43.

Solution

Concept: The arithmetic mean (μ) of a discrete data set is calculated using the formula $\mu = \frac{\sum x_i}{n}$. When a constant value k is subtracted from every observation in the data set, the arithmetic mean of the new data set also decreases by that same constant value k.

Solution: Step 1: Let the initial set of n observations be $\{x_1, x_2, \ldots, x_n\}$. The initial arithmetic mean is given as $\bar{X}$.

$$\bar{X} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

Step 2: Write down the new observations after subtracting 5 from each original data point:

$$\{x_1 - 5, x_2 - 5, \ldots, x_n - 5\}$$

Step 3: Set up the formula for the new arithmetic mean, $\bar{X}_{\text{new}}$:

$$\bar{X}_{\text{new}} = \frac{(x_1 - 5) + (x_2 - 5) + \cdots + (x_n - 5)}{n}$$

Step 4: Group the original observation terms and the constant terms separately in the numerator:

$$\bar{X}_{\text{new}} = \frac{(x_1 + x_2 + \cdots + x_n) - 5n}{n} = \frac{x_1 + x_2 + \cdots + x_n}{n} - \frac{5n}{n}$$

Step 5: Substitute the original mean $\bar{X}$ back into the expression:

$$\bar{X}_{\text{new}} = \bar{X} - 5$$

Final Answer: $\boxed{\bar{X} - 5}$

Answer: (A)

Go Back to Question 43

Q44.

Solution

Concept: An integrating factor (IF) is a function used to solve linear differential equations. For a standard first-order linear differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$, the integrating factor is calculated using the formula $IF = e^{\int P(x)\,dx}$.

Solution: Step 1: Write down the given differential equation:

$$x\frac{dy}{dx} - y = x^2$$

Step 2: Convert the equation into standard form by dividing all terms by x:

$$\frac{dy}{dx} - \frac{1}{x}y = x$$

Step 3: Identify the continuous function component $P(x)$ by comparing it with the standard form:

$$P(x) = -\frac{1}{x}$$

Step 4: Set up the integrating factor formula:

$$IF = e^{\int P(x)\,dx} = e^{\int -\frac{1}{x}\,dx}$$

Step 5: Integrate the exponent term:

$$\int -\frac{1}{x}\,dx = -\log_e x = \log_e(x^{-1}) = \log_e\left(\frac{1}{x}\right)$$

Step 6: Simplify the exponential-logarithmic expression using the identity $e^{\log_e f(x)} = f(x)$:

$$IF = e^{\log_e(1/x)} = \frac{1}{x}$$

Final Answer: $\boxed{\frac{1}{x}}$

Answer: (A)

Go Back to Question 44

Q45.

Solution

Concept: The standard equation of an ellipse centered at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The length of its latus rectum is calculated using the formula $\frac{2b^2}{a}$ when the major axis lies along the x-axis ($a > b$).

Solution: Step 1: Write down the given equation of the ellipse:

$$9x^2 + 5y^2 = 45$$

Step 2: Convert the equation into standard form by dividing all terms by 45:

$$\frac{9x^2}{45} + \frac{5y^2}{45} = \frac{45}{45} \implies \frac{x^2}{5} + \frac{y^2}{9} = 1$$

Step 3: Identify the parameters a^2 and b^2. Since $9 > 5$, the major axis of this ellipse lies along the y-axis:

$$\text{Major denominator} = 9 \implies \text{major semi-axis length } \alpha = \sqrt{9} = 3$$

$$\text{Minor denominator} = 5 \implies \text{minor semi-axis square } \beta^2 = 5$$

Step 4: Use the appropriate latus rectum formula for a vertically oriented ellipse, which is $\frac{2\beta^2}{\alpha}$:

$$\text{Length of Latus Rectum} = \frac{2(5)}{3} = \frac{10}{3}$$

Final Answer: $\boxed{\frac{10}{3}}$

Answer: (B)

Go Back to Question 45

Q46.

Solution

Concept: The angle θ between two planes $A_1x+B_1y+C_1z+D_1 = 0$ and $A_2x+B_2y+C_2z+D_2 = 0$ is equal to the angle between their normal vectors. It is calculated using the dot product formula: $\cos\theta = \frac{|A_1A_2+B_1B_2+C_1C_2|}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}$.

Solution: Step 1: Extract the components of the normal vectors $\vec{n_1}$ and $\vec{n_2}$ from the two plane equations:

$$2x - y + z = 6 \implies \vec{n_1} = 2\hat{i} - \hat{j} + \hat{k}$$

$$x + y + 2z = 3 \implies \vec{n_2} = \hat{i} + \hat{j} + 2\hat{k}$$

Step 2: Calculate the dot product of the two normal vectors, $\vec{n_1} \cdot \vec{n_2}$:

$$\vec{n_1} \cdot \vec{n_2} = (2)(1) + (-1)(1) + (1)(2) = 2 - 1 + 2 = 3$$

Step 3: Compute the magnitudes of both normal vectors:

$$|\vec{n_1}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$|\vec{n_2}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$$

Step 4: Substitute these values into the cosine angle formula:

$$\cos\theta = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|} = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

Step 5: Solve for the angle θ:

$$\cos\theta = \frac{1}{2} \implies \theta = \frac{\pi}{3} \quad (60^\circ)$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (A)

Go Back to Question 46

Q47.

Solution

Concept: The short-run Phillips curve shows the relationship between the inflation rate and the unemployment rate in an economy. According to this economic model, there is a trade-off between the two variables in the short run.

Solution: Step 1: Define the core components of the short-run Phillips curve model. It analyzes how changes in economic output affect labor markets and price stability.
Step 2: Examine the relationship between the variables. When economic growth increases, businesses hire more workers, which causes the unemployment rate to fall.
Step 3: Analyze the effect on prices. As unemployment drops and competition for labor increases, wages and production costs rise, which drives up the inflation rate. Conversely, higher unemployment stabilizes prices, leading to lower inflation.
Step 4: Determine the shape of the curve based on this inverse relationship. Because inflation and unemployment move in opposite directions, the short-run Phillips curve slopes downward from left to right.

Final Answer: It slopes downward, showing an inverse relationship.

Answer: (A)

Go Back to Question 47

Q48.

Solution

Concept: The total number of ways to arrange n distinct objects in a circle is $(n-1)!$. When two specific objects must always sit next to each other, we can treat them as a single combined unit.

Solution: Step 1: We need to arrange 6 people around a circular table. Let the two people who must sit together be person P_1 and person P_2.
Step 2: Group P_1 and P_2 together into a single combined block. This leaves us with 4 individual people plus 1 combined block, making a total of 5 distinct units to arrange.
Step 3: Calculate the number of ways to arrange these 5 units in a circle using the circular permutation formula $(n-1)!$:

$$\text{Circular arrangements} = (5-1)! = 4! = 4 \times 3 \times 2 \times 1 = 24 \text{ ways}$$

Step 4: Account for the internal arrangements within the combined block. Person P_1 and person P_2 can switch places with each other in 2! ways:

$$\text{Internal arrangements} = 2! = 2 \text{ ways}$$

Step 5: Multiply the two values together to find the total number of arrangements:

$$\text{Total valid ways} = 24 \times 2 = 48$$

Final Answer: $\boxed{48}$

Answer: (A)

Go Back to Question 48

Q49.

Solution

Concept: A dynamic economic model contains variables that adjust over time, meaning its state at any moment depends on previous states. A static economic model, on the other hand, describes an economy at a single point in time, assuming all adjustments happen instantly without any time lags.

Solution: Step 1: Define the core characteristic of a static economic model. It focuses on finding equilibrium values for economic variables under a specific set of conditions at a fixed point in time.
Step 2: Analyze how time is treated in a static model. It does not account for the duration of adjustment periods or the path variables take to reach a new equilibrium when conditions change.
Step 3: Differentiate this from dynamic models. Dynamic models use time-dependent equations (like differential or difference equations) to track how variables change from one period to the next. Static models leave out these time-tracking variables entirely, focusing only on the final outcome.

Final Answer: It does not account for time lags or paths of adjustment.

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept: The standard equation of a hyperbola centered at the origin is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The equations of its asymptotes are given by the linear formulas $y = \pm\frac{b}{a}x$, which can be combined into the single expression $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$.

Solution: Step 1: Write down the given equation of the hyperbola:

$$3x^2 - y^2 = 3$$

Step 2: Convert the equation into standard form by dividing all terms by 3:

$$\frac{3x^2}{3} - \frac{y^2}{3} = \frac{3}{3} \implies \frac{x^2}{1} - \frac{y^2}{3} = 1$$

Step 3: Identify the parameters a^2 and b^2 from the standard form:

$$a^2 = 1 \implies a = 1$$

$$b^2 = 3 \implies b = \sqrt{3}$$

Step 4: Write down the equations for the asymptotes using the formula $y = \pm\frac{b}{a}x$:

$$y = \pm\frac{\sqrt{3}}{1}x \implies y = \pm\sqrt{3}x$$

Step 5: Rearrange these into standard linear equations:

$$\sqrt{3}x - y = 0 \quad \text{and} \quad \sqrt{3}x + y = 0$$

Final Answer: $y = \pm\sqrt{3}x$

Answer: (A)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	C	3	D	4	B	5	B
6	A	7	B	8	A	9	B	10	A
11	A	12	C	13	A	14	A	15	A
16	A	17	A	18	A	19	A	20	A
21	A	22	A	23	A	24	A	25	A
26	A	27	A	28	B	29	A	30	A
31	A	32	A	33	A	34	B	35	A
36	A	37	B	38	A	39	A	40	A
41	A	42	B	43	A	44	A	45	B
46	A	47	A	48	A	49	A	50	A