NIMCET Mathematics Sample Paper-9

Duration: 70 Minutes **Maximum Marks: 600**

Instructions

- This paper contains **50** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+12 marks**.
- Each incorrect answer carries: **−3** marks.
- Unattempted questions carry **0** marks.
- Only one option is correct for each question.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. Let A and B be two sets such that $n(A \cup B) = 50$, $n(A \setminus B) = 15$, and $n(B \setminus A) = 22$. Find the value of $n(A \cap B)$.

(A) 11

(B) 13

(C) 15

(D) 17

Q2. What is the value of $\lim_{x\to 0} \frac{\ln(1+3x)}{\sin(2x)}$?

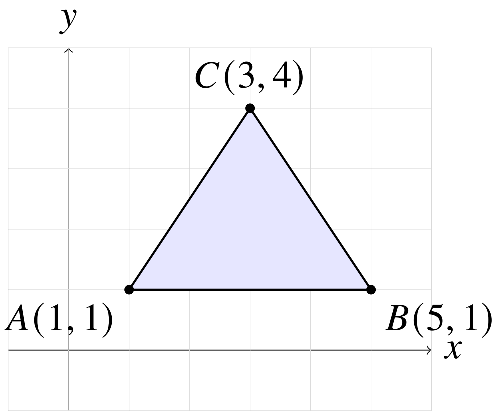
(A) $\frac{2}{3}$

(B) 1

(C) $\frac{3}{2}$

(D) Does not exist

Q3. Consider the triangle ABC with vertices $A(1,1)$, $B(5,1)$, and $C(3,4)$. An analytical representation of its boundary and region can be analyzed through its coordinate graph:



Find the area of the triangle ABC.

(A) 6 sq. units

(B) 8 sq. units

(C) 12 sq. units

(D) 16 sq. units

Q4. If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$, then find the unit vector in the direction of $\vec{a} + \vec{b}$.

(A) $\frac{3\hat{i}+4\hat{j}+4\hat{k}}{\sqrt{41}}$

(B) $\frac{3\hat{i}+4\hat{j}+4\hat{k}}{\sqrt{33}}$

(C) $\frac{\hat{i}-2\hat{j}-6\hat{k}}{\sqrt{41}}$

(D) $\frac{3\hat{i}+4\hat{j}+4\hat{k}}{\sqrt{50}}$

Q5. A box contains 4 red balls and 6 black balls. Three balls are drawn at random one by one without replacement. What is the probability that the first two are red and the third one is black?

(A) $\frac{1}{10}$

(B) $\frac{1}{12}$

(C) $\frac{3}{20}$

(D) $\frac{1}{15}$

Q6. If α and β are the roots of the quadratic equation $x^2 - 5x + 6 = 0$, find the value of $\alpha^3 + \beta^3$.

(A) 35

(B) 45

(C) 65

(D) 95

Q7. Solve the inequality $\frac{x-1}{x+2} \geq 0$.

(A) $(-\infty, -2) \cup [1, \infty)$

(B) $(-\infty, -2] \cup [1, \infty)$

(C) $(-2, 1]$

(D) $(-\infty, \infty)$

Q8. Find the general solution of the trigonometric equation $\sin^2\theta - 2\sin\theta = 0$.

(A) $\theta = n\pi, n \in \mathbb{Z}$

(B) $\theta = 2n\pi, n \in \mathbb{Z}$

(C) $\theta = n\pi + (-1)^n\frac{\pi}{2}, n \in \mathbb{Z}$

(D) No solution exists

Q9. Find the equation of the line passing through the point $(2, 3)$ and perpendicular to the line $3x - 4y + 7 = 0$.

(A) $4x + 3y - 17 = 0$

(B) $4x - 3y + 1 = 0$

(C) $3x + 4y - 18 = 0$

(D) $4x + 3y + 17 = 0$

Q10. Evaluate the definite integral $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx$.

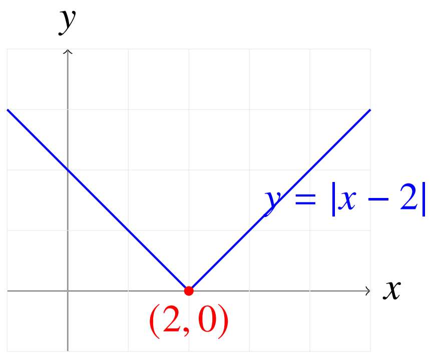
(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) π

(D) 0

Q11. Let $f(x) = |x - 2|$. The behavior of this function around its critical point can be interpreted via its geometric graph:



Which of the following statements is true for $f(x)$ at $x = 2$?

(A) It is differentiable but not continuous.

(B) It is continuous but not differentiable.

(C) It is both continuous and differentiable.

(D) It is neither continuous nor differentiable.

Q12. In how many different ways can the letters of the word 'NIMCET' be arranged such that the vowels always come together?

(A) 120

(B) 240

(C) 360

(D) 720

Q13. If the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, and $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar, find the value of λ.

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Q14. The standard deviation of 5 observations is 4. If each observation is multiplied by 3, what will be the new variance?

(A) 12

(B) 36

(C) 48

(D) 144

Q15. Find the value of $\cos 15° - \sin 15°$.

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

(D) $\sqrt{2}$

Q16. Find the sum to infinity of the geometric progression $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$.

(A) $\frac{4}{3}$

(B) $\frac{3}{2}$

(C) 2

(D) $\frac{5}{3}$

Q17. Let R be a relation on the set of integers $\mathbb{Z}$ defined by aRb if and only if $a - b$ is divisible by 5. The relation R is:

(A) Reflexive and symmetric but not transitive

(B) Symmetric and transitive but not reflexive

(C) An equivalence relation

(D) A partial order relation

Q18. What is the value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$?

(A) 0

(B) $a + b + c$

(C) abc

(D) 1

Q19. Find the focus of the parabola $y^2 - 4y - 8x + 4 = 0$.

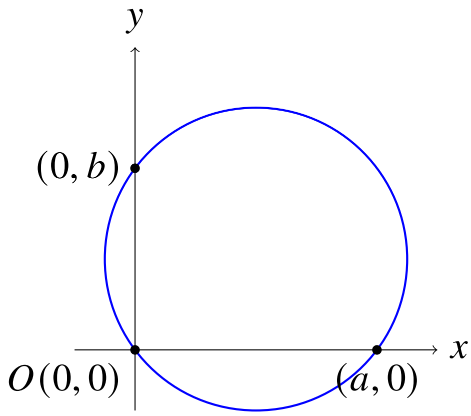
(A) $(0, 2)$

(B) $(2, 2)$

(C) $(2, 0)$

(D) $(0, 0)$

Q20. Consider a circle passing through the origin with intercepts on the axes being a and b respectively as modeled below:



What is the equation of this circle?

(A) $x^2 + y^2 - ax - by = 0$

(B) $x^2 + y^2 + ax + by = 0$

(C) $x^2 + y^2 - ax + by = 0$

(D) $x^2 + y^2 + ax - by = 0$

Q21. Find the value of $\frac{d}{dx}\left(\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)\right)$.

(A) 1

(B) -1

(C) $\frac{1}{1+x^2}$

(D) $\frac{-1}{1+x^2}$

Q22. Out of 7 men and 4 women, a committee of 5 members is to be formed. In how many ways can this be done so that the committee contains at least 3 women?

(A) 91

(B) 105

(C) 112

(D) 140

Q23. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then find the value of $\cos^{-1} x + \cos^{-1} y$.

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) π

(D) $\frac{\pi}{6}$

Q24. Evaluate the limit $\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x$.

(A) e

(B) e^2

(C) $\frac{1}{e}$

(D) 1

Q25. Find the angle between the two vectors $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$.

(A) 30°

(B) 45°

(C) 60°

(D) 90°

Q26. If the local maximum value of a function occurs at a point where the curve switches its path, evaluate the local minimum value of the function $f(x) = 2x^3 - 9x^2 + 12x + 3$.

(A) 3

(B) 7

(C) 8

(D) 10

Q27. A coin is tossed 6 times. What is the probability of getting exactly 4 heads?

(A) $\frac{15}{64}$

(B) $\frac{5}{16}$

(C) $\frac{3}{32}$

(D) $\frac{21}{64}$

Q28. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, find the number of digits in 6^{20}.

(A) 15

(B) 16

(C) 17

(D) 18

Q29. Find the length of the latus rectum of the ellipse $9x^2 + 25y^2 = 225$.

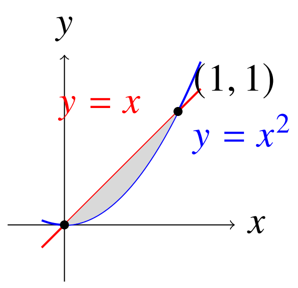
(A) $\frac{18}{5}$

(B) $\frac{9}{5}$

(C) $\frac{25}{3}$

(D) $\frac{50}{3}$

Q30. Find the area enclosed between the parabola $y = x^2$ and the line $y = x$, shown as the shaded region in the plot below:



Calculate the exact area of this bounded region.

(A) $\frac{1}{2}$ sq. units

(B) $\frac{1}{3}$ sq. units

(C) $\frac{1}{6}$ sq. units

(D) $\frac{1}{12}$ sq. units

Q31. If A is a square matrix of order 3 such that $|A| = 4$, then find the value of $|\text{adj}(A)|$.

(A) 4

(B) 8

(C) 16

(D) 64

Q32. Find the value of $\tan\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)$.

(A) $\frac{3}{4}$

(B) $\frac{4}{3}$

(C) $\frac{3}{5}$

(D) $\frac{1}{2}$

Q33. Find the value of $\int \frac{1}{x(x^4+1)}\, dx$.

(A) $\ln\left|\frac{x^4}{x^4+1}\right| + C$

(B) $\frac{1}{4}\ln\left|\frac{x^4}{x^4+1}\right| + C$

(C) $\frac{1}{4}\ln\left|\frac{x^4+1}{x^4}\right| + C$

(D) $4\ln\left|\frac{x^4}{x^4+1}\right| + C$

Q34. Let A and B be two independent events such that $P(A) = 0.3$ and $P(B) = 0.4$. Find $P(A \cup B)$.

(A) 0.7

(B) 0.12

(C) 0.58

(D) 0.46

Q35. Find the value of k for which the system of equations $x+y+z = 2$, $2x+3y+2z = 5$, $2x + 3y + (k^2 - 1)z = k + 1$ has infinitely many solutions.

(A) $k = \sqrt{3}$

(B) $k = -\sqrt{3}$

(C) $k = 3$

(D) No such value of k exists

Q36. Find the derivative of $e^{\sin x}$ with respect to $\cos x$.

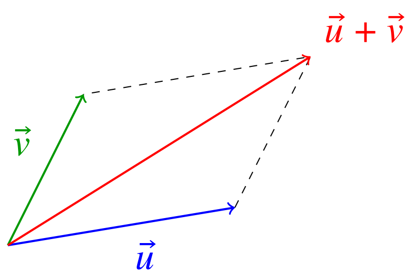
(A) $-e^{\sin x} \tan x$

(B) $-e^{\sin x} \cot x$

(C) $e^{\sin x} \cot x$

(D) $e^{\sin x} \tan x$

Q37. Consider the vectors forming two adjacent sides of a parallelogram, $\vec{u}$ and $\vec{v}$, originating from the same point:



If $\vec{u} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{v} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$, find the area of the parallelogram.

(A) $\sqrt{35}$ sq. units

(B) $\sqrt{59}$ sq. units

(C) $\sqrt{74}$ sq. units

(D) $\sqrt{83}$ sq. units

Q38. In a class of 100 students, 60 like Mathematics, 45 like Physics, and 20 like both. How many students like neither Mathematics nor Physics?

(A) 15

(B) 20

(C) 25

(D) 30

Q39. Find the shortest distance between the parallel lines $y = 2x + 3$ and $y = 2x - 7$.

(A) $2\sqrt{5}$

(B) $\sqrt{5}$

(C) $\frac{10}{\sqrt{5}}$

(D) 4

Q40. Find the maximum value of xe^{-x} for $x > 0$.

(A) e

(B) $\frac{1}{e}$

(C) 1

(D) $\frac{2}{e^2}$

Q41. Find the sum of the coefficients in the expansion of $(1 - 3x + 2x^2)^{50}$.

(A) 0

(B) 1

(C) 2^{50}

(D) -1

Q42. If $\tan\theta + \sec\theta = 1.5$, then find the value of $\sin\theta$.

(A) $\frac{5}{13}$

(B) $\frac{12}{13}$

(C) $\frac{7}{13}$

(D) $\frac{9}{13}$

Q43. The mean of 100 items was found to be 40. If at the time of calculation, two items were wrongly taken as 30 and 20 instead of 40 and 50, find the correct mean.

(A) 40.2

(B) 40.4

(C) 41.2

(D) 41.5

Q44. Evaluate the limit $\lim_{x\to 0} \frac{1-\cos 4x}{x^2}$.

(A) 2

(B) 4

(C) 8

(D) 16

Q45. If $A = \{x \in \mathbb{R} : x^2 - 3x + 2 = 0\}$ and $B = \{x \in \mathbb{R} : x^2 - 4x + 3 = 0\}$, then find $A \Delta B$ (Symmetric difference of A and B).

(A) $\{1\}$

(B) $\{2, 3\}$

(C) $\{1, 2, 3\}$

(D) $\emptyset$

Q46. Find the value of $\int_{-1}^{1} x|x|\, dx$.

(A) 0

(B) $\frac{2}{3}$

(C) $-\frac{2}{3}$

(D) 1

Q47. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$, then find the value of $(2\vec{a} - 5\vec{b}) \cdot (3\vec{a} + \vec{b})$.

(A) -2

(B) $-\frac{11}{2}$

(C) 4

(D) $-\frac{7}{2}$

Q48. Find the eccentricity of the hyperbola $16x^2 - 9y^2 = 144$.

(A) $\frac{4}{3}$

(B) $\frac{5}{4}$

(C) $\frac{5}{3}$

(D) $\frac{7}{4}$

Q49. In how many ways can 5 prizes be distributed among 4 students if any student can receive any number of prizes?

(A) 5^4

(B) 4^5

(C) $P(5, 4)$

(D) $C(5, 4)$

Q50. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$, find the value of $\cos(\alpha - \beta)$.

(A) $\frac{a^2+b^2-2}{2}$

(B) $\frac{a^2+b^2+2}{2}$

(C) $a^2 + b^2 - 1$

(D) $\frac{a^2-b^2}{2}$

Detailed Solutions

Q1.

Solution

Concept: Set theory principles, specifically the partitioning of elements within a universal Venn diagram framework. The identity for the union of two sets is $n(A \cup B) = n(A \setminus B) + n(B \setminus A) + n(A \cap B)$.

Solution:

(a) The universal union region of two sets can be fundamentally split into three disjoint subsets: elements purely in A, elements purely in B, and elements common to both sets.

(b) Expressing this relation mathematically yields the absolute cardinal formula: $n(A \cup B) = n(A \setminus B) + n(B \setminus A) + n(A \cap B)$.

(c) Substitute the values provided in the problem statement into the cardinal equation: $50 = 15 + 22 + n(A \cap B)$.

(d) Consolidate the independent values on the right-hand side of the expression to isolate the parameter: $50 = 37 + n(A \cap B)$.

(e) Subtract 37 from both sides to evaluate the intersection cardinality: $n(A \cap B) = 50 - 37 = 13$.

Final Answer: The intersection contains 13 elements.

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept: Evaluation of standard limits using fundamental trigonometric and logarithmic asymptotic expansions as $x \to 0$, or applying L'Hôpital's Rule for $\frac{0}{0}$ indeterminate forms.

Solution:

(a) Substituting $x = 0$ directly into the expression gives $\frac{\ln(1)}{\sin(0)} = \frac{0}{0}$, confirming an indeterminate structural state.

(b) Apply the standard limits $\lim_{x\to 0} \frac{\ln(1+kx)}{kx} = 1$ and $\lim_{x\to 0} \frac{\sin(kx)}{kx} = 1$ by manipulating the expression.

(c) Multiply and divide the numerator by $3x$ and the denominator by $2x$: $\frac{\ln(1+3x)}{3x} \cdot 3x \cdot \frac{2x}{\sin(2x)} \cdot \frac{1}{2x}$.

(d) Cancel the common variable x from the numerator and denominator factors to isolate the scaling constants: $\frac{3}{2} \cdot \left[\frac{\ln(1+3x)}{3x}\right] \cdot \left[\frac{2x}{\sin(2x)}\right]$.

(e) Evaluate individual limits as x approaches zero: $\frac{3}{2} \cdot 1 \cdot 1 = \frac{3}{2}$.

Final Answer: The value of the limit is $\frac{3}{2}$.

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept: Coordinate geometry metrics for finding a standard polygon region using coordinate matrices, or calculating base times perpendicular height measurements from a visual plot.

Solution:

(a) Identify coordinates $A(1, 1)$ and $B(5, 1)$. Because their ordinates match ($y = 1$), line segment ABC has a perfectly horizontal base line AB.

(b) Calculate base length AB using absolute horizontal displacement: Base $= 5 - 1 = 4$ units.

(c) Determine vertex altitude height by calculating vertical distance from $C(3, 4)$ to line $y = 1$: Height $= 4 - 1 = 3$ units.

(d) Use the standard Euclidean geometric computation formula: Area $= \frac{1}{2} \times$ base $\times$ height.

(e) Perform the numerical computation using these values: Area $= \frac{1}{2} \times 4 \times 3 = 6$ square units.

Final Answer: The area of the triangle is 6 sq. units.

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept: Vector algebra properties involving linear combinations, vector addition, absolute magnitude derivation, and normalized unit vector definition $\hat{u} = \frac{\vec{u}}{|\vec{u}|}$.

Solution:

(a) Combine vectors $\vec{a}$ and $\vec{b}$ component-wise to compute their linear vector sum: $\vec{a} + \vec{b} = (2+1)\hat{i} + (1+3)\hat{j} + (-1+5)\hat{k}$.

(b) Simplify the components to get the resultant vector layout: $\vec{a} + \vec{b} = 3\hat{i} + 4\hat{j} + 4\hat{k}$.

(c) Calculate total vector magnitude using the sum of squared orthogonal components: $|\vec{a}+\vec{b}| = \sqrt{3^2 + 4^2 + 4^2}$.

(d) Evaluate the arithmetic values under the radical sign: $|\vec{a} + \vec{b}| = \sqrt{9 + 16 + 16} = \sqrt{41}$.

(e) Normalize the vector by dividing the resultant vector by its absolute magnitude: $\frac{3\hat{i}+4\hat{j}+4\hat{k}}{\sqrt{41}}$.

Final Answer: The normalized vector is $\frac{3\hat{i}+4\hat{j}+4\hat{k}}{\sqrt{41}}$.

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: Conditional compound probability and dependent events calculation using sequential multi-stage drawing without tracking back or replacements.

Solution:

(a) The total number of initial entities is 10. The sample space drops by exactly 1 sample item after every draw.

(b) Probability of choosing a red ball on the first attempt is total red count over total pool size: $P(R_1) = \frac{4}{10}$.

(c) Since no ball is replaced, 3 red and 6 black items remain. Probability for a second red selection is: $P(R_2|R_1) = \frac{3}{9}$.

(d) With 2 red balls gone, 8 balls remain in total. Probability of drawing a black ball on the third attempt is: $P(B_3|R_1 \cap R_2) = \frac{6}{8}$.

(e) Compute the combined probability product along the dependent chain: $P = \frac{4}{10} \times \frac{3}{9} \times \frac{6}{8} = \frac{2}{5} \times \frac{1}{3} \times \frac{3}{4} = \frac{1}{10}$.

Final Answer: The target probability equals $\frac{1}{10}$.

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: Algebraic identities paired with Vieta's formulas for quadratic polynomial equations, using symmetric functions of roots $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.

Solution:

(a) Extract the coefficients from the quadratic equation $x^2 - 5x + 6 = 0$: $a = 1, b = -5, c = 6$.

(b) Apply Vieta's relations to find the sum and product of the roots: $\alpha + \beta = 5$ and $\alpha\beta = 6$.

(c) Use the symmetric algebraic identity for cubes: $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$.

(d) Substitute the known parameters directly into the factored formula: $\alpha^3 + \beta^3 = (5)^3 - 3(6)(5)$.

(e) Compute the final arithmetic values: $\alpha^3 + \beta^3 = 125 - 90 = 35$.

Final Answer: The value of the expression is 35.

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: Rational function inequalities solved via boundary criteria mapping and sign schemes (the wavy curve method).

Solution:

(a) Set the numerator and denominator terms to zero to find the critical critical boundary roots: $x - 1 = 0 \implies x = 1$, and $x + 2 = 0 \implies x = -2$.

(b) Plot these points on a real number line to divide it into three distinct open sub-intervals: $(-\infty, -2)$, $(-2, 1)$, and $(1, \infty)$.

(c) Test signs across the regions: for $x > 1$, both components are positive $(+)$. For x between -2 and 1, the expression yields a negative value $(-)$. For $x < -2$, both factors are negative, making the product positive $(+)$.

(d) The problem statement specifies a non-negative inequality constraint (≥ 0). This matches the positive intervals.

(e) Include the root $x = 1$ because it satisfies equality. Exclude $x = -2$ to avoid a division-by-zero undefined state, giving: $(-\infty, -2) \cup [1, \infty)$.

Final Answer: The solution set is $(-\infty, -2) \cup [1, \infty)$.

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: Factorization of trigonometric equations and application of general solution periodic rules for primary circular functions.

Solution:

(a) Factor out the common trigonometric term from the expression: $\sin\theta(\sin\theta - 2) = 0$.

(b) This splits the equation into two independent possible equations: $\sin\theta = 0$ or $\sin\theta - 2 = 0$.

(c) Analyze the second equation: $\sin\theta = 2$. This has no real solution because the range of the sine function is bounded by $[-1, 1]$.

(d) Analyze the first equation: $\sin\theta = 0$. The sine function equals zero at integer multiples of π.

(e) Write the complete solution using standard periodic notation: $\theta = n\pi$ where $n \in \mathbb{Z}$.

Final Answer: The general solution is $\theta = n\pi, n \in \mathbb{Z}$.

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: Straight line constraints using slope-intercept equations. Orthogonal perpendicular lines satisfy the slope condition $m_1 \cdot m_2 = -1$.

Solution:

(a) Convert the given line equation $3x - 4y + 7 = 0$ into standard slope form to find its gradient: $y = \frac{3}{4}x + \frac{7}{4} \implies m_1 = \frac{3}{4}$.

(b) Calculate the perpendicular slope using the orthogonality condition: $m_2 = -\frac{1}{m_1} = -\frac{4}{3}$.

(c) Write the equation of the line using point-slope form with coordinates $(2, 3)$: $y - 3 = -\frac{4}{3}(x - 2)$.

(d) Multiply both sides by 3 to clear the fraction and expand the terms: $3y - 9 = -4x + 8$.

(e) Move all terms to one side to express the equation in standard form: $4x + 3y - 17 = 0$.

Final Answer: The equation of the line is $4x + 3y - 17 = 0$.

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: Definite integrals and calculus properties, specifically using the definite integral reflection identity $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$.

Solution:

(a) Define the integral equation: $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx$.

(b) Apply the reflection property by replacing the variable x with $(\frac{\pi}{2} - x)$: $I = \int_0^{\pi/2} \frac{\sin(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)}\,dx$.

(c) Simplify using standard co-function trigonometric properties: $I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\,dx$.

(d) Add the two equations for I together to combine their identical denominators: $2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx$.

(e) Simplify the integrand to 1 and integrate: $2I = \int_0^{\pi/2} 1\,dx = [x]_0^{\pi/2} = \frac{\pi}{2} \implies I = \frac{\pi}{4}$.

Final Answer: The value of the definite integral is $\frac{\pi}{4}$.

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: Mathematical analysis of real functions involving absolute values. Continuity requires identical directional limits, while differentiability fails at sharp turn corners due to conflicting tangent lines.

Solution:

(a) Evaluate the left-hand limit and right-hand limit of the function as x approaches 2: $\lim_{x\to 2^-} |x-2| = 0$ and $\lim_{x\to 2^+} |x-2| = 0$. Since both match $f(2) = 0$, the function is continuous.

(b) Determine the derivative for values smaller than the turning point: for $x < 2$, $f(x) = -(x-2)$, which gives a constant left derivative $f'(x) = -1$.

(c) Determine the derivative for values larger than the turning point: for $x > 2$, $f(x) = x - 2$, which gives a constant right derivative $f'(x) = 1$.

(d) Compare the two directional derivative limits at the boundary vertex point: the left-hand derivative (-1) does not match the right-hand derivative (1).

(e) This mismatch shows that the function graph forms a sharp corner point at $(2, 0)$, meaning it cannot be differentiated at that exact point.

Final Answer: The function is continuous but not differentiable.

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept: Combinatorics and permutation arrangements of linear strings with grouping constraints where specific grouped elements must remain together as a single item.

Solution:

(a) Identify the specific letters in the target word NIMCET: the vowels are I and E, while the remaining consonants are N, M, C, and T.

(b) Group the vowels together into a single block item (IE). Treat this entire unit as one individual object for the overall arrangement.

(c) Count the total number of items to arrange: the single vowel block plus the 4 distinct consonants gives 5 items in total.

(d) Calculate the arrangements of these 5 independent elements using standard factorial notation: $5! = 120$ permutations.

(e) Calculate internal permutations within the vowel unit: the 2 distinct vowels can swap positions in $2! = 2$ ways.

(f) Multiply the external and internal permutations together to find the total combinations: $120 \times 2 = 240$.

Final Answer: The letters can be arranged in 240 different ways.

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: Vector space dependency constraints. Three dimensional vectors are coplanar if and only if their scalar triple product matrix determinant equals zero.

Solution:

(a) Set up a 3×3 matrix using the vector components: row one is $[1, -1, 1]$, row two is $[2, 1, -1]$, and row three is $[\lambda, -1, \lambda]$.

(b) Set the scalar triple product matrix determinant equation to zero to solve for coplanarity:
$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix} = 0.$$

(c) Expand the determinant along the first row: $1(\lambda - 1) - (-1)(2\lambda - (-\lambda)) + 1(-2 - \lambda) = 0.$

(d) Simplify individual terms: $(\lambda - 1) + (2\lambda + \lambda) + (-2 - \lambda) = 0.$

(e) Combine like variable terms and constants to simplify the linear equation: $\lambda - 1 + 3\lambda - 2 - \lambda = 0 \implies 3\lambda - 3 = 0.$

(f) Isolate the variable to find the missing coefficient: $3\lambda = 3 \implies \lambda = 1.$

Final Answer: The value of λ is 1.

Answer: (B)

Go Back to Question 13

Q14.

Solution

Concept: Statistical scaling transformations. Multiplying data observations by a constant factor k changes the standard deviation by $|k|$ and the variance by k^2.

Solution:

(a) Find the initial data variance by squaring the given standard deviation: $\text{Variance}_{\text{old}} = \sigma^2 = 4^2 = 16$.

(b) State the variance scaling rule: if every observation x_i is transformed to $k \cdot x_i$, the variance scales by a factor of k^2.

(c) Identify the constant scaling factor from the problem statement: $k = 3$.

(d) Calculate the new variance scaling multiplier: $k^2 = 3^2 = 9$.

(e) Multiply the original variance by this scaling factor to calculate the new variance: $\text{Variance}_{\text{new}} = 9 \times 16 = 144$.

Final Answer: The new variance value is 144.

Answer: (D)

Go Back to Question 14

Q15.

Solution

Concept: Trigonometric structural identities involving compound angle formulas for sine and cosine differences: $\cos(A + B)$ and $\sin(A - B)$.

Solution:

(a) Square the given expression to transform it into a simpler form using basic identities: $(\cos 15^\circ - \sin 15^\circ)^2$.

(b) Expand the squared expression: $\cos^2 15^\circ + \sin^2 15^\circ - 2 \sin 15^\circ \cos 15^\circ$.

(c) Substitute standard identities: $\cos^2 \theta + \sin^2 \theta = 1$ and $2 \sin \theta \cos \theta = \sin(2\theta)$.

(d) Simplify the expression using these values: $1 - \sin(2 \times 15^\circ) = 1 - \sin 30^\circ$.

(e) Use the standard value $\sin 30^\circ = 0.5$: $1 - 0.5 = 0.5$.

(f) Take the positive square root because $\cos 15^\circ > \sin 15^\circ$: $\sqrt{0.5} = \frac{1}{\sqrt{2}}$.

Final Answer: The value of the expression is $\frac{1}{\sqrt{2}}$.

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept: Infinite geometric series summation. The sum exists if the common ratio satisfies $|r| < 1$, using the formula $S_\infty = \frac{a}{1-r}$.

Solution:

(a) Identify the first term of the progression from the series: $a = 1$.

(b) Determine the common ratio by dividing consecutive terms: $r = \frac{1/3}{1} = \frac{1}{3}$.

(c) Verify that the common ratio satisfies the convergence condition: $|\frac{1}{3}| < 1$, so the series converges.

(d) Write out the infinite geometric series sum formula: $S_\infty = \frac{a}{1-r}$.

(e) Substitute the terms into the formula and simplify: $S_\infty = \frac{1}{1-1/3} = \frac{1}{2/3} = \frac{3}{2}$.

Final Answer: The sum to infinity is $\frac{3}{2}$.

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: Relation classification properties. A relation is an equivalence relation if it is simultaneously reflexive, symmetric, and transitive.

Solution:

(a) Test for reflexivity: $a - a = 0$, which is divisible by 5. Therefore, aRa holds true for all integers, confirming the relation is reflexive.

(b) Test for symmetry: assume aRb is true, meaning $a - b = 5k$ for some integer k. It follows that $b - a = -5k = 5(-k)$, so bRa holds true, confirming the relation is symmetric.

(c) Test for transitivity: assume aRb and bRc are true. This means $a - b = 5k$ and $b - c = 5m$ for integers k and m.

(d) Add the two equations together: $(a - b) + (b - c) = 5k + 5m \implies a - c = 5(k + m)$. Since this is divisible by 5, aRc holds true, confirming the relation is transitive.

(e) Because the relation satisfies all three properties, it is classified as an equivalence relation.

Final Answer: The relation is an equivalence relation.

Answer: (C)

Go Back to Question 17

Q18.

Solution

Concept: Matrix determinant operations. Applying linear combinations across rows or columns can simplify expressions without changing the value of the determinant.

Solution:

(a) Set up the initial matrix layout: column one contains all ones, column two contains $[a, b, c]^T$, and column three contains $[b + c, c + a, a + b]^T$.

(b) Apply a column addition operation to simplify column three: add column two to column three ($C_3 \rightarrow C_3 + C_2$).

(c) Write out the modified third column elements: each entry becomes the identical expression $a + b + c$.

(d) Factor out the common term $(a + b + c)$ from the third column: $(a + b + c) \cdot \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$.

(e) Analyze the remaining matrix: column one and column three are identical. A determinant with two identical columns equals zero, so $(a + b + c) \times 0 = 0$.

Final Answer: The value of the determinant is 0.

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: Conic sections analysis. Transform general quadratic parabola forms into standard coordinate vertex equations $(y-k)^2 = 4p(x-h)$ using completing the square.

Solution:

(a) Group the dependent variable terms on one side: $y^2 - 4y = 8x - 4$.

(b) Complete the square on the left side by adding 4 to both sides of the equation: $y^2 - 4y + 4 = 8x - 4 + 4$.

(c) Simplify into standard binomial form: $(y-2)^2 = 8x$, which matches the standard parabola format $(y-k)^2 = 4p(x-h)$.

(d) Extract the core features of the parabola from this form: the vertex is $(h, k) = (0, 2)$ and the focal multiplier is $4p = 8 \implies p = 2$.

(e) Calculate the focus coordinates for a horizontal parabola opening to the right: Focus $= (h + p, k) = (0 + 2, 2) = (2, 2)$.

Final Answer: The focus coordinates are $(2, 2)$.

Answer: (B)

Go Back to Question 19

Q20.

Solution

Concept: Circle coordinate geometry. Circles passing through the origin with given axial intercepts can be modeled using standard diameter equations or right triangle geometry.

Solution:

(a) Identify the three points the circle passes through on the coordinate plane: the origin $O(0,0)$, the x-intercept $(a,0)$, and the y-intercept $(0,b)$.

(b) Since the axes meet at a right angle (90°) at the origin, Thales's theorem states that the line segment connecting $(a,0)$ and $(0,b)$ forms a diameter of the circle.

(c) Find the center of the circle by calculating the midpoint of this diameter line segment: Center $= (\frac{a}{2}, \frac{b}{2})$.

(d) Calculate the radius squared of the circle using the distance formula from the center to the origin: $R^2 = (\frac{a}{2})^2 + (\frac{b}{2})^2 = \frac{a^2+b^2}{4}$.

(e) Write out the standard equation of the circle: $(x - \frac{a}{2})^2 + (y - \frac{b}{2})^2 = \frac{a^2+b^2}{4}$. Expand and simplify to get: $x^2 + y^2 - ax - by = 0$.

Final Answer: The equation of the circle is $x^2 + y^2 - ax - by = 0$.

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: Trigonometric expression simplification paired with basic calculus chain rule differentiation for inverse circular functions.

Solution:

(a) Simplify the algebraic term inside the inverse tangent function by dividing both its numerator and denominator by $\cos x$: $\frac{1-\tan x}{1+\tan x}$.

(b) Express this fractional tangent format using the standard compound angle identity: $\frac{\tan(\pi/4)-\tan x}{1+\tan(\pi/4)\tan x} = \tan(\frac{\pi}{4} - x)$.

(c) Substitute this simplified identity back into the original inverse function: $f(x) = \tan^{-1}(\tan(\frac{\pi}{4} - x))$.

(d) Apply inverse cancellation properties over the domain to simplify the function expression to a linear form: $f(x) = \frac{\pi}{4} - x$.

(e) Differentiate this linear expression with respect to x to calculate the derivative: $\frac{d}{dx}(\frac{\pi}{4} - x) = 0 - 1 = -1$.

Final Answer: The derivative value is -1.

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept: Combinatorics addition and multiplication principles using combinations $C(n, r) = \frac{n!}{r!(n-r)!}$ to solve group selection problems with constraints.

Solution:

(a) Break down the committee requirements based on the given constraint of having at least 3 women in a 5-member team chosen from 7 men and 4 women.

(b) Case 1: Select exactly 3 women and 2 men. Calculate the combinations using the multiplication rule: $C(4, 3) \times C(7, 2) = 4 \times 21 = 84$.

(c) Case 2: Select exactly 4 women and 1 man. Calculate the combinations using the multiplication rule: $C(4, 4) \times C(7, 1) = 1 \times 7 = 7$.

(d) Case 3: Select 5 women. This case is impossible because there are only 4 women available in total.

(e) Add the valid combinations from both cases together to find the total number of ways to form the committee: $84 + 7 = 91$.

Final Answer: The committee can be formed in 91 ways.

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: Inverse trigonometric identities. Complementary function pairs satisfy the constant identity relation $\sin^{-1} z + \cos^{-1} z = \frac{\pi}{2}$.

Solution:

(a) Write out the standard complementary identities for both variables x and y: $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ and $\sin^{-1} y + \cos^{-1} y = \frac{\pi}{2}$.

(b) Add these two independent equations together to group the inverse terms: $(\sin^{-1} x + \sin^{-1} y) + (\cos^{-1} x + \cos^{-1} y) = \frac{\pi}{2} + \frac{\pi}{2} = \pi$.

(c) Substitute the known sum given in the problem statement $(\frac{2\pi}{3})$ into this combined equation: $\frac{2\pi}{3} + (\cos^{-1} x + \cos^{-1} y) = \pi$.

(d) Isolate the required target sum on one side of the equation: $\cos^{-1} x + \cos^{-1} y = \pi - \frac{2\pi}{3}$.

(e) Subtract the values to find the final result: $\frac{3\pi - 2\pi}{3} = \frac{\pi}{3}$.

Final Answer: The value of the expression is $\frac{\pi}{3}$.

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: Asymptotic calculus limits. Evaluating the indeterminate form 1^∞ using the exponential transformation identity $\lim f(x)^{g(x)} = e^{\lim[f(x)-1]g(x)}$.

Solution:

(a) Substitute $x = \infty$ into the limit expression to identify its form: $(1+0)^\infty = 1^\infty$, which is an indeterminate form.

(b) Apply the standard exponential limit transformation rule for this form: $e^{\lim_{x\to\infty}[(1+\frac{2}{x})-1]\cdot x}$.

(c) Simplify the expression inside the exponent layer: $e^{\lim_{x\to\infty}(\frac{2}{x}\cdot x)}$.

(d) Cancel out the common variable x in the numerator and denominator to simplify the exponent limit: $e^{\lim_{x\to\infty} 2}$.

(e) Since the remaining term is a constant, evaluating the limit gives the final value: e^2.

Final Answer: The value of the limit is e^2.

Answer: (B)

Go Back to Question 24

Q25.

Solution

Concept: Vector geometry dot products. The angle θ between two vectors is calculated using their scalar product divided by their magnitudes: $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$.

Solution:

(a) Compute the scalar dot product by multiplying the corresponding components of vectors $\vec{a}$ and $\vec{b}$: $\vec{a}\cdot\vec{b} = (1\times 0) + (1\times 1) + (0\times 1) = 1$.

(b) Calculate the absolute magnitude of the first vector $\vec{a}$: $|\vec{a}| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$.

(c) Calculate the absolute magnitude of the second vector $\vec{b}$: $|\vec{b}| = \sqrt{0^2+1^2+1^2} = \sqrt{2}$.

(d) Substitute these calculated values into the standard cosine angle formula: $\cos\theta = \frac{1}{\sqrt{2}\times\sqrt{2}} = \frac{1}{2}$.

(e) Find the angle whose cosine value is 0.5: $\theta = \cos^{-1}(\frac{1}{2}) = 60^\circ$.

Final Answer: The angle between the two vectors is 60°.

Answer: (C)

Go Back to Question 25

Q26.

Solution

Concept: Calculus optimization. Local extrema are found by solving for critical points where $f'(x) = 0$ and checking the sign of the second derivative $f''(x)$.

Solution:

(a) Differentiate the function $f(x) = 2x^3 - 9x^2 + 12x + 3$ to find its first derivative: $f'(x) = 6x^2 - 18x + 12$.

(b) Set this derivative to zero and factor the equation to find the critical points: $6(x^2 - 3x + 2) = 0 \implies 6(x-1)(x-2) = 0$, giving $x = 1$ and $x = 2$.

(c) Differentiate a second time to find the second derivative for the inflection test: $f''(x) = 12x - 18$.

(d) Test the critical point $x = 1$: $f''(1) = -6 < 0$, which confirms a local maximum occurs here. Test $x = 2$: $f''(2) = 6 > 0$, confirming a local minimum.

(e) Substitute the local minimum point $x = 2$ back into the original function to calculate its value: $f(2) = 2(8) - 9(4) + 12(2) + 3 = 16 - 36 + 24 + 3 = 7$.

Final Answer: The local minimum value of the function is 7.

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept: Probability theory distributions. Binomial probability formula for independent trials: $P(X = k) = C(n, k) \cdot p^k \cdot q^{n-k}$.

Solution:

(a) Identify the binomial parameters for flipping a fair coin 6 times: total trials $n = 6$ and target successes $k = 4$.

(b) State the individual success and failure probabilities for a fair coin: $p = 0.5$ and $q = 0.5$.

(c) Set up the standard binomial probability distribution formula using these parameters: $P(X = 4) = C(6, 4) \times (0.5)^4 \times (0.5)^{6-4}$.

(d) Calculate the combination factor and group the exponential terms: $C(6, 4) = 15$, so the expression becomes $15 \times (0.5)^6$.

(e) Compute the final fractional value: $15 \times \frac{1}{64} = \frac{15}{64}$.

Final Answer: The probability of getting exactly 4 heads is $\frac{15}{64}$.

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: Logarithmic properties applied to large numbers. The number of digits in an integer value N is calculated using its characteristic value: $\lfloor \log_{10} N \rfloor + 1$.

Solution:

(a) Define the target value as an equation: $N = 6^{20}$. Take the base-10 logarithm of both sides: $\log_{10} N = \log_{10}(6^{20})$.

(b) Apply logarithmic power rules to bring down the exponent as a multiplier: $\log_{10} N = 20 \times \log_{10}(2 \times 3)$.

(c) Split the product inside the logarithm into a sum of log terms: $20 \times (\log_{10} 2 + \log_{10} 3)$.

(d) Substitute the decimal values given in the problem statement into the equation: $20 \times (0.3010 + 0.4771) = 20 \times 0.7781$.

(e) Multiply the terms to find the final value: 15.562. Add 1 to the integer part ($\lfloor 15.562 \rfloor = 15$) to get the number of digits: $15 + 1 = 16$.

Final Answer: The total number of digits is 16.

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: Conic sections analysis. Standardizing ellipse equations to the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to calculate chord attributes like the latus rectum length $\frac{2b^2}{a}$.

Solution:

(a) Divide both sides of the given equation $9x^2 + 25y^2 = 225$ by 225 to normalize it: $\frac{9x^2}{225} + \frac{25y^2}{225} = 1$.

(b) Simplify the fractions to match the standard ellipse format: $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

(c) Extract the semi-major and semi-minor axis values from the denominators: $a^2 = 25 \implies a = 5$, and $b^2 = 9 \implies b = 3$.

(d) State the standard formula for the length of the latus rectum of a horizontal ellipse: Length $= \frac{2b^2}{a}$.

(e) Substitute the values into this formula to calculate the final length: Length $= \frac{2\times 9}{5} = \frac{18}{5}$.

Final Answer: The length of the latus rectum is $\frac{18}{5}$.

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: Integral calculus applications. The area bounded between two intersecting curves is calculated using the single variable definite integral formula $\int_a^b [f_{\text{top}}(x) - f_{\text{bottom}}(x)]\, dx$.

Solution:

(a) Find the intersection boundary points by setting the line equation equal to the parabola equation: $x^2 = x \implies x(x-1) = 0$, which gives limits at $x = 0$ and $x = 1$.

(b) Set up the area definite integral over this interval, noting from the graph that the line $y = x$ lies above the curve $y = x^2$: Area $= \int_0^1 (x - x^2)\, dx$.

(c) Find the antiderivative of each term inside the integral using the power rule: $[\frac{x^2}{2} - \frac{x^3}{3}]_0^1$.

(d) Substitute the upper integration limit (1) into this integrated expression: $(\frac{1}{2} - \frac{1}{3}) = \frac{3-2}{6} = \frac{1}{6}$.

(e) Substitute the lower integration limit (0), which leaves the calculated fractional value unchanged: $\frac{1}{6}$ square units.

Final Answer: The exact bounded area is $\frac{1}{6}$ sq. units.

Answer: (C)

Go Back to Question 30

Q31.

Solution

Concept: Matrix algebra determinants. The determinant of an adjoint matrix scales according to the matrix size order via the mathematical property $|\text{adj}(A)| = |A|^{n-1}$.

Solution:

(a) Identify the given matrix parameters from the problem: the matrix determinant value is $|A| = 4$ and its square order dimension is $n = 3$.

(b) State the standard determinant identity for adjoint transformations of square matrices: $|\text{adj}(A)| = |A|^{n-1}$.

(c) Substitute the order value into the exponent of the formula: $n - 1 = 3 - 1 = 2$.

(d) Substitute the base determinant value into the formula to find the expression: $|\text{adj}(A)| = 4^2$.

(e) Compute the final power value: $4 \times 4 = 16$.

Final Answer: The value of $|\text{adj}(A)|$ is 16.

Answer: (C)

Go Back to Question 31

Q32.

Solution

Concept: Inverse trigonometric identities. Combining elements requires the double angle tangent transformation formula: $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$.

Solution:

(a) Define the inner angle component as a variable: let $\theta = \tan^{-1}(\frac{1}{3})$, which means $\tan\theta = \frac{1}{3}$.

(b) Rewrite the overall target expression in terms of this new angle variable: $\tan(2\theta)$.

(c) State the standard double angle trigonometric identity for the tangent function: $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$.

(d) Substitute the fractional value of $\tan\theta$ into the numerator and denominator: $\frac{2(1/3)}{1-(1/3)^2} = \frac{2/3}{1-1/9}$.

(e) Simplify the denominator and compute the final combined fraction: $\frac{2/3}{8/9} = \frac{2}{3} \times \frac{9}{8} = \frac{3}{4}$.

Final Answer: The value of the expression is $\frac{3}{4}$.

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: Integral calculus techniques. Evaluate non-trivial rational functions by factoring variables out of brackets or using substitution methods to simplify the integrand.

Solution:

(a) Factor out x^4 from the denominator parentheses to rewrite the integral form: $\int \frac{1}{x \cdot x^4(1+x^{-4})}\,dx = \int \frac{1}{x^5(1+x^{-4})}\,dx$.

(b) Move the power term to the numerator to prepare for variable substitution: $\int \frac{x^{-5}}{1+x^{-4}}\,dx$.

(c) Substitute a new variable for the denominator expression: let $u = 1 + x^{-4}$. Differentiating gives $du = -4x^{-5}\,dx$, which means $x^{-5}\,dx = -\frac{1}{4}\,du$.

(d) Substitute these terms back into the integral: $-\frac{1}{4}\int \frac{1}{u}\,du = -\frac{1}{4}\ln|u| + C$.

(e) Substitute the original expression back in and use log rules to simplify: $-\frac{1}{4}\ln|1 + \frac{1}{x^4}| + C = \frac{1}{4}\ln|\frac{x^4}{x^4+1}| + C$.

Final Answer: The integral equals $\frac{1}{4}\ln\left|\frac{x^4}{x^4+1}\right| + C$.

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept: Probability theory rules. For statistically independent events, the probability of their intersection simplifies to a direct product: $P(A \cap B) = P(A) \times P(B)$.

Solution:

(a) State the general addition rule for the union of two events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(b) Apply the independence condition to rewrite the intersection term as a product: $P(A \cap B) = P(A) \times P(B)$.

(c) Calculate this intersection value using the given probabilities: $P(A \cap B) = 0.3 \times 0.4 = 0.12$.

(d) Substitute these numerical values back into the main union formula: $P(A \cup B) = 0.3 + 0.4 - 0.12$.

(e) Add and subtract the terms to find the final probability: $0.7 - 0.12 = 0.58$.

Final Answer: The probability $P(A \cup B)$ is 0.58.

Answer: (C)

Go Back to Question 34

Q35.

Solution

Concept: Linear system conditions. Systems of equations have infinitely many solutions when their matrix determinants match, satisfying consistency conditions via Cramer's Rule.

Solution:

(a) Write out the coefficient matrix determinant equation Δ and set it to zero: $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & k^2-1 \end{vmatrix} = 0.$

(b) Apply row operations to simplify the matrix columns: perform $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 2R_1$.

(c) Write out the simplified determinant structure: $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & k^2-3 \end{vmatrix} = 0.$

(d) Expand along the first column to solve for the critical parameter value: $1(k^2 - 3 - 0) = 0 \implies k^2 = 3$, giving $k = \pm\sqrt{3}$.

(e) Test $k = \sqrt{3}$ in the third equation: $2x + 3y + 2z = \sqrt{3} + 1$. This contradicts the second equation $(2x + 3y + 2z = 5)$. Since the equations are inconsistent, no solution exists.

Final Answer: No such value of k exists due to system inconsistency.

Answer: (D)

Go Back to Question 35

Q36.

Solution

Concept: Calculus derivative properties. Parametric differentiation allows finding derivatives relative to a secondary function using the chain rule formula $\frac{du}{dv} = \frac{du/dx}{dv/dx}$.

Solution:

(a) Define the two parametric expressions as distinct variables: let $u = e^{\sin x}$ and let $v = \cos x$.

(b) Differentiate the first function u with respect to x using the chain rule: $\frac{du}{dx} = e^{\sin x} \cdot \cos x$.

(c) Differentiate the second function v with respect to x: $\frac{dv}{dx} = -\sin x$.

(d) Combine these two derivatives using the parametric quotient formula: $\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{e^{\sin x} \cdot \cos x}{-\sin x}$.

(e) Simplify the trigonometric ratio into a single cotangent function: $-e^{\sin x} \cdot \cot x$.

Final Answer: The derivative value is $-e^{\sin x} \cot x$.

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept: Vector geometry operations. The geometric area of a parallelogram spanned by two vectors equals the absolute magnitude of their vector cross product $|\vec{u} \times \vec{v}|$.
Solution:

(a) Set up a 3×3 matrix determinant equation to calculate the vector cross product: $\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & -1 \end{vmatrix}$.

(b) Expand the determinant along the first row components: $\hat{i}(1-4) - \hat{j}(-3-2) + \hat{k}(6-(-1))$.

(c) Simplify the terms to find the cross product vector: $-3\hat{i} + 5\hat{j} + 7\hat{k}$.

(d) Calculate the total magnitude of this cross product vector: $|\vec{u} \times \vec{v}| = \sqrt{(-3)^2 + 5^2 + 7^2}$.

(e) Evaluate the arithmetic sum under the radical sign to find the final area: $\sqrt{9 + 25 + 49} = \sqrt{83}$ square units.

Final Answer: The area of the parallelogram is $\sqrt{83}$ sq. units.

Answer: (D)

Go Back to Question 37

Q38.

Solution

Concept: Set theory applications. Calculating overlapping group criteria using the inclusion-exclusion principle: $n(M \cup P) = n(M) + n(P) - n(M \cap P)$.
Solution:

(a) Identify the individual set sizes from the problem statement: total universe $U = 100$, $n(M) = 60$, $n(P) = 45$, and $n(M \cap P) = 20$.

(b) Apply the inclusion-exclusion identity to calculate the total union size: $n(M \cup P) = 60 + 45 - 20$.

(c) Simplify the arithmetic expression: $n(M \cup P) = 105 - 20 = 85$. This means 85 students like at least one subject.

(d) Calculate the remaining students who like neither subject by subtracting the union size from the total universe: $n(\text{neither}) = n(U) - n(M \cup P)$.

(e) Compute the final value: $100 - 85 = 15$.

Final Answer: There are 15 students who like neither subject.

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: Coordinate geometry metrics. The shortest perpendicular distance between two parallel lines $yx + c_1$ and $yx + c_2$ is given by the formula $d = \frac{|c_1 - c_2|}{\sqrt{1+m^2}}$.

Solution:

(a) Express both line equations in standard slope format to identify their coefficients: $y = 2x + 3$ and $y = 2x - 7$.

(b) Extract the common slope gradient and individual intercept values from the formulas: $m = 2$, $c_1 = 3$, and $c_2 = -7$.

(c) Write out the standard parallel distance metric equation: $d = \frac{|c_1 - c_2|}{\sqrt{1+m^2}}$.

(d) Substitute the values into the formula: $d = \frac{|3-(-7)|}{\sqrt{1+2^2}} = \frac{|3+7|}{\sqrt{1+4}}$.

(e) Simplify the expression and rationalize the fraction: $d = \frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$.

Final Answer: The shortest distance between the lines is $2\sqrt{5}$.

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: Calculus optimization. Find local maximum values by locating where the first derivative equals zero and confirming a negative second derivative value.

Solution:

(a) Differentiate the function $f(x) = xe^{-x}$ with respect to x using the calculus product rule: $f'(x) = 1 \cdot e^{-x} + x \cdot (-e^{-x}) = e^{-x}(1 - x)$.

(b) Set this first derivative expression to zero to find the critical turning points: $e^{-x}(1 - x) = 0$. Since e^{-x} is never zero, this gives $1 - x = 0 \implies x = 1$.

(c) Find the second derivative to test the nature of this critical point: $f''(x) = -e^{-x}(1 - x) + e^{-x}(-1) = e^{-x}(x - 2)$.

(d) Evaluate the second derivative at the critical point $x = 1$: $f''(1) = e^{-1}(1 - 2) = -\frac{1}{e} < 0$, which confirms a local maximum.

(e) Substitute $x = 1$ back into the original function to calculate the maximum value: $f(1) = 1 \cdot e^{-1} = \frac{1}{e}$.

Final Answer: The maximum value of the function is $\frac{1}{e}$.

Answer: (B)

Go Back to Question 40

Q41.

Solution

Concept: Binomial and multinomial polynomial theorems. The sum of all coefficients in any algebraic expansion is found by setting all independent variable terms to 1.

Solution:

(a) Let the given polynomial expansion function be defined as $f(x) = (1 - 3x + 2x^2)^{50}$.

(b) The sum of the coefficients of a polynomial expression equals the functional value when the variable x is substituted with 1.

(c) Substitute $x = 1$ directly into the multinomial expression base: $f(1) = (1 - 3(1) + 2(1)^2)^{50}$.

(d) Simplify the basic arithmetic operations inside the parentheses: $1 - 3 + 2 = 0$.

(e) Evaluate the final exponential power calculation: $0^{50} = 0$.

Final Answer: The sum of the coefficients is 0.

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept: Trigonometric identity relationships. Reciprocal circular functions are bounded by fundamental Pythagorean identity constraints like $\sec^2\theta - \tan^2\theta = 1$.

Solution:

(a) State the given equation in clear mathematical terms: $\sec\theta + \tan\theta = 1.5 = \frac{3}{2}$.

(b) Express the standard Pythagorean identity in its factored difference format: $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$.

(c) Substitute the given sum fraction into this identity to find the difference value: $(\sec\theta - \tan\theta) \times \frac{3}{2} = 1 \implies \sec\theta - \tan\theta = \frac{2}{3}$.

(d) Add the sum equation and difference equation together to eliminate the tangent variable: $2\sec\theta = \frac{3}{2} + \frac{2}{3} = \frac{13}{6} \implies \sec\theta = \frac{13}{12}$.

(e) Invert the secant value to find the cosine: $\cos\theta = \frac{12}{13}$. Use the right triangle identity to calculate sine: $\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{144}{169}} = \frac{5}{13}$.

Final Answer: The value of $\sin\theta$ is $\frac{5}{13}$.

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: Statistical mean corrections. Adjust an incorrect calculated average value by subtracting the wrong data entries and adding the true correct data items.

Solution:

(a) Calculate the original incorrect sum of all entries by multiplying the sample size by the initial mean: $\text{Sum}_{\text{old}} = 100 \times 40 = 4000$.

(b) Identify the miscalculated values and the true values from the problem statement: incorrect entries are 30 and 20; true entries are 40 and 50.

(c) Set up the linear correction arithmetic equation: $\text{Sum}_{\text{correct}} = \text{Sum}_{\text{old}} - (\text{Incorrect}) + (\text{Correct})$.

(d) Substitute the values into this equation and simplify: $\text{Sum}_{\text{correct}} = 4000 - (30 + 20) + (40 + 50) = 4000 - 50 + 90 = 4040$.

(e) Divide this correct total sum by the invariant sample size to evaluate the true average: $\text{Mean}_{\text{correct}} = \frac{4040}{100} = 40.4$.

Final Answer: The correct mean is 40.4.

Answer: (B)

Go Back to Question 43

Q44.

Solution

Concept: Trigonometric limits in calculus. Use double-angle transformations or identity reductions such as $1 - \cos(2\theta) = 2\sin^2\theta$ to resolve indeterminate forms.

Solution:

(a) Substitute $x = 0$ into the expression to identify its indeterminate form: $\frac{1-\cos(0)}{0} = \frac{0}{0}$.

(b) Apply the double-angle cosine identity transformation to modify the numerator: $1 - \cos 4x = 2\sin^2(2x)$.

(c) Substitute this identity back into the limit expression: $\lim_{x\to 0} \frac{2\sin^2(2x)}{x^2}$.

(d) Multiply and divide the expression by 4 to group and match the internal angle variable components: $2 \times 4 \times \lim_{x\to 0} \left(\frac{\sin(2x)}{2x}\right)^2$.

(e) Apply the standard fundamental limit identity $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$: $8 \times (1)^2 = 8$.

Final Answer: The value of the limit is 8.

Answer: (C)

Go Back to Question 44

Q45.

Solution

Concept: Set theory symmetric differences. The symmetric difference operation isolates non-overlapping elements: $A\Delta B = (A \setminus B) \cup (B \setminus A)$.

Solution:

(a) Find the elements of set A by solving its quadratic equation $x^2 - 3x + 2 = 0$: $(x-1)(x-2) = 0 \implies A = \{1, 2\}$.

(b) Find the elements of set B by solving its quadratic equation $x^2 - 4x + 3 = 0$: $(x-1)(x-3) = 0 \implies B = \{1, 3\}$.

(c) Determine the intersection set containing the common elements: $A \cap B = \{1\}$.

(d) Find the unique elements in each set by removing the intersection: $A \setminus B = \{2\}$ and $B \setminus A = \{3\}$.

(e) Combine these unique elements to find the symmetric difference set: $A\Delta B = \{2, 3\}$.

Final Answer: The symmetric difference set is $\{2, 3\}$.

Answer: (B)

Go Back to Question 45

Q46.

Solution

Concept: Definite integrals of absolute functions. Split the integration interval into piecewise smooth sub-domains based on the absolute value sign transitions.

Solution:

(a) Identify the absolute value change behavior over the given interval: $|x| = -x$ for $x < 0$, and $|x| = x$ for $x \geq 0$.

(b) Split the full definite integral across its origin boundary point: $\int_{-1}^{0} x(-x)\,dx + \int_{0}^{1} x(x)\,dx$.

(c) Simplify the independent integrand terms before computing: $\int_{-1}^{0} -x^2\,dx + \int_{0}^{1} x^2\,dx$.

(d) Find the antiderivative for each section using power rules: $[-\frac{x^3}{3}]_{-1}^{0} + [\frac{x^3}{3}]_{0}^{1}$.

(e) Substitute the integration limits and compute the values: $(0 - \frac{1}{3}) + (\frac{1}{3} - 0) = -\frac{1}{3} + \frac{1}{3} = 0$.

Final Answer: The definite integral value is 0.

Answer: (A)

Go Back to Question 46

Q47.

Solution

Concept: Vector dot product algebra. Square vector binomial equations to expand them into scalar expressions using the dot product expansion rule $|\vec{u}|^2 = \vec{u} \cdot \vec{u}$.

Solution:

(a) Square both sides of the given magnitude equation to expand it: $|\vec{a} + \vec{b}|^2 = (\sqrt{3})^2 \implies |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a} \cdot \vec{b}) = 3$.

(b) Substitute the given unit vector attributes $|\vec{a}| = 1$ and $|\vec{b}| = 1$: $1 + 1 + 2(\vec{a} \cdot \vec{b}) = 3 \implies 2(\vec{a} \cdot \vec{b}) = 1 \implies \vec{a} \cdot \vec{b} = \frac{1}{2}$.

(c) Expand the required target dot product expression linearly: $(2\vec{a} - 5\vec{b}) \cdot (3\vec{a} + \vec{b}) = 6|\vec{a}|^2 + 2(\vec{a} \cdot \vec{b}) - 15(\vec{a} \cdot \vec{b}) - 5|\vec{b}|^2$.

(d) Group the common scalar variables and simplify: $6|\vec{a}|^2 - 13(\vec{a} \cdot \vec{b}) - 5|\vec{b}|^2$.

(e) Substitute the calculated unit magnitudes and dot product values: $6(1) - 13(\frac{1}{2}) - 5(1) = 1 - \frac{13}{2} = -\frac{11}{2}$.

Final Answer: The value of the dot product expression is $-\frac{11}{2}$.

Answer: (B)

Go Back to Question 47

Q48.

Solution

Concept: Hyperbola conic attributes. Normalize general hyperbola forms to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to compute eccentricity metrics using the formula $e = \sqrt{1 + \frac{b^2}{a^2}}$.

Solution:

(a) Normalize the given equation $16x^2 - 9y^2 = 144$ by dividing both sides by 144: $\frac{16x^2}{144} - \frac{9y^2}{144} = 1$.

(b) Simplify the fractions to find the axis coefficients: $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

(c) Extract the metric denominator parameters from this standard form: $a^2 = 9$ and $b^2 = 16$.

(d) State the standard eccentricity equation for a horizontal hyperbola: $e = \sqrt{1 + \frac{b^2}{a^2}}$.

(e) Substitute the parameters and evaluate the final value: $e = \sqrt{1 + \frac{16}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$.

Final Answer: The eccentricity value is $\frac{5}{3}$.

Answer: (C)

Go Back to Question 48

Q49.

Solution

Concept: Combinatorics counting distribution paths. Distributing n distinct objects among r distinct boxes with no capacity limits yields r^n total options.

Solution:

(a) Identify the elements involved in the distribution process: there are 5 unique prizes to distribute among 4 unique students.

(b) Analyze the distribution choice path from the perspective of each individual prize.

(c) The first prize can be given to any of the 4 students, creating 4 options. Similarly, the second prize has 4 options since students can receive multiple prizes.

(d) Every remaining independent prize has exactly 4 candidate choices.

(e) Apply the fundamental counting multiplication rule across all 5 prizes: $4 \times 4 \times 4 \times 4 \times 4 = 4^5$.

Final Answer: The prizes can be distributed in 4^5 ways.

Answer: (B)

Go Back to Question 49

Q50.

Solution

Concept: Trigonometric coordinate systems. Squaring and adding compound function pairs uses standard circular identity reductions to isolate difference factors.

Solution:

(a) Write down the two given equations: $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$.

(b) Square both sides of the first sine equation: $\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta = a^2$.

(c) Square both sides of the second cosine equation: $\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = b^2$.

(d) Add these two expanded equations together to group the components: $(\sin^2\alpha + \cos^2\alpha) + (\sin^2\beta + \cos^2\beta) + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = a^2 + b^2$.

(e) Substitute basic trigonometric identities $(1 + 1 + 2\cos(\alpha - \beta) = a^2 + b^2)$ and isolate the target expression: $2\cos(\alpha - \beta) = a^2 + b^2 - 2 \implies \cos(\alpha - \beta) = \frac{a^2+b^2-2}{2}$.

Final Answer: The expression value is $\frac{a^2+b^2-2}{2}$.

Answer: (A)

Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	A	4	A	5	A
6	A	7	A	8	A	9	A	10	B
11	B	12	B	13	B	14	D	15	A
16	B	17	C	18	A	19	B	20	A
21	B	22	A	23	A	24	B	25	C
26	B	27	A	28	B	29	A	30	C
31	C	32	A	33	B	34	C	35	D
36	B	37	D	38	A	39	A	40	B
41	A	42	A	43	B	44	C	45	B
46	A	47	B	48	C	49	B	50	A