NIOS Class 12 Mathematics Sample Paper – 10

Duration: 180 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **45** Questions. The paper is divided into two sections: **Section A – 50** marks, **Section B – 50** marks.
- **Section A** consists of
  -**Q.No. 1 to 20** – Multiple Choice type questions (MCQs) carrying **+1 mark** each. Select and write the most appropriate option out of the four options given in each of these questions.
  -**Q.No. 21 to 29** – **Objective type questions**.
  -**Q.No. 21 to 24** carry **02 marks** each (with 2 sub-parts of 1 mark each).
  -**Q.No. 25 to 28** carry **04 marks** each (with 4 sub-parts of 1 mark each).
  -**Q.No. 29** carries **06 marks** (with 6 sub-parts of 1 mark each). Attempt these questions as per the instructions given for each of the questions 21–29.
- **Section B** consists of
  -**Q.No. 30 to 38**– Very Short questions carrying **02 marks** each.
  -**Q.No. 39 to 43** – Short Answer type questions carrying **04 marks** each.
  -**Q.No. 44 to 45** – Long Answer type questions carrying **06 marks** each.
  (An internal choice has been provided in some of the questions in Section B. You have to attempt only one of the given choices in such questions.)
- There is **No Negative marking**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Section: A

Q1. The value of $\lim_{x \to 0} \dfrac{1 - \cos 4x}{x \tan 2x}$ is: **(1)**

(A) 4

(B) 2

(C) 8

(D) 1

Q2. If A is a 2×2 matrix with $|A| = 5$, then $|3A|$ equals: **(1)**

(A) 45

(B) 15

(C) 30

(D) 9

Q3. A circle has $(2, 1)$ and $(6, 5)$ as endpoints of a diameter. Its radius is: **(1)**

(A) $2\sqrt{2}$

(B) $4\sqrt{2}$

(C) 4

(D) $\sqrt{2}$

Q4. If $f(x) = 5x - 2$, then $f^{-1}(8)$ is: **(1)**

(A) 1

(B) 2

(C) 3

(D) 10

Q5. The angle between $\vec{a} = \hat{i} + \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is: **(1)**

(A) 30°

(B) 45°

(C) 60°

(D) 90°

Q6. If $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, then A^3 is: **(1)**

(A) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$

Q7. On $\mathbb{Z}$, define aRb if $a - b$ is divisible by 4. Then R is: **(1)**

(A) Reflexive only

(B) Symmetric only

(C) An equivalence relation

(D) Not transitive

Q8. The directrix of the parabola $y^2 = 12x$ is: **(1)**

(A) $x = 3$

(B) $x = -3$

(C) $y = 3$

(D) $y = -3$

Q9. $\int_0^{\pi/4} \sec^2 x \, dx$ equals: **(1)**

(A) 1

(B) $\frac{\pi}{4}$

(C) $\sqrt{3}$

(D) 0

Q10. The negation of $p \wedge q$ is: **(1)**

(A) $\neg p \wedge \neg q$

(B) $\neg p \vee \neg q$

(C) $p \vee q$

(D) $p \rightarrow q$

Q11. Direction cosines of a line whose direction ratios are $2, -2, 1$ are: **(1)**

(A) $\left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$

(B) $\left(\frac{1}{2}, -\frac{1}{2}, 1\right)$

(C) $(2, -2, 1)$

(D) $\left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right)$

Q12. For $x > 0$, the minimum value of $x + \frac{9}{x}$ is: **(1)**

(A) 3

(B) 6

(C) 9

(D) 12

Q13. The matrix $\begin{pmatrix} 2 & k \\ k & 8 \end{pmatrix}$ is singular for positive k equal to: **(1)**

(A) 2

(B) 3

(C) 4

(D) 6

Q14. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ equals: **(1)**

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) 0

Q15. The equation of the line making intercepts 3 and 4 on the positive x-axis and y-axis respectively is: **(1)**

(A) $3x + 4y = 12$

(B) $4x + 3y = 12$

(C) $x + y = 7$

(D) $4x - 3y = 12$

Q16. The order of $\left(\frac{d^3y}{dx^3}\right)^2 + \frac{dy}{dx} = \cos x$ is: **(1)**

(A) 1

(B) 2

(C) 3

(D) Not defined

Q17. In a linear programming problem, the restrictions $x \geq 0$ and $y \geq 0$ are called: **(1)**

(A) Objective restrictions

(B) Non-negativity restrictions

(C) Slack equations

(D) Profit conditions

Q18. The distance of the point $(3, -1, 2)$ from the plane $x + 2y + 2z - 9 = 0$ is: **(1)**

(A) $\frac{4}{3}$

(B) $\frac{2}{3}$

(C) 4

(D) $\frac{3}{4}$

Q19. $\int 2x \cos(x^2)\, dx$ equals: **(1)**

(A) $\sin(x^2) + C$

(B) $\cos(x^2) + C$

(C) $2\sin x + C$

(D) $x^2 \sin(x^2) + C$

Q20. The eccentricity of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is: **(1)**

(A) $\frac{5}{3}$

(B) $\frac{4}{3}$

(C) $\frac{3}{5}$

(D) $\sqrt{2}$

Q21. Match Column-I with Column-II: **(2)**

Column-I	**Column-II**
(i) $\det(I_3)$	(A) 2
(ii) $\int_0^{\pi} \sin x \, dx$	(B) 1

(A) (i)→(A), (ii)→(B)

(B) (i)→(B), (ii)→(A)

Q22. Fill in the blanks: **(2)**

(i) The principal value range of $\tan^{-1} x$ is ____________.

(ii) The number of elements in a 4×3 matrix is ________.

Q23. Write TRUE for correct statement and FALSE for incorrect statement: **(2)**

(i) Every skew-symmetric matrix has all diagonal entries equal to zero.

(ii) A relation which is symmetric and transitive is necessarily reflexive.

Q24. Write the negation of each of the following statements: **(2)**

(i) Every differentiable function is continuous.

(ii) There exists a real number x such that $x^2 + 1 = 0$.

Q25. Fill in the blanks: **(4)**

(i) $\lim_{x \to 0} \frac{\sin 4x}{\tan 2x} = $ ________.

(ii) $\frac{d}{dx}\left(x^2 e^x\right) = $ ____________.

(iii) $\int \sec^2(5x)\, dx = $ ____________.

(iv) $\int_0^1 (2x + 1)\, dx = $ ________.

Q26. Fill in the blanks: **(4)**

(i) The slope of the line through $(1, 2)$ and $(5, 10)$ is ________.

(ii) The centre of $x^2 + y^2 + 8x - 6y + 12 = 0$ is ________.

(iii) $\det\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{pmatrix} = $ ________.

(iv) The length of the minor axis of $\frac{x^2}{36} + \frac{y^2}{16} = 1$ is ________.

Q27. Write TRUE for correct statement and FALSE for incorrect statement: **(4)**

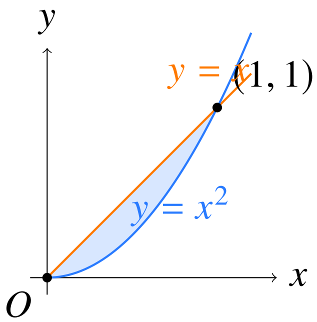
(i) If A and B are invertible matrices of the same order, then $(AB)^{-1} = B^{-1}A^{-1}$.

(ii) For every square matrix A, $|A^T| = -|A|$.

(iii) If l, m, n are direction cosines of a line, then $l^2 + m^2 + n^2 = 1$.

(iv) For $-1 \leq x \leq 1$, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

Q28. Carefully study the figure showing the line $y = x$ and the parabola $y = x^2$ in the first quadrant, and answer the following: **(4)**



(i) Write the x-coordinates of the points of intersection.

(ii) Identify the upper curve on $[0, 1]$.

(iii) Write the integral representing the shaded area.

(iv) Find the shaded area.

Q29. Read the passage and answer (i)–(vi):
A bakery prepares two types of packs, A and B. One pack of type A uses 2 kg of flour and 1 hour of baking time, and gives a profit of Rs. 30. One pack of type B uses 1 kg of flour and 2 hours of baking time, and gives a profit of Rs. 40. The bakery has 80 kg of flour and 70 hours of baking time available. Let x and y denote the numbers of packs of type A and B respectively. **(6)**

(i) The flour constraint is:

(A) $x + 2y \leq 80$

(B) $2x + y \leq 80$

(C) $30x + 40y \leq 80$

(D) $x + y \leq 80$

(ii) The baking-time constraint is:

(A) $2x + y \leq 70$

(B) $2x + 2y \leq 70$

(C) $x + 2y \leq 70$

(D) $30x + 40y \leq 70$

(iii) The objective function is:

(A) Maximize $Z = 30x + 40y$

(B) Maximize $Z = 2x + y$

(C) Minimize $Z = 30x + 40y$

(D) Maximize $Z = x + 2y$

(iv) The intersection of $2x + y = 80$ and $x + 2y = 70$ is:

(A) $(20, 30)$

(B) $(25, 25)$

(C) $(35, 10)$

(D) $(30, 20)$

(v) The profit at $(40, 0)$ is:

(A) Rs. 1000

(B) Rs. 1200

(C) Rs. 1400

(D) Rs. 1600

(vi) The maximum profit is:

(A) Rs. 1200

(B) Rs. 1400

(C) Rs. 1700

(D) Rs. 2000

Section: B

Q30. Find the range of $f(x) = 2 + \sin x$. **(2)**

Q31. Evaluate $\det\begin{pmatrix} 2 & 1 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & -1 \end{pmatrix}$. **(2)**

Q32. Find $\dfrac{dy}{dx}$ if $y = \tan^{-1}(x^2)$. **(2)**

Q33. Evaluate $\int xe^{x^2}\,dx$. **(2)**

Q34. Find a unit vector in the direction of $2\hat{i} + \hat{j} - 2\hat{k}$.

OR

Find $\vec{a} \cdot \vec{b}$ if $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - \hat{k}$. **(2)**

Q35. Find the equation of the plane passing through $(1, 1, 1)$ and normal to $2\hat{i} - \hat{j} + 3\hat{k}$. **(2)**

Q36. Solve the differential equation $\dfrac{dy}{dx} = 3y$. **(2)**

Q37. Evaluate $\displaystyle\int_0^{\pi/6} \sec^2 x \, dx$. **(2)**

Q38. Find the midpoint of the segment joining $A(2, -1, 3)$ and $B(4, 3, -1)$. **(2)**

Q39. Using the inverse matrix method, solve the system $2x + y = 7$, $3x + 2y = 12$. **(4)**

Q40. Prove that $2\tan^{-1}\left(\dfrac{1}{3}\right) + \tan^{-1}\left(\dfrac{1}{7}\right) = \dfrac{\pi}{4}$. **(4)**

Q41. Find the equation of the plane through $P(2, -1, 1)$ and parallel to the vectors $\hat{i} + 2\hat{j}$ and $\hat{j} + 3\hat{k}$. **(4)**

Q42. Evaluate $\displaystyle\int_0^{2} |x - 1| \, dx$. **(4)**

Q43. Solve graphically: Maximize $Z = 3x + 2y$ subject to $x + 2y \le 8$, $3x + y \le 9$, $x \ge 0$, $y \ge 0$. **(4)**

Q44. Find the area bounded by the curves $y = 2x$ and $y = x^2$ using integration. **(6)**

Q45. A rectangular field is to be fenced on three sides using 120 m of fencing, the fourth side being along a straight wall. Find the dimensions of the field for maximum area and the maximum area. **(6)**

Detailed Solutions

Q1.

Solution

Concept: The standard limits $\lim_{u\to 0} \frac{1-\cos u}{u^2} = \frac{1}{2}$ and $\lim_{u\to 0} \frac{\tan u}{u} = 1$ help evaluate trigonometric limits. The coefficients inside the angles must be handled carefully.

Solution:

Step 1: Write the limit as

$$L = \lim_{x\to 0} \frac{1-\cos 4x}{x\tan 2x}.$$

Step 2: Use the small-angle equivalents $1-\cos 4x \sim \frac{(4x)^2}{2} = 8x^2$ and $\tan 2x \sim 2x$.

Step 3: Then the denominator behaves like $x(2x) = 2x^2$.

Step 4: Therefore,

$$L = \frac{8x^2}{2x^2} = 4.$$

Step 5: The cancellation is valid after replacing each factor by its limiting equivalent near $x = 0$.

Final Answer: $\boxed{4}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: If A is an $n \times n$ matrix, then $|kA| = k^n|A|$. Since this is a 2×2 matrix, multiplying the matrix by 3 multiplies its determinant by 3^2.

Solution:

Step 1: Given $|A| = 5$ and the order of A is 2.

Step 2: Apply the determinant property:

$$|3A| = 3^2|A|.$$

Step 3: Substitute the given determinant:

$$|3A| = 9 \times 5 = 45.$$

Step 4: Hence the determinant becomes 45, not merely 3×5, because both rows are multiplied by 3.

Final Answer: $\boxed{45}$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept: If endpoints of a diameter of a circle are known, the diameter length is the distance between those points. The radius is half of that distance.

Solution:

Step 1: Let the endpoints be $A(2, 1)$ and $B(6, 5)$.

Step 2: Distance AB is

$$AB = \sqrt{(6-2)^2 + (5-1)^2} = \sqrt{16+16} = 4\sqrt{2}.$$

Step 3: Since AB is the diameter, radius is

$$r = \frac{AB}{2} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}.$$

Step 4: The radius is positive, so the required value is $2\sqrt{2}$.

Final Answer: $\boxed{2\sqrt{2}}$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept: The value $f^{-1}(8)$ means the input whose image under f is 8. So we solve $f(x) = 8$ for x.

Solution:

Step 1: Given $f(x) = 5x - 2$.

Step 2: Put $f(x) = 8$:

$$5x - 2 = 8.$$

Step 3: Add 2 to both sides:

$$5x = 10.$$

Step 4: Divide by 5:

$$x = 2.$$

Step 5: Hence $f^{-1}(8) = 2$.

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: The angle between two vectors is found from $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$. Dot product measures the component of one vector along another.

Solution:

Step 1: Here $\vec{a} = (1, 0, 1)$ and $\vec{b} = (0, 1, 1)$.

Step 2: Dot product:

$$\vec{a} \cdot \vec{b} = 1 \cdot 0 + 0 \cdot 1 + 1 \cdot 1 = 1.$$

Step 3: Magnitudes:

$$|\vec{a}| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}, \quad |\vec{b}| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2}.$$

Step 4: Therefore,

$$\cos\theta = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}.$$

Step 5: Hence $\theta = 60^\circ$.

Final Answer: $\boxed{60^\circ}$

Answer: (C)

Go Back to Question 5

Q6.

Solution

Concept: Matrix powers are obtained by repeated multiplication. For $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, the upper right entry increases by one with each multiplication.

Solution:

Step 1: Compute A^2:

$$A^2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}.$$

Step 2: Now multiply by A again:

$$A^3 = A^2 A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Step 3: Row-column multiplication gives

$$A^3 = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}.$$

Step 4: All entries are checked by ordinary matrix multiplication.

Final Answer: $\boxed{\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}}$

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept: A relation is an equivalence relation if it is reflexive, symmetric, and transitive. Congruence modulo a fixed positive integer satisfies all these three properties.

Solution:

Step 1: Reflexive: for every integer a, $a - a = 0$, which is divisible by 4.

Step 2: Symmetric: if $a - b$ is divisible by 4, then $b - a = -(a - b)$ is also divisible by 4.

Step 3: Transitive: if $a - b$ and $b - c$ are divisible by 4, then

$$a - c = (a - b) + (b - c)$$

is also divisible by 4.

Step 4: Thus R is reflexive, symmetric, and transitive.

Final Answer: $\boxed{\text{An equivalence relation}}$

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: The standard form $y^2 = 4ax$ represents a parabola opening towards the positive x-axis. Its focus is $(a, 0)$ and directrix is $x = -a$.

Solution:

Step 1: Compare $y^2 = 12x$ with $y^2 = 4ax$.

Step 2: We get

$$4a = 12 \Rightarrow a = 3.$$

Step 3: The directrix is

$$x = -a = -3.$$

Final Answer: $\boxed{x = -3}$

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept: The derivative of $\tan x$ is $\sec^2 x$, so the antiderivative of $\sec^2 x$ is $\tan x$. Then apply the upper and lower limits.

Solution:

Step 1: Evaluate

$$\int_0^{\pi/4} \sec^2 x \, dx = [\tan x]_0^{\pi/4}.$$

Step 2: Substitute the limits:

$$\tan\frac{\pi}{4} - \tan 0 = 1 - 0 = 1.$$

Step 3: Hence the definite integral is 1.

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: De Morgan's laws give the negation of compound statements. The negation of a conjunction changes "and" into "or" and negates both simple statements.

Solution:

Step 1: The given statement is $p \wedge q$.

Step 2: Its negation is written as

$$\neg(p \wedge q).$$

Step 3: By De Morgan's law,

$$\neg(p \wedge q) = \neg p \vee \neg q.$$

Final Answer: $\boxed{\neg p \vee \neg q}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: Direction cosines are found by dividing each direction ratio by the magnitude of the direction-ratio vector. If direction ratios are a, b, c, then the denominator is $\sqrt{a^2 + b^2 + c^2}$.

Solution:

Step 1: Direction ratios are $2, -2, 1$.

Step 2: Their magnitude is

$$\sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{4 + 4 + 1} = 3.$$

Step 3: Therefore direction cosines are

$$\left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right).$$

Step 4: Check: $\left(\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = 1.$

Final Answer: $\boxed{\left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: For positive x, an expression of the form $x + \frac{a}{x}$ may be minimized by differentiation or by AM-GM. The minimum occurs when the two positive terms are equal.

Solution:

Step 1: Let

$$f(x) = x + \frac{9}{x}, \quad x > 0.$$

Step 2: Differentiate:

$$f'(x) = 1 - \frac{9}{x^2}.$$

Step 3: Put $f'(x) = 0$:

$$1 - \frac{9}{x^2} = 0 \Rightarrow x^2 = 9 \Rightarrow x = 3,$$

since $x > 0$.

Step 4: The value at $x = 3$ is

$$f(3) = 3 + \frac{9}{3} = 6.$$

Step 5: Also $f''(x) = \frac{18}{x^3} > 0$, so it is a minimum.

Final Answer: $\boxed{6}$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: A matrix is singular when its determinant is zero. For a 2×2 matrix, the determinant is obtained by subtracting the product of the off-diagonal entries from the product of the diagonal entries.

Solution:

Step 1: For

$$\begin{pmatrix} 2 & k \\ k & 8 \end{pmatrix},$$

the determinant is

$$2 \cdot 8 - k^2 = 16 - k^2.$$

Step 2: For singularity,

$$16 - k^2 = 0.$$

Step 3: Hence

$$k^2 = 16 \Rightarrow k = \pm 4.$$

Step 4: Since k is positive, $k = 4$.

Final Answer: $\boxed{4}$

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept: Inverse trigonometric functions return principal values. Here $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ and $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$.

Solution:

Step 1: Evaluate the sine inverse term:

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}.$$

Step 2: Evaluate the cosine inverse term:

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}.$$

Step 3: Subtract:

$$\frac{\pi}{3} - \frac{\pi}{6} = \frac{2\pi - \pi}{6} = \frac{\pi}{6}.$$

Final Answer: $\boxed{\frac{\pi}{6}}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: The intercept form of a line is $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are the intercepts on the x-axis and y-axis respectively.

Solution:

Step 1: The given intercepts are $a = 3$ and $b = 4$.

Step 2: Therefore,

$$\frac{x}{3} + \frac{y}{4} = 1.$$

Step 3: Multiply by 12:

$$4x + 3y = 12.$$

Step 4: This is the required equation of the line.

Final Answer: $\boxed{4x + 3y = 12}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: The order of a differential equation is the order of the highest derivative present in it. We do not look at the exponent first; we first identify the highest derivative.

Solution:

Step 1: The equation contains $\frac{d^3y}{dx^3}$ and $\frac{dy}{dx}$.

Step 2: The highest derivative is the third derivative.

Step 3: Therefore the order is 3.

Final Answer: $\boxed{3}$

Answer: (C)

Go Back to Question 16

Q17.

Solution

Concept: In linear programming, variables representing quantities cannot usually be negative. Conditions such as $x \geq 0$ and $y \geq 0$ are therefore called non-negativity restrictions.

Solution:

Step 1: The restrictions given are $x \geq 0$ and $y \geq 0$.

Step 2: These ensure that the decision variables are not negative.

Step 3: Therefore these are called non-negativity restrictions.

Final Answer: Non-negativity restrictions

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept: Distance from (x_1, y_1, z_1) to plane $ax + by + cz + d = 0$ is $\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$. Absolute value is used because distance is always non-negative.

Solution:

Step 1: Here the plane is $x + 2y + 2z - 9 = 0$.

Step 2: Substitute $(3, -1, 2)$ into the numerator:

$$|3 + 2(-1) + 2(2) - 9| = |3 - 2 + 4 - 9| = | - 4| = 4.$$

Step 3: Denominator is

$$\sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3.$$

Step 4: Hence distance is

$$\frac{4}{3}.$$

Final Answer: $\boxed{\frac{4}{3}}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: Substitution is useful when the derivative of the inner function appears outside. Since $\frac{d}{dx}(x^2) = 2x$, put $u = x^2$.

Solution:

Step 1: Let $u = x^2$.

Step 2: Then $du = 2x\, dx$.

Step 3: The integral becomes

$$\int 2x \cos(x^2)\, dx = \int \cos u\, du.$$

Step 4: Integrate:

$$\int \cos u\, du = \sin u + C.$$

Step 5: Substitute back $u = x^2$:

$$\sin(x^2) + C.$$

Final Answer: $\boxed{\sin(x^2) + C}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, eccentricity is $e = \sqrt{1 + \frac{b^2}{a^2}}$.

Solution:

Step 1: From

$$\frac{x^2}{9} - \frac{y^2}{16} = 1,$$

we have $a^2 = 9$ and $b^2 = 16$.

Step 2: Use the formula:

$$e = \sqrt{1 + \frac{16}{9}}.$$

Step 3: Simplify:

$$e = \sqrt{\frac{25}{9}} = \frac{5}{3}.$$

Final Answer: $\boxed{\frac{5}{3}}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: The determinant of an identity matrix is always 1. Also, $\int_0^\pi \sin x\,dx$ is evaluated using the antiderivative $-\cos x$.

Solution:

Step 1: Since I_3 is an identity matrix,

$$\det(I_3) = 1.$$

So (i) matches (B).

Step 2: Evaluate the integral:

$$\int_0^\pi \sin x\,dx = [-\cos x]_0^\pi = -\cos\pi + \cos 0 = 1 + 1 = 2.$$

So (ii) matches (A).

Step 3: Therefore the correct matching is (i)→(B), (ii)→(A).

Final Answer: $\boxed{(i) \to (B),\ (ii) \to (A)}$

Go Back to Question 21

Q22.

Solution

Concept: The inverse tangent function is defined for all real numbers and its principal values lie in an open interval. The number of elements in a matrix is the product of its number of rows and columns.

Solution:

Step 1: The principal value range of $\tan^{-1} x$ is

$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$

Step 2: A 4×3 matrix has 4 rows and 3 columns.

Step 3: Thus the number of elements is

$$4 \times 3 = 12.$$

Final Answer: $\boxed{\text{(i) } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \quad \text{(ii) } 12}$

Go Back to Question 22

Q23.

Solution

Concept: A skew-symmetric matrix satisfies $A^T = -A$, which forces each diagonal element to be its own negative. For relations, symmetry and transitivity alone do not guarantee reflexivity on the whole set.

Solution:

Step 1: If A is skew-symmetric, then for a diagonal entry a_{ii} we get $a_{ii} = -a_{ii}$.

Step 2: Therefore $2a_{ii} = 0$, so $a_{ii} = 0$. Statement (i) is TRUE.

Step 3: Statement (ii) is FALSE because a relation may be symmetric and transitive without containing every pair (a, a) required for reflexivity.

Step 4: For example, the empty relation on a non-empty set is symmetric and transitive, but not reflexive.

Final Answer: $\boxed{\text{(i) TRUE, (ii) FALSE}}$

Go Back to Question 23

Q24.

Solution

Concept: The negation of a universal statement begins with "There exists at least one". The negation of an existential statement begins with "For every" or "There does not exist".

Solution:

Step 1: Statement (i) says: Every differentiable function is continuous.

Step 2: Its negation is: There exists at least one differentiable function which is not continuous.

Step 3: Statement (ii) says: There exists a real number x such that $x^2 + 1 = 0$.

Step 4: Its negation is: For every real number x, $x^2 + 1 \neq 0$.

Final Answer:

$$\boxed{\text{(i) Some differentiable function is not continuous.}}$$

$$\boxed{\text{(ii) For every real } x,\ x^2 + 1 \neq 0.}$$

Go Back to Question 24

Q25.

Solution

Concept: This question uses standard trigonometric limits, product rule, basic integration of trigonometric derivatives, and the fundamental theorem of calculus.

Solution:

Step 1: For part (i), use $\sin 4x \sim 4x$ and $\tan 2x \sim 2x$ as $x \to 0$:

$$\lim_{x\to 0} \frac{\sin 4x}{\tan 2x} = \frac{4}{2} = 2.$$

Step 2: For part (ii), apply product rule:

$$\frac{d}{dx}(x^2 e^x) = 2xe^x + x^2 e^x = e^x(x^2 + 2x).$$

Step 3: For part (iii), since $\frac{d}{dx}\tan(5x) = 5\sec^2(5x)$,

$$\int \sec^2(5x)\,dx = \frac{1}{5}\tan(5x) + C.$$

Step 4: For part (iv),

$$\int_0^1 (2x + 1)\,dx = [x^2 + x]_0^1 = 2.$$

Final Answer: $\boxed{\text{(i) } 2,\ \text{(ii) } e^x(x^2 + 2x),\ \text{(iii) } \frac{1}{5}\tan(5x) + C,\ \text{(iv) } 2}$

Go Back to Question 25

Q26.

Solution

Concept: Coordinate geometry and algebra require careful substitution into standard formulas: slope formula, circle centre formula, triangular determinant rule for diagonal matrices, and axis length of an ellipse.

Solution:

Step 1: Slope through $(1, 2)$ and $(5, 10)$ is

$$m = \frac{10-2}{5-1} = \frac{8}{4} = 2.$$

Step 2: Compare $x^2 + y^2 + 8x - 6y + 12 = 0$ with $x^2 + y^2 + 2gx + 2fy + c = 0$.

$$2g = 8 \Rightarrow g = 4, \quad 2f = -6 \Rightarrow f = -3.$$

Centre is $(-g, -f) = (-4, 3)$.

Step 3: Since the matrix is diagonal, determinant is the product of diagonal elements:

$$1 \cdot (-2) \cdot 5 = -10.$$

Step 4: In $\frac{x^2}{36} + \frac{y^2}{16} = 1$, the semi-minor axis is 4. Minor axis length is $2 \times 4 = 8$.

Final Answer: $\boxed{\text{(i) } 2,\ \text{(ii) } (-4, 3),\ \text{(iii) } -10,\ \text{(iv) } 8}$

Go Back to Question 26

Q27.

Solution

Concept: This question checks standard facts from matrices, vectors, and inverse trigonometric functions. The order of factors reverses when taking inverse of a product, determinant is unchanged by transpose, direction cosines satisfy a unit relation, and inverse sine plus inverse cosine equals $\pi/2$.

Solution:

Step 1: Statement (i) is TRUE because $(AB)^{-1} = B^{-1}A^{-1}$.

Step 2: Statement (ii) is FALSE because $|A^T| = |A|$, not $-|A|$.

Step 3: Statement (iii) is TRUE because direction cosines are components of a unit direction vector.

Step 4: Statement (iv) is TRUE for all $x \in [-1, 1]$:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}.$$

Final Answer: $\boxed{\text{(i) TRUE, (ii) FALSE, (iii) TRUE, (iv) TRUE}}$

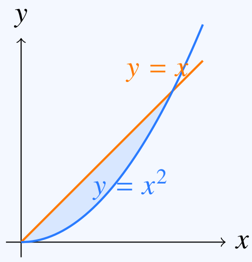
Go Back to Question 27

Q28.

Solution

Concept: The area between two curves is computed by integrating upper curve minus lower curve over the interval of intersection. First solve the equations together, then decide which curve is above the other.

Solution:



Step 1: Intersections are found from $x = x^2$:

$$x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1.$$

Step 2: On $[0, 1]$, take $x = \frac{1}{2}$. Then $x = \frac{1}{2}$ and $x^2 = \frac{1}{4}$, so $y = x$ is the upper curve.

Step 3: The required integral is

$$\int_0^1 (x - x^2)\, dx.$$

Step 4: Evaluate:

$$\int_0^1 (x - x^2)\, dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

Final Answer: $\boxed{\text{(i) } 0, 1, \ \text{(ii) } y = x, \ \text{(iii) } \int_0^1 (x - x^2) dx, \ \text{(iv) } \frac{1}{6}}$

Go Back to Question 28

Q29.

Solution

Concept: A linear programming passage is converted into inequalities by reading the resource use per unit. The objective function represents total profit. The maximum profit occurs at a corner point of the feasible region.

Solution:

Step 1: Flour used is 2 kg for each A and 1 kg for each B, so

$$2x + y \leq 80.$$

Thus part (i) is option B.

Step 2: Baking time used is 1 hour for each A and 2 hours for each B, so

$$x + 2y \leq 70.$$

Thus part (ii) is option C.

Step 3: Total profit is

$$Z = 30x + 40y.$$

Thus part (iii) is option A.

Step 4: Solve the boundary equations:

$$2x + y = 80, \quad x + 2y = 70.$$

From the first, $y = 80 - 2x$. Substitute:

$$x + 2(80 - 2x) = 70 \Rightarrow -3x = -90 \Rightarrow x = 30.$$

Then $y = 20$. Thus part (iv) is option D.

Step 5: Profit at $(40, 0)$ is

$$Z = 30(40) + 40(0) = 1200.$$

Thus part (v) is option B.

Step 6: Check main corner values: $(40, 0)$ gives 1200, $(0, 35)$ gives 1400, and $(30, 20)$ gives

$$30(30) + 40(20) = 900 + 800 = 1700.$$

Thus the maximum profit is Rs. 1700, option C.

Final Answer: (i) B, (ii) C, (iii) A, (iv) D, (v) B, (vi) C

Go Back to Question 29

Q30.

Solution

Concept: The mathematical range of a modified trigonometric expression can be determined by analyzing the behavior of its core foundational components. The standard continuous sine function, $y = \sin x$, possesses a well-defined bounded domain map where its outputs strictly alternate and remain enclosed between the critical scalar values of -1 and 1 for all possible real values of x. Adding a fixed uniform real constant to this function serves to apply a vertical translation, shifting the entire functional output structure upward or downward without modifying its period or base properties.

Solution:

Step 1: State the universal boundary inequality rule that governs the output values of the core trigonometric sine function:

$$-1 \leq \sin x \leq 1$$

Step 2: Apply a uniform algebraic shift by adding the scalar integer value of 2 across all three parts of the inequality expression:

$$-1 + 2 \leq 2 + \sin x \leq 1 + 2$$

Step 3: Simplify the lower and upper bounds to determine the minimum and maximum boundaries of the translated function:

$$1 \leq 2 + \sin x \leq 3$$

Step 4: Observe that the output values are bounded between 1 and 3, meaning the complete range of the modified function $f(x) = 2 + \sin x$ is exactly given by the closed interval $[1, 3]$.

Final Answer: $\boxed{[1, 3]}$

Go Back to Question 30

Q31.

Solution

Concept: Evaluating the determinant of a large square matrix can be simplified if the matrix exhibits a special structural layout. An upper triangular matrix is defined as a square matrix where all the numerical entries positioned strictly below the main diagonal line are equal to zero. According to matrix algebra theorems, the determinant of any upper triangular or lower triangular matrix is equal to the product of its main diagonal entries, eliminating the need to perform long Laplace row or column expansions.

Solution:

Step 1: Inspect the arrangement of entries within the given third-order square matrix:

$$A = \begin{pmatrix} 2 & 1 & 5 \\ 0 & 4 & 3 \\ 0 & 0 & -1 \end{pmatrix}$$

Observe that all numerical coefficients situated beneath the principal diagonal elements are zero, confirming that the given structure is classified as an upper triangular matrix.

Step 2: Isolate the specific diagonal coefficients from the matrix: $a_{11} = 2$, $a_{22} = 4$, and $a_{33} = -1$.

Step 3: Apply the diagonal multiplication property to compute the scalar value of the determinant directly:

$$\det(A) = 2 \cdot 4 \cdot (-1)$$

Step 4: Evaluate the arithmetic product to find the final scalar determinant result:

$$\det(A) = 8 \cdot (-1) = -8$$

Final Answer: $\boxed{-8}$

Go Back to Question 31

Q32.

Solution

Concept: Differentiating a composite function requires the application of the calculus chain rule. For an expression where an algebraic function is nested inside an inverse trigonometric function, such as $y = \tan^{-1} u$, the derivative with respect to x is computed by taking the derivative of the outer inverse tangent function and multiplying it by the inner derivative u'. This is expressed as $\frac{dy}{dx} = \frac{1}{1+u^2} \cdot \frac{du}{dx} = \frac{u'}{1+u^2}$.

Solution:

Step 1: State the core compound functional equation provided in the problem statement:

$$y = \tan^{-1}\left(x^2\right)$$

Step 2: Identify the nested inner algebraic component and define it as a variable function u:

$$u = x^2$$

Step 3: Differentiate this inner component with respect to x using the standard algebraic power rule:

$$u' = \frac{du}{dx} = 2x$$

Step 4: Substitute both expressions for u and u' back into the standard inverse tangent derivative formula:

$$\frac{dy}{dx} = \frac{2x}{1 + \left(x^2\right)^2}$$

Step 5: Simplify the algebraic power term in the denominator to write down the final derivative expression:

$$\frac{dy}{dx} = \frac{2x}{1 + x^4}$$

Final Answer: $\boxed{\frac{2x}{1 + x^4}}$

Go Back to Question 32

Q33.

Solution

Concept: Evaluating an indefinite integral involving nested functions can often be simplified by choosing an appropriate variable substitution. When the integrand features a composite exponential function along with an extra factor that matches the derivative of the inner function up to a constant scalar multiplier, substituting a new variable for that inner function transforms the expression into a standard exponential integral.

Solution:

Step 1: Write down the indefinite integral expression provided in the prompt:

$$I = \int xe^{x^2}\,dx$$

Step 2: Define a new integration variable u by setting it equal to the inner quadratic exponent:

$$u = x^2$$

Step 3: Differentiate this substitution equation to relate the differentials du and dx:

$$du = 2x\,dx \implies x\,dx = \frac{1}{2}\,du$$

Step 4: Substitute these expressions back into the original integrand to rewrite the integral in terms of u:

$$I = \int e^u \cdot \left(\frac{1}{2}\,du\right) = \frac{1}{2}\int e^u\,du$$

Step 5: Evaluate the standard exponential integral, remembering to include the constant of integration C:

$$I = \frac{1}{2}e^u + C$$

Step 6: Perform the back-substitution step by replacing the variable u with its original expression x^2:

$$I = \frac{1}{2}e^{x^2} + C$$

Final Answer: $\boxed{\frac{1}{2}e^{x^2} + C}$

Go Back to Question 33

Q34.

Solution

Concept: A unit vector is defined as a vector that points in the exact same spatial direction as a given vector but has a geometric length or norm equal to exactly one. It is computed by dividing the original vector by its absolute scalar magnitude. For the alternative choice question, the scalar dot product between two vectors is calculated by multiplying their corresponding directional components together and summing those individual products.

Solution:

Step 1: For the primary question, isolate the three directional component coefficients from the vector $\vec{v} = 2\hat{i} + \hat{j} - 2\hat{k}$:

$$x = 2, \quad y = 1, \quad z = -2$$

Step 2: Compute the absolute geometric magnitude of this vector using the spatial Pythagorean formula:

$$|\vec{v}| = \sqrt{2^2 + 1^2 + (-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

Step 3: Divide the vector components by this magnitude to construct the required unit vector:

$$\hat{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{2\hat{i} + \hat{j} - 2\hat{k}}{3} = \frac{2}{3}\hat{i} + \frac{1}{1}3\hat{j} - \frac{2}{3}\hat{k}$$

Step 4: For the alternative choice question, take the scalar components of the two vectors:

$$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}, \quad \vec{b} = \hat{i} + 4\hat{j} - \hat{k}$$

Step 5: Evaluate their dot product by multiplying and summing the corresponding coefficients:

$$\vec{a} \cdot \vec{b} = (2)(1) + (-1)(4) + (3)(-1) = 2 - 4 - 3 = -5$$

Final Answer: $\boxed{\frac{2\hat{i} + \hat{j} - 2\hat{k}}{3}}$ OR $\boxed{-5}$

Go Back to Question 34

Q35.

Solution

Concept: In three-dimensional coordinate geometry, the spatial equation of a flat plane can be uniquely determined if a specific point lying on that plane and a vector perpendicular to its surface are known. If a plane passes through a fixed coordinate point (x_1, y_1, z_1) and has a perpendicular direction defined by the normal vector components $\vec{n} = (a, b, c)$, its point-normal equation is expressed as $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$.

Solution:

Step 1: Identify and extract the structural parameters provided in the problem statement. The fixed passing point is given as $(x_1, y_1, z_1) = (1, 1, 1)$, and the perpendicular normal direction vector is $\vec{n} = (2, -1, 3)$.

Step 2: Substitute these coordinates and directional coefficients into the point-normal vector formula:

$$2(x - 1) - 1(y - 1) + 3(z - 1) = 0$$

Step 3: Expand the algebraic terms by distributing the scalar coefficients over each binomial term:

$$2x - 2 - y + 1 + 3z - 3 = 0$$

Step 4: Group the variable terms together and combine the remaining constant integers to simplify the equation:

$$2x - y + 3z + (-2 + 1 - 3) = 0$$

$$2x - y + 3z - 4 = 0$$

This matches the standard Cartesian form of a plane.

Final Answer: $\boxed{2x - y + 3z - 4 = 0}$

Go Back to Question 35

Q36.

Solution

Concept: A first-order ordinary differential equation is classified as a separable differential equation if its algebraic structure allows all expressions involving the dependent variable y to be grouped together on one side with the differential dy, while all expressions involving the independent variable x are grouped together on the opposite side with the differential dx. Once the variables are separated, the general solution can be found by integrating both sides independently.

Solution:

Step 1: State the first-order ordinary differential equation given in the problem prompt:

$$\frac{dy}{dx} = 3y$$

Step 2: Separate the variables by dividing both sides of the equation by y and multiplying by dx:

$$\frac{1}{y}\,dy = 3\,dx$$

Step 3: Apply integration operators to both sides of the separated differential expression:

$$\int \frac{1}{y}\,dy = \int 3\,dx$$

Step 4: Evaluate the integrals, using the natural logarithm for the left side and introducing an arbitrary integration constant C_1:

$$\log|y| = 3x + C_1$$

Step 5: Transform the logarithmic equation into an exponential function by raising e to the power of both sides:

$$|y| = e^{3x+C_1} = e^{C_1} \cdot e^{3x}$$

Step 6: Define a new arbitrary constant scalar $C = \pm e^{C_1}$ to express the general solution cleanly:

$$y = Ce^{3x}$$

Final Answer: $\boxed{y = Ce^{3x}}$

Go Back to Question 36

Q37.

Solution

Concept: Evaluating a definite integral involves applying the fundamental theorem of calculus, which states that if a function $f(x)$ is continuous, the integral can be computed using its antiderivative function $F(x)$ evaluated at the boundaries. Because the standard derivative of the trigonometric tangent function is equal to the squared secant function, $\frac{d}{dx}(\tan x) = \sec^2 x$, the antiderivative of $\sec^2 x$ is exactly $\tan x$.

Solution:

Step 1: Write down the definite integral defined over the limits from 0 to $\frac{\pi}{6}$:

$$I = \int_0^{\pi/6} \sec^2 x \, dx$$

Step 2: Substitute the standard trigonometric antiderivative function into the boundary evaluation brackets:

$$I = [\tan x]_0^{\pi/6}$$

Step 3: Apply the limits of integration by calculating the difference between the antiderivative values at the upper limit and the lower limit:

$$I = \tan\left(\frac{\pi}{6}\right) - \tan(0)$$

Step 4: Substitute the exact trigonometric values for these specific angles into the equation:

$$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}, \quad \tan(0) = 0$$

Step 5: Subtract the values to arrive at the final evaluated definite scalar result:

$$I = \frac{1}{\sqrt{3}} - 0 = \frac{1}{\sqrt{3}}$$

Final Answer: $\boxed{\frac{1}{\sqrt{3}}}$

Go Back to Question 37

Q38.

Solution

Concept: In three-dimensional Cartesian coordinate space, the midpoint of a line segment connecting two distinct points represents the exact geometric center point of that segment. The coordinates of this midpoint are calculated using the mid-point formula, which takes the arithmetic average of the corresponding individual coordinate values from the two endpoints: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

Solution:

Step 1: Identify and extract the individual coordinate components from the two given endpoints A and B:

$$A(x_1, y_1, z_1) = A(2, -1, 3)$$

$$B(x_2, y_2, z_2) = B(4, 3, -1)$$

Step 2: Substitute these coordinate values into the three-dimensional midpoint formula components:

$$\text{Midpoint} = \left(\frac{2+4}{2}, \frac{-1+3}{2}, \frac{3+(-1)}{2}\right)$$

Step 3: Simplify the numerator additions within each of the three fractional parts:

$$\text{Midpoint} = \left(\frac{6}{2}, \frac{2}{2}, \frac{2}{2}\right)$$

Step 4: Perform the final division steps to obtain the coordinate triplet for the midpoint:

$$\text{Midpoint} = (3, 1, 1)$$

This point sits exactly halfway along the spatial segment connecting A and B.

Final Answer: $\boxed{(3, 1, 1)}$

Go Back to Question 38

Q39.

Solution

Concept: A system of independent linear equations can be written in the standard matrix form $AX = B$, where A represents the coefficient matrix, X is the column vector containing the unknown variables, and B is the column vector of constants. If the matrix A is non-singular, meaning its determinant is non-zero, its unique inverse matrix A^{-1} exists, and the system can be solved using the inverse matrix method: $X = A^{-1}B$.

Solution:

Step 1: Express the given pair of simultaneous linear equations as a single matrix equation:

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 12 \end{pmatrix}$$

Step 2: Define the coefficient matrix A and evaluate its determinant to check for invertibility:

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \implies |A| = (2)(2) - (1)(3) = 4 - 3 = 1$$

Since $|A| = 1 \neq 0$, the inverse matrix exists.

Step 3: Construct the inverse matrix A^{-1} using the formula $A^{-1} = \frac{1}{|A|}\text{adj}(A)$:

$$A^{-1} = \frac{1}{1} \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$$

Step 4: Set up the multiplication step to isolate the variable vector X:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} 7 \\ 12 \end{pmatrix}$$

Step 5: Multiply rows by columns to compute the numeric values for x and y:

$$x = (2)(7) + (-1)(12) = 14 - 12 = 2$$

$$y = (-3)(7) + (2)(12) = -21 + 24 = 3$$

Step 6: Conclude that the unique solutions for the system are $x = 2$ and $y = 3$.

Final Answer: $\boxed{x = 2,\ y = 3}$

Go Back to Question 39

Q40.

Solution

Concept: Simplifying expressions that contain multiple inverse tangent terms requires the application of trigonometric addition identities. The standard arctangent sum formula states that $\tan^{-1} u + \tan^{-1} v = \tan^{-1}\left(\frac{u+v}{1-uv}\right)$, which is valid as long as the product of the inputs satisfies $uv < 1$. Additionally, a doubled inverse tangent term can be rewritten using the identity $2\tan^{-1} t = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$, provided the argument remains within the proper boundaries.

Solution:

Step 1: Simplify the leading doubled term by applying the arctangent duplication identity with $t = \frac{1}{3}$:

$$2\tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{2(1/3)}{1-(1/3)^2}\right) = \tan^{-1}\left(\frac{2/3}{1-1/9}\right)$$

Step 2: Evaluate the fractional expression inside the argument:

$$\frac{2/3}{8/9} = \frac{2}{3}\cdot\frac{9}{8} = \frac{3}{4} \implies 2\tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

Step 3: Substitute this result back into the full expression and add the remaining term $\tan^{-1}\left(\frac{1}{7}\right)$:

$$\text{Sum} = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

Step 4: Apply the standard arctangent addition formula, noting that the product $uv = \frac{3}{28} < 1$:

$$\text{Sum} = \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}}\right) = \tan^{-1}\left(\frac{\frac{21+4}{28}}{1-\frac{3}{28}}\right)$$

Step 5: Simplify the complex fraction to find the final single argument value:

$$\text{Sum} = \tan^{-1}\left(\frac{25/28}{25/28}\right) = \tan^{-1}(1)$$

Step 6: Determine the angle, noting that $\tan\left(\frac{\pi}{4}\right) = 1$:

$$\text{Sum} = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

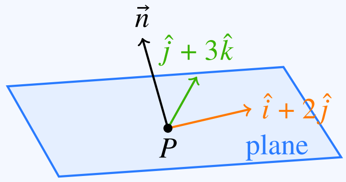
Go Back to Question 40

Q41.

Solution

Concept: A plane parallel to two non-parallel vectors has a normal vector perpendicular to both. The cross product of the two parallel direction vectors gives such a normal vector.

TikZ Diagram Explanation: The diagram shows a plane through P containing two direction vectors. Their cross product gives the normal direction used in the plane equation.



Solution:

Step 1: The two vectors parallel to the plane are

$$\vec{a} = (1, 2, 0), \qquad \vec{b} = (0, 1, 3).$$

Step 2: A normal vector is

$$\vec{n} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 0 & 1 & 3 \end{vmatrix}.$$

Step 3: Expand the determinant:

$$\vec{n} = \hat{i}(6-0) - \hat{j}(3-0) + \hat{k}(1-0) = 6\hat{i} - 3\hat{j} + \hat{k}.$$

Step 4: Thus normal vector is $(6, -3, 1)$.

Step 5: The plane passes through $P(2, -1, 1)$, so

$$6(x-2) - 3(y+1) + (z-1) = 0.$$

Step 6: Simplify:

$$6x - 12 - 3y - 3 + z - 1 = 0,$$

$$6x - 3y + z - 16 = 0.$$

Final Answer: $\boxed{6x - 3y + z - 16 = 0}$

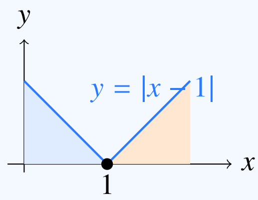
Go Back to Question 41

Q42.

Solution

Concept: The absolute value expression $|x-1|$ changes definition at $x=1$. Therefore the integral must be split at $x=1$.

TikZ Diagram Explanation: The graph is a V-shaped graph with vertex at $(1,0)$. The required integral is the total area of two right triangles from 0 to 1 and from 1 to 2.



Solution:

Step 1: On $0 \le x \le 1$, $x-1 \le 0$, so $|x-1| = 1-x$.

Step 2: On $1 \le x \le 2$, $x-1 \ge 0$, so $|x-1| = x-1$.

Step 3: Split the integral:

$$\int_0^2 |x-1|\,dx = \int_0^1 (1-x)\,dx + \int_1^2 (x-1)\,dx.$$

Step 4: Evaluate the first part:

$$\int_0^1 (1-x)\,dx = \left[x - \frac{x^2}{2}\right]_0^1 = \frac{1}{2}.$$

Step 5: Evaluate the second part:

$$\int_1^2 (x-1)\,dx = \left[\frac{x^2}{2} - x\right]_1^2 = 0 - \left(\frac{1}{2} - 1\right) = \frac{1}{2}.$$

Step 6: Total value is $\frac{1}{2} + \frac{1}{2} = 1$.

Final Answer: $\boxed{1}$

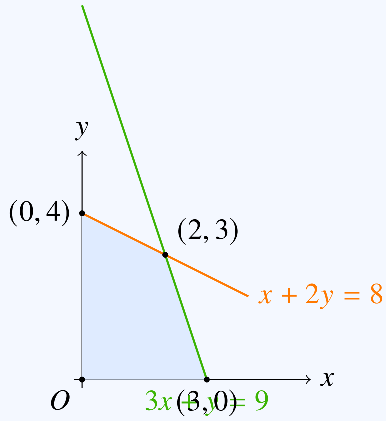
Go Back to Question 42

Q43.

Solution

Concept: A linear programming problem is solved graphically by drawing the boundary lines, finding the feasible region, identifying corner points, and evaluating the objective function at those points.

TikZ Diagram Explanation: The shaded polygon is the feasible region in the first quadrant. The maximum of a linear objective function occurs at one of its vertices.



Solution:

Step 1: Boundary lines are

$$x + 2y = 8, \qquad 3x + y = 9.$$

Step 2: Their intercepts help draw the lines: $x + 2y = 8$ has intercepts $(8, 0)$ and $(0, 4)$; $3x + y = 9$ has intercepts $(3, 0)$ and $(0, 9)$.

Step 3: Find intersection:

$$x + 2y = 8, \quad 3x + y = 9.$$

From the second equation $y = 9 - 3x$. Substitute:

$$x + 2(9 - 3x) = 8 \Rightarrow -5x = -10 \Rightarrow x = 2.$$

Then $y = 3$.

Step 4: Corner points are

$$(0, 0), \ (3, 0), \ (2, 3), \ (0, 4).$$

Step 5: Evaluate $Z = 3x + 2y$:

$$Z(0, 0) = 0, \quad Z(3, 0) = 9, \quad Z(2, 3) = 12, \quad Z(0, 4) = 8.$$

Step 6: Maximum value is 12 at $(2, 3)$.

Final Answer: $\boxed{Z_{max} = 12 \text{ at } (2, 3)}$

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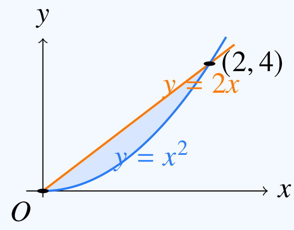
Q44.

Solution

Concept: The area between two curves is found by solving their intersection points and integrating the difference between the upper and lower curves. The upper curve must be identified on the required interval.

TikZ Diagram Explanation: The line $y = 2x$ lies above the parabola $y = x^2$ between $x = 0$ and $x = 2$. The shaded region represents the required area.



Solution:

Step 1: Find points of intersection:

$$x^2 = 2x \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0.$$

Thus $x = 0$ and $x = 2$.

Step 2: On $(0, 2)$, take $x = 1$. Then $2x = 2$ and $x^2 = 1$, so the line $y = 2x$ is above the parabola.

Step 3: Required area is

$$A = \int_0^2 (2x - x^2)\,dx.$$

Step 4: Integrate:

$$A = \left[x^2 - \frac{x^3}{3}\right]_0^2.$$

Step 5: Substitute limits:

$$A = \left(4 - \frac{8}{3}\right) - 0 = \frac{12-8}{3} = \frac{4}{3}.$$

Step 6: The area is positive because upper curve minus lower curve was used.

Final Answer: $\boxed{\frac{4}{3} \text{ square units}}$

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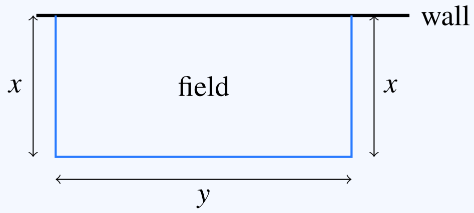
Q45.

Solution

Concept:
The wall forms one side of the rectangle, so fencing is needed only for two widths and one length. If each width is x and the length is y, then $2x + y = 120$.



Solution:

Step 1: Let the two equal sides perpendicular to the wall be x metres each, and the side parallel to the wall be y metres.

Step 2: Fencing is needed for three sides only, so

$$2x + y = 120.$$

Step 3: Express y in terms of x:

$$y = 120 - 2x.$$

Step 4: Area of the rectangle is

$$A = xy = x(120 - 2x) = 120x - 2x^2.$$

Step 5: Differentiate:

$$\frac{dA}{dx} = 120 - 4x.$$

Step 6: Put derivative equal to zero:

$$120 - 4x = 0 \Rightarrow x = 30.$$

Step 7: Check maximum:

$$\frac{d^2A}{dx^2} = -4 < 0,$$

so the area is maximum.

Step 8: Maximum area is

$$A_{max} = 30 \times 60 = 1800 \text{ m}^2.$$

Final Answer: $\boxed{x = 30 \text{ m}, \ y = 60 \text{ m}, \ A_{max} = 1800 \text{ m}^2}$

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	A	3	A	4	B	5	C
6	B	7	C	8	B	9	A	10	B
11	A	12	B	13	C	14	A	15	B
16	C	17	B	18	A	19	A	20	A