NIOS Class 12 Mathematics Sample Paper – 2

Duration: 180 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **45** Questions. The paper is divided into two sections: **Section A – 50** marks, **Section B – 50** marks.
- **Section A** consists of
  -**Q.No. 1 to 20** – Multiple Choice type questions (MCQs) carrying **+1 mark** each. Select and write the most appropriate option out of the four options given in each of these questions.
  -**Q.No. 21 to 29** – **Objective type questions**.
  -**Q.No. 21 to 24** carry **02 marks** each (with 2 sub-parts of 1 mark each).
  -**Q.No. 25 to 28** carry **04 marks** each (with 4 sub-parts of 1 mark each).
  -**Q.No. 29** carries **06 marks** (with 6 sub-parts of 1 mark each). Attempt these questions as per the instructions given for each of the questions 21–29.
- **Section B** consists of
  -**Q.No. 30 to 38**– Very Short questions carrying **02 marks** each.
  -**Q.No. 39 to 43** – Short Answer type questions carrying **04 marks** each.
  -**Q.No. 44 to 45** – Long Answer type questions carrying **06 marks** each.
  (An internal choice has been provided in some of the questions in Section B. You have to attempt only one of the given choices in such questions.)
- There is **No Negative marking**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Section: A

Q1. $\lim_{x\to 0} \dfrac{e^{3x}-1}{\sin 2x}$ equals: **(1)**

(A) 1

(B) 3

(C) $\frac{3}{2}$

(D) $\frac{2}{3}$

Q2. If A is a 2×2 matrix with $|A| = 5$, then $|3A|$ equals: **(1)**

(A) 15

(B) 45

(C) 5

(D) 30

Q3. The equation of the line through $(1, 2)$ parallel to $2x + 3y = 6$ is: **(1)**

(A) $2x + 3y - 8 = 0$

(B) $3x - 2y + 1 = 0$

(C) $2x + 3y + 8 = 0$

(D) $3x + 2y - 7 = 0$

Q4. $\frac{d}{dx}\left[\sin\left(x^2\right)\right]$ equals: **(1)**

(A) $\cos\left(x^2\right)$

(B) $2x\sin\left(x^2\right)$

(C) $-2x\cos\left(x^2\right)$

(D) $2x\cos\left(x^2\right)$

Q5. The direction cosines of a line whose direction ratios are $6, -2, 3$ are: **(1)**

(A) $\langle 6, -2, 3\rangle$

(B) $\left\langle \frac{6}{7}, -\frac{2}{7}, \frac{3}{7}\right\rangle$

(C) $\left\langle \frac{6}{11}, -\frac{2}{11}, \frac{3}{11}\right\rangle$

(D) $\left\langle \frac{2}{7}, -\frac{6}{7}, \frac{3}{7}\right\rangle$

Q6. The principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ is: **(1)**

(A) $\frac{2\pi}{3}$

(B) $-\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $-\frac{\pi}{3}$

Q7. The radius of the circle $x^2 + y^2 - 6x + 8y = 0$ is: **(1)**

(A) 5

(B) 25

(C) $\sqrt{7}$

(D) 10

Q8. $\int_0^{\pi/4} \sec^2 x \, dx$ equals: **(1)**

(A) 0

(B) $\frac{\pi}{4}$

(C) 1

(D) $\sqrt{2}$

Q9. If the matrix $\begin{pmatrix} 2 & 4 \\ 3 & k \end{pmatrix}$ is singular, then k equals: **(1)**

(A) 3

(B) 6

(C) -6

(D) 12

Q10. The angle between the vectors $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is: **(1)**

(A) 60°

(B) 30°

(C) 90°

(D) 45°

Q11. The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y = 0$ is: **(1)**

(A) 2

(B) 3

(C) 4

(D) 1

Q12. The negation of the compound statement $p \wedge q$ is: **(1)**

(A) $\neg p \wedge \neg q$

(B) $p \vee q$

(C) $\neg p \vee \neg q$

(D) $\neg(p \vee q)$

Q13. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 2x + 3$, then $f^{-1}(x)$ equals: **(1)**

(A) $\frac{x+3}{2}$

(B) $2x - 3$

(C) $\frac{1}{2x+3}$

(D) $\frac{x-3}{2}$

Q14. If A is a skew-symmetric matrix of order 3, then $|A|$ equals: **(1)**

(A) 1

(B) 0

(C) -1

(D) 3

Q15. The eccentricity of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ is: **(1)**

(A) $\frac{4}{5}$

(B) $\frac{5}{3}$

(C) $\frac{3}{5}$

(D) $\frac{9}{25}$

Q16. The maximum value of $f(x) = -x^2 + 6x - 5$ is: **(1)**

(A) 3

(B) -4

(C) 5

(D) 4

Q17. In a Linear Programming Problem, the objective function is always: **(1)**

(A) Linear

(B) Quadratic

(C) Cubic

(D) Constant

Q18. The integrating factor of the differential equation $\frac{dy}{dx} + y\tan x = \sec x$ is: **(1)**

(A) $\cos x$

(B) $\sec x$

(C) $\tan x$

(D) $e^{\tan x}$

Q19. The directrix of the parabola $y^2 = 12x$ is: **(1)**

(A) $x = 3$

(B) $y = -3$

(C) $y = 3$

(D) $x = -3$

Q20. The projection of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is: **(1)**

(A) $\frac{\sqrt{6}}{3}$

(B) $\frac{10}{3}$

(C) $\frac{5\sqrt{6}}{3}$

(D) $\sqrt{6}$

Q21. Match Column-I with Column-II: **(2)**

Column-I	**Column-II**
(i) If A is a 2×2 matrix with $\lvert A\rvert = 3$, then $\lvert \operatorname{adj} A\rvert$ equals	(A) **3**
(ii) $\lvert 5I_2\rvert$, where I_2 is the 2×2 identity matrix, equals	(B) **25**

(A) (i)→(A), (ii)→(B)

(B) (i)→(B), (ii)→(A)

Q22. Fill in the blanks: **(2)**

(i) The magnitude of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ is ________.

(ii) The distance of the plane $3x - 4y + 12z = 26$ from the origin is ________.

Q23. Write TRUE or FALSE: **(2)**

(i) $\sin^{-1}\left(\sin \frac{2\pi}{3}\right) = \frac{2\pi}{3}$.

(ii) The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^3$ is one-one.

Q24. Answer as directed: **(2)**

(i) Write the converse of: "If a triangle is equilateral, then it is isosceles."

(ii) Write the negation of: "All birds can fly."

Q25. Fill in the blanks (Differentiation): **(4)**

(i) $\frac{d}{dx}[\log(\cos x)] =$ ________.

(ii) $\frac{d}{dx}(x\,e^x) =$ ________.

(iii) The value of $\frac{d}{dx}\left(\tan^{-1} x\right)$ at $x = 1$ is ________.

(iv) If $y = 5^x$, then $\frac{dy}{dx} =$ ________.

Q26. Fill in the blanks (Coordinate Geometry): **(4)**

(i) The distance between the points $(2, 3)$ and $(5, 7)$ is ________.

(ii) The x-intercept of the line $2x - 3y = 6$ is ________.

(iii) The radius of the circle $x^2 + y^2 = 49$ is ________.

(iv) The eccentricity of a parabola is ________.

Q27. Write TRUE or FALSE: **(4)**

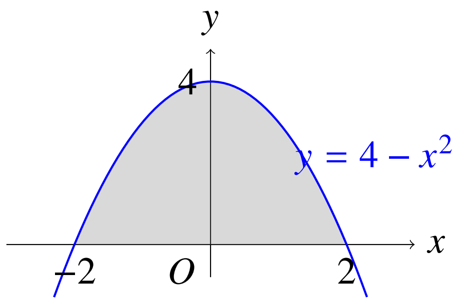
(i) $(A + B)' = A' + B'$ for matrices A, B of the same order.

(ii) Matrix multiplication is always commutative.

(iii) If $|A| \neq 0$, then the matrix A is invertible.

(iv) $\text{adj}(AB) = (\text{adj } A)(\text{adj } B)$ for all square matrices A, B of the same order.

Q28. Study the figure showing the curve $y = 4 - x^2$ and the x-axis: **(4)**



(i) Find the points where the curve meets the x-axis.

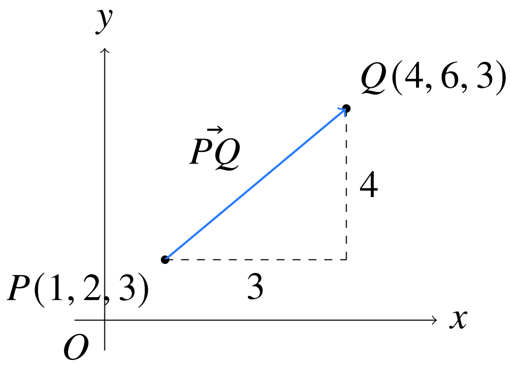
(ii) Write the definite integral for the shaded area.

(iii) Rewrite the integral using the symmetry of the region.

(iv) Compute the shaded area.

Q29. Read and answer (i)–(vi):

A survey drone takes off and flies in a straight line from point $P(1, 2, 3)$ to point $Q(4, 6, 3)$, where coordinates are in metres with respect to the base station O. **(6)**



(i) The vector $\vec{PQ}$ is:

(A) $3\hat{i} + 4\hat{j}$

(B) $5\hat{i} + 8\hat{j} + 6\hat{k}$

(C) $-3\hat{i} - 4\hat{j}$

(D) $3\hat{i} + 4\hat{j} + 6\hat{k}$

(ii) The distance $|\vec{PQ}|$ (in metres) is:

(A) 7

(B) $\sqrt{7}$

(C) 5

(D) 25

(iii) The unit vector along $\vec{PQ}$ is:

(A) $\frac{3\hat{i}+4\hat{j}+3\hat{k}}{5}$

(B) $\frac{3\hat{i}+4\hat{j}}{5}$

(C) $\frac{4\hat{i}+3\hat{j}}{5}$

(D) $3\hat{i} + 4\hat{j}$

(iv) The position vector of the midpoint of PQ is:

(A) $5\hat{i} + 8\hat{j} + 6\hat{k}$

(B) $3\hat{i} + 4\hat{j} + 3\hat{k}$

(C) $\frac{5}{2}\hat{i} + 4\hat{j}$

(D) $\frac{5}{2}\hat{i} + 4\hat{j} + 3\hat{k}$

(v) Since $\vec{PQ} \cdot \hat{k} = 0$, the flight path is:

(A) Horizontal (perpendicular to $\hat{k}$)

(B) Vertical

(C) Parallel to $\hat{k}$

(D) Along the y-axis

(vi) If the drone continues from Q along $2\vec{PQ}$, it reaches the point:

(A) $(7, 10, 3)$

(B) $(10, 14, 3)$

(C) $(6, 8, 0)$

(D) $(10, 14, 6)$

Section: B

Q30. Find k if $f(x) = \begin{cases} \dfrac{\sin 5x}{3x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$. **(2)**

Q31. Find the inverse of the matrix $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$. **(2)**

Q32. If $f(x) = x^2$ and $g(x) = 2x + 1$, find $(f \circ g)(x)$ and $(g \circ f)(x)$. **(2)**

Q33. Evaluate $\displaystyle\int \frac{(\log x)^2}{x}\, dx$. **(2)**

Q34. Find the equation of the circle with centre $(2, -3)$ and radius 4. **OR** Find the centre and radius of the circle $x^2 + y^2 + 4x - 6y - 3 = 0$. **(2)**

Q35. Find λ if the vectors $2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular. **(2)**

Q36. Evaluate $\displaystyle\int_0^{\pi/2} \frac{\cos x}{1 + \sin^2 x}\, dx$. **(2)**

Q37. If $x^2 + y^2 = 25$, find $\dfrac{dy}{dx}$ at the point $(3, 4)$. **(2)**

Q38. Evaluate $\cos^{-1}\left(\cos \dfrac{7\pi}{6}\right)$. **(2)**

Q39. Prove that $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$. **(4)**

Q40. If $y = x^{\sin x}$, $x > 0$, find $\frac{dy}{dx}$. **(4)**

Q41. Find the foot of the perpendicular drawn from the point $P(1, 2, 3)$ to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also find the perpendicular distance. **(4)**

Q42. Evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx$. **OR** Evaluate $\int \frac{dx}{x^2 - 6x + 13}$. **(4)**

Q43. Solve the LPP graphically: Minimize $Z = 3x + 5y$ subject to $x + 3y \geq 3, x + y \geq 2$, $x, y \geq 0$. **(4)**

Q44. Find the equation of the ellipse whose foci are $(\pm 4, 0)$ and vertices are $(\pm 5, 0)$. Also find its eccentricity, length of the latus rectum and length of the minor axis.

OR

Find the equation of the hyperbola whose vertices are $(0, \pm 3)$ and foci are $(0, \pm 5)$. Also find its eccentricity, length of the latus rectum and length of the conjugate axis. **(6)**

Q45. If $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$, find A^{-1} and hence solve the system: $x + 2y + z = 8$, $2x + y + 3z = 13$, $x + y + z = 6$.

OR

If $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$. **(6)**

Detailed Solutions

Q1.

Solution

Concept: Two standard limits govern this evaluation: the exponential limit $\lim_{x\to 0} \frac{e^{mx}-1}{mx} = 1$ and the trigonometric limit $\lim_{x\to 0} \frac{\sin nx}{nx} = 1$. Dividing numerator and denominator by x converts the ratio into a quotient of these two standard forms.

Solution:

(a) Divide the numerator and denominator by x:

$$L = \lim_{x\to 0} \frac{\frac{e^{3x}-1}{x}}{\frac{\sin 2x}{x}}$$

(b) Rescale each standard form by its own coefficient:

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} = 3 \cdot \lim_{x\to 0} \frac{e^{3x}-1}{3x} = 3 \cdot 1 = 3$$

(c) Similarly for the denominator:

$$\lim_{x\to 0} \frac{\sin 2x}{x} = 2 \cdot \lim_{x\to 0} \frac{\sin 2x}{2x} = 2 \cdot 1 = 2$$

(d) Take the quotient of the two evaluated limits:

$$L = \frac{3}{2}$$

Why other options are wrong:

- **Option A:** 1 ignores the coefficients 3 and 2 of x in the two functions.
- **Option B:** 3 uses only the numerator coefficient and forgets $\sin 2x \sim 2x$.
- **Option D:** $\frac{2}{3}$ inverts the ratio (denominator coefficient over numerator coefficient).

Final Answer: $\frac{3}{2}$

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept: For a square matrix of order n and a scalar k, the determinant scaling law states $|kA| = k^n|A|$, because the scalar multiplies every one of the n rows of the matrix.

Solution:

(a) Record the given data: A is of order $n = 2$ and $|A| = 5$.

(b) Apply the scaling law with $k = 3$:

$$|3A| = 3^2|A|$$

(c) Substitute the known determinant value:

$$|3A| = 9 \times 5 = 45$$

Why other options are wrong:

- **Option A:** 15 uses $k|A|$ instead of $k^n|A|$ — the scalar must be squared for order 2.
- **Option C:** 5 assumes scalar multiplication leaves the determinant unchanged.
- **Option D:** 30 multiplies by $2k$ rather than k^2.

Final Answer: 45

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: Parallel lines share the same slope. A line parallel to $ax + by + c = 0$ can be written in the family form $ax + by + k = 0$, where the constant k is fixed by forcing the line to pass through the given point.

Solution:

(a) Write the required line in the parallel family form:

$$2x + 3y + k = 0$$

(b) Substitute the given point $(1, 2)$ into this equation:

$$2(1) + 3(2) + k = 0 \implies 2 + 6 + k = 0$$

(c) Solve for the constant:

$$k = -8$$

(d) State the final equation of the required line:

$$2x + 3y - 8 = 0$$

Why other options are wrong:

- **Option B:** $3x - 2y + 1 = 0$ has slope $\frac{3}{2}$ — this is a perpendicular direction, not parallel.
- **Option C:** $2x + 3y + 8 = 0$ uses the wrong sign of k; the point $(1, 2)$ does not satisfy it.
- **Option D:** $3x + 2y - 7 = 0$ swaps the coefficients, changing the slope to $-\frac{3}{2}$.

Final Answer: $2x + 3y - 8 = 0$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept: The chain rule states that for a composite function $y = f(g(x))$, the derivative is $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$ — differentiate the outer function first, keep the inner function intact, then multiply by the derivative of the inner function.

Solution:

(a) Identify the layers of the composition: outer function $\sin(u)$ and inner function $u = x^2$.

(b) Differentiate the outer layer while preserving the inner argument:

$$\frac{d}{du}(\sin u) = \cos u = \cos\left(x^2\right)$$

(c) Differentiate the inner layer:

$$\frac{du}{dx} = \frac{d}{dx}\left(x^2\right) = 2x$$

(d) Multiply the two results by the chain rule:

$$\frac{d}{dx}\left[\sin\left(x^2\right)\right] = 2x\cos\left(x^2\right)$$

Why other options are wrong:

- **Option A:** $\cos(x^2)$ forgets to multiply by the inner derivative $2x$.
- **Option B:** $2x\sin(x^2)$ fails to differentiate the outer sine into cosine.
- **Option C:** $-2x\cos(x^2)$ introduces a spurious negative sign (that belongs to $\frac{d}{dx}\cos$, not sin).

Final Answer: $2x\cos\left(x^2\right)$

Answer: (D)

Go Back to Question 4

Q5.

Solution

Concept: Direction cosines are obtained from direction ratios $\langle a, b, c\rangle$ by dividing each ratio by the magnitude $\sqrt{a^2+b^2+c^2}$. The resulting triplet $\langle l, m, n\rangle$ always satisfies $l^2+m^2+n^2=1$.

Solution:

(a) Record the direction ratios: $a = 6$, $b = -2$, $c = 3$.

(b) Compute the magnitude of the ratio vector:

$$\sqrt{6^2+(-2)^2+3^2} = \sqrt{36+4+9} = \sqrt{49} = 7$$

(c) Divide each ratio by 7:

$$l = \frac{6}{7}, \quad m = -\frac{2}{7}, \quad n = \frac{3}{7}$$

(d) Verify the normalisation check: $\frac{36+4+9}{49} = 1$. ✓

Why other options are wrong:

- **Option A:** lists the raw direction ratios without normalising by the magnitude.
- **Option C:** divides by 11 (the sum $6+2+3$) instead of $\sqrt{49} = 7$.
- **Option D:** swaps the first two components, so the cosines correspond to a different line.

Final Answer: $\left\langle \frac{6}{7}, -\frac{2}{7}, \frac{3}{7} \right\rangle$

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept: The principal value branch of the inverse tangent function is the open interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. For a negative argument, the identity $\tan^{-1}(-x) = -\tan^{-1}(x)$ applies because $\tan^{-1}$ is an odd function.

Solution:

(a) Apply the odd-function identity to extract the negative sign:

$$\tan^{-1}\left(-\sqrt{3}\right) = -\tan^{-1}\left(\sqrt{3}\right)$$

(b) Recall the standard angle whose tangent equals $\sqrt{3}$:

$$\tan\frac{\pi}{3} = \sqrt{3} \implies \tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$$

(c) Combine the results:

$$\tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$$

(d) Confirm this angle lies inside the principal branch $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. ✓

Why other options are wrong:

- **Option A:** $\frac{2\pi}{3}$ has $\tan\frac{2\pi}{3} = -\sqrt{3}$, but it lies outside the principal branch of $\tan^{-1}$.
- **Option B:** $-\frac{\pi}{6}$ corresponds to argument $-\frac{1}{\sqrt{3}}$, not $-\sqrt{3}$.
- **Option C:** $\frac{\pi}{3}$ drops the negative sign of the argument.

Final Answer: $-\dfrac{\pi}{3}$

Answer: (D)

Go Back to Question 6

Q7.

Solution

Concept: For the general circle $x^2 + y^2 + 2gx + 2fy + c = 0$, the centre is $(-g, -f)$ and the radius is $r = \sqrt{g^2 + f^2 - c}$.

Solution:

(a) Compare $x^2 + y^2 - 6x + 8y = 0$ with the general form:

$$2g = -6 \implies g = -3, \qquad 2f = 8 \implies f = 4, \qquad c = 0$$

(b) Substitute into the radius formula:

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{(-3)^2 + 4^2 - 0}$$

(c) Evaluate the radical:

$$r = \sqrt{9 + 16} = \sqrt{25} = 5$$

Why other options are wrong:

- **Option B:** 25 is r^2; the square root was not taken.
- **Option C:** $\sqrt{7}$ comes from wrongly computing $g^2 + f^2$ as $16 - 9$.
- **Option D:** 10 is the diameter, not the radius.

Final Answer: 5

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: The antiderivative of $\sec^2 x$ is $\tan x$, since $\frac{d}{dx}(\tan x) = \sec^2 x$. The definite integral is then evaluated with the fundamental theorem of calculus.

Solution:

(a) Write the antiderivative and attach the limits:

$$\int_0^{\pi/4} \sec^2 x \, dx = [\tan x]_0^{\pi/4}$$

(b) Substitute the upper and lower limits:

$$= \tan \frac{\pi}{4} - \tan 0$$

(c) Evaluate the standard values:

$$= 1 - 0 = 1$$

Why other options are wrong:

- **Option A:** 0 would require identical values at both limits, which is false here.
- **Option B:** $\frac{\pi}{4}$ confuses the integral of $\sec^2 x$ with the length of the interval.
- **Option D:** $\sqrt{2}$ is $\sec \frac{\pi}{4}$, i.e. the wrong antiderivative was used.

Final Answer: 1

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept: A square matrix is singular precisely when its determinant equals zero. For a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant is $ad - bc$.

Solution:

(a) Impose the singularity condition on the given matrix:

$$\begin{vmatrix} 2 & 4 \\ 3 & k \end{vmatrix} = 0$$

(b) Expand the determinant:

$$2k - (4)(3) = 0 \implies 2k - 12 = 0$$

(c) Solve the linear equation:

$$2k = 12 \implies k = 6$$

Why other options are wrong:

- **Option A:** 3 gives $|A| = 6 - 12 = -6 \neq 0$; the matrix stays invertible.
- **Option C:** -6 arises from the sign error $2k + 12 = 0$.
- **Option D:** 12 solves $k - 12 = 0$, forgetting the factor 2 from the diagonal product.

Final Answer: 6

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: The angle between two vectors is obtained from the dot product identity $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$.

Solution:

(a) Write the component forms: $\vec{a} = \hat{i} + \hat{j} = (1, 1, 0)$ and $\vec{b} = \hat{j} + \hat{k} = (0, 1, 1)$.

(b) Compute the dot product:

$$\vec{a}\cdot\vec{b} = (1)(0) + (1)(1) + (0)(1) = 1$$

(c) Compute the magnitudes:

$$|\vec{a}| = \sqrt{1+1} = \sqrt{2}, \qquad |\vec{b}| = \sqrt{1+1} = \sqrt{2}$$

(d) Substitute into the cosine formula:

$$\cos\theta = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}$$

(e) Identify the principal angle:

$$\theta = \cos^{-1}\frac{1}{2} = 60^\circ$$

Why other options are wrong:

- **Option B:** 30° would need $\cos\theta = \frac{\sqrt{3}}{2}$, not $\frac{1}{2}$.
- **Option C:** 90° requires a zero dot product, but $\vec{a}\cdot\vec{b} = 1 \neq 0$.
- **Option D:** 45° would need $\cos\theta = \frac{1}{\sqrt{2}}$ — a miscount of the magnitudes.

Final Answer: 60°

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: The order of a differential equation is the order of the highest derivative present in the equation. It is unaffected by the powers (exponents) to which the derivatives are raised — powers determine the degree, not the order.

Solution:

(a) List the derivatives appearing in the equation:

$$\left(\frac{d^2y}{dx^2}\right)^3 \quad \text{and} \quad \left(\frac{dy}{dx}\right)^4$$

(b) The highest derivative present is the second derivative $\frac{d^2y}{dx^2}$.

(c) Hence the order equals 2, regardless of the cube on the second derivative.

(d) (Separately, the degree would be 3 — the power of the highest-order derivative.)

Why other options are wrong:

- **Option B:** 3 is the degree (the power of $\frac{d^2y}{dx^2}$), not the order.
- **Option C:** 4 is merely the exponent of the first derivative.
- **Option D:** 1 ignores the second derivative term entirely.

Final Answer: 2

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: De Morgan's laws in mathematical logic state that the negation of a conjunction is the disjunction of the negations: $\neg(p \wedge q) \equiv \neg p \vee \neg q$; similarly $\neg(p \vee q) \equiv \neg p \wedge \neg q$.

Solution:

(a) Start with the compound statement $p \wedge q$ ("both p and q are true").

(b) Its negation asserts that they are not both true — i.e. at least one of them fails.

(c) "At least one fails" translates symbolically to $\neg p \vee \neg q$.

(d) This is exactly De Morgan's first law:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Why other options are wrong:

- **Option A:** $\neg p \wedge \neg q$ says *both* fail — too strong; it is the negation of $p \vee q$.
- **Option B:** $p \vee q$ is not a negation at all; it can be true simultaneously with $p \wedge q$.
- **Option D:** $\neg(p \vee q)$ equals option A by De Morgan, hence wrong for the same reason.

Final Answer: $\neg p \vee \neg q$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: For a one-one and onto function, the inverse is found by writing $y = f(x)$, solving the equation for x in terms of y, and then interchanging the roles of the variables.

Solution:

(a) Set y equal to the function output:

$$y = 2x + 3$$

(b) Solve this equation for x:

$$y - 3 = 2x \implies x = \frac{y-3}{2}$$

(c) Express the inverse function by renaming the variable:

$$f^{-1}(x) = \frac{x-3}{2}$$

(d) Verify: $f\left(\frac{x-3}{2}\right) = 2 \cdot \frac{x-3}{2} + 3 = x$. ✓

Why other options are wrong:

- **Option A:** $\frac{x+3}{2}$ adds 3 instead of subtracting; composition gives $x + 6 \neq x$.
- **Option B:** $2x - 3$ reverses the operations in the wrong order.
- **Option C:** $\frac{1}{2x+3}$ confuses the inverse function with the reciprocal.

Final Answer: $\dfrac{x-3}{2}$

Answer: (D)

Go Back to Question 13

Q14.

Solution

Concept: A skew-symmetric matrix satisfies $A^T = -A$. Taking determinants of both sides and using $|A^T| = |A|$ together with $|-A| = (-1)^n|A|$ for order n yields a constraint on $|A|$ when n is odd.

Solution:

(a) Start from the defining property and take determinants:

$$|A^T| = |-A|$$

(b) Use the transpose rule on the left and the scalar rule on the right with $n = 3$:

$$|A| = (-1)^3|A| = -|A|$$

(c) Move all terms to one side:

$$2|A| = 0$$

(d) Conclude:

$$|A| = 0$$

Every skew-symmetric matrix of odd order is therefore singular.

Why other options are wrong:

- **Option A:** 1 holds for special orthogonal matrices, not skew-symmetric ones of odd order.
- **Option C:** -1 cannot occur since the derivation forces the determinant to vanish.
- **Option D:** 3 confuses the order of the matrix with its determinant.

Final Answer: 0

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept: For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a^2 > b^2$, the eccentricity is $e = \sqrt{1 - \frac{b^2}{a^2}}$, a number strictly between 0 and 1.

Solution:

(a) Read off the parameters from $\frac{x^2}{25} + \frac{y^2}{16} = 1$:

$$a^2 = 25, \qquad b^2 = 16$$

(b) Confirm the major axis is horizontal since $25 > 16$.

(c) Substitute into the eccentricity formula:

$$e = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25 - 16}{25}} = \sqrt{\frac{9}{25}}$$

(d) Simplify the radical:

$$e = \frac{3}{5}$$

Why other options are wrong:

- **Option A:** $\frac{4}{5}$ is $\frac{b}{a}$, not the eccentricity.
- **Option B:** $\frac{5}{3}$ exceeds 1, impossible for an ellipse (that is hyperbola territory).
- **Option D:** $\frac{9}{25}$ is e^2; the square root was omitted.

Final Answer: $\frac{3}{5}$

Answer: (C)

Go Back to Question 15

Q16.

Solution

Concept: For a differentiable function, extrema occur where $f'(x) = 0$. The second derivative test classifies the point: $f'' < 0$ confirms a local maximum.

Solution:

(a) Differentiate the function $f(x) = -x^2 + 6x - 5$:

$$f'(x) = -2x + 6$$

(b) Set the derivative to zero to locate the critical point:

$$-2x + 6 = 0 \implies x = 3$$

(c) Apply the second derivative test:

$$f''(x) = -2 < 0 \implies \text{maximum at } x = 3$$

(d) Evaluate the function at the maximiser:

$$f(3) = -(3)^2 + 6(3) - 5 = -9 + 18 - 5 = 4$$

Why other options are wrong:

- **Option A:** 3 is the location x of the maximum, not the maximum *value*.
- **Option B:** -4 carries a sign error in evaluating $f(3)$.
- **Option C:** 5 ignores the -9 term arising from $-x^2$ at $x = 3$.

Final Answer: 4

Answer: (D)

Go Back to Question 16

Q17.

Solution

Concept: A Linear Programming Problem, by definition, optimises a *linear* objective function $Z = ax + by$ subject to a system of linear inequality constraints. Every defining component — objective and constraints — is of first degree in the decision variables.

Solution:

(a) Recall the standard form of an LPP: optimise $Z = ax + by$ subject to linear constraints and non-negativity conditions.

(b) The decision variables x and y appear only to the first power with constant coefficients.

(c) No product, square, or higher power of the variables is permitted — otherwise the problem is no longer "linear" programming.

(d) Hence the objective function is always linear.

Why other options are wrong:

- **Option B:** a quadratic objective defines quadratic programming, a different class of problem.
- **Option C:** cubic terms violate the linearity that gives LPP its name.
- **Option D:** a constant objective cannot be optimised meaningfully — every feasible point would be optimal.

Final Answer: Linear

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: For the first-order linear differential equation $\frac{dy}{dx} + P(x)\,y = Q(x)$, the integrating factor is IF $= e^{\int P\,dx}$.

Solution:

(a) Compare the given equation with the standard linear form:

$$\frac{dy}{dx} + y\tan x = \sec x \implies P(x) = \tan x$$

(b) Integrate $P(x)$:

$$\int \tan x\,dx = \log|\sec x|$$

(c) Exponentiate to build the integrating factor:

$$\text{IF} = e^{\log|\sec x|} = \sec x$$

Why other options are wrong:

- **Option A:** $\cos x$ is the reciprocal of the correct factor — a sign slip in $\int \tan x\,dx = -\log|\cos x|$ misread as $+\log|\cos x|$.
- **Option C:** $\tan x$ is $P(x)$ itself, not its exponentiated integral.
- **Option D:** $e^{\tan x}$ exponentiates $P(x)$ directly without integrating first.

Final Answer: $\sec x$

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept: For the right-opening parabola $y^2 = 4ax$ with $a > 0$, the focus is at $(a, 0)$ and the directrix is the vertical line $x = -a$, equidistant from the vertex on the opposite side of the focus.

Solution:

(a) Compare the given equation with the standard form:

$$y^2 = 12x \quad \text{vs} \quad y^2 = 4ax$$

(b) Equate the coefficients:

$$4a = 12 \implies a = 3$$

(c) Write the directrix for this orientation:

$$x = -a \implies x = -3$$

Why other options are wrong:

- **Option A:** $x = 3$ is the vertical line through the *focus* $(3, 0)$, not the directrix.
- **Option B:** $y = -3$ is horizontal — that shape of directrix belongs to $x^2 = 4ay$ parabolas.
- **Option C:** $y = 3$ is likewise horizontal and on the wrong side.

Final Answer: $x = -3$

Answer: (D)

Go Back to Question 19

Q20.

Solution

Concept: The scalar projection of $\vec{a}$ on $\vec{b}$ is the component of $\vec{a}$ along the direction of $\vec{b}$, computed as proj $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Solution:

(a) Compute the dot product of the two vectors:

$$\vec{a}\cdot\vec{b} = (2)(1) + (3)(2) + (2)(1) = 2 + 6 + 2 = 10$$

(b) Compute the magnitude of $\vec{b}$:

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$$

(c) Form the projection ratio:

$$\text{proj} = \frac{10}{\sqrt{6}}$$

(d) Rationalise the denominator:

$$\text{proj} = \frac{10\sqrt{6}}{6} = \frac{5\sqrt{6}}{3}$$

Why other options are wrong:

- **Option A:** $\frac{\sqrt{6}}{3}$ inverts the ratio, dividing $|\vec{b}|$ by a partial dot product.
- **Option B:** $\frac{10}{3}$ divides by $|\vec{b}|^2 = 6$ then doubles — i.e. the vector-projection coefficient, not the scalar projection.
- **Option D:** $\sqrt{6}$ is just $|\vec{b}|$, ignoring the dot product.

Final Answer: $\frac{5\sqrt{6}}{3}$

Answer: (C)

Go Back to Question 20

Q21.

Solution

Concept: Part (i) uses the adjugate determinant identity $|\operatorname{adj} A| = |A|^{n-1}$ for a matrix of order n. Part (ii) uses the scalar determinant law $|kI_n| = k^n$.

Solution:

(a) For sub-question (i), apply the adjugate identity with $n = 2$ and $|A| = 3$:

$$|\operatorname{adj} A| = |A|^{2-1} = |A|^1 = 3$$

This matches option (A).

(b) For sub-question (ii), apply the scalar law to the 2×2 identity matrix with $k = 5$:

$$|5I_2| = 5^2|I_2| = 25 \times 1 = 25$$

This matches option (B).

(c) Note the trap in (i): for order 3 the answer would have been $|A|^2 = 9$; the exponent $n - 1$ depends on the order.

Final Answer: (i)→(A), (ii)→(B)

Go Back to Question 21

Q22.

Solution

Concept: Part (i) uses the vector magnitude formula $|\vec{v}| = \sqrt{x^2 + y^2 + z^2}$. Part (ii) uses the point-to-plane distance formula from the origin: for the plane $ax + by + cz = d$, distance $= \frac{|d|}{\sqrt{a^2+b^2+c^2}}$.

Solution:

(a) For sub-question (i), substitute the components $(1, -2, 2)$:

$$|\vec{v}| = \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

(b) For sub-question (ii), identify the plane parameters: $a = 3$, $b = -4$, $c = 12$, $d = 26$.

(c) Compute the normal magnitude:

$$\sqrt{3^2 + (-4)^2 + 12^2} = \sqrt{9 + 16 + 144} = \sqrt{169} = 13$$

(d) Form the distance ratio:

$$\text{Distance} = \frac{|26|}{13} = 2$$

Final Answer: (i) 3, (ii) 2

Go Back to Question 22

Q23.

Solution

Concept: Part (i) tests the principal value branch of $\sin^{-1}$, which is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$: the identity $\sin^{-1}(\sin\theta) = \theta$ holds only when θ lies inside this branch. Part (ii) tests injectivity: a function is one-one if $f(x_1) = f(x_2) \implies x_1 = x_2$.

Solution:

(a) For statement (i), note that $\frac{2\pi}{3}$ lies *outside* the principal branch $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(b) Reduce the angle: $\sin\frac{2\pi}{3} = \sin\left(\pi - \frac{2\pi}{3}\right) = \sin\frac{\pi}{3}$, and $\frac{\pi}{3}$ is inside the branch. Hence

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \frac{\pi}{3} \neq \frac{2\pi}{3}$$

The statement is FALSE.

(c) For statement (ii), suppose $f(x_1) = f(x_2)$, i.e. $x_1^3 = x_2^3$.

(d) The real cube function is strictly increasing, so taking real cube roots gives $x_1 = x_2$. Hence $f(x) = x^3$ is one-one on $\mathbb{R}$: the statement is TRUE.

Final Answer: (i) FALSE, (ii) TRUE

Go Back to Question 23

Q24.

Solution

Concept: The converse of the implication $p \rightarrow q$ is $q \rightarrow p$: hypothesis and conclusion are interchanged without negation. The negation of a universally quantified statement ("All X are Y") is the existential statement "Some X are not Y".

Solution:

(a) For sub-question (i), identify the components: p = "a triangle is equilateral", q = "it is isosceles".

(b) Interchange them to form the converse $q \rightarrow p$: "If a triangle is isosceles, then it is equilateral." (Note the converse need not be true even though the original statement is.)

(c) For sub-question (ii), the original statement uses the universal quantifier "All".

(d) Apply the quantifier negation rule to obtain: "Some birds cannot fly." (Equivalently: "There exists at least one bird that cannot fly.")

Final Answer: (i) If a triangle is isosceles, then it is equilateral. (ii) Some birds cannot fly.

Go Back to Question 24

Q25.

Solution

Concept: Four standard derivative rules apply: the chain rule for $\log(\cos x)$, the product rule for $x\,e^x$, the inverse tangent derivative $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$, and the exponential rule $\frac{d}{dx}a^x = a^x \log a$.

Solution:

(a) For sub-question (i), apply the chain rule with outer $\log u$ and inner $u = \cos x$:

$$\frac{d}{dx}\left[\log(\cos x)\right] = \frac{1}{\cos x}\cdot(-\sin x) = -\tan x$$

(b) For sub-question (ii), apply the product rule:

$$\frac{d}{dx}(x\,e^x) = (1)e^x + x\,e^x = e^x(1+x)$$

(c) For sub-question (iii), evaluate the inverse tangent derivative at $x = 1$:

$$\left.\frac{1}{1+x^2}\right|_{x=1} = \frac{1}{1+1} = \frac{1}{2}$$

(d) For sub-question (iv), apply the general exponential rule with base $a = 5$:

$$\frac{dy}{dx} = 5^x \log 5$$

Final Answer: (i) $-\tan x$, (ii) $e^x(1+x)$, (iii) $\frac{1}{2}$, (iv) $5^x \log 5$

Go Back to Question 25

Q26.

Solution

Concept: This question reviews four coordinate geometry basics: the two-point distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$; the x-intercept found by setting $y = 0$; the circle $x^2 + y^2 = r^2$ centred at the origin; and the defining property that every parabola has eccentricity exactly 1.

Solution:

(a) For sub-question (i), apply the distance formula to $(2, 3)$ and $(5, 7)$:

$$d = \sqrt{(5-2)^2 + (7-3)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

(b) For sub-question (ii), set $y = 0$ in $2x - 3y = 6$:

$$2x = 6 \implies x = 3$$

(c) For sub-question (iii), compare $x^2 + y^2 = 49$ with $x^2 + y^2 = r^2$:

$$r^2 = 49 \implies r = 7$$

(d) For sub-question (iv), recall the conic classification by eccentricity: ellipse $e < 1$, parabola $e = 1$, hyperbola $e > 1$. Hence the eccentricity of a parabola is 1.

Final Answer: (i) 5, (ii) 3, (iii) 7, (iv) 1

Go Back to Question 26

Q27.

Solution

Concept: This question tests structural properties of matrix algebra: linearity of the transpose, non-commutativity of matrix multiplication, the invertibility criterion $|A| \neq 0$, and the reversal law for adjugates $\text{adj}(AB) = (\text{adj}\,B)(\text{adj}\,A)$.

Solution:

(a) For statement (i), the transpose distributes over addition entry-by-entry: $(A + B)' = A' + B'$ always holds. TRUE.

(b) For statement (ii), matrix multiplication is generally not commutative. Counterexample:

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \neq \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

FALSE.

(c) For statement (iii), when $|A| \neq 0$ the inverse exists explicitly as $A^{-1} = \frac{1}{|A|}\,\text{adj}\,A$. TRUE.

(d) For statement (iv), the correct reversal law is $\text{adj}(AB) = (\text{adj}\,B)(\text{adj}\,A)$ — the order reverses, just as for inverses and transposes. The claim $(\text{adj}\,A)(\text{adj}\,B)$ keeps the original order, which fails for general non-commuting matrices. FALSE.

Final Answer: (i) TRUE, (ii) FALSE, (iii) TRUE, (iv) FALSE

Go Back to Question 27

Q28.

Solution

Concept: The area under a curve $y = f(x)$ lying above the x-axis between $x = a$ and $x = b$ is $\int_a^b f(x)\,dx$. When the region is symmetric about the y-axis (an even function), the computation halves: $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

Solution:

(a) For sub-question (i), set $y = 0$ in $y = 4 - x^2$:

$$4 - x^2 = 0 \implies x^2 = 4 \implies x = -2 \text{ and } x = 2$$

The curve meets the x-axis at $(-2, 0)$ and $(2, 0)$.

(b) For sub-question (ii), the shaded region lies between these roots and above the axis:

$$\text{Area} = \int_{-2}^{2} \left(4 - x^2\right) dx$$

(c) For sub-question (iii), the integrand $4 - x^2$ is an even function, so by symmetry:

$$\text{Area} = 2\int_0^2 \left(4 - x^2\right) dx$$

(d) For sub-question (iv), evaluate using antiderivatives:

$$\text{Area} = 2\left[4x - \frac{x^3}{3}\right]_0^2 = 2\left(8 - \frac{8}{3}\right) = 2 \cdot \frac{16}{3} = \frac{32}{3} \text{ sq. units}$$

Final Answer: (i) $(\pm 2, 0)$, (ii) $\int_{-2}^{2}(4 - x^2)dx$, (iii) $2\int_0^2(4 - x^2)dx$, (iv) $\frac{32}{3}$

Go Back to Question 28

Q29.

Solution

Concept: This case study applies core vector algebra: the displacement vector between two points equals the difference of their position vectors; distance is its magnitude; the unit vector is the displacement divided by its magnitude; and the midpoint has position vector equal to the average of the endpoints.

Solution:

(a) For sub-question (i), subtract position vectors:

$$\vec{PQ} = (4-1)\hat{i} + (6-2)\hat{j} + (3-3)\hat{k} = 3\hat{i} + 4\hat{j}$$

Option (A).

(b) For sub-question (ii), take the magnitude:

$$|\vec{PQ}| = \sqrt{3^2 + 4^2 + 0^2} = \sqrt{25} = 5 \text{ m}$$

Option (C).

(c) For sub-question (iii), divide by the magnitude:

$$\hat{u} = \frac{3\hat{i} + 4\hat{j}}{5}$$

Option (B).

(d) For sub-question (iv), average the endpoints:

$$\vec{m} = \frac{(1+4)}{2}\hat{i} + \frac{(2+6)}{2}\hat{j} + \frac{(3+3)}{2}\hat{k} = \frac{5}{2}\hat{i} + 4\hat{j} + 3\hat{k}$$

Option (D).

(e) For sub-question (v), a zero $\hat{k}$-component of displacement means no change in height: the path is horizontal, perpendicular to $\hat{k}$. Option (A).

(f) For sub-question (vi), add twice the displacement to Q:

$$Q + 2\vec{PQ} = (4+6,\ 6+8,\ 3+0) = (10, 14, 3)$$

Option (B).

Final Answer: (i) A, (ii) C, (iii) B, (iv) D, (v) A, (vi) B

Go Back to Question 29

Q30.

Solution

Concept: A function is continuous at a point when the limit of the function at that point equals the function value there. The standard limit $\lim_{x\to 0} \frac{\sin mx}{mx} = 1$ evaluates the trigonometric ratio.

Solution:

(a) Impose the continuity condition at $x = 0$:

$$k = \lim_{x\to 0} \frac{\sin 5x}{3x}$$

(b) Rewrite the ratio to expose the standard form:

$$\frac{\sin 5x}{3x} = \frac{5}{3} \cdot \frac{\sin 5x}{5x}$$

(c) Apply the standard limit as $x \longrightarrow 0$:

$$\lim_{x\to 0} \frac{\sin 5x}{5x} = 1$$

(d) Conclude:

$$k = \frac{5}{3} \cdot 1 = \frac{5}{3}$$

Final Answer: $k = \frac{5}{3}$

Go Back to Question 30

Q31.

Solution

Concept: For a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ with $|A| = ad - bc \neq 0$, the inverse is $A^{-1} = \frac{1}{|A|}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$: swap the diagonal entries, negate the off-diagonal entries, divide by the determinant.

Solution:

(a) Compute the determinant of $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$:

$$|A| = (2)(3) - (5)(1) = 6 - 5 = 1$$

(b) Since $|A| = 1 \neq 0$, the inverse exists.

(c) Form the adjugate by swapping and negating:

$$\text{adj } A = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

(d) Divide by the determinant:

$$A^{-1} = \frac{1}{1}\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$$

(e) Verify: $AA^{-1} = \begin{pmatrix} 6-5 & -10+10 \\ 3-3 & -5+6 \end{pmatrix} = I$. ✓

Final Answer: $A^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$

Go Back to Question 31

Q32.

Solution

Concept: The composition $(f \circ g)(x)$ means $f(g(x))$: feed the output of g into f. Order matters — $(f \circ g)$ and $(g \circ f)$ are generally different functions.

Solution:

(a) Recall the two functions: $f(x) = x^2$ and $g(x) = 2x + 1$.

(b) Build $(f \circ g)(x)$ by substituting $g(x)$ into f:

$$(f \circ g)(x) = f(2x + 1) = (2x + 1)^2 = 4x^2 + 4x + 1$$

(c) Build $(g \circ f)(x)$ by substituting $f(x)$ into g:

$$(g \circ f)(x) = g\left(x^2\right) = 2x^2 + 1$$

(d) Observe that the two compositions differ, confirming that composition is not commutative.

Final Answer: $(f \circ g)(x) = (2x + 1)^2; \quad (g \circ f)(x) = 2x^2 + 1$

Go Back to Question 32

Q33.

Solution

Concept: When an integrand contains a function together with its derivative as a factor, the substitution t = (inner function) reduces the integral to a power rule. Here $\frac{d}{dx}(\log x) = \frac{1}{x}$ appears alongside $(\log x)^2$.

Solution:

(a) Choose the substitution:

$$t = \log x \implies dt = \frac{1}{x}\,dx$$

(b) Rewrite the integral entirely in terms of t:

$$\int \frac{(\log x)^2}{x}\,dx = \int t^2\,dt$$

(c) Apply the power rule of integration:

$$\int t^2\,dt = \frac{t^3}{3} + C$$

(d) Revert to the original variable:

$$\int \frac{(\log x)^2}{x}\,dx = \frac{(\log x)^3}{3} + C$$

Final Answer: $\frac{(\log x)^3}{3} + C$

Go Back to Question 33

Q34.

Solution

Concept: The circle with centre (h, k) and radius r has equation $(x - h)^2 + (y - k)^2 = r^2$. Conversely, for the general form $x^2 + y^2 + 2gx + 2fy + c = 0$, the centre is $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

Solution:

(a) For the primary question, substitute centre $(2, -3)$ and radius 4 into the standard form:

$$(x - 2)^2 + (y + 3)^2 = 16$$

(b) Expand the squares:

$$x^2 - 4x + 4 + y^2 + 6y + 9 = 16$$

(c) Collect terms into general form:

$$x^2 + y^2 - 4x + 6y - 3 = 0$$

(d) For the alternate (OR) question, compare $x^2 + y^2 + 4x - 6y - 3 = 0$ with the general form:

$$2g = 4 \implies g = 2, \qquad 2f = -6 \implies f = -3, \qquad c = -3$$

(e) Read the centre and compute the radius:

$$\text{Centre} = (-2, 3), \qquad r = \sqrt{4 + 9 + 3} = \sqrt{16} = 4$$

Final Answer: $x^2 + y^2 - 4x + 6y - 3 = 0 \quad$ (OR: centre $(-2, 3)$, radius 4)

Go Back to Question 34

Q35.

Solution

Concept: Two non-zero vectors are perpendicular precisely when their scalar dot product vanishes: $\vec{a} \cdot \vec{b} = 0$.

Solution:

(a) Write the two vectors in component form:

$$\vec{a} = (2, \lambda, 1), \qquad \vec{b} = (1, -2, 3)$$

(b) Impose the perpendicularity condition:

$$\vec{a} \cdot \vec{b} = 0$$

(c) Expand the dot product:

$$(2)(1) + (\lambda)(-2) + (1)(3) = 0$$

(d) Simplify the linear equation:

$$2 - 2\lambda + 3 = 0 \implies 5 = 2\lambda$$

(e) Solve for the parameter:

$$\lambda = \frac{5}{2}$$

Final Answer: $\lambda = \frac{5}{2}$

Go Back to Question 35

Q36.

Solution

Concept: When the numerator is the derivative of an inner function, substitute t for that inner function. Here $\frac{d}{dx}(\sin x) = \cos x$ sits in the numerator, and the resulting integrand $\frac{1}{1+t^2}$ integrates to $\tan^{-1} t$.

Solution:

(a) Choose the substitution:

$$t = \sin x \implies dt = \cos x \, dx$$

(b) Transform the limits: when $x = 0$, $t = \sin 0 = 0$; when $x = \frac{\pi}{2}$, $t = \sin \frac{\pi}{2} = 1$.

(c) Rewrite the integral in terms of t:

$$\int_0^{\pi/2} \frac{\cos x}{1 + \sin^2 x} \, dx = \int_0^1 \frac{dt}{1 + t^2}$$

(d) Integrate using the inverse tangent antiderivative:

$$= \left[\tan^{-1} t\right]_0^1 = \tan^{-1}(1) - \tan^{-1}(0)$$

(e) Evaluate the standard values:

$$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

Final Answer: $\frac{\pi}{4}$

Go Back to Question 36

Q37.

Solution

Concept: Implicit differentiation treats y as a function of x: differentiate both sides of the relation term-by-term, applying the chain rule to every occurrence of y, and then solve for $\frac{dy}{dx}$.

Solution:

(a) Differentiate both sides of $x^2 + y^2 = 25$ with respect to x:

$$2x + 2y\frac{dy}{dx} = 0$$

(b) Isolate the derivative:

$$\frac{dy}{dx} = -\frac{x}{y}$$

(c) Substitute the point $(3, 4)$:

$$\left.\frac{dy}{dx}\right|_{(3,4)} = -\frac{3}{4}$$

(d) Sanity check: $(3, 4)$ satisfies $9 + 16 = 25$, so the point genuinely lies on the circle, and the negative slope matches the tangent direction in the first quadrant. ✓

Final Answer: $-\frac{3}{4}$

Go Back to Question 37

Q38.

Solution

Concept: The identity $\cos^{-1}(\cos\theta) = \theta$ is valid only when θ lies in the principal branch $[0, \pi]$. For angles outside this interval, use the periodicity and evenness of cosine, $\cos\theta = \cos(2\pi - \theta)$, to fold the angle back into $[0, \pi]$.

Solution:

(a) Observe that $\frac{7\pi}{6} > \pi$, so it lies outside the principal branch $[0, \pi]$.

(b) Fold the angle using $\cos\theta = \cos(2\pi - \theta)$:

$$\cos\frac{7\pi}{6} = \cos\left(2\pi - \frac{7\pi}{6}\right) = \cos\frac{5\pi}{6}$$

(c) Confirm that $\frac{5\pi}{6}$ lies inside $[0, \pi]$. ✓

(d) Apply the principal value identity:

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6}$$

Final Answer: $\frac{5\pi}{6}$

Go Back to Question 38

Q39.

Solution

Concept: Set $A = \cos^{-1}\frac{4}{5}$ and $B = \cos^{-1}\frac{12}{13}$, so that $\cos A$ and $\cos B$ are known. The compound angle formula $\cos(A+B) = \cos A\cos B - \sin A\sin B$ then produces the cosine of the sum, with the sines recovered from the Pythagorean identity $\sin\theta = \sqrt{1-\cos^2\theta}$ (positive since $A, B \in [0, \pi/2]$).

Solution:

(a) Define the angles and record their cosines:

$$\cos A = \frac{4}{5}, \qquad \cos B = \frac{12}{13}$$

(b) Recover the sines from the Pythagorean identity:

$$\sin A = \sqrt{1-\frac{16}{25}} = \frac{3}{5}, \qquad \sin B = \sqrt{1-\frac{144}{169}} = \frac{5}{13}$$

(c) Apply the compound angle formula:

$$\cos(A+B) = \cos A\cos B - \sin A\sin B = \frac{4}{5}\cdot\frac{12}{13} - \frac{3}{5}\cdot\frac{5}{13}$$

(d) Simplify the fractions over the common denominator 65:

$$\cos(A+B) = \frac{48}{65} - \frac{15}{65} = \frac{33}{65}$$

(e) Since $A, B \in \left[0, \frac{\pi}{2}\right]$ and $\cos(A+B) > 0$, the sum $A+B$ lies in $\left[0, \frac{\pi}{2}\right] \subset [0, \pi]$, the principal branch of $\cos^{-1}$. Therefore:

$$A + B = \cos^{-1}\frac{33}{65} \implies \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65} \qquad \blacksquare$$

Final Answer: Proved

Go Back to Question 39

Q40.

Solution

Concept: When both the base and the exponent contain the variable, use logarithmic differentiation: take natural logarithms of both sides, differentiate implicitly (product rule on the right), and multiply back by the original function.

Solution:

(a) Take logarithms of both sides of $y = x^{\sin x}$:

$$\log y = \sin x \cdot \log x$$

(b) Differentiate both sides with respect to x; the left side uses the chain rule, the right side the product rule:

$$\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \log x + \sin x \cdot \frac{1}{x}$$

(c) Multiply through by y:

$$\frac{dy}{dx} = y\left(\cos x \log x + \frac{\sin x}{x}\right)$$

(d) Replace y with its original expression:

$$\frac{dy}{dx} = x^{\sin x}\left(\cos x \log x + \frac{\sin x}{x}\right)$$

Final Answer: $\frac{dy}{dx} = x^{\sin x}\left(\cos x \log x + \frac{\sin x}{x}\right)$

Go Back to Question 40

Q41.

Solution

Concept: Any point on the line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \lambda$ can be parametrised as $(x_1 + a\lambda,\ y_1 + b\lambda,\ z_1 + c\lambda)$. The foot of the perpendicular F from an external point P is the parameter value at which the vector $\vec{PF}$ is orthogonal to the direction vector of the line, i.e. their dot product vanishes.

Solution:

(a) Parametrise a general point F on the line with parameter λ:

$$F = (6 + 3\lambda,\ 7 + 2\lambda,\ 7 - 2\lambda)$$

(b) Form the vector from $P(1, 2, 3)$ to F:

$$\vec{PF} = (5 + 3\lambda,\ 5 + 2\lambda,\ 4 - 2\lambda)$$

(c) Impose orthogonality with the direction vector $(3, 2, -2)$:

$$3(5 + 3\lambda) + 2(5 + 2\lambda) - 2(4 - 2\lambda) = 0$$

(d) Expand and collect terms:

$$15 + 9\lambda + 10 + 4\lambda - 8 + 4\lambda = 0 \implies 17 + 17\lambda = 0 \implies \lambda = -1$$

(e) Substitute $\lambda = -1$ to find the foot of the perpendicular:

$$F = (6 - 3,\ 7 - 2,\ 7 + 2) = (3, 5, 9)$$

(f) Compute the perpendicular distance $|PF|$:

$$|PF| = \sqrt{(3-1)^2 + (5-2)^2 + (9-3)^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

Final Answer: Foot $= (3, 5, 9)$; distance $= 7$ units

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Q42.

Solution

Concept: King's property of definite integrals states $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. Applying it to an integrand of the form $x\,g(\sin x, \cos^2 x)$ over $[0,\pi]$ eliminates the awkward factor x and leaves a symmetric integral that yields to substitution.

Solution:

(a) Name the integral and apply King's property with $a = \pi$:

$$I = \int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,dx = \int_0^\pi \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}\,dx = \int_0^\pi \frac{(\pi - x)\sin x}{1+\cos^2 x}\,dx$$

(b) Add the two expressions for I:

$$2I = \pi\int_0^\pi \frac{\sin x}{1+\cos^2 x}\,dx$$

(c) Substitute $t = \cos x$, $dt = -\sin x\,dx$; limits: $x = 0 \Rightarrow t = 1$, $x = \pi \Rightarrow t = -1$:

$$2I = \pi\int_{-1}^1 \frac{dt}{1+t^2}$$

(d) Integrate:

$$2I = \pi\left[\tan^{-1} t\right]_{-1}^{1} = \pi\left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right) = \frac{\pi^2}{2}$$

(e) Solve for I:

$$I = \frac{\pi^2}{4}$$

(f) **(OR variant)** Complete the square in the denominator:

$$x^2 - 6x + 13 = (x-3)^2 + 4$$

Then apply the standard form $\int \frac{dx}{u^2+a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$ with $u = x - 3$, $a = 2$:

$$\int \frac{dx}{x^2 - 6x + 13} = \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$$

Final Answer: $\frac{\pi^2}{4}$ (OR: $\frac{1}{2}\tan^{-1}\frac{x-3}{2} + C$)

Go Back to Question 42

Q43.

Solution

Concept: By the corner point theorem, a linear objective attains its optimum at a vertex of the feasible region. For an *unbounded* region and a minimisation problem, the candidate minimum at a corner is confirmed by checking that the open half-plane $ax + by < Z_{\min}$ shares no point with the feasible region.

Solution:

(a) Draw the boundary lines using intercepts:

$$x + 3y = 3 \implies (3, 0) \text{ and } (0, 1), \qquad x + y = 2 \implies (2, 0) \text{ and } (0, 2)$$

Both constraints are of the "≥" type, so the feasible region lies above/right of both lines in the first quadrant, and is unbounded.

(b) Find the intersection of the two boundary lines by elimination:

$$(x + 3y) - (x + y) = 3 - 2 \implies 2y = 1 \implies y = \frac{1}{2}, \qquad x = 2 - \frac{1}{2} = \frac{3}{2}$$

(c) List the corner points of the feasible region:

$$(3, 0), \qquad \left(\frac{3}{2}, \frac{1}{2}\right), \qquad (0, 2)$$

(d) Evaluate $Z = 3x + 5y$ at each vertex:

$$Z(3, 0) = 9, \qquad Z\left(\frac{3}{2}, \frac{1}{2}\right) = \frac{9}{2} + \frac{5}{2} = 7, \qquad Z(0, 2) = 10$$

(e) The smallest value is 7. Because the region is unbounded, test the open half-plane $3x + 5y < 7$: it lies entirely below both constraint lines near the origin, hence has no point in common with the feasible region.

(f) Therefore the minimum is genuinely attained:

$$Z_{\min} = 7 \text{ at } \left(\frac{3}{2}, \frac{1}{2}\right)$$

Final Answer: Min $Z = 7$ at $x = \frac{3}{2}$, $y = \frac{1}{2}$

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Q44.

Solution

Concept: For a horizontal ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$: vertices $(\pm a, 0)$, foci $(\pm c, 0)$ with $c^2 = a^2 - b^2$, eccentricity $e = \frac{c}{a}$, latus rectum $\frac{2b^2}{a}$, minor axis $2b$. For a vertical hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$: vertices $(0, \pm a)$, foci $(0, \pm c)$ with $c^2 = a^2 + b^2$, eccentricity $e = \frac{c}{a}$, latus rectum $\frac{2b^2}{a}$, conjugate axis $2b$.

Solution:

(a) From the vertices $(\pm 5, 0)$ read $a = 5$; from the foci $(\pm 4, 0)$ read $c = 4$. Both lie on the x-axis, so the ellipse is horizontal.

(b) Determine b^2 from the focal relation:

$$b^2 = a^2 - c^2 = 25 - 16 = 9$$

(c) Write the equation of the ellipse:

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

(d) Compute the requested measurements:

$$e = \frac{c}{a} = \frac{4}{5}, \qquad \text{Latus rectum} = \frac{2b^2}{a} = \frac{2 \cdot 9}{5} = \frac{18}{5}, \qquad \text{Minor axis} = 2b = 6$$

(e) **(OR variant)** From the vertices $(0, \pm 3)$ read $a = 3$; from the foci $(0, \pm 5)$ read $c = 5$. Both lie on the y-axis, so the hyperbola is vertical.

(f) Determine b^2 from the hyperbola relation:

$$b^2 = c^2 - a^2 = 25 - 9 = 16$$

(g) Write the equation and its measurements:

$$\frac{y^2}{9} - \frac{x^2}{16} = 1, \qquad e = \frac{5}{3}, \qquad \text{Latus rectum} = \frac{2 \cdot 16}{3} = \frac{32}{3}, \qquad \text{Conjugate axis} = 2b = 8$$

Final Answer: $\frac{x^2}{25} + \frac{y^2}{9} = 1$; $e = \frac{4}{5}$, LR $= \frac{18}{5}$, minor axis $= 6$ (OR: $\frac{y^2}{9} - \frac{x^2}{16} = 1$; $e = \frac{5}{3}$, LR $= \frac{32}{3}$, conjugate axis $= 8$)

Go Back to Question 44

Q45.

Solution

Concept: A linear system $AX = B$ with $|A| \neq 0$ has the unique solution $X = A^{-1}B$, where $A^{-1} = \frac{1}{|A|}$ adj A and the adjugate is the transpose of the cofactor matrix.

Solution:

(a) Compute the determinant of $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$ by expansion along the first row:

$$|A| = 1(1\cdot1-3\cdot1)-2(2\cdot1-3\cdot1)+1(2\cdot1-1\cdot1) = 1(-2)-2(-1)+1(1) = -2+2+1 = 1$$

Since $|A| = 1 \neq 0$, the inverse exists.

(b) Compute the nine cofactors:

$$C_{11} = -2, \quad C_{12} = 1, \quad C_{13} = 1$$

$$C_{21} = -1, \quad C_{22} = 0, \quad C_{23} = 1$$

$$C_{31} = 5, \quad C_{32} = -1, \quad C_{33} = -3$$

(c) Transpose the cofactor matrix to form the adjugate, and divide by $|A| = 1$:

$$A^{-1} = \text{adj } A = \begin{pmatrix} -2 & -1 & 5 \\ 1 & 0 & -1 \\ 1 & 1 & -3 \end{pmatrix}$$

(d) Write the system in matrix form $AX = B$ with $B = \begin{pmatrix} 8 \\ 13 \\ 6 \end{pmatrix}$ and apply $X = A^{-1}B$:

$$x = (-2)(8) + (-1)(13) + (5)(6) = -16 - 13 + 30 = 1$$

$$y = (1)(8) + (0)(13) + (-1)(6) = 8 - 6 = 2$$

$$z = (1)(8) + (1)(13) + (-3)(6) = 8 + 13 - 18 = 3$$

(e) Verify in all three original equations:

$$1 + 4 + 3 = 8 \checkmark, \qquad 2 + 2 + 9 = 13 \checkmark, \qquad 1 + 2 + 3 = 6 \checkmark$$

Solution

(OR variant)

(a) Compute AB:

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 5 & 13 \\ 3 & 8 \end{pmatrix}, \qquad |AB| = 40 - 39 = 1$$

(b) Invert AB directly:

$$(AB)^{-1} = \begin{pmatrix} 8 & -13 \\ -3 & 5 \end{pmatrix}$$

(c) Invert A and B separately ($|A| = 1, |B| = 1$):

$$A^{-1} = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}, \qquad B^{-1} = \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$$

(d) Multiply in reversed order:

$$B^{-1}A^{-1} = \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 6+2 & -9-4 \\ -2-1 & 3+2 \end{pmatrix} = \begin{pmatrix} 8 & -13 \\ -3 & 5 \end{pmatrix}$$

(e) The two results agree, verifying $(AB)^{-1} = B^{-1}A^{-1}$. ■

Final Answer: $x = 1,\ y = 2,\ z = 3$ (OR: verified)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	B	3	A	4	D	5	B
6	D	7	A	8	C	9	B	10	A
11	A	12	C	13	D	14	B	15	C
16	D	17	A	18	B	19	D	20	C