NIOS Class 12 Mathematics Sample Paper – 3

Duration: 180 Minutes | **Maximum Marks: 100**

Instructions

- This paper contains **45** Questions. The paper is divided into two sections: **Section A – 50** marks, **Section B – 50** marks.
- **Section A** consists of
  -**Q.No. 1 to 20** – Multiple Choice type questions (MCQs) carrying **+1 mark** each. Select and write the most appropriate option out of the four options given in each of these questions.
  -**Q.No. 21 to 29** – **Objective type questions**.
  -**Q.No. 21 to 24** carry **02 marks** each (with 2 sub-parts of 1 mark each).
  -**Q.No. 25 to 28** carry **04 marks** each (with 4 sub-parts of 1 mark each).
  -**Q.No. 29** carries **06 marks** (with 6 sub-parts of 1 mark each). Attempt these questions as per the instructions given for each of the questions 21–29.
- **Section B** consists of
  -**Q.No. 30 to 38**– Very Short questions carrying **02 marks** each.
  -**Q.No. 39 to 43** – Short Answer type questions carrying **04 marks** each.
  -**Q.No. 44 to 45** – Long Answer type questions carrying **06 marks** each.
  (An internal choice has been provided in some of the questions in Section B. You have to attempt only one of the given choices in such questions.)
- There is **No Negative marking**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Section: A

Q1. $\lim_{x\to 2} \frac{x^4 - 16}{x - 2}$ equals: **(1)**

(A) 8

(B) 16

(C) 32

(D) 4

Q2. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $A + B = \begin{pmatrix} 4 & 6 \\ 8 & 10 \end{pmatrix}$, then B equals: **(1)**

(A) $\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$

(B) $\begin{pmatrix} 5 & 8 \\ 11 & 14 \end{pmatrix}$

(C) $\begin{pmatrix} 4 & 3 \\ 6 & 5 \end{pmatrix}$

(D) $\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$

Q3. The angle that the line $\sqrt{3}x - y + 4 = 0$ makes with the positive x-axis is: **(1)**

(A) 30°

(B) 45°

(C) 120°

(D) 60°

Q4. If $y = \tan x + \sec x$, then $\frac{dy}{dx}$ equals: **(1)**

(A) $\sec^2 x + \sec x \tan x$

(B) $\sec^2 x + \tan^2 x$

(C) $\sec x \tan x + \tan^2 x$

(D) $\sec^2 x - \sec x \tan x$

Q5. The point $(2, -3, 4)$ lies in which octant? **(1)**

(A) First

(B) Fourth

(C) Second

(D) Eighth

Q6. The value of $\sin\left(2\sin^{-1}\frac{3}{5}\right)$ is: **(1)**

(A) $\frac{6}{5}$

(B) $\frac{24}{25}$

(C) $\frac{12}{25}$

(D) $\frac{7}{25}$

Q7. The two circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 8x + 12 = 0$ have: **(1)**

(A) No common point

(B) Exactly one common point

(C) Two common points

(D) Infinitely many common points

Q8. $\int x\cos x\,dx$ equals: **(1)**

(A) $x\sin x - \cos x + C$

(B) $-x\sin x + \cos x + C$

(C) $x\sin x + \cos x + C$

(D) $\sin x - x\cos x + C$

Q9. If $A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, then AB equals: **(1)**

(A) $\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$

(C) $\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$

Q10. If $\vec{a}$ and $\vec{b}$ are unit vectors and $\vec{a} + \vec{b}$ is also a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is: **(1)**

(A) $60°$

(B) $90°$

(C) $120°$

(D) $150°$

Q11. The general solution of $\frac{dy}{dx} = \frac{x}{y}$ is: **(1)**

(A) $y = x + C$

(B) $xy = C$

(C) $y^2 + x^2 = C$

(D) $y^2 - x^2 = C$

Q12. Which of the following sentences is a statement (in mathematical logic)? **(1)**

(A) Please open the door.

(B) What a beautiful day!

(C) The number 7 is prime.

(D) Where are you going?

Q13. If $f(x) = \frac{x-1}{x+1}$, $x \neq -1$, then $f\left(\frac{1}{x}\right)$ equals: **(1)**

(A) $\frac{x-1}{x+1}$

(B) $\frac{x+1}{x-1}$

(C) $-\frac{x+1}{x-1}$

(D) $\frac{1-x}{1+x}$

Q14. Using properties of determinants, the value of $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{vmatrix}$ is: **(1)**

(A) 2

(B) -2

(C) 6

(D) 0

Q15. The vertex of the parabola $x^2 = 8(y - 2)$ is: **(1)**

(A) $(0, 0)$

(B) $(2, 0)$

(C) $(0, 2)$

(D) $(0, -2)$

Q16. The function $f(x) = x^3 - 6x^2 + 9x + 8$ is decreasing on the interval: **(1)**

(A) $(-\infty, 1)$

(B) $(1, 3)$

(C) $(3, \infty)$

(D) $(-1, 3)$

Q17. In an LPP, if the feasible region is bounded, then the objective function Z: **(1)**

(A) Attains both a maximum and a minimum value

(B) Attains only a maximum value

(C) Attains only a minimum value

(D) Need not attain any optimal value

Q18. The differential equation whose general solution is $y = A\cos x + B\sin x$ is: **(1)**

(A) $\dfrac{d^2y}{dx^2} - y = 0$

(B) $\frac{d^2y}{dx^2} + y = 0$

(C) $\frac{dy}{dx} + y = 0$

(D) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Q19. The foci of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ are: **(1)**

(A) $(\pm 4, 0)$

(B) $(0, \pm 5)$

(C) $(\pm\sqrt{7}, 0)$

(D) $(\pm 5, 0)$

Q20. If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 5\hat{k}$, then $\vec{a} - 2\vec{b}$ equals: **(1)**

(A) $-5\hat{i} + 9\hat{k}$

(B) $-5\hat{i} - 9\hat{k}$

(C) $5\hat{i} - 9\hat{k}$

(D) $-5\hat{i} + 4\hat{j} + 9\hat{k}$

Q21. Match Column-I with Column-II: **(2)**

Column-I	**Column-II**
(i) $\int_{-1}^{1} x^3\, dx$ equals	(A) **0**
(ii) $\int_{0}^{2} x\, dx$ equals	(B) **2**

(A) (i)→(A), (ii)→(B)

(B) (i)→(B), (ii)→(A)

Q22. Fill in the blanks: **(2)**

(i) If $\vec{a} \times \vec{b} = \vec{0}$ and $\vec{a}, \vec{b}$ are non-zero vectors, then $\vec{a}$ and $\vec{b}$ are ________.

(ii) The yz-plane divides the line segment joining $(2, 4, 5)$ and $(-4, 3, 2)$ in the ratio ________.

Q23. Write TRUE or FALSE: **(2)**

(i) The domain of $\sin^{-1} x$ is $[-1, 1]$.

(ii) Every relation on a set is a function.

Q24. Answer as directed: **(2)**

(i) Write the contrapositive of: "If it rains, then the match is cancelled."

(ii) Identify the quantifier in: "There exists a real number x such that $x^2 = 2$."

Q25. Fill in the blanks (Integration): **(4)**

(i) $\int e^{5x}\,dx =$ ________.

(ii) $\int \frac{dx}{1+x^2} =$ ________.

(iii) $\int_1^e \frac{dx}{x} =$ ________.

(iv) $\int \sec x \tan x\,dx =$ ________.

Q26. Fill in the blanks (Straight Lines): **(4)**

(i) The slope of the line joining $(1, 2)$ and $(3, 8)$ is ________.

(ii) The distance of the point $(3, 4)$ from the line $3x + 4y - 10 = 0$ is ________.

(iii) The equation of the line with slope 2 and y-intercept -3 is ________.

(iv) Two lines with slopes m_1 and m_2 are perpendicular if $m_1 m_2 =$ ________.

Q27. Write TRUE or FALSE: **(4)**

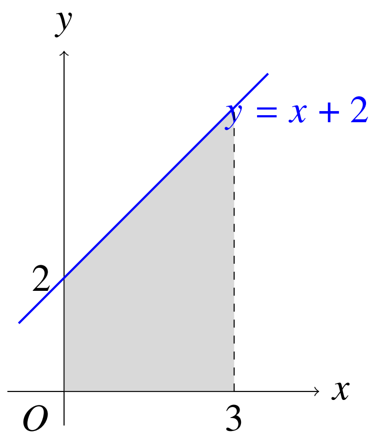
(i) The function $f(x) = |x|$ is differentiable at $x = 0$.

(ii) If $y = e^{3x}$, then $\frac{d^2y}{dx^2} = 9e^{3x}$.

(iii) $\frac{d}{dx}\left(\sin^{-1} x + \cos^{-1} x\right) = 0$ for $x \in (-1, 1)$.

(iv) The derivative of $\log(x^2)$ with respect to x is $\frac{1}{x^2}$.

Q28. Study the figure showing the line $y = x + 2$ and the region bounded by it, the x-axis and the ordinates $x = 0$, $x = 3$: **(4)**



(i) Write the value of y at $x = 0$ and at $x = 3$.

(ii) Write the definite integral representing the shaded area.

(iii) Name the geometric shape of the shaded region.

(iv) Compute the shaded area and verify it using the area formula of that shape.

Q29. Read and answer (i)–(vi):

A tailoring unit stitches shirts and trousers. Each shirt needs 1 hour of cutting and 3 hours of stitching; each trouser needs 2 hours of cutting and 2 hours of stitching. The unit has at most 10 hours of cutting time and 18 hours of stitching time per day. The profit is ₹ 150 per shirt and ₹ 200 per trouser. Let x = number of shirts and y = number of trousers stitched per day. **(6)**

(i) The cutting-time constraint is:

(A) $x + 2y \leq 10$

(B) $3x + 2y \leq 10$

(C) $x + 2y \geq 10$

(D) $2x + y \leq 10$

(ii) The stitching-time constraint is:

(A) $x + 2y \leq 18$

(B) $2x + 3y \leq 18$

(C) $3x + 2y \leq 18$

(D) $3x + 2y \geq 18$

(iii) The objective function is:

(A) Max $Z = 150x + 200y$

(B) Min $Z = 150x + 200y$

(C) Max $Z = 200x + 150y$

(D) Max $Z = x + 2y$

(iv) The corner point obtained by solving $x + 2y = 10$ and $3x + 2y = 18$ is:

(A) $(3, 4)$

(B) $(4, 3)$

(C) $(2, 4)$

(D) $(4, 2)$

(v) The value of Z at the corner point $(0, 5)$ is:

(A) 750

(B) 1000

(C) 900

(D) 1250

(vi) The maximum daily profit is:

(A) ₹ 900

(B) ₹ 1000

(C) ₹ 1200

(D) ₹ 1400

Section: B

Q30. Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$. **(2)**

Q31. Solve using Cramer's rule: $2x + 3y = 7, \ x - y = 1$. **(2)**

Q32. Show that the relation R on $\mathbb{Z}$ defined by $R = \{(a, b) : a - b \text{ is divisible by } 3\}$ is symmetric and transitive. **(2)**

Q33. Evaluate $\int \frac{2x}{1+x^2}\,dx$. **(2)**

Q34. Find the equation of the ellipse with vertices $(\pm 6, 0)$ and eccentricity $\frac{1}{3}$. **OR** Find the eccentricity and foci of the hyperbola $4x^2 - 9y^2 = 36$. **(2)**

Q35. Find the value of p so that the lines $\frac{x-1}{-3} = \frac{y-2}{2p} = \frac{z-3}{2}$ and $\frac{x-1}{3p} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular to each other. **(2)**

Q36. Evaluate $\int_0^{\pi/2} \sin 2x\,dx$. **(2)**

Q37. Find the rate of change of the area of a circle with respect to its radius when the radius is 5 cm. **(2)**

Q38. If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, find $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$. **(2)**

Q39. Using elementary row operations, find the inverse of $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$. **(4)**

Q40. Solve for x: $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\frac{8}{31}$. **(4)**

Q41. Find the equation of the plane passing through the three points $A(1, 1, 0)$, $B(1, 2, 1)$ and $C(-2, 2, -1)$. **(4)**

Q42. Evaluate $\int e^{2x} \sin x\,dx$. **OR** Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$. **(4)**

Q43. A firm manufactures two products A and B on which the profits are ₹ 5 and ₹ 3 per unit respectively. Each product is processed on machines M_1 and M_2. Product A requires 1 minute on M_1 and 2 minutes on M_2; product B requires 1 minute on each machine. M_1 is available for at most 5 hours and M_2 for at most 6 hours per day. Formulate and solve the LPP graphically to maximise profit. **(4)**

Q44. Using integration, find the area of the region bounded by the parabola $y^2 = 8x$ and its latus rectum.

OR

Using integration, find the area of the circle $x^2 + y^2 = 16$. **(6)**

Q45. A wire of length 36 cm is cut into two pieces. One piece is bent into a square and the other into an equilateral triangle. Find the lengths of the two pieces so that the combined area of the square and the triangle is minimum.

OR

Show that the right circular cylinder of given surface area (including the two ends) and maximum volume is such that its height is equal to the diameter of its base. **(6)**

Detailed Solutions

Q1.

Solution

Concept: The algebraic limit theorem states $\lim_{x\to a} \frac{x^n-a^n}{x-a} = na^{n-1}$. Alternatively, the numerator can be factorised so that the troublesome factor $(x-a)$ cancels before substitution.

Solution:

(a) Identify the standard form with $n = 4$ and $a = 2$:

$$\lim_{x\to 2} \frac{x^4 - 2^4}{x - 2}$$

(b) Apply the theorem directly:

$$= n\,a^{n-1} = 4 \cdot 2^3$$

(c) Evaluate:

$$= 4 \cdot 8 = 32$$

(d) Cross-check by factorising: $x^4 - 16 = (x-2)(x+2)(x^2+4)$, so the limit is $(2+2)(4+4) = 4 \cdot 8 = 32$. ✓

Why other options are wrong:

- **Option A:** 8 is only 2^3 — the multiplier $n = 4$ was dropped.
- **Option B:** 16 is x^4 evaluated at 2, confusing the limit with the numerator value.
- **Option D:** 4 is just the exponent n, ignoring a^{n-1}.

Final Answer: 32

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept: Matrix addition and subtraction are performed entry-by-entry on matrices of the same order. From $A + B = C$ it follows that $B = C - A$.

Solution:

(a) Rearrange the matrix equation:

$$B = (A + B) - A = \begin{pmatrix} 4 & 6 \\ 8 & 10 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

(b) Subtract corresponding entries:

$$B = \begin{pmatrix} 4 - 1 & 6 - 2 \\ 8 - 3 & 10 - 4 \end{pmatrix}$$

(c) Simplify:

$$B = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$$

(d) Verify: $A + B = \begin{pmatrix} 1 + 3 & 2 + 4 \\ 3 + 5 & 4 + 6 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 8 & 10 \end{pmatrix}$. ✓

Why other options are wrong:

- **Option B:** $\begin{pmatrix} 5 & 8 \\ 11 & 14 \end{pmatrix}$ *adds* A instead of subtracting it.
- **Option C:** $\begin{pmatrix} 4 & 3 \\ 6 & 5 \end{pmatrix}$ swaps entries within each row.
- **Option D:** $\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$ is the transpose of the correct matrix.

Final Answer: $B = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept: The inclination θ of a line is related to its slope by $m = \tan\theta$, where $0^\circ \le \theta < 180^\circ$. For $ax + by + c = 0$ the slope is $-\frac{a}{b}$.

Solution:

(a) Compute the slope of $\sqrt{3}x - y + 4 = 0$:

$$m = -\frac{a}{b} = -\frac{\sqrt{3}}{-1} = \sqrt{3}$$

(b) Set the slope equal to the tangent of the inclination:

$$\tan\theta = \sqrt{3}$$

(c) Identify the standard angle in $[0^\circ, 180^\circ)$:

$$\theta = 60^\circ$$

Why other options are wrong:

- **Option A:** 30° corresponds to slope $\frac{1}{\sqrt{3}}$, the reciprocal of the correct slope.
- **Option B:** 45° corresponds to slope 1, not $\sqrt{3}$.
- **Option C:** 120° has $\tan 120^\circ = -\sqrt{3}$ — a sign error when computing $-a/b$.

Final Answer: 60°

Answer: (D)

Go Back to Question 3

Q4.

Solution

Concept: Differentiation is linear: the derivative of a sum is the sum of the derivatives. The standard results are $\frac{d}{dx}(\tan x) = \sec^2 x$ and $\frac{d}{dx}(\sec x) = \sec x \tan x$.

Solution:

(a) Split the derivative across the sum:

$$\frac{dy}{dx} = \frac{d}{dx}(\tan x) + \frac{d}{dx}(\sec x)$$

(b) Apply the two standard results:

$$\frac{dy}{dx} = \sec^2 x + \sec x \tan x$$

(c) (Optional factored form: $\sec x(\sec x + \tan x)$, which also shows $\frac{dy}{dx} = y \sec x$.)

Why other options are wrong:

- **Option B:** $\sec^2 x + \tan^2 x$ wrongly differentiates $\sec x$ as $\tan^2 x$.
- **Option C:** $\sec x \tan x + \tan^2 x$ wrongly differentiates $\tan x$ as $\tan^2 x$.
- **Option D:** the minus sign belongs to derivatives of co-functions ($\csc x$, $\cot x$), not these.

Final Answer: $\sec^2 x + \sec x \tan x$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: The eight octants of three-dimensional space are numbered by the signs of (x, y, z): the first four (I–IV) have $z > 0$ and follow the anticlockwise order of quadrants $(+,+), (-,+), (-,-), (+,-)$ in the xy-plane; octants V–VIII repeat the same order with $z < 0$.

Solution:

(a) Record the sign pattern of the point $(2, -3, 4)$:

$$x > 0, \quad y < 0, \quad z > 0$$

(b) Since $z > 0$, the point lies in one of the upper octants I–IV.

(c) Within the upper set, the sign pair $(x > 0,\ y < 0)$ matches the fourth quadrant of the xy-plane.

(d) Hence the point lies in the fourth octant.

Why other options are wrong:

- **Option A:** the first octant needs all three coordinates positive, but $y = -3 < 0$.
- **Option C:** the second octant needs $x < 0,\ y > 0$ — both signs are reversed here.
- **Option D:** the eighth octant needs $z < 0$, but $z = 4 > 0$.

Final Answer: Fourth octant

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept: Set $\theta = \sin^{-1} x$ so that $\sin\theta = x$ and $\cos\theta = \sqrt{1 - x^2}$ (positive on the principal branch for $x \in [0, 1]$). The double angle identity then gives $\sin 2\theta = 2\sin\theta\cos\theta$.

Solution:

(a) Let $\theta = \sin^{-1}\frac{3}{5}$, so that:

$$\sin\theta = \frac{3}{5}$$

(b) Recover the cosine from the Pythagorean identity:

$$\cos\theta = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

(c) Apply the double angle formula:

$$\sin 2\theta = 2\sin\theta\cos\theta = 2 \cdot \frac{3}{5} \cdot \frac{4}{5}$$

(d) Simplify:

$$\sin\left(2\sin^{-1}\frac{3}{5}\right) = \frac{24}{25}$$

Why other options are wrong:

- **Option A:** $\frac{6}{5}$ doubles the sine directly ($2 \times \frac{3}{5}$) — and exceeds 1, impossible for a sine.
- **Option C:** $\frac{12}{25}$ forgets the factor 2 in the double angle formula.
- **Option D:** $\frac{7}{25}$ is $\cos 2\theta$, not $\sin 2\theta$.

Final Answer: $\frac{24}{25}$

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept: Two circles with centres C_1, C_2 and radii r_1, r_2 touch externally when the distance between centres equals the sum of the radii: $d = r_1 + r_2$. In that case they share exactly one common point.

Solution:

(a) Read the data of the first circle $x^2 + y^2 = 4$:

$$C_1 = (0, 0), \qquad r_1 = 2$$

(b) Convert the second circle $x^2 + y^2 - 8x + 12 = 0$ to centre–radius form:

$$2g = -8 \implies g = -4, \quad c = 12 \implies C_2 = (4, 0), \quad r_2 = \sqrt{16 - 12} = 2$$

(c) Compute the distance between the centres:

$$d = \sqrt{(4-0)^2 + 0^2} = 4$$

(d) Compare with the radii:

$$r_1 + r_2 = 2 + 2 = 4 = d$$

(e) Since $d = r_1 + r_2$, the circles touch externally — exactly one common point.

Why other options are wrong:

- **Option A:** no common point requires $d > r_1 + r_2$; here equality holds.
- **Option C:** two common points require $|r_1 - r_2| < d < r_1 + r_2$, a strict inequality that fails.
- **Option D:** infinitely many common points would mean identical circles, clearly false.

Final Answer: Exactly one common point

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept: Integration by parts follows $\int u\,dv = uv - \int v\,du$. By the ILATE priority rule, the algebraic factor x is chosen as u and the trigonometric factor $\cos x\,dx$ as dv.

Solution:

(a) Assign the parts:

$$u = x \implies du = dx, \qquad dv = \cos x\,dx \implies v = \sin x$$

(b) Apply the by-parts formula:

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$

(c) Integrate the remaining term:

$$= x\sin x - (-\cos x) + C$$

(d) Simplify:

$$= x\sin x + \cos x + C$$

(e) Check by differentiating: $\frac{d}{dx}(x\sin x + \cos x) = \sin x + x\cos x - \sin x = x\cos x$. ✓

Why other options are wrong:

- **Option A:** $x\sin x - \cos x$ has the wrong sign on $\cos x$ (from $-\int \sin x\,dx$ mishandled).
- **Option B:** differentiates to $-x\cos x$, the negative of the integrand.
- **Option D:** $\sin x - x\cos x$ is $\int x\sin x\,dx$, a different integral.

Final Answer: $x\sin x + \cos x + C$

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept: The product of two matrices is formed row-by-column: the (i, j) entry of AB is the dot product of row i of A with column j of B.

Solution:

(a) Set up the product:

$$AB = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

(b) Compute each entry:

$$\text{Row 1, Col 1} = (1)(1) + (0)(0) = 1, \qquad \text{Row 1, Col 2} = (1)(1) + (0)(1) = 1$$

$$\text{Row 2, Col 1} = (2)(1) + (1)(0) = 2, \qquad \text{Row 2, Col 2} = (2)(1) + (1)(1) = 3$$

(c) Assemble:

$$AB = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

Why other options are wrong:

- **Option B:** bottom-right entry 2 misses the $(1)(1)$ contribution from the second row of A.
- **Option C:** is BA, not AB — matrix multiplication is order-sensitive.
- **Option D:** is the transpose $(AB)'$, not AB itself.

Final Answer: $\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: The magnitude-square of a vector sum expands as $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}\cdot\vec{b}$, and for unit vectors $\vec{a}\cdot\vec{b} = \cos\theta$.

Solution:

(a) Impose the given condition $|\vec{a}+\vec{b}| = 1$ and square it:

$$1 = |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}\cdot\vec{b}$$

(b) Insert the unit magnitudes $|\vec{a}| = |\vec{b}| = 1$:

$$1 = 1 + 1 + 2\cos\theta$$

(c) Solve for the cosine:

$$2\cos\theta = -1 \implies \cos\theta = -\frac{1}{2}$$

(d) Identify the principal angle:

$$\theta = 120^\circ$$

Why other options are wrong:

- **Option A:** 60° has $\cos\theta = +\frac{1}{2}$ — a sign slip; it makes $|\vec{a}+\vec{b}| = \sqrt{3}$, not 1.
- **Option B:** 90° gives $|\vec{a}+\vec{b}| = \sqrt{2} \neq 1$.
- **Option D:** 150° needs $\cos\theta = -\frac{\sqrt{3}}{2}$, which does not satisfy the equation.

Final Answer: 120°

Answer: (C)

Go Back to Question 10

Q11.

Solution

Concept: A separable differential equation is solved by collecting all y-terms with dy and all x-terms with dx, then integrating each side independently.

Solution:

(a) Separate the variables in $\frac{dy}{dx} = \frac{x}{y}$:

$$y\,dy = x\,dx$$

(b) Integrate both sides:

$$\int y\,dy = \int x\,dx \implies \frac{y^2}{2} = \frac{x^2}{2} + C_1$$

(c) Multiply through by 2 and absorb the constant:

$$y^2 - x^2 = C$$

Why other options are wrong:

- **Option A:** $y = x + C$ would need $\frac{dy}{dx} = 1$; here the slope is $\frac{x}{y}$, not constant.
- **Option B:** $xy = C$ solves $\frac{dy}{dx} = -\frac{y}{x}$, a different equation.
- **Option C:** $y^2 + x^2 = C$ solves $\frac{dy}{dx} = -\frac{x}{y}$ — the sign of the right side is opposite.

Final Answer: $y^2 - x^2 = C$

Answer: (D)

Go Back to Question 11

Q12.

Solution

Concept: In mathematical logic, a statement (proposition) is a declarative sentence that is definitively either true or false — never both, and never neither. Commands, exclamations and questions carry no truth value and are not statements.

Solution:

(a) Classify each sentence by its grammatical/logical type:

(b) "Please open the door." — an imperative (command); it cannot be true or false.

(c) "What a beautiful day!" — an exclamation; subjective, no definite truth value.

(d) "The number 7 is prime." — a declarative sentence with a definite truth value (in fact, true).

(e) "Where are you going?" — an interrogative (question); no truth value.

(f) Only the third sentence qualifies as a logical statement.

Why other options are wrong:

- **Option A:** a command has no truth value.
- **Option B:** an exclamation expresses feeling, not a verifiable claim.
- **Option D:** a question asserts nothing, so it cannot be assigned true or false.

Final Answer: The number 7 is prime.

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: Evaluating f at a transformed argument means substituting that expression for every occurrence of x in the formula, then simplifying — here by clearing the complex fraction.

Solution:

(a) Substitute $\frac{1}{x}$ into $f(x) = \frac{x-1}{x+1}$:

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x} - 1}{\frac{1}{x} + 1}$$

(b) Multiply numerator and denominator by x to clear the inner fractions:

$$= \frac{1 - x}{1 + x}$$

(c) Note the neat relation $f\left(\frac{1}{x}\right) = -f(x)$, since $\frac{1-x}{1+x} = -\frac{x-1}{x+1}$.

Why other options are wrong:

- **Option A:** $\frac{x-1}{x+1}$ is $f(x)$ unchanged — the substitution was never made.
- **Option B:** $\frac{x+1}{x-1}$ is $\frac{1}{f(x)}$, the reciprocal instead of the substituted value.
- **Option C:** $-\frac{x+1}{x-1}$ is $-\frac{1}{f(x)}$, combining two separate errors.

Final Answer: $\dfrac{1 - x}{1 + x}$

Answer: (D)

Go Back to Question 13

Q14.

Solution

Concept: A key property of determinants: if one row (or column) of a matrix is a scalar multiple of another row (or column), the determinant is zero, because the rows are linearly dependent.

Solution:

(a) Inspect the rows of the determinant:

$$R_1 = (1, 2, 3), \qquad R_2 = (2, 4, 6), \qquad R_3 = (3, 5, 7)$$

(b) Observe the proportionality:

$$R_2 = 2R_1$$

(c) Since two rows are proportional, the rows are linearly dependent, and by the determinant property:

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{vmatrix} = 0$$

(d) (Verification by the row operation $R_2 \to R_2 - 2R_1$ produces a zero row, confirming the value 0.)

Why other options are wrong:

- **Option A:** 2 misreads the proportionality factor as the determinant value.
- **Option B:** -2 likewise, with a sign error added.
- **Option C:** 6 multiplies diagonal entries only, which is valid only for triangular matrices.

Final Answer: 0

Answer: (D)

Go Back to Question 14

Q15.

Solution

Concept: The parabola $(x-h)^2 = 4a(y-k)$ is the standard upward parabola $x^2 = 4ay$ translated so that its vertex moves from the origin to (h, k).

Solution:

(a) Compare the given equation with the shifted standard form:

$$x^2 = 8(y-2) \quad \text{vs} \quad (x-h)^2 = 4a(y-k)$$

(b) Match the pieces:

$$h = 0, \qquad k = 2, \qquad 4a = 8 \implies a = 2$$

(c) Read off the vertex:

$$\text{Vertex} = (h, k) = (0, 2)$$

Why other options are wrong:

- **Option A:** $(0, 0)$ ignores the translation $y \to y - 2$ entirely.
- **Option B:** $(2, 0)$ places the shift on the wrong axis.
- **Option D:** $(0, -2)$ reverses the direction of the shift; $y - 2 = 0$ gives $y = +2$.

Final Answer: $(0, 2)$

Answer: (C)

Go Back to Question 15

Q16.

Solution

Concept: A differentiable function decreases exactly where its first derivative is negative. Factorising $f'(x)$ and testing the sign in each interval between the critical points locates the decreasing interval.

Solution:

(a) Differentiate the function:

$$f'(x) = 3x^2 - 12x + 9$$

(b) Factorise:

$$f'(x) = 3(x^2 - 4x + 3) = 3(x-1)(x-3)$$

(c) The critical points are $x = 1$ and $x = 3$. Test the sign of f' in each interval:

$$x < 1 : \ (+)(-)(-) > 0, \qquad 1 < x < 3 : \ (+)(+)(-) < 0, \qquad x > 3 : \ (+)(+)(+) > 0$$

(d) $f'(x) < 0$ exactly on $(1, 3)$, so the function decreases there.

Why other options are wrong:

- **Option A:** on $(-\infty, 1)$ the derivative is positive — the function is increasing.
- **Option C:** on $(3, \infty)$ the derivative is again positive.
- **Option D:** $(-1, 3)$ wrongly includes $(-1, 1)$, where f increases.

Final Answer: $(1, 3)$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: The fundamental theorem of linear programming: if the feasible region of an LPP is non-empty, closed and bounded, a continuous linear objective function attains both its maximum and its minimum, and each optimum occurs at a corner point of the region.

Solution:

(a) A bounded feasible region can be enclosed in some large circle — it does not extend to infinity in any direction.

(b) A linear function is continuous everywhere, and a continuous function on a closed and bounded region attains both extreme values (extreme value theorem).

(c) By the corner point theorem, these extremes occur at vertices of the region.

(d) Hence, for a bounded feasible region, Z attains both a maximum and a minimum.

Why other options are wrong:

- **Option B:** "only a maximum" can fail — the minimum also exists on a bounded region.
- **Option C:** "only a minimum" fails symmetrically.
- **Option D:** optima may fail to exist only when the region is *unbounded* (or empty), not bounded.

Final Answer: Attains both a maximum and a minimum value

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: To find the differential equation of a family of curves with two arbitrary constants, differentiate twice and eliminate the constants. For $y = A\cos x + B\sin x$, both basis functions reproduce their own negatives after two differentiations.

Solution:

(a) Start with the general solution:

$$y = A\cos x + B\sin x$$

(b) Differentiate once:

$$\frac{dy}{dx} = -A\sin x + B\cos x$$

(c) Differentiate again:

$$\frac{d^2y}{dx^2} = -A\cos x - B\sin x = -(A\cos x + B\sin x)$$

(d) Recognise the bracket as y itself:

$$\frac{d^2y}{dx^2} = -y \implies \frac{d^2y}{dx^2} + y = 0$$

Both constants are eliminated, and the order (2) matches the number of constants. ✓

Why other options are wrong:

- **Option A:** $y'' - y = 0$ is solved by e^x, e^{-x}, not sines and cosines.
- **Option C:** $y' + y = 0$ is first order — it cannot absorb two arbitrary constants.
- **Option D:** $y'' + y' = 0$ is solved by $A + Be^{-x}$, a different family.

Final Answer: $\frac{d^2y}{dx^2} + y = 0$

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept: For the horizontal hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the foci lie on the transverse (x) axis at $(\pm c, 0)$, where $c^2 = a^2 + b^2$ — note the *plus* sign, unlike the ellipse.

Solution:

(a) Read the parameters from $\frac{x^2}{16} - \frac{y^2}{9} = 1$:

$$a^2 = 16, \qquad b^2 = 9$$

(b) Apply the hyperbola focal relation:

$$c^2 = a^2 + b^2 = 16 + 9 = 25 \implies c = 5$$

(c) Place the foci on the transverse axis:

$$\text{Foci} = (\pm 5, 0)$$

Why other options are wrong:

- **Option A:** $(\pm 4, 0)$ are the *vertices*, located at distance a, not c.
- **Option B:** $(0, \pm 5)$ puts the foci on the conjugate axis — wrong orientation.
- **Option C:** $(\pm\sqrt{7}, 0)$ uses the ellipse relation $c^2 = a^2 - b^2$ by mistake.

Final Answer: $(\pm 5, 0)$

Answer: (D)

Go Back to Question 19

Q20.

Solution

Concept: Linear combinations of vectors are computed component-wise: multiply each component of $\vec{b}$ by the scalar 2, then subtract from the corresponding component of $\vec{a}$.

Solution:

(a) Scale $\vec{b}$ by 2:

$$2\vec{b} = 6\hat{i} + 2\hat{j} - 10\hat{k}$$

(b) Subtract component-wise from $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$:

$$\vec{a} - 2\vec{b} = (1-6)\hat{i} + (2-2)\hat{j} + (-1+10)\hat{k}$$

(c) Simplify each component:

$$\vec{a} - 2\vec{b} = -5\hat{i} + 0\hat{j} + 9\hat{k} = -5\hat{i} + 9\hat{k}$$

Why other options are wrong:

- **Option B:** $-5\hat{i} - 9\hat{k}$ computes $-1 - 10$ for the $\hat{k}$ component instead of $-1 - (-10)$.
- **Option C:** $5\hat{i} - 9\hat{k}$ evaluates $2\vec{b} - \vec{a}$, the negative of the required vector.
- **Option D:** keeps a spurious $4\hat{j}$ from adding the $\hat{j}$ components instead of subtracting.

Final Answer: $-5\hat{i} + 9\hat{k}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: Part (i) uses the odd-function property of definite integrals: $\int_{-a}^{a} f(x)\,dx = 0$ whenever $f(-x) = -f(x)$. Part (ii) is a direct power rule evaluation.

Solution:

(a) For sub-question (i), note $f(x) = x^3$ satisfies $f(-x) = -x^3 = -f(x)$: it is an odd function.

(b) Over the symmetric interval $[-1, 1]$, the odd-function property gives:

$$\int_{-1}^{1} x^3\,dx = 0$$

This matches option (A).

(c) For sub-question (ii), integrate by the power rule:

$$\int_{0}^{2} x\,dx = \left[\frac{x^2}{2}\right]_0^2 = \frac{4}{2} - 0 = 2$$

This matches option (B).

Final Answer: (i)→(A), (ii)→(B)

Go Back to Question 21

Q22.

Solution

Concept: Part (i): the cross product of two non-zero vectors vanishes exactly when $\sin\theta = 0$, i.e. the vectors are parallel (collinear). Part (ii): the yz-plane is $x = 0$; the section ratio follows from the section formula applied to the x-coordinates.

Solution:

(a) For sub-question (i), use $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$. With non-zero magnitudes, $\vec{a} \times \vec{b} = \vec{0}$ forces $\sin\theta = 0$, i.e. $\theta = 0$ or π. Hence the vectors are **parallel (collinear)**.

(b) For sub-question (ii), let the yz-plane divide the segment joining $(2, 4, 5)$ and $(-4, 3, 2)$ in ratio $k : 1$.

(c) The x-coordinate of the dividing point must be zero:

$$\frac{k(-4) + 1(2)}{k + 1} = 0 \implies -4k + 2 = 0 \implies k = \frac{1}{2}$$

(d) Hence the ratio is $\frac{1}{2} : 1 = \mathbf{1} : \mathbf{2}$ (internal division).

Final Answer: (i) parallel (collinear), (ii) $1 : 2$ internally

Go Back to Question 22

Q23.

Solution

Concept: Part (i) recalls the domain of the inverse sine: $\sin^{-1} x$ is defined only where a sine value can exist, namely $[-1, 1]$. Part (ii) distinguishes relations from functions: a function must assign exactly one image to every element of the domain.

Solution:

(a) For statement (i), since $-1 \leq \sin\theta \leq 1$ for all θ, only numbers in $[-1, 1]$ possess an inverse sine. The domain of $\sin^{-1} x$ is exactly $[-1, 1]$: TRUE.

(b) For statement (ii), consider the relation $R = \{(1, 2), (1, 3)\}$ on the set $\{1, 2, 3\}$.

(c) The element 1 is related to two different images (2 and 3), violating the uniqueness requirement of a function.

(d) Hence not every relation is a function: FALSE. (Every function *is* a relation, but not conversely.)

Final Answer: (i) TRUE, (ii) FALSE

Go Back to Question 23

Q24.

Solution

Concept: The contrapositive of $p \to q$ is $\neg q \to \neg p$: negate both parts and reverse the arrow. Quantifiers are the phrases "for all" ($\forall$) and "there exists" ($\exists$) that specify how many objects satisfy a predicate.

Solution:

(a) For sub-question (i), identify the components: p = "it rains", q = "the match is cancelled".

(b) Negate both and reverse the implication to form $\neg q \to \neg p$: "If the match is not cancelled, then it does not rain."

(c) For sub-question (ii), examine the sentence: "There exists a real number x such that $x^2 = 2$."

(d) The quantifier is the phrase **"There exists"** — the existential quantifier, denoted $\exists$.

Final Answer: (i) If the match is not cancelled, then it does not rain. (ii) "There exists" (existential quantifier $\exists$)

Go Back to Question 24

Q25.

Solution

Concept: Four standard antiderivatives: $\int e^{kx} dx = \frac{e^{kx}}{k} + C$; $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$; $\int \frac{dx}{x} = \log|x| + C$ evaluated between limits; and $\int \sec x \tan x \, dx = \sec x + C$ since $\frac{d}{dx} \sec x = \sec x \tan x$.

Solution:

(a) For sub-question (i), apply the exponential rule with $k = 5$:

$$\int e^{5x} \, dx = \frac{e^{5x}}{5} + C$$

(b) For sub-question (ii), recognise the inverse tangent form:

$$\int \frac{dx}{1 + x^2} = \tan^{-1} x + C$$

(c) For sub-question (iii), evaluate the logarithmic integral between the limits:

$$\int_1^e \frac{dx}{x} = [\log x]_1^e = \log e - \log 1 = 1 - 0 = 1$$

(d) For sub-question (iv), recall the derivative of the secant:

$$\int \sec x \tan x \, dx = \sec x + C$$

Final Answer: (i) $\frac{e^{5x}}{5} + C$, (ii) $\tan^{-1} x + C$, (iii) 1, (iv) $\sec x + C$

Go Back to Question 25

Q26.

Solution

Concept: Four straight-line basics: slope = $\frac{y_2-y_1}{x_2-x_1}$; point-to-line distance = $\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$; slope–intercept form $y = mx + c$; and the perpendicularity condition $m_1m_2 = -1$.

Solution:

(a) For sub-question (i), apply the slope formula to $(1, 2)$ and $(3, 8)$:

$$m = \frac{8-2}{3-1} = \frac{6}{2} = 3$$

(b) For sub-question (ii), apply the distance formula for $(3, 4)$ and $3x + 4y - 10 = 0$:

$$d = \frac{|3(3) + 4(4) - 10|}{\sqrt{3^2 + 4^2}} = \frac{|9 + 16 - 10|}{5} = \frac{15}{5} = 3$$

(c) For sub-question (iii), substitute $m = 2$, $c = -3$ into $y = mx + c$:

$$y = 2x - 3$$

(d) For sub-question (iv), state the perpendicularity condition:

$$m_1m_2 = -1$$

Final Answer: (i) 3, (ii) 3, (iii) $y = 2x - 3$, (iv) -1

Go Back to Question 26

Q27.

Solution

Concept: This set tests differentiability versus continuity, repeated differentiation of exponentials, the constancy of $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$, and the chain rule on logarithms of powers.

Solution:

(a) For statement (i), the left-hand derivative of $|x|$ at 0 is -1 while the right-hand derivative is $+1$. Since they differ, $|x|$ is *not* differentiable at $x = 0$ (though it is continuous). FALSE.

(b) For statement (ii), differentiate twice:

$$y = e^{3x} \implies \frac{dy}{dx} = 3e^{3x} \implies \frac{d^2y}{dx^2} = 9e^{3x}$$

TRUE.

(c) For statement (iii), by the identity $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ (a constant) on $(-1, 1)$, the derivative of the sum is 0. TRUE.

(d) For statement (iv), apply the chain rule (for $x \neq 0$):

$$\frac{d}{dx} \log\left(x^2\right) = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$$

The claimed value $\frac{1}{x^2}$ omits the inner derivative $2x$. FALSE.

Final Answer: (i) FALSE, (ii) TRUE, (iii) TRUE, (iv) FALSE

Go Back to Question 27

Q28.

Solution

Concept: The definite integral $\int_a^b y\,dx$ measures the area between a curve and the x-axis. When the curve is a straight line, the region is a trapezium, and the integral must agree with the mensuration formula Area $= \frac{1}{2}$(sum of parallel sides) $\times$ (distance between them).

Solution:

(a) For sub-question (i), evaluate $y = x + 2$ at the two ordinates:

$$y(0) = 2, \qquad y(3) = 5$$

(b) For sub-question (ii), the shaded area under the line from $x = 0$ to $x = 3$ is:

$$\text{Area} = \int_0^3 (x + 2)\,dx$$

(c) For sub-question (iii), the region has two parallel vertical sides (lengths 2 and 5) joined by the x-axis and the slanted line: it is a **trapezium**.

(d) For sub-question (iv), evaluate the integral:

$$\int_0^3 (x + 2)\,dx = \left[\frac{x^2}{2} + 2x\right]_0^3 = \frac{9}{2} + 6 = \frac{21}{2}$$

Verify with the trapezium formula:

$$\text{Area} = \frac{1}{2}(2 + 5) \times 3 = \frac{21}{2}\checkmark$$

Final Answer: (i) 2 and 5, (ii) $\int_0^3 (x + 2)dx$, (iii) trapezium, (iv) $\frac{21}{2}$ sq. units

Go Back to Question 28

Q29.

Solution

Concept: An LPP is formulated by translating each limited resource into a linear inequality and the profit into a linear objective. The optimum of the objective over the feasible region occurs at a corner point.

Solution:

(a) For sub-question (i), cutting time: shirts use 1 hour and trousers 2 hours from at most 10 hours:

$$x + 2y \leq 10 \implies \text{Option (A)}$$

(b) For sub-question (ii), stitching time: shirts use 3 hours and trousers 2 hours from at most 18 hours:

$$3x + 2y \leq 18 \implies \text{Option (C)}$$

(c) For sub-question (iii), profit per unit is ₹ 150 and ₹ 200:

$$\text{Max } Z = 150x + 200y \implies \text{Option (A)}$$

(d) For sub-question (iv), solve the boundary system by subtraction:

$$(3x + 2y) - (x + 2y) = 18 - 10 \implies 2x = 8 \implies x = 4, \qquad y = \frac{10 - 4}{2} = 3$$

The corner point is $(4, 3)$: Option (B).

(e) For sub-question (v), evaluate Z at $(0, 5)$:

$$Z = 150(0) + 200(5) = 1000 \implies \text{Option (B)}$$

(f) For sub-question (vi), compare all corner points $(0, 0)$, $(6, 0)$, $(4, 3)$, $(0, 5)$:

$$Z(6, 0) = 900, \qquad Z(4, 3) = 600 + 600 = 1200, \qquad Z(0, 5) = 1000$$

The maximum profit is ₹ 1200 at $(4, 3)$: Option (C).

Final Answer: (i) A, (ii) C, (iii) A, (iv) B, (v) B, (vi) C

Go Back to Question 29

Q30.

Solution

Concept: Limits of the indeterminate form $\frac{0}{0}$ containing radicals are evaluated by rationalisation: multiply numerator and denominator by the conjugate of the radical expression, so the difference of squares removes the root.

Solution:

(a) Multiply by the conjugate $\sqrt{1+x}+1$ over itself:

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

(b) Apply the difference of squares in the numerator:

$$= \lim_{x\to 0} \frac{(1+x)-1}{x\left(\sqrt{1+x}+1\right)} = \lim_{x\to 0} \frac{x}{x\left(\sqrt{1+x}+1\right)}$$

(c) Cancel the common factor x (valid since $x \neq 0$ in the limit process):

$$= \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1}$$

(d) Substitute $x = 0$:

$$= \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

Final Answer: $\frac{1}{2}$

Go Back to Question 30

Q31.

Solution

Concept: Cramer's rule solves the system $a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$ via determinants: $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$, where D is the coefficient determinant and D_x, D_y replace the respective columns by the constants.

Solution:

(a) Form the coefficient determinant for $2x + 3y = 7$, $x - y = 1$:

$$D = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = (2)(-1) - (3)(1) = -5$$

(b) Replace the x-column with the constants:

$$D_x = \begin{vmatrix} 7 & 3 \\ 1 & -1 \end{vmatrix} = -7 - 3 = -10$$

(c) Replace the y-column with the constants:

$$D_y = \begin{vmatrix} 2 & 7 \\ 1 & 1 \end{vmatrix} = 2 - 7 = -5$$

(d) Apply Cramer's rule:

$$x = \frac{D_x}{D} = \frac{-10}{-5} = 2, \qquad y = \frac{D_y}{D} = \frac{-5}{-5} = 1$$

(e) Verify: $2(2) + 3(1) = 7$ ✓ and $2 - 1 = 1$ ✓.

Final Answer: $x = 2, \ y = 1$

Go Back to Question 31

Q32.

Solution

Concept: A relation R is symmetric if $(a, b) \in R \implies (b, a) \in R$, and transitive if $(a, b), (b, c) \in R \implies (a, c) \in R$. Divisibility arguments rest on the closure of integers: if $3 \mid m$ then $3 \mid (-m)$, and if $3 \mid m,\ 3 \mid n$ then $3 \mid (m + n)$.

Solution:

(a) **Symmetry:** let $(a, b) \in R$, so $a - b = 3k$ for some integer k.

(b) Then $b - a = -(a - b) = 3(-k)$, and $-k$ is an integer, so 3 divides $b - a$; hence $(b, a) \in R$. Symmetric. ✓

(c) **Transitivity:** let $(a, b) \in R$ and $(b, c) \in R$, so $a - b = 3k$ and $b - c = 3m$ for integers k, m.

(d) Add the two equations:

$$a - c = (a - b) + (b - c) = 3k + 3m = 3(k + m)$$

Since $k + m$ is an integer, 3 divides $a - c$; hence $(a, c) \in R$. Transitive. ✓

Final Answer: R is symmetric and transitive (shown)

Go Back to Question 32

Q33.

Solution

Concept: Whenever the numerator is exactly the derivative of the denominator, the integral is the logarithm of the denominator: $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$.

Solution:

(a) Identify the denominator and its derivative:

$$f(x) = 1 + x^2 \implies f'(x) = 2x$$

(b) The numerator $2x$ matches $f'(x)$ exactly, so the log rule applies directly:

$$\int \frac{2x}{1+x^2}\,dx = \log\left|1 + x^2\right| + C$$

(c) Since $1 + x^2 > 0$ always, the absolute value may be dropped:

$$= \log\left(1 + x^2\right) + C$$

Final Answer: $\log\left(1 + x^2\right) + C$

Go Back to Question 33

Q34.

Solution

Concept: For a horizontal ellipse, vertices $(\pm a, 0)$ give a; the eccentricity gives $c = ae$; and $b^2 = a^2 - c^2$ completes the equation. For a hyperbola in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the relations are $c^2 = a^2 + b^2$ and $e = \frac{c}{a}$.

Solution:

(a) From the vertices $(\pm 6, 0)$, read $a = 6$.

(b) From the eccentricity:

$$c = ae = 6 \cdot \frac{1}{3} = 2$$

(c) Compute b^2:

$$b^2 = a^2 - c^2 = 36 - 4 = 32$$

(d) Write the equation of the ellipse:

$$\frac{x^2}{36} + \frac{y^2}{32} = 1$$

(e) **(OR variant)** Divide $4x^2 - 9y^2 = 36$ by 36:

$$\frac{x^2}{9} - \frac{y^2}{4} = 1 \implies a^2 = 9,\ b^2 = 4$$

(f) Compute the focal data:

$$c^2 = 9 + 4 = 13 \implies c = \sqrt{13}, \qquad e = \frac{\sqrt{13}}{3}, \qquad \text{Foci} = \left(\pm\sqrt{13}, 0\right)$$

Final Answer: $\frac{x^2}{36} + \frac{y^2}{32} = 1$ (OR: $e = \frac{\sqrt{13}}{3}$, foci $(\pm\sqrt{13}, 0)$)

Go Back to Question 34

Q35.

Solution

Concept: Two lines with direction ratios $\langle a_1, b_1, c_1\rangle$ and $\langle a_2, b_2, c_2\rangle$ are perpendicular exactly when $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

Solution:

(a) Extract the direction ratios of the two lines:

$$\vec{d}_1 = \langle -3,\ 2p,\ 2\rangle, \qquad \vec{d}_2 = \langle 3p,\ 1,\ -5\rangle$$

(b) Impose the perpendicularity condition:

$$(-3)(3p) + (2p)(1) + (2)(-5) = 0$$

(c) Expand:

$$-9p + 2p - 10 = 0$$

(d) Collect terms and solve:

$$-7p = 10 \implies p = -\frac{10}{7}$$

Final Answer: $p = -\frac{10}{7}$

Go Back to Question 35

Q36.

Solution

Concept: The antiderivative of $\sin kx$ is $-\frac{\cos kx}{k}$, followed by evaluation between the limits using the fundamental theorem of calculus.

Solution:

(a) Write the antiderivative with $k = 2$:

$$\int_0^{\pi/2} \sin 2x \, dx = \left[-\frac{\cos 2x}{2}\right]_0^{\pi/2}$$

(b) Substitute the limits:

$$= -\frac{\cos \pi}{2} + \frac{\cos 0}{2}$$

(c) Evaluate the cosines ($\cos \pi = -1$, $\cos 0 = 1$):

$$= -\frac{(-1)}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$$

Final Answer: 1

Go Back to Question 36

Q37.

Solution

Concept: The rate of change of a quantity A with respect to a variable r is the derivative $\frac{dA}{dr}$, evaluated at the given instant. For a circle, $A = \pi r^2$.

Solution:

(a) Write the area of a circle as a function of the radius:

$$A = \pi r^2$$

(b) Differentiate with respect to r:

$$\frac{dA}{dr} = 2\pi r$$

(c) Evaluate at $r = 5$ cm:

$$\left.\frac{dA}{dr}\right|_{r=5} = 2\pi(5) = 10\pi$$

(d) Attach the units: area changes at 10π square centimetres per centimetre of radius growth.

Final Answer: $10\pi\ \text{cm}^2/\text{cm}$

Go Back to Question 37

Q38.

Solution

Concept: The dot product distributes over vector addition, giving the algebraic identity $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$, since the cross terms $\vec{a}\cdot\vec{b}$ and $-\vec{b}\cdot\vec{a}$ cancel.

Solution:

(a) Expand using distributivity:

$$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = |\vec{a}|^2 - |\vec{b}|^2$$

(b) Compute $|\vec{a}|^2$ for $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$:

$$|\vec{a}|^2 = 4 + 1 + 4 = 9$$

(c) Compute $|\vec{b}|^2$ for $\vec{b} = \hat{i} + \hat{j} + \hat{k}$:

$$|\vec{b}|^2 = 1 + 1 + 1 = 3$$

(d) Subtract:

$$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 9 - 3 = 6$$

Final Answer: 6

Go Back to Question 38

Q39.

Solution

Concept: The row-operation method writes the augmented pair $[A \mid I]$ and applies elementary row operations until the left block becomes I; the right block is then A^{-1}. Each operation must be applied to the entire row of both blocks simultaneously.

Solution:

(a) Set up the augmented pair:

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right]$$

(b) Eliminate below the pivot with $R_2 \to R_2 - 2R_1$:

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

(c) Eliminate above the second pivot with $R_1 \to R_1 - 2R_2$:

$$\left[\begin{array}{cc|cc} 1 & 0 & 5 & -2 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

(d) The left block is now I, so:

$$A^{-1} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$$

(e) Verify: $AA^{-1} = \begin{pmatrix} 5-4 & -2+2 \\ 10-10 & -4+5 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \checkmark$

Final Answer: $A^{-1} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$

Go Back to Question 39

Q40.

Solution

Concept: The arctangent addition formula $\tan^{-1} u + \tan^{-1} v = \tan^{-1} \frac{u+v}{1-uv}$ (valid for $uv < 1$) converts the equation into an equality of arctangents, whose arguments may then be equated.

Solution:

(a) Apply the addition formula with $u = x + 1$, $v = x - 1$:

$$\tan^{-1} \frac{(x+1)+(x-1)}{1-(x+1)(x-1)} = \tan^{-1} \frac{8}{31}$$

(b) Simplify numerator and denominator:

$$\frac{2x}{1-(x^2-1)} = \frac{2x}{2-x^2}$$

(c) Equate the arguments:

$$\frac{2x}{2-x^2} = \frac{8}{31}$$

(d) Cross-multiply:

$$62x = 16 - 8x^2 \implies 8x^2 + 62x - 16 = 0 \implies 4x^2 + 31x - 8 = 0$$

(e) Factorise the quadratic:

$$4x^2 + 32x - x - 8 = 0 \implies 4x(x+8) - 1(x+8) = 0 \implies (4x-1)(x+8) = 0$$

(f) So $x = \frac{1}{4}$ or $x = -8$. Check validity in the addition formula (need $uv = (x^2 - 1) < 1$): for $x = \frac{1}{4}$, $uv = \frac{1}{16} - 1 < 1$ ✓; for $x = -8$, $uv = 63 > 1$, and the left side becomes negative while the right side is positive — rejected.

Final Answer: $x = \dfrac{1}{4}$

Go Back to Question 40

Q41.

Solution

Concept: The plane through three non-collinear points contains the two edge vectors $\vec{AB}$ and $\vec{AC}$; their cross product provides the normal vector $\vec{n}$, and the plane is then $\vec{n} \cdot (\vec{r} - \vec{a}) = 0$ through any one of the points.

Solution:

(a) Form the two edge vectors from $A(1, 1, 0)$:

$$\vec{AB} = (1 - 1,\ 2 - 1,\ 1 - 0) = (0, 1, 1), \qquad \vec{AC} = (-2 - 1,\ 2 - 1,\ -1 - 0) = (-3, 1, -1)$$

(b) Compute the normal via the cross product:

$$\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix}$$

(c) Expand along the first row:

$$\vec{n} = \hat{i}\,((1)(-1) - (1)(1)) - \hat{j}\,((0)(-1) - (1)(-3)) + \hat{k}\,((0)(1) - (1)(-3))$$

$$\vec{n} = -2\hat{i} - 3\hat{j} + 3\hat{k}$$

(d) Write the plane through $A(1, 1, 0)$ with this normal:

$$-2(x - 1) - 3(y - 1) + 3(z - 0) = 0$$

(e) Expand and simplify:

$$-2x + 2 - 3y + 3 + 3z = 0 \implies 2x + 3y - 3z = 5$$

(f) Verify with $B(1, 2, 1)$: $2 + 6 - 3 = 5$ ✓; and $C(-2, 2, -1)$: $-4 + 6 + 3 = 5$ ✓.

Final Answer: $2x + 3y - 3z = 5$

Go Back to Question 41

Q42.

Solution

Concept: For $\int e^{ax}\sin bx\,dx$, integrating by parts twice returns a multiple of the original integral, which can then be solved algebraically. The standard result is $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$.

Solution:

(a) Name the integral and integrate by parts with $u = \sin x$, $dv = e^{2x}dx$:

$$I = \int e^{2x}\sin x\,dx = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\int e^{2x}\cos x\,dx$$

(b) Integrate by parts again on the remaining integral with $u = \cos x$:

$$\int e^{2x}\cos x\,dx = \frac{e^{2x}\cos x}{2} + \frac{1}{2}\int e^{2x}\sin x\,dx = \frac{e^{2x}\cos x}{2} + \frac{I}{2}$$

(c) Substitute back into the first equation:

$$I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{I}{4}$$

(d) Collect the I terms:

$$\frac{5I}{4} = \frac{e^{2x}(2\sin x - \cos x)}{4} \implies I = \frac{e^{2x}(2\sin x - \cos x)}{5} + C$$

(e) **(OR variant)** Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$. Apply King's property $x \to \frac{\pi}{2} - x$:

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx$$

(f) Add the two forms of I: the numerators sum to the common denominator:

$$2I = \int_0^{\pi/2} 1\,dx = \frac{\pi}{2} \implies I = \frac{\pi}{4}$$

Final Answer: $\frac{e^{2x}(2\sin x - \cos x)}{5} + C$ (OR: $\frac{\pi}{4}$)

Go Back to Question 42

Q43.

Solution

Concept: Formulate the resource limits as linear constraints (converting hours to minutes for consistency), then apply the corner point method: evaluate the objective at every vertex of the feasible region and select the largest value.

Solution:

(a) Define variables: let x = units of product A and y = units of product B per day.

(b) Convert machine availability to minutes: M_1: $5 \times 60 = 300$ minutes; M_2: $6 \times 60 = 360$ minutes. The constraints are:

$$x + y \leq 300 \quad (M_1), \qquad 2x + y \leq 360 \quad (M_2), \qquad x, y \geq 0$$

(c) The objective is to maximise:

$$Z = 5x + 3y$$

(d) Find the intersection of the boundary lines by subtraction:

$$(2x + y) - (x + y) = 360 - 300 \implies x = 60, \qquad y = 300 - 60 = 240$$

(e) List the corner points and evaluate Z:

$$Z(0, 0) = 0, \qquad Z(180, 0) = 900, \qquad Z(60, 240) = 300 + 720 = 1020 = 900$$

(f) The maximum is at $(60, 240)$:

$$Z_{\max} = | \, 1020 \text{ per day (60 units of A and 240 units of B)}$$

Final Answer: Max profit = | 1020 at $x = 60$, $y = 240$

Go Back to Question 43

Q44.

Solution

Concept: For the parabola $y^2 = 4ax$, the latus rectum is the vertical chord through the focus $(a, 0)$. The enclosed region is symmetric about the x-axis, so its area is twice the integral of the upper branch $y = \sqrt{4ax}$ from the vertex to the focus. For a circle, area is likewise assembled from $\int \sqrt{r^2 - x^2}\, dx$ over a quarter and multiplied by 4.

Solution:

(a) Compare $y^2 = 8x$ with $y^2 = 4ax$:

$$4a = 8 \implies a = 2$$

The latus rectum is the line $x = 2$.

(b) By symmetry about the x-axis, the required area is:

$$\text{Area} = 2\int_0^2 \sqrt{8x}\, dx = 2 \cdot 2\sqrt{2}\int_0^2 x^{1/2}\, dx$$

(c) Integrate the power:

$$= 4\sqrt{2}\left[\frac{2x^{3/2}}{3}\right]_0^2 = \frac{8\sqrt{2}}{3} \cdot 2^{3/2}$$

(d) Simplify $2^{3/2} = 2\sqrt{2}$:

$$\text{Area} = \frac{8\sqrt{2} \cdot 2\sqrt{2}}{3} = \frac{32}{3} \text{ sq. units}$$

(e) **(OR variant)** For the circle $x^2 + y^2 = 16$ ($r = 4$), the total area is four times the first-quadrant area:

$$\text{Area} = 4\int_0^4 \sqrt{16 - x^2}\, dx$$

(f) Apply the standard result $\int \sqrt{r^2 - x^2}\, dx = \frac{x}{2}\sqrt{r^2 - x^2} + \frac{r^2}{2}\sin^{-1}\frac{x}{r}$:

$$\int_0^4 \sqrt{16 - x^2}\, dx = \left[\frac{x}{2}\sqrt{16 - x^2} + 8\sin^{-1}\frac{x}{4}\right]_0^4 = 0 + 8 \cdot \frac{\pi}{2} = 4\pi$$

(g) Multiply by 4:

$$\text{Area} = 16\pi \text{ sq. units} \quad (= \pi r^2 \text{ with } r = 4 \checkmark)$$

Final Answer: $\frac{32}{3}$ sq. units (OR: 16π sq. units)

Go Back to Question 44

Q45.

Solution

Concept: Optimisation with a constraint: express the total area as a single-variable function using the length constraint, find the critical point via the first derivative, and confirm the minimum with the second derivative test. A square of perimeter p has area $\frac{p^2}{16}$; an equilateral triangle of perimeter q has side $\frac{q}{3}$ and area $\frac{\sqrt{3}}{4}\left(\frac{q}{3}\right)^2$.

Solution:

(a) Let x cm be used for the square, so $36 - x$ cm forms the triangle.

(b) Write the combined area:

$$A(x) = \frac{x^2}{16} + \frac{\sqrt{3}}{4}\left(\frac{36-x}{3}\right)^2 = \frac{x^2}{16} + \frac{\sqrt{3}}{36}(36-x)^2$$

(c) Differentiate:

$$A'(x) = \frac{x}{8} - \frac{\sqrt{3}}{18}(36-x)$$

(d) Set $A'(x) = 0$ and clear denominators (multiply by 72):

$$9x = 4\sqrt{3}(36-x) \implies 9x + 4\sqrt{3}x = 144\sqrt{3} \implies x = \frac{144\sqrt{3}}{9+4\sqrt{3}}$$

(e) Confirm the minimum:

$$A''(x) = \frac{1}{8} + \frac{\sqrt{3}}{18} > 0 \implies \text{minimum}\checkmark$$

(f) State the two pieces:

$$\text{Square piece} = \frac{144\sqrt{3}}{9+4\sqrt{3}}\text{ cm} \approx 15.65\text{ cm}$$

$$\text{Triangle piece} = 36 - x = \frac{324}{9+4\sqrt{3}}\text{ cm} \approx 20.35\text{ cm}$$

Solution

(OR variant)

(a) Let the cylinder have radius r and height h, with fixed total surface area S:

$$S = 2\pi r^2 + 2\pi r h \implies h = \frac{S - 2\pi r^2}{2\pi r}$$

(b) Express the volume in terms of r alone:

$$V = \pi r^2 h = \pi r^2 \cdot \frac{S - 2\pi r^2}{2\pi r} = \frac{Sr}{2} - \pi r^3$$

(c) Differentiate and set to zero:

$$\frac{dV}{dr} = \frac{S}{2} - 3\pi r^2 = 0 \implies S = 6\pi r^2$$

(d) Confirm the maximum:

$$\frac{d^2V}{dr^2} = -6\pi r < 0 \implies \text{maximum}\checkmark$$

(e) Substitute $S = 6\pi r^2$ back into the height expression:

$$h = \frac{6\pi r^2 - 2\pi r^2}{2\pi r} = \frac{4\pi r^2}{2\pi r} = 2r$$

(f) Hence at maximum volume the height equals the diameter of the base. ■

Final Answer: Square piece = $\frac{144\sqrt{3}}{9+4\sqrt{3}}$ cm, triangle piece = $\frac{324}{9+4\sqrt{3}}$ cm (OR: $h = 2r$ shown)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	D	4	A	5	B
6	B	7	B	8	C	9	A	10	C
11	D	12	C	13	D	14	D	15	C
16	B	17	A	18	B	19	D	20	A