NIOS Class 12 Mathematics Sample Paper – 4

Duration: 180 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **45** Questions. The paper is divided into two sections: **Section A – 50** marks, **Section B – 50** marks.
- **Section A** consists of
  -**Q.No. 1 to 20** – Multiple Choice type questions (MCQs) carrying **+1 mark** each. Select and write the most appropriate option out of the four options given in each of these questions.
  -**Q.No. 21 to 29** – **Objective type questions**.
  -**Q.No. 21 to 24** carry **02 marks** each (with 2 sub-parts of 1 mark each).
  -**Q.No. 25 to 28** carry **04 marks** each (with 4 sub-parts of 1 mark each).
  -**Q.No. 29** carries **06 marks** (with 6 sub-parts of 1 mark each). Attempt these questions as per the instructions given for each of the questions 21–29.
- **Section B** consists of
  -**Q.No. 30 to 38**– Very Short questions carrying **02 marks** each.
  -**Q.No. 39 to 43** – Short Answer type questions carrying **04 marks** each.
  -**Q.No. 44 to 45** – Long Answer type questions carrying **06 marks** each.
  (An internal choice has been provided in some of the questions in Section B. You have to attempt only one of the given choices in such questions.)
- There is **No Negative marking**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Section: A

Q1. $\lim_{x\to 0} \frac{\tan 3x}{\tan 5x}$ equals: **(1)**

(A) $\frac{5}{3}$

(B) $\frac{3}{5}$

(C) 1

(D) $\frac{15}{1}$

Q2. If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, then $A^T A$ has trace (sum of diagonal entries) equal to: **(1)**

(A) 5

(B) 10

(C) 15

(D) 13

Q3. The midpoint of the portion of the line $\frac{x}{4} + \frac{y}{6} = 1$ intercepted between the axes is: **(1)**

(A) $(4, 6)$

(B) $(2, 3)$

(C) $(3, 2)$

(D) $(2, -3)$

Q4. If $y = \cos^2 x$, then $\frac{dy}{dx}$ equals: **(1)**

(A) $-\sin 2x$

(B) $\sin 2x$

(C) $-2\cos x$

(D) $2 \sin x \cos^2 x$

Q5. The distance of the point $(3, -4, 5)$ from the xy-plane is: **(1)**

(A) 3

(B) 4

(C) 5

(D) $\sqrt{50}$

Q6. The value of $\tan\left(\cos^{-1}\frac{3}{5}\right)$ is: **(1)**

(A) $\frac{3}{4}$

(B) $\frac{5}{4}$

(C) $\frac{4}{5}$

(D) $\frac{4}{3}$

Q7. The length of the chord which the circle $x^2 + y^2 = 25$ cuts on the line $y = 3$ is: **(1)**

(A) 4

(B) 8

(C) 10

(D) 6

Q8. $\int \frac{dx}{x \log x}$ equals: **(1)**

(A) $\frac{(\log x)^2}{2} + C$

(B) $\log x + C$

(C) $\log|\log x| + C$

(D) $\frac{1}{\log x} + C$

Q9. If $A = \begin{pmatrix} k & 2 \\ 8 & k \end{pmatrix}$ and $|A^2| = 225$, then a possible value of k is: **(1)**

(A) ± 1

(B) ± 5

(C) ± 3

(D) ± 4

Q10. If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then $|\vec{a} \cdot \vec{b}|$ equals: **(1)**

(A) 8

(B) 10

(C) 4

(D) 6

Q11. The general solution of the differential equation $x\,dy + y\,dx = 0$ is: **(1)**

(A) $x + y = C$

(B) $x - y = C$

(C) $\frac{x}{y} = C$

(D) $xy = C$

Q12. The statement $p \lor \neg p$ is: **(1)**

(A) A tautology

(B) A contradiction

(C) Sometimes true, sometimes false

(D) Not a statement

Q13. Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 + 1$. The range of f is: **(1)**

(A) $\mathbb{R}$

(B) $(1, \infty)$

(C) $[1, \infty)$

(D) $[0, \infty)$

Q14. If $\begin{pmatrix} x+y & 2 \\ 5 & xy \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 5 & 6 \end{pmatrix}$, then x and y are roots of: **(1)**

(A) $t^2 - 5t + 6 = 0$

(B) $t^2 + 5t + 6 = 0$

(C) $t^2 - 6t + 5 = 0$

(D) $t^2 - 5t - 6 = 0$

Q15. The centre of the ellipse $\frac{(x-2)^2}{16} + \frac{(y+3)^2}{9} = 1$ is: **(1)**

(A) $(2, 3)$

(B) $(-2, 3)$

(C) $(2, -3)$

(D) $(-2, -3)$

Q16. The slope of the tangent to the curve $y = \sqrt{4x - 3}$ at $x = 3$ is: **(1)**

(A) 3

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) $\frac{4}{3}$

Q17. The corner points of a feasible region are $(0, 0)$, $(5, 0)$, $(3, 4)$ and $(0, 6)$. The maximum of $Z = 2x + 3y$ is: **(1)**

(A) 10

(B) 12

(C) 20

(D) 18

Q18. The number of arbitrary constants in the general solution of a third order differential equation is: **(1)**

(A) 0

(B) 2

(C) 4

(D) 3

Q19. The length of the latus rectum of the parabola $y^2 = -12x$ is: **(1)**

(A) 12

(B) 3

(C) 6

(D) −12

Q20. If the position vectors of points P and Q are $3\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$, then the vector $\vec{QP}$ is: **(1)**

(A) $-2\hat{i} - 3\hat{j} + 3\hat{k}$

(B) $2\hat{i} + 3\hat{j} - 3\hat{k}$

(C) $4\hat{i} + \hat{j} + \hat{k}$

(D) $2\hat{i} - 3\hat{j} + 3\hat{k}$

Q21. Match Column-I with Column-II: **(2)**

Column-I	**Column-II**
(i) The derivative of e^{2x} at $x = 0$ equals	(A) **2**
(ii) $\lim_{x\to 0} \frac{\log(1+4x)}{x}$ equals	(B) **4**

(A) (i)→(A), (ii)→(B)

(B) (i)→(B), (ii)→(A)

Q22. Fill in the blanks: **(2)**

(i) If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, then $\vec{a} \cdot \vec{b} =$ ________.

(ii) The direction ratios of the line $\frac{x-2}{4} = \frac{1-y}{2} = \frac{z+3}{5}$ are ________.

Q23. Write TRUE or FALSE: **(2)**

(i) The principal value branch of $\cos^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(ii) If a function is differentiable at a point, then it is continuous at that point.

Q24. Answer as directed: **(2)**

(i) Write the inverse of: "If a number is even, then it is divisible by 2."

(ii) Write the component statements of: "$\sqrt{2}$ is irrational and 2 is prime."

Q25. Fill in the blanks (Limits and Continuity): **(4)**

(i) $\lim_{x\to 3} \frac{x^2-9}{x-3} =$ ________.

(ii) $\lim_{x\to 0} \frac{e^x-1}{x} =$ ________.

(iii) $\lim_{x\to 0} \frac{1-\cos x}{x^2} =$ ________.

(iv) If $f(x) = \frac{x^2-4}{x-2}$ for $x \neq 2$ is continuous at $x = 2$, then $f(2) =$ ________.

Q26. Fill in the blanks (Vectors): **(4)**

(i) The unit vector in the direction of $3\hat{i} + 4\hat{j}$ is ________.

(ii) If $\vec{a}$ and $\vec{b}$ represent two adjacent sides of a triangle, its area equals ________.

(iii) The value of $\hat{i} \cdot (\hat{j} \times \hat{k})$ is ________.

(iv) The position vector of the point dividing the join of $\vec{a}$ and $\vec{b}$ internally in the ratio 1 : 1 is ________.

Q27. Write TRUE or FALSE: **(4)**

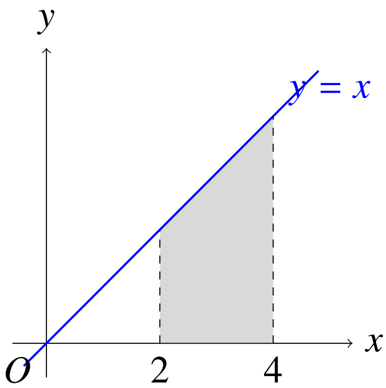
(i) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx.$

(ii) The degree of the differential equation $\left(\frac{dy}{dx}\right)^2 + \frac{1}{\frac{dy}{dx}} = 2$ is 2.

(iii) $\int_{-2}^{2} \sin^3 x\,dx = 0.$

(iv) The order of the differential equation of the family $y = Ce^x$ is 2.

Q28. Study the figure showing the lines $y = x$, $x = 2$, $x = 4$ and the x-axis: **(4)**



(i) Write the values of y on the line $y = x$ at $x = 2$ and $x = 4$.

(ii) Write the definite integral representing the shaded area.

(iii) Evaluate the integral.

(iv) Verify the result using the area formula of a trapezium.

Q29. Read and answer (i)–(vi):

The population $P(t)$ (in thousands) of a town t years after the year 2020 is modelled by the function $P(t) = 2t^3 - 15t^2 + 36t + 50$, valid for $0 \le t \le 8$. **(6)**

(i) The rate of change of population, $P'(t)$, is:

(A) $6t^2 - 30t + 36$

(B) $2t^2 - 15t + 36$

(C) $6t^2 - 15t + 36$

(D) $6t^2 - 30t + 50$

(ii) The critical points of $P(t)$ are:

(A) $t = 1, 6$

(B) $t = 2, 3$

(C) $t = 3, 6$

(D) $t = 1, 5$

(iii) $P(t)$ is decreasing on the interval:

(A) $(0, 2)$

(B) $(2, 3)$

(C) $(3, 8)$

(D) $(0, 8)$

(iv) $P''(t)$ equals:

(A) $12t - 30$

(B) $6t - 30$

(C) $12t - 15$

(D) $12t + 30$

(v) The population at the local maximum ($t = 2$) is (in thousands):

(A) 78

(B) 82

(C) 72

(D) 66

(vi) At $t = 3$ the second derivative test gives:

(A) $P''(3) > 0$, local minimum

(B) $P''(3) < 0$, local maximum

(C) $P''(3) = 0$, no conclusion

(D) $P''(3) > 0$, local maximum

Section: B

Q30. Differentiate $y = \dfrac{x^2 + 1}{x^2 - 1}$ with respect to x. **(2)**

Q31. If $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$, find $|A|$ and adj A, and verify $|\text{adj } A| = |A|^{n-1}$ for $n = 2$. **(2)**

Q32. Let $f : \mathbb{R} \to \mathbb{R}$ be $f(x) = 4x + 3$. Show that f is one-one and onto. **(2)**

Q33. Evaluate $\displaystyle\int \sin^2 x \, dx$. **(2)**

Q34. Find the equation of the parabola with vertex at the origin, axis along the y-axis, and passing through the point $(4, 2)$. **OR** Find the coordinates of the foci and the eccentricity of the ellipse $16x^2 + 25y^2 = 400$. **(2)**

Q35. Find the angle between the planes $x + y + 2z = 9$ and $2x - y + z = 15$. **(2)**

Q36. Evaluate $\int_0^1 x\, e^{x^2}\, dx$. **(2)**

Q37. The side of a square sheet is increasing at the rate of 4 cm per minute. At what rate is the area increasing when the side is 8 cm long? **(2)**

Q38. Show that the points with position vectors $2\hat{i} + 3\hat{j} + 4\hat{k}$, $3\hat{i} + 4\hat{j} + 5\hat{k}$ and $5\hat{i} + 6\hat{j} + 7\hat{k}$ are collinear. **(2)**

Q39. Solve the system by the matrix method: $3x + 2y = 11,\ 2x - 3y = 3$. **(4)**

Q40. Prove that $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \sin^{-1}\frac{77}{85}$. **(4)**

Q41. Find the image (reflection) of the point $(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. **(4)**

Q42. Evaluate $\int \frac{x}{(x-1)(x-2)}\, dx$. **OR** Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$. **(4)**

Q43. A dietician mixes two foods F_1 and F_2. One kg of F_1 contains 2 units of vitamin A and 1 unit of vitamin C; one kg of F_2 contains 1 unit of vitamin A and 2 units of vitamin C. The mixture must contain at least 8 units of vitamin A and at least 10 units of vitamin C. One kg of F_1 costs ₹ 50 and one kg of F_2 costs ₹ 70. Formulate and solve the LPP graphically to minimise the cost of the mixture. **(4)**

Q44. Using integration, find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -1$ and $x = 1$.

OR

Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$. **(6)**

Q45. Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$, given that $y = 1$ when $x = 1$.

OR

Show that the semi-vertical angle of a right circular cone of given slant height and maximum volume is $\tan^{-1}\sqrt{2}$. **(6)**

Detailed Solutions

Q1.

Solution

Concept: The standard limit $\lim_{x\to 0} \frac{\tan kx}{kx} = 1$ lets each tangent be replaced by its linear coefficient. Dividing numerator and denominator by x converts the ratio into a quotient of two such standard forms.

Solution:

(a) Divide numerator and denominator by x:

$$L = \lim_{x\to 0} \frac{\frac{\tan 3x}{x}}{\frac{\tan 5x}{x}}$$

(b) Rescale each standard form by its own coefficient:

$$\lim_{x\to 0} \frac{\tan 3x}{x} = 3 \cdot \lim_{x\to 0} \frac{\tan 3x}{3x} = 3, \qquad \lim_{x\to 0} \frac{\tan 5x}{x} = 5$$

(c) Take the quotient:

$$L = \frac{3}{5}$$

Why other options are wrong:

- **Option A:** $\frac{5}{3}$ inverts the ratio — coefficients swapped between numerator and denominator.
- **Option C:** 1 ignores the different coefficients 3 and 5.
- **Option D:** 15 multiplies the coefficients instead of dividing them.

Final Answer: $\frac{3}{5}$

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept: The transpose of a matrix swaps rows and columns. The trace of a product $A^T A$ equals the sum of squares of all entries of A, because each diagonal entry of $A^T A$ is the dot product of a column of A with itself.

Solution:

(a) Write the transpose of $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$:

$$A^T = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}$$

(b) Multiply $A^T A$:

$$A^T A = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 9+1 & 3-2 \\ 3-2 & 1+4 \end{pmatrix} = \begin{pmatrix} 10 & 1 \\ 1 & 5 \end{pmatrix}$$

(c) Sum the diagonal entries:

$$\text{trace}(A^T A) = 10 + 5 = 15$$

(d) Cross-check with the sum-of-squares shortcut: $3^2 + 1^2 + (-1)^2 + 2^2 = 9 + 1 + 1 + 4 = 15$. ✓

Why other options are wrong:

- **Option A:** 5 counts only the second diagonal entry of $A^T A$.
- **Option B:** 10 counts only the first diagonal entry.
- **Option D:** 13 misses the two squared entries of one row of A.

Final Answer: 15

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept: The intercept form $\frac{x}{a} + \frac{y}{b} = 1$ meets the axes at $(a, 0)$ and $(0, b)$. The midpoint of the segment joining two points averages their coordinates.

Solution:

(a) Read the intercepts from $\frac{x}{4} + \frac{y}{6} = 1$:

$$\text{On the } x\text{-axis: } (4, 0), \qquad \text{on the } y\text{-axis: } (0, 6)$$

(b) Apply the midpoint formula:

$$M = \left(\frac{4+0}{2}, \frac{0+6}{2}\right)$$

(c) Simplify:

$$M = (2, 3)$$

Why other options are wrong:

- **Option A:** $(4, 6)$ pairs the two intercept lengths without halving.
- **Option C:** $(3, 2)$ swaps the two coordinates.
- **Option D:** $(2, -3)$ misplaces the y-intercept below the axis; the intercept is $+6$.

Final Answer: $(2, 3)$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: The chain rule applied to a squared trigonometric function: $\frac{d}{dx}\left[u^2\right] = 2u\,u'$ with $u = \cos x$. The double angle identity $2\sin x\cos x = \sin 2x$ compacts the result.

Solution:

(a) Apply the chain rule with outer square and inner cosine:

$$\frac{dy}{dx} = 2\cos x \cdot \frac{d}{dx}(\cos x) = 2\cos x \cdot (-\sin x)$$

(b) Simplify the product:

$$\frac{dy}{dx} = -2\sin x\cos x$$

(c) Compress with the double angle identity:

$$\frac{dy}{dx} = -\sin 2x$$

Why other options are wrong:

- **Option B:** $\sin 2x$ drops the minus sign produced by $\frac{d}{dx}\cos x = -\sin x$.
- **Option C:** $-2\cos x$ differentiates only the outer square, forgetting the inner derivative.
- **Option D:** $2\sin x\cos^2 x$ has the wrong sign and an extra factor of $\cos x$.

Final Answer: $-\sin 2x$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: The perpendicular distance of a point (x_0, y_0, z_0) from the xy-plane ($z = 0$) is simply $|z_0|$, since the z-axis is normal to that plane.

Solution:

(a) Identify the coordinates of the point: $(3, -4, 5)$.

(b) The xy-plane consists of all points with $z = 0$; the foot of the perpendicular from the point is $(3, -4, 0)$.

(c) The distance is the absolute value of the z-coordinate:

$$d = |5| = 5$$

Why other options are wrong:

- **Option A:** 3 is the distance from the yz-plane, i.e. $|x_0|$.
- **Option B:** 4 is the distance from the xz-plane, i.e. $|y_0|$.
- **Option D:** $\sqrt{50}$ is the distance from the origin, not from the plane.

Final Answer: 5

Answer: (C)

Go Back to Question 5

Q6.

Solution

Concept: Set $\theta = \cos^{-1} x$ so that $\cos\theta = x$; the remaining trigonometric ratios follow from a right triangle (or the identity $\sin\theta = \sqrt{1 - x^2}$ on the principal branch $[0, \pi]$ where sine is non-negative).

Solution:

(a) Let $\theta = \cos^{-1} \frac{3}{5}$, so:

$$\cos\theta = \frac{3}{5}$$

(b) Compute the sine on the principal branch:

$$\sin\theta = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

(c) Form the tangent:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4/5}{3/5} = \frac{4}{3}$$

Why other options are wrong:

- **Option A:** $\frac{3}{4}$ is $\cot\theta$, the reciprocal of the required tangent.
- **Option B:** $\frac{5}{4}$ is $\sec\theta \cdot \frac{3}{4} \cdot \frac{5}{3}$ — a hypotenuse-based mix-up ($\csc\theta$).
- **Option C:** $\frac{4}{5}$ is $\sin\theta$, not $\tan\theta$.

Final Answer: $\frac{4}{3}$

Answer: (D)

Go Back to Question 6

Q7.

Solution

Concept: A chord of a circle at perpendicular distance d from the centre has half-length $\sqrt{r^2 - d^2}$, by dropping a perpendicular from the centre to the chord and applying Pythagoras.

Solution:

(a) Identify the circle data: $x^2 + y^2 = 25$ has centre $(0, 0)$ and radius $r = 5$.

(b) The line $y = 3$ is horizontal; its perpendicular distance from the origin is:

$$d = 3$$

(c) Compute the half-chord:

$$\text{half-chord} = \sqrt{r^2 - d^2} = \sqrt{25 - 9} = \sqrt{16} = 4$$

(d) Double it for the full chord length:

$$\text{Chord} = 2 \times 4 = 8$$

(e) (Directly: substituting $y = 3$ gives $x^2 = 16$, $x = \pm 4$, endpoints $(\pm 4, 3)$, length 8. ✓)

Why other options are wrong:

- **Option A:** 4 is only the half-chord.
- **Option C:** 10 is the diameter, attained only by a chord through the centre.
- **Option D:** 6 doubles the distance d instead of the half-chord.

Final Answer: 8

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept: With the substitution $t = \log x$, the factor $\frac{1}{x}\,dx$ becomes dt, reducing the integrand to $\frac{1}{t}$, whose antiderivative is $\log|t|$.

Solution:

(a) Choose the substitution:

$$t = \log x \implies dt = \frac{1}{x}\,dx$$

(b) Rewrite the integral:

$$\int \frac{dx}{x \log x} = \int \frac{dt}{t}$$

(c) Integrate:

$$= \log|t| + C$$

(d) Revert to x:

$$= \log|\log x| + C$$

Why other options are wrong:

- **Option A:** $\frac{(\log x)^2}{2}$ is $\int \frac{\log x}{x} dx$ — the $\log x$ belongs in the numerator there, not the denominator.
- **Option B:** $\log x$ is $\int \frac{dx}{x}$, ignoring the extra $\log x$ factor.
- **Option D:** $\frac{1}{\log x}$ differentiates to $-\frac{1}{x(\log x)^2}$, not the integrand.

Final Answer: $\log|\log x| + C$

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept: The determinant is multiplicative: $|A^2| = |A|^2$. Setting $|A|^2$ equal to the given value produces an equation for k.

Solution:

(a) Compute the determinant of $A = \begin{pmatrix} k & 2 \\ 8 & k \end{pmatrix}$:

$$|A| = k^2 - 16$$

(b) Apply the multiplicative property:

$$|A^2| = |A|^2 = \left(k^2 - 16\right)^2 = 225$$

(c) Take square roots:

$$k^2 - 16 = \pm 15$$

(d) Solve both cases:

$$k^2 = 31 \quad \text{or} \quad k^2 = 1$$

The integer solutions come from $k^2 = 1$, giving $k = \pm 1$.

Why other options are wrong:

- **Option B:** ± 5 gives $|A| = 9$, so $|A^2| = 81 \neq 225$.
- **Option C:** ± 3 gives $|A| = -7$, so $|A^2| = 49 \neq 225$.
- **Option D:** ± 4 gives $|A| = 0$, so $|A^2| = 0 \neq 225$.

Final Answer: ± 1

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: Lagrange's identity connects the two vector products: $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$. Knowing any three of the four quantities determines the fourth.

Solution:

(a) Record the data: $|\vec{a}| = 2$, $|\vec{b}| = 5$, $|\vec{a} \times \vec{b}| = 8$.

(b) Substitute into Lagrange's identity:

$$8^2 + (\vec{a} \cdot \vec{b})^2 = 2^2 \cdot 5^2$$

(c) Simplify:

$$64 + (\vec{a} \cdot \vec{b})^2 = 100 \implies (\vec{a} \cdot \vec{b})^2 = 36$$

(d) Take the absolute value:

$$|\vec{a} \cdot \vec{b}| = 6$$

Why other options are wrong:

- **Option A:** 8 repeats the cross product magnitude without using the identity.
- **Option B:** 10 is $|\vec{a}||\vec{b}|$, attained only when the vectors are parallel.
- **Option C:** 4 comes from the arithmetic slip $100 - 64 = 16$, $\sqrt{16} = 4$.

Final Answer: 6

Answer: (D)

Go Back to Question 10

Q11.

Solution

Concept: The combination $x\,dy + y\,dx$ is an exact differential: it equals $d(xy)$ by the product rule. Integrating an exact differential immediately yields the general solution.

Solution:

(a) Recognise the left side as a product-rule expansion:

$$x\,dy + y\,dx = d(xy)$$

(b) The equation becomes:

$$d(xy) = 0$$

(c) Integrate both sides:

$$xy = C$$

(d) (Check by separation: $\frac{dy}{y} = -\frac{dx}{x} \implies \log|y| = -\log|x| + c \implies xy = C$. ✓)

Why other options are wrong:

- **Option A:** $x + y = C$ solves $dx + dy = 0$, which lacks the coefficients x and y.
- **Option B:** $x - y = C$ solves $dx - dy = 0$, again a different equation.
- **Option C:** $\frac{x}{y} = C$ solves $y\,dx - x\,dy = 0$ — the minus sign matters.

Final Answer: $xy = C$

Answer: (D)

Go Back to Question 11

Q12.

Solution

Concept: A tautology is a compound statement that is true for every assignment of truth values to its components; a contradiction is false for every assignment. The law of the excluded middle asserts that $p \vee \neg p$ is always true.

Solution:

(a) Build the truth table for $p \vee \neg p$:

(b) If p is true, then $\neg p$ is false, and $T \vee F = T$.

(c) If p is false, then $\neg p$ is true, and $F \vee T = T$.

(d) The compound statement is true in every case — it is a tautology (the law of the excluded middle).

Why other options are wrong:

- **Option B:** a contradiction is *always false*; that describes $p \wedge \neg p$, not $p \vee \neg p$.
- **Option C:** the truth table shows no case in which the statement is false.
- **Option D:** it is a well-formed compound statement with a definite truth value.

Final Answer: A tautology

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept: The range of a function is the set of all output values actually attained. For $f(x) = x^2 + 1$, analyse the minimum of the squared term and how outputs grow from there.

Solution:

(a) Note that $x^2 \geq 0$ for every real x, with equality exactly at $x = 0$.

(b) Add 1 throughout:

$$f(x) = x^2 + 1 \geq 1$$

(c) The value 1 *is* attained (at $x = 0$), and every value $y > 1$ is attained by $x = \pm\sqrt{y - 1}$.

(d) Hence the range is the closed ray:

$$[1, \infty)$$

Why other options are wrong:

- **Option A:** $\mathbb{R}$ would include negative outputs, impossible since $x^2 + 1 \geq 1$.
- **Option B:** $(1, \infty)$ wrongly excludes 1, which is attained at $x = 0$.
- **Option D:** $[0, \infty)$ includes values in $[0, 1)$ that are never attained.

Final Answer: $[1, \infty)$

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept: Equality of matrices means equality of corresponding entries. Two numbers with known sum s and product p are the roots of the quadratic $t^2 - st + p = 0$ (Vieta's relations).

Solution:

(a) Equate corresponding entries of the two matrices:

$$x + y = 5, \qquad xy = 6$$

(b) Apply Vieta's construction with sum $s = 5$ and product $p = 6$:

$$t^2 - st + p = 0$$

(c) Substitute:

$$t^2 - 5t + 6 = 0$$

(d) (Indeed $t = 2, 3$ satisfy both conditions: $2 + 3 = 5$, $2 \cdot 3 = 6$. ✓)

Why other options are wrong:

- **Option B:** $t^2 + 5t + 6 = 0$ has roots $-2, -3$ whose sum is -5, not 5.
- **Option C:** $t^2 - 6t + 5 = 0$ swaps the roles of sum and product.
- **Option D:** $t^2 - 5t - 6 = 0$ has product -6, contradicting $xy = 6$.

Final Answer: $t^2 - 5t + 6 = 0$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: The ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ is the standard ellipse translated so its centre sits at (h, k): the centre is read from the values that make each squared bracket zero.

Solution:

(a) Compare the given equation with the shifted standard form:

$$\frac{(x-2)^2}{16} + \frac{(y+3)^2}{9} = 1$$

(b) Set each bracket to zero to locate the centre:

$$x - 2 = 0 \implies x = 2, \qquad y + 3 = 0 \implies y = -3$$

(c) Hence:

$$\text{Centre} = (2, -3)$$

Why other options are wrong:

- **Option A:** $(2, 3)$ misreads $(y + 3)$ as $(y - 3)$.
- **Option B:** $(-2, 3)$ reverses both signs.
- **Option D:** $(-2, -3)$ misreads $(x - 2)$ as $(x + 2)$.

Final Answer: $(2, -3)$

Answer: (C)

Go Back to Question 15

Q16.

Solution

Concept: The slope of the tangent to $y = f(x)$ at a point is the derivative evaluated there. For a square root composite, the chain rule gives $\frac{d}{dx}\sqrt{u} = \frac{u'}{2\sqrt{u}}$.

Solution:

(a) Differentiate $y = \sqrt{4x-3}$ by the chain rule:

$$\frac{dy}{dx} = \frac{4}{2\sqrt{4x-3}} = \frac{2}{\sqrt{4x-3}}$$

(b) Evaluate at $x = 3$:

$$\left.\frac{dy}{dx}\right|_{x=3} = \frac{2}{\sqrt{12-3}} = \frac{2}{\sqrt{9}}$$

(c) Simplify:

$$= \frac{2}{3}$$

Why other options are wrong:

- **Option A:** 3 is the y-value $\sqrt{9}$ at the point, not the slope.
- **Option C:** $\frac{1}{3}$ halves once more — the factor 4 from the inner derivative was halved twice.
- **Option D:** $\frac{4}{3}$ forgets the 2 in the denominator of the square root rule.

Final Answer: $\frac{2}{3}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: By the corner point theorem, the maximum of a linear objective over a bounded feasible region is found by evaluating the objective at every vertex and selecting the largest value.

Solution:

(a) Evaluate $Z = 2x + 3y$ at each corner point:

$$Z(0, 0) = 0$$

$$Z(5, 0) = 10$$

$$Z(3, 4) = 6 + 12 = 18$$

$$Z(0, 6) = 18$$

(b) The largest value is 18, attained at both $(3, 4)$ and $(0, 6)$.

(c) (When two adjacent vertices tie, every point of the edge joining them is optimal — the maximum value itself is still 18.)

Why other options are wrong:

- **Option A:** 10 is the value at $(5, 0)$, not the maximum.
- **Option B:** 12 counts only $3y$ at $(0, 4)$ — not even a vertex.
- **Option C:** 20 is attained at no corner point of this region.

Final Answer: 18

Answer: (D)

Go Back to Question 17

Q18.

Solution

Concept: The general solution of an n-th order ordinary differential equation contains exactly n independent arbitrary constants — one constant of integration for each integration performed.

Solution:

(a) A third order differential equation involves the third derivative $\frac{d^3y}{dx^3}$ as its highest derivative.

(b) Recovering y requires three successive integrations.

(c) Each integration introduces one arbitrary constant, giving constants C_1, C_2, C_3.

(d) Hence the general solution carries exactly 3 arbitrary constants.

Why other options are wrong:

- **Option A:** 0 constants describes a particular solution, not the general one.
- **Option B:** 2 matches a second order equation.
- **Option C:** 4 matches a fourth order equation.

Final Answer: 3

Answer: (D)

Go Back to Question 18

Q19.

Solution

Concept: For any parabola of the form $y^2 = \pm 4ax$ or $x^2 = \pm 4ay$, the length of the latus rectum is $4a$ — a length, hence always positive, regardless of the direction of opening.

Solution:

(a) Compare $y^2 = -12x$ with the standard left-opening form $y^2 = -4ax$:

$$4a = 12 \implies a = 3$$

(b) The latus rectum has length:

$$4a = 12$$

(c) The negative sign only fixes the direction of opening (towards negative x); it does not affect the length.

Why other options are wrong:

- **Option B:** 3 is the focal distance a, not the latus rectum $4a$.
- **Option C:** 6 is $2a$, the semi-latus rectum doubled incorrectly.
- **Option D:** -12 is impossible — a length cannot be negative.

Final Answer: 12

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: The vector from point Q to point P is the difference of position vectors $\vec{QP} = \vec{p} - \vec{q}$ — terminal point minus initial point, computed component-wise.

Solution:

(a) Record the position vectors:

$$\vec{p} = 3\hat{i} + 2\hat{j} - \hat{k}, \qquad \vec{q} = \hat{i} - \hat{j} + 2\hat{k}$$

(b) Subtract in the correct order (terminal minus initial):

$$\vec{QP} = \vec{p} - \vec{q} = (3-1)\hat{i} + (2-(-1))\hat{j} + (-1-2)\hat{k}$$

(c) Simplify each component:

$$\vec{QP} = 2\hat{i} + 3\hat{j} - 3\hat{k}$$

Why other options are wrong:

- **Option A:** $-2\hat{i} - 3\hat{j} + 3\hat{k}$ is $\vec{PQ}$, the reverse direction.
- **Option C:** $4\hat{i} + \hat{j} + \hat{k}$ *adds* the position vectors instead of subtracting.
- **Option D:** $2\hat{i} - 3\hat{j} + 3\hat{k}$ mishandles the signs of the $\hat{j}$ and $\hat{k}$ components.

Final Answer: $2\hat{i} + 3\hat{j} - 3\hat{k}$

Answer: (B)

Go Back to Question 20

Q21.

Solution

Concept: Part (i) evaluates the derivative of e^{kx}, namely ke^{kx}, at a point. Part (ii) uses the standard logarithmic limit $\lim_{x\to 0} \frac{\log(1+kx)}{kx} = 1$, rescaled by the coefficient k.

Solution:

(a) For sub-question (i), differentiate and evaluate at $x = 0$:

$$\frac{d}{dx}e^{2x} = 2e^{2x} \implies 2e^0 = 2$$

This matches option (A).

(b) For sub-question (ii), rescale the standard limit with $k = 4$:

$$\lim_{x\to 0} \frac{\log(1+4x)}{x} = 4 \cdot \lim_{x\to 0} \frac{\log(1+4x)}{4x} = 4 \cdot 1 = 4$$

This matches option (B).

Final Answer: (i)→(A), (ii)→(B)

Go Back to Question 21

Q22.

Solution

Concept: Part (i): the dot product of component vectors is the sum of products of corresponding components. Part (ii): direction ratios are the denominators of the symmetric form, but each numerator must first be arranged as (variable − constant); a numerator like $1 - y$ hides a sign change.

Solution:

(a) For sub-question (i), multiply corresponding components and add:

$$\vec{a} \cdot \vec{b} = (3)(1) + (-2)(1) + (1)(1) = 3 - 2 + 1 = 2$$

(b) For sub-question (ii), rewrite the middle fraction in standard form:

$$\frac{1-y}{2} = \frac{-(y-1)}{2} = \frac{y-1}{-2}$$

(c) The symmetric form becomes:

$$\frac{x-2}{4} = \frac{y-1}{-2} = \frac{z+3}{5}$$

(d) Read the direction ratios from the denominators:

$$\langle 4,\ -2,\ 5 \rangle$$

Final Answer: (i) 2, (ii) $\langle 4, -2, 5 \rangle$

Go Back to Question 22

Q23.

Solution

Concept: Part (i) recalls the principal value branches of the inverse trigonometric functions: $\cos^{-1}$ uses $[0, \pi]$, while $[-\frac{\pi}{2}, \frac{\pi}{2}]$ belongs to $\sin^{-1}$ and $\tan^{-1}$. Part (ii) is the classical theorem that differentiability implies continuity (the converse fails, e.g. $|x|$).

Solution:

(a) For statement (i), the range of $\cos^{-1} x$ must cover angles whose cosines span $[-1, 1]$ injectively; that branch is $[0, \pi]$, *not* $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (on which cosine is not one-one). FALSE.

(b) For statement (ii), suppose f is differentiable at a. Then:

$$\lim_{x \to a} [f(x) - f(a)] = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \cdot (x - a) = f'(a) \cdot 0 = 0$$

(c) Hence $\lim_{x \to a} f(x) = f(a)$, which is precisely continuity at a. TRUE.

Final Answer: (i) FALSE, (ii) TRUE

Go Back to Question 23

Q24.

Solution

Concept: The inverse of the implication $p \rightarrow q$ is $\neg p \rightarrow \neg q$: negate both parts but keep the direction (contrast with the converse $q \rightarrow p$ and contrapositive $\neg q \rightarrow \neg p$). A compound statement joined by "and" splits into its two component statements.

Solution:

(a) For sub-question (i), identify p = "a number is even", q = "it is divisible by 2".

(b) Negate both while keeping the direction to form the inverse $\neg p \rightarrow \neg q$: "If a number is not even, then it is not divisible by 2."

(c) For sub-question (ii), the connective "and" joins two independent claims. The component statements are:

$$p: \sqrt{2} \text{ is irrational.} \qquad q: 2 \text{ is prime.}$$

Final Answer: (i) If a number is not even, then it is not divisible by 2. (ii) p: $\sqrt{2}$ is irrational; q: 2 is prime.

Go Back to Question 24

Q25.

Solution

Concept: Four standard limit results: factorisation of a difference of squares; the exponential limit $\lim_{x\to 0}\frac{e^x-1}{x}=1$; the half-angle limit $\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$; and continuity forcing $f(2)$ to equal the limiting value.

Solution:

(a) For sub-question (i), factorise and cancel:

$$\lim_{x\to 3}\frac{(x-3)(x+3)}{x-3}=\lim_{x\to 3}(x+3)=6$$

(b) For sub-question (ii), quote the standard exponential limit:

$$\lim_{x\to 0}\frac{e^x-1}{x}=1$$

(c) For sub-question (iii), use $1-\cos x=2\sin^2\frac{x}{2}$:

$$\lim_{x\to 0}\frac{2\sin^2(x/2)}{x^2}=2\cdot\frac{1}{4}\lim_{x\to 0}\left(\frac{\sin(x/2)}{x/2}\right)^2=\frac{1}{2}$$

(d) For sub-question (iv), continuity at $x=2$ demands:

$$f(2)=\lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}=\lim_{x\to 2}(x+2)=4$$

Final Answer: (i) 6, (ii) 1, (iii) $\frac{1}{2}$, (iv) 4

Go Back to Question 25

Q26.

Solution

Concept: Four vector basics: the unit vector $\hat{u} = \frac{\vec{v}}{|\vec{v}|}$; the triangle area $\frac{1}{2}|\vec{a} \times \vec{b}|$; the scalar triple product of the orthonormal basis $\hat{i} \cdot (\hat{j} \times \hat{k}) = 1$; and the midpoint (section) formula.

Solution:

(a) For sub-question (i), compute the magnitude and divide:

$$|3\hat{i} + 4\hat{j}| = \sqrt{9 + 16} = 5 \implies \hat{u} = \frac{3\hat{i} + 4\hat{j}}{5}$$

(b) For sub-question (ii), the parallelogram on $\vec{a}, \vec{b}$ has area $|\vec{a} \times \vec{b}|$; the triangle is half of it:

$$\text{Area} = \frac{1}{2}\left|\vec{a} \times \vec{b}\right|$$

(c) For sub-question (iii), since $\hat{j} \times \hat{k} = \hat{i}$:

$$\hat{i} \cdot (\hat{j} \times \hat{k}) = \hat{i} \cdot \hat{i} = 1$$

(d) For sub-question (iv), the ratio 1 : 1 gives the midpoint:

$$\vec{r} = \frac{\vec{a} + \vec{b}}{2}$$

Final Answer: (i) $\frac{3\hat{i}+4\hat{j}}{5}$, (ii) $\frac{1}{2}|\vec{a} \times \vec{b}|$, (iii) 1, (iv) $\frac{\vec{a}+\vec{b}}{2}$

Go Back to Question 26

Q27.

Solution

Concept: This set tests King's property of definite integrals, the polynomial requirement in the definition of degree, the odd-function integral over symmetric limits, and the rule that the order of a family's differential equation equals its number of arbitrary constants.

Solution:

(a) For statement (i), substituting $x \to a - x$ maps the interval $[0, a]$ onto itself, so $\int_0^a f(x)dx = \int_0^a f(a - x)dx$ always holds. TRUE.

(b) For statement (ii), clear the fraction by multiplying through by $\frac{dy}{dx}$:

$$\left(\frac{dy}{dx}\right)^3 + 1 = 2\frac{dy}{dx}$$

Now the equation is polynomial in the derivative, and the highest power of the highest-order derivative is 3, not 2. FALSE.

(c) For statement (iii), $\sin^3 x$ is an odd function ($\sin^3(-x) = -\sin^3 x$), so its integral over the symmetric interval $[-2, 2]$ vanishes. TRUE.

(d) For statement (iv), the family $y = Ce^x$ has *one* arbitrary constant; differentiating once eliminates it ($y' = y$), giving a first order equation. The order is 1, not 2. FALSE.

Final Answer: (i) TRUE, (ii) FALSE, (iii) TRUE, (iv) FALSE

Go Back to Question 27

Q28.

Solution

Concept: The area under $y = x$ between two ordinates is a definite integral; geometrically the region is a trapezium whose parallel sides are the ordinates and whose width is the gap between them, so the integral must agree with $\frac{1}{2}$(sum of parallel sides) × width.

Solution:

(a) For sub-question (i), evaluate $y = x$ at the ordinates:

$$y(2) = 2, \qquad y(4) = 4$$

(b) For sub-question (ii), the shaded area is:

$$\text{Area} = \int_2^4 x\,dx$$

(c) For sub-question (iii), integrate:

$$\int_2^4 x\,dx = \left[\frac{x^2}{2}\right]_2^4 = \frac{16}{2} - \frac{4}{2} = 8 - 2 = 6$$

(d) For sub-question (iv), verify with the trapezium formula (parallel sides 2 and 4, width 2):

$$\text{Area} = \frac{1}{2}(2 + 4) \times 2 = 6\checkmark$$

Final Answer: (i) 2 and 4, (ii) $\int_2^4 x\,dx$, (iii) 6, (iv) verified, 6 sq. units

Go Back to Question 28

Q29.

Solution

Concept: This case study applies the derivative toolkit to a cubic model: the first derivative gives the rate of change and critical points; the sign of the first derivative locates increasing/decreasing intervals; the second derivative test classifies each critical point.

Solution:

(a) For sub-question (i), differentiate term by term:

$$P'(t) = 6t^2 - 30t + 36 \implies \text{Option (A)}$$

(b) For sub-question (ii), set $P'(t) = 0$ and factorise:

$$6(t^2 - 5t + 6) = 0 \implies 6(t-2)(t-3) = 0 \implies t = 2,\ 3 \implies \text{Option (B)}$$

(c) For sub-question (iii), test the sign of P' between the roots: for $2 < t < 3$, $(t-2) > 0$ and $(t-3) < 0$, so $P' < 0$. The population decreases on $(2,3)$: Option (B).

(d) For sub-question (iv), differentiate again:

$$P''(t) = 12t - 30 \implies \text{Option (A)}$$

(e) For sub-question (v), evaluate the population at $t = 2$:

$$P(2) = 2(8) - 15(4) + 36(2) + 50 = 16 - 60 + 72 + 50 = 78 \implies \text{Option (A)}$$

(f) For sub-question (vi), apply the second derivative test at $t = 3$:

$$P''(3) = 36 - 30 = 6 > 0 \implies \text{local minimum} \implies \text{Option (A)}$$

Final Answer: (i) A, (ii) B, (iii) B, (iv) A, (v) A, (vi) A

Go Back to Question 29

Q30.

Solution

Concept: The quotient rule: $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$, applied with $u = x^2 + 1$ and $v = x^2 - 1$.

Solution:

(a) Identify the parts and their derivatives:

$$u = x^2 + 1 \implies u' = 2x, \qquad v = x^2 - 1 \implies v' = 2x$$

(b) Apply the quotient rule:

$$\frac{dy}{dx} = \frac{2x(x^2 - 1) - (x^2 + 1)(2x)}{(x^2 - 1)^2}$$

(c) Expand the numerator:

$$= \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2 - 1)^2} = \frac{-4x}{(x^2 - 1)^2}$$

Final Answer: $\frac{dy}{dx} = \frac{-4x}{(x^2 - 1)^2}$

Go Back to Question 30

Q31.

Solution

Concept: For a 2×2 matrix, the adjugate swaps the diagonal entries and negates the off-diagonal entries. The identity $|\operatorname{adj} A| = |A|^{n-1}$ links the two determinants for a matrix of order n.

Solution:

(a) Compute the determinant of $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$:

$$|A| = (1)(3) - (2)(4) = 3 - 8 = -5$$

(b) Form the adjugate:

$$\operatorname{adj} A = \begin{pmatrix} 3 & -2 \\ -4 & 1 \end{pmatrix}$$

(c) Compute its determinant:

$$|\operatorname{adj} A| = (3)(1) - (-2)(-4) = 3 - 8 = -5$$

(d) Verify the identity for $n = 2$:

$$|A|^{n-1} = (-5)^1 = -5 = |\operatorname{adj} A| \checkmark$$

Final Answer: $|A| = -5$, $\operatorname{adj} A = \begin{pmatrix} 3 & -2 \\ -4 & 1 \end{pmatrix}$, identity verified

Go Back to Question 31

Q32.

Solution

Concept: A function is one-one if equal outputs force equal inputs, and onto if every element of the codomain is attained. For a linear function with non-zero slope both properties follow by direct algebra.

Solution:

(a) **One-one:** suppose $f(x_1) = f(x_2)$:

$$4x_1 + 3 = 4x_2 + 3 \implies 4x_1 = 4x_2 \implies x_1 = x_2$$

Hence f is injective. ✓

(b) **Onto:** let $y \in \mathbb{R}$ be arbitrary. Solve $y = 4x + 3$ for x:

$$x = \frac{y-3}{4} \in \mathbb{R}$$

(c) Check: $f\left(\frac{y-3}{4}\right) = 4 \cdot \frac{y-3}{4} + 3 = y$. Every real y has a pre-image, so f is surjective. ✓

(d) Being both one-one and onto, f is a bijection.

Final Answer: f is one-one and onto (shown)

Go Back to Question 32

Q33.

Solution

Concept: Powers of sine are integrated by first lowering the power with the identity $\sin^2 x = \frac{1-\cos 2x}{2}$, converting the integrand into terms with known antiderivatives.

Solution:

(a) Apply the power-reduction identity:

$$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx$$

(b) Split the integral:

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos 2x\,dx$$

(c) Integrate each piece:

$$= \frac{x}{2} - \frac{1}{2}\cdot\frac{\sin 2x}{2} + C$$

(d) Combine:

$$= \frac{x}{2} - \frac{\sin 2x}{4} + C$$

Final Answer: $\frac{x}{2} - \frac{\sin 2x}{4} + C$

Go Back to Question 33

Q34.

Solution

Concept: A parabola with vertex at the origin and axis along the y-axis has the form $x^2 = 4ay$ (opening up if $a > 0$); the constant a is fixed by substituting the known point. For an ellipse, dividing by the constant brings it to standard form, after which $c^2 = a^2 - b^2$ and $e = \frac{c}{a}$.

Solution:

(a) Assume the form $x^2 = 4ay$ and substitute the point $(4, 2)$:

$$16 = 4a(2) = 8a \implies a = 2$$

(b) Write the equation:

$$x^2 = 8y$$

(c) **(OR variant)** Divide $16x^2 + 25y^2 = 400$ by 400:

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \implies a^2 = 25,\ b^2 = 16$$

(d) Compute the focal data:

$$c^2 = 25 - 16 = 9 \implies c = 3, \qquad \text{Foci} = (\pm 3, 0), \qquad e = \frac{3}{5}$$

Final Answer: $x^2 = 8y$ (OR: foci $(\pm 3, 0)$, $e = \frac{3}{5}$)

Go Back to Question 34

Q35.

Solution

Concept: The angle between two planes equals the angle between their normal vectors: $\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$.

Solution:

(a) Extract the normals:

$$\vec{n}_1 = (1, 1, 2), \qquad \vec{n}_2 = (2, -1, 1)$$

(b) Compute the dot product:

$$\vec{n}_1 \cdot \vec{n}_2 = 2 - 1 + 2 = 3$$

(c) Compute the magnitudes:

$$|\vec{n}_1| = \sqrt{1 + 1 + 4} = \sqrt{6}, \qquad |\vec{n}_2| = \sqrt{4 + 1 + 1} = \sqrt{6}$$

(d) Form the cosine:

$$\cos\theta = \frac{3}{\sqrt{6} \cdot \sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

(e) Identify the angle:

$$\theta = 60^\circ = \frac{\pi}{3}$$

Final Answer: 60°

Go Back to Question 35

Q36.

Solution

Concept: The factor x is (up to a constant) the derivative of the exponent x^2, so the substitution $t = x^2$ turns the integrand into a pure exponential.

Solution:

(a) Substitute:

$$t = x^2 \implies dt = 2x\,dx \implies x\,dx = \frac{dt}{2}$$

(b) Transform the limits: $x = 0 \Rightarrow t = 0$; $x = 1 \Rightarrow t = 1$.

(c) Rewrite and integrate:

$$\int_0^1 x\,e^{x^2}\,dx = \frac{1}{2}\int_0^1 e^t\,dt = \frac{1}{2}\left[e^t\right]_0^1$$

(d) Evaluate:

$$= \frac{1}{2}(e - 1)$$

Final Answer: $\frac{e-1}{2}$

Go Back to Question 36

Q37.

Solution

Concept: Related rates: differentiate the area formula with respect to time using the chain rule, then substitute the instantaneous values. For a square of side s, $A = s^2$.

Solution:

(a) Write the area and differentiate with respect to time t:

$$A = s^2 \implies \frac{dA}{dt} = 2s\frac{ds}{dt}$$

(b) Record the given rates and values:

$$\frac{ds}{dt} = 4 \text{ cm/min}, \qquad s = 8 \text{ cm}$$

(c) Substitute:

$$\frac{dA}{dt} = 2(8)(4) = 64$$

(d) Attach units: the area grows at 64 cm^2 per minute.

Final Answer: 64 cm^2/min

Go Back to Question 37

Q38.

Solution

Concept: Three points A, B, C are collinear when the connecting vectors $\vec{AB}$ and $\vec{AC}$ are parallel, i.e. one is a scalar multiple of the other (equivalently their cross product vanishes).

Solution:

(a) Name the position vectors:

$$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \quad \vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}, \quad \vec{c} = 5\hat{i} + 6\hat{j} + 7\hat{k}$$

(b) Form the joining vectors:

$$\vec{AB} = \vec{b} - \vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{AC} = \vec{c} - \vec{a} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

(c) Compare:

$$\vec{AC} = 3\,\vec{AB}$$

(d) Since $\vec{AC}$ is a scalar multiple of $\vec{AB}$ and both emanate from the same point A, the three points lie on one straight line. ■

Final Answer: Collinear (shown, $\vec{AC} = 3\vec{AB}$)

Go Back to Question 38

Q39.

Solution

Concept: The matrix method writes the system as $AX = B$ and solves $X = A^{-1}B$, with $A^{-1} = \frac{1}{|A|}\,\text{adj}\,A$ for the coefficient matrix.

Solution:

(a) Write the system $3x + 2y = 11$, $2x - 3y = 3$ in matrix form:

$$A = \begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix}, \qquad X = \begin{pmatrix} x \\ y \end{pmatrix}, \qquad B = \begin{pmatrix} 11 \\ 3 \end{pmatrix}$$

(b) Compute the determinant:

$$|A| = (3)(-3) - (2)(2) = -9 - 4 = -13 \neq 0$$

so a unique solution exists.

(c) Form the inverse:

$$A^{-1} = \frac{1}{-13}\begin{pmatrix} -3 & -2 \\ -2 & 3 \end{pmatrix}$$

(d) Multiply $X = A^{-1}B$:

$$X = \frac{1}{-13}\begin{pmatrix} -3(11) - 2(3) \\ -2(11) + 3(3) \end{pmatrix} = \frac{1}{-13}\begin{pmatrix} -39 \\ -13 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

(e) Verify: $3(3) + 2(1) = 11$ ✓ and $2(3) - 3(1) = 3$ ✓.

Final Answer: $x = 3,\ y = 1$

Go Back to Question 39

Q40.

Solution

Concept: Set $A = \sin^{-1}\frac{3}{5}$ and $B = \sin^{-1}\frac{8}{17}$ so the sines are known; the cosines follow from the Pythagorean identity (both positive on the principal branch). The compound angle formula $\sin(A+B) = \sin A\cos B + \cos A\sin B$ then gives the sine of the sum.

Solution:

(a) Record the sines:

$$\sin A = \frac{3}{5}, \qquad \sin B = \frac{8}{17}$$

(b) Recover the cosines:

$$\cos A = \sqrt{1-\frac{9}{25}} = \frac{4}{5}, \qquad \cos B = \sqrt{1-\frac{64}{289}} = \frac{15}{17}$$

(c) Apply the compound angle formula:

$$\sin(A+B) = \frac{3}{5}\cdot\frac{15}{17} + \frac{4}{5}\cdot\frac{8}{17}$$

(d) Simplify over the common denominator 85:

$$\sin(A+B) = \frac{45}{85} + \frac{32}{85} = \frac{77}{85}$$

(e) Both A, B lie in $\left(0, \frac{\pi}{2}\right)$ and $\cos(A+B) = \frac{4}{5}\cdot\frac{15}{17} - \frac{3}{5}\cdot\frac{8}{17} = \frac{36}{85} > 0$, so $A+B \in \left(0, \frac{\pi}{2}\right)$, inside the principal branch of $\sin^{-1}$. Therefore:

$$A + B = \sin^{-1}\frac{77}{85} \qquad \blacksquare$$

Final Answer: Proved

Go Back to Question 40

Q41.

Solution

Concept: The image of a point P in a line is found in two stages: first locate the foot of the perpendicular F by parametrising the line and imposing orthogonality with the direction vector; then use the midpoint property — F is the midpoint of P and its image P', so $P' = 2F - P$.

Solution:

(a) Parametrise a general point on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$:

$$F = (\lambda,\ 1 + 2\lambda,\ 2 + 3\lambda)$$

(b) Form the vector from $P(1, 6, 3)$ to F:

$$\vec{PF} = (\lambda - 1,\ 2\lambda - 5,\ 3\lambda - 1)$$

(c) Impose orthogonality with the direction vector $(1, 2, 3)$:

$$(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$$

(d) Expand and solve:

$$\lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0 \implies 14\lambda = 14 \implies \lambda = 1$$

(e) Substitute $\lambda = 1$ to find the foot:

$$F = (1,\ 3,\ 5)$$

(f) Apply the midpoint relation $P' = 2F - P$:

$$P' = (2 \cdot 1 - 1,\ 2 \cdot 3 - 6,\ 2 \cdot 5 - 3) = (1,\ 0,\ 7)$$

Final Answer: Image = $(1, 0, 7)$

Go Back to Question 41

Q42.

Solution

Concept: A proper rational function with distinct linear factors decomposes as $\frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$; the cover-up method evaluates each constant by substituting the corresponding root.

Solution:

(a) Set up the decomposition:

$$\frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

(b) Clear denominators:

$$x = A(x-2) + B(x-1)$$

(c) Substitute $x = 1$:

$$1 = A(1-2) \implies A = -1$$

(d) Substitute $x = 2$:

$$2 = B(2-1) \implies B = 2$$

(e) Integrate term by term:

$$\int \frac{x\,dx}{(x-1)(x-2)} = -\log|x-1| + 2\log|x-2| + C = \log\frac{(x-2)^2}{|x-1|} + C$$

(f) **(OR variant)** Let $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$. Apply King's property with $a + b = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$, using $\tan\left(\frac{\pi}{2} - x\right) = \cot x$:

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+1}\,dx$$

(g) Add the two forms: the integrands sum to 1:

$$2I = \int_{\pi/6}^{\pi/3} 1\,dx = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} \implies I = \frac{\pi}{12}$$

Final Answer: $\log\frac{(x-2)^2}{|x-1|} + C \quad$ (OR: $\frac{\pi}{12}$)

Go Back to Question 42

Q43.

Solution

Concept: A minimisation LPP with "at least" constraints has ≥ inequalities and an unbounded feasible region. The corner point method still applies, but the candidate minimum must be confirmed by checking that the half-plane with smaller objective values does not meet the feasible region.

Solution:

(a) Formulate with x kg of F_1 and y kg of F_2:

$$\text{Vitamin A: } 2x + y \geq 8, \qquad \text{Vitamin C: } x + 2y \geq 10, \qquad x, y \geq 0$$

$$\text{Minimise } Z = 50x + 70y$$

(b) Draw the boundary lines: $2x+y = 8$ through $(4, 0)$, $(0, 8)$; $x+2y = 10$ through $(10, 0)$, $(0, 5)$. The feasible region lies above both lines (unbounded).

(c) Find the intersection of the two boundaries:

$$2x + y = 8 \implies y = 8 - 2x; \quad x + 2(8 - 2x) = 10 \implies 16 - 3x = 10 \implies x = 2, \ y = 4$$

(d) Evaluate Z at the corner points $(10, 0)$, $(2, 4)$, $(0, 8)$:

$$Z(10, 0) = 500, \qquad Z(2, 4) = 100 + 280 = 380, \qquad Z(0, 8) = 560$$

(e) The smallest value is 380. Since the region is unbounded, check the open half-plane $50x + 70y < 380$: it lies entirely below at least one constraint line, sharing no point with the feasible region, so the minimum is genuine.

(f) Conclusion: the cheapest mixture uses 2 kg of F_1 and 4 kg of F_2.

Final Answer: Min cost = | 380 at $x = 2$ kg, $y = 4$ kg

Go Back to Question 43

Q44.

Solution

Concept: When a line crosses the x-axis inside the interval of integration, the area between the line and the axis must be split at the zero-crossing, taking the absolute value of the negative part. The ellipse–line problem instead subtracts the triangle under the chord from the quarter-ellipse.

Solution:

(a) Locate the zero of $y = 3x + 2$:

$$3x + 2 = 0 \implies x = -\frac{2}{3} \in [-1, 1]$$

(b) Split the area at $x = -\frac{2}{3}$, negating the sub-axis part:

$$\text{Area} = \left| \int_{-1}^{-2/3} (3x + 2)\,dx \right| + \int_{-2/3}^{1} (3x + 2)\,dx$$

(c) Evaluate the first piece with antiderivative $\frac{3x^2}{2} + 2x$:

$$\left[\frac{3x^2}{2} + 2x \right]_{-1}^{-2/3} = \left(\frac{2}{3} - \frac{4}{3} \right) - \left(\frac{3}{2} - 2 \right) = -\frac{2}{3} + \frac{1}{2} = -\frac{1}{6} \implies \left| -\frac{1}{6} \right| = \frac{1}{6}$$

(d) Evaluate the second piece:

$$\left[\frac{3x^2}{2} + 2x \right]_{-2/3}^{1} = \left(\frac{3}{2} + 2 \right) - \left(-\frac{2}{3} \right) = \frac{7}{2} + \frac{2}{3} = \frac{25}{6}$$

(e) Add the two parts:

$$\text{Area} = \frac{1}{6} + \frac{25}{6} = \frac{26}{6} = \frac{13}{3} \text{ sq. units}$$

Solution

(OR variant)

(a) The smaller region lies in the first quadrant between the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the chord $\frac{x}{3} + \frac{y}{2} = 1$ joining $(3, 0)$ and $(0, 2)$:

$$\text{Area} = \int_0^3 \left[\frac{2}{3}\sqrt{9 - x^2} - \left(2 - \frac{2x}{3}\right)\right] dx$$

(b) Evaluate the ellipse part with the standard result $\int_0^3 \sqrt{9 - x^2}\, dx = \frac{9\pi}{4}$:

$$\frac{2}{3} \cdot \frac{9\pi}{4} = \frac{3\pi}{2}$$

(c) Evaluate the line part (a right triangle of legs 3 and 2):

$$\int_0^3 \left(2 - \frac{2x}{3}\right) dx = \left[2x - \frac{x^2}{3}\right]_0^3 = 6 - 3 = 3$$

(d) Subtract:

$$\text{Area} = \frac{3\pi}{2} - 3 = \frac{3}{2}(\pi - 2) \text{ sq. units}$$

Final Answer: $\frac{13}{3}$ sq. units (OR: $\frac{3}{2}(\pi - 2)$ sq. units)

Go Back to Question 44

Q45.

Solution

Concept: The equation $\frac{dy}{dx} + P(x)y = Q(x)$ is linear first order: multiply through by the integrating factor $e^{\int P\,dx}$, after which the left side becomes the exact derivative of ($y\times$ IF), ready for direct integration; the initial condition then fixes the constant.

Solution:

(a) Identify the linear form:

$$P(x) = \frac{1}{x}, \qquad Q(x) = x^2$$

(b) Compute the integrating factor:

$$\text{IF} = e^{\int \frac{dx}{x}} = e^{\log x} = x$$

(c) Multiply the equation by x:

$$x\frac{dy}{dx} + y = x^3 \implies \frac{d}{dx}(xy) = x^3$$

(d) Integrate both sides:

$$xy = \frac{x^4}{4} + C$$

(e) Apply the initial condition $y(1) = 1$:

$$1 \cdot 1 = \frac{1}{4} + C \implies C = \frac{3}{4}$$

(f) Write the particular solution:

$$xy = \frac{x^4}{4} + \frac{3}{4} \quad \text{i.e.} \quad y = \frac{x^3}{4} + \frac{3}{4x}$$

Solution

(OR variant)

(a) Let the cone have slant height l (fixed) and semi-vertical angle θ, so radius $r = l \sin\theta$ and height $h = l \cos\theta$.

(b) Express the volume as a function of θ:

$$V = \frac{1}{3}\pi r^2 h = \frac{\pi l^3}{3} \sin^2\theta \cos\theta$$

(c) Differentiate with respect to θ:

$$\frac{dV}{d\theta} = \frac{\pi l^3}{3}\left(2\sin\theta\cos^2\theta - \sin^3\theta\right) = \frac{\pi l^3}{3}\sin\theta\left(2\cos^2\theta - \sin^2\theta\right)$$

(d) Set $\frac{dV}{d\theta} = 0$ (with $\sin\theta \neq 0$ for a genuine cone):

$$2\cos^2\theta = \sin^2\theta \implies \tan^2\theta = 2 \implies \tan\theta = \sqrt{2}$$

(e) Confirm the maximum: for $\tan\theta < \sqrt{2}$ the factor $2\cos^2\theta - \sin^2\theta > 0$ (volume rising) and for $\tan\theta > \sqrt{2}$ it is negative (volume falling), so $\theta = \tan^{-1}\sqrt{2}$ gives the maximum volume.

(f) Hence the semi-vertical angle of the cone of maximum volume is:

$$\theta = \tan^{-1}\sqrt{2} \qquad \blacksquare$$

Final Answer: $y = \frac{x^3}{4} + \frac{3}{4x}$ (OR: $\theta = \tan^{-1}\sqrt{2}$ shown)

Go Back to Question 45

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	B	4	A	5	C
6	D	7	B	8	C	9	A	10	D
11	D	12	A	13	C	14	A	15	C
16	B	17	D	18	D	19	A	20	B