NIOS Class 12 Mathematics Sample Paper – 5

Duration: 180 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **45** Questions. The paper is divided into two sections: **Section A – 50** marks, **Section B – 50** marks.
- **Section A** consists of
  -**Q.No. 1 to 20** – Multiple Choice type questions (MCQs) carrying **+1 mark** each. Select and write the most appropriate option out of the four options given in each of these questions.
  -**Q.No. 21 to 29** – **Objective type questions**.
  -**Q.No. 21 to 24** carry **02 marks** each (with 2 sub-parts of 1 mark each).
  -**Q.No. 25 to 28** carry **04 marks** each (with 4 sub-parts of 1 mark each).
  -**Q.No. 29** carries **06 marks** (with 6 sub-parts of 1 mark each). Attempt these questions as per the instructions given for each of the questions 21–29.
- **Section B** consists of
  -**Q.No. 30 to 38**– Very Short questions carrying **02 marks** each.
  -**Q.No. 39 to 43** – Short Answer type questions carrying **04 marks** each.
  -**Q.No. 44 to 45** – Long Answer type questions carrying **06 marks** each.
  (An internal choice has been provided in some of the questions in Section B. You have to attempt only one of the given choices in such questions.)
- There is **No Negative marking**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Section: A

Q1. $\lim_{x\to\pi/2} \dfrac{\cos x}{\pi/2 - x}$ equals: **(1)**

(A) 0

(B) -1

(C) 1

(D) Does not exist

Q2. If $A = \text{diag}(2, 3, 5)$, then $|A^{-1}|$ equals: **(1)**

(A) 30

(B) $\frac{1}{30}$

(C) $\frac{1}{10}$

(D) 10

Q3. The value of k for which the lines $kx + 2y = 5$ and $3x + y = 1$ are parallel is: **(1)**

(A) $\frac{3}{2}$

(B) $\frac{2}{3}$

(C) -6

(D) 6

Q4. If $x = at^2$, $y = 2at$, then $\dfrac{dy}{dx}$ equals: **(1)**

(A) t

(B) $\frac{1}{t}$

(C) $2t$

(D) $\frac{1}{2t}$

Q5. The point on the y-axis equidistant from $(2, 3, 1)$ and $(0, -1, 3)$ has y-coordinate: **(1)**

(A) $\frac{1}{2}$

(B) 2

(C) $-\frac{1}{2}$

(D) 1

Q6. $\sec^2(\tan^{-1} 2)$ equals: **(1)**

(A) 2

(B) 3

(C) 4

(D) 5

Q7. The equation of the circle whose ends of a diameter are $(1, 2)$ and $(3, -4)$ is: **(1)**

(A) $x^2 + y^2 + 4x - 2y - 5 = 0$

(B) $x^2 + y^2 - 4x + 2y + 5 = 0$

(C) $x^2 + y^2 - 2x + 4y - 5 = 0$

(D) $x^2 + y^2 - 4x + 2y - 5 = 0$

Q8. $\int_1^2 \frac{x^3 - 1}{x^2}\, dx$ equals: **(1)**

(A) 1

(B) 2

(C) $\frac{5}{2}$

(D) $\frac{3}{2}$

Q9. If $A\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}5\\4\end{pmatrix}$ where $A = \begin{pmatrix}1 & a\\2 & b\end{pmatrix}$, then (a, b) equals: **(1)**

(A) $(1, 2)$

(B) $(2, -1)$

(C) $(-2, 1)$

(D) $(2, 1)$

Q10. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then the angle between $\vec{a}$ and $\vec{b}$ is: **(1)**

(A) 30°

(B) 45°

(C) 60°

(D) 90°

Q11. The differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ is: **(1)**

(A) Homogeneous

(B) Not homogeneous

(C) Of second order

(D) Of degree two

Q12. If the truth values of p and q are T and F respectively, then the truth value of $p \rightarrow q$ is: **(1)**

(A) T

(B) F

(C) Both T and F

(D) Cannot be determined

Q13. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in \mathbb{R}$, then $(f \circ g)(7)$ equals: **(1)**

(A) 14

(B) 0

(C) 7

(D) −7

Q14. The cofactor of the element 6 in the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ is: **(1)**

(A) 6

(B) −6

(C) 22

(D) −22

Q15. The equation $\frac{x^2}{9-k} + \frac{y^2}{k-4} = 1$ represents an ellipse if: **(1)**

(A) $k < 4$

(B) $k > 9$

(C) $4 < k < 9$ and $k \neq \frac{13}{2}$

(D) $k = \frac{13}{2}$ only

Q16. The maximum value of $\sin x + \cos x$ is: **(1)**

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Q17. The region represented by the inequality $x \geq 0$, $y \geq 0$ is: **(1)**

(A) First quadrant

(B) Second quadrant

(C) Third quadrant

(D) Entire plane

Q18. The solution of $\dfrac{dy}{dx} = 2^{y-x}$ is: **(1)**

(A) $2^x + 2^y = C$

(B) $2^{-x} - 2^{-y} = C$

(C) $2^x - 2^y = C$

(D) $2^{-x} + 2^{-y} = C$

Q19. The distance between the foci of the hyperbola $\dfrac{x^2}{25} - \dfrac{y^2}{11} = 1$ is: **(1)**

(A) 6

(B) 12

(C) 10

(D) $2\sqrt{11}$

Q20. If the vectors $2\hat{i} + 3\hat{j} - 6\hat{k}$ and $4\hat{i} + m\hat{j} - 12\hat{k}$ are parallel, then m equals: **(1)**

(A) 3

(B) 4

(C) 12

(D) 6

Q21. Match Column-I with Column-II: **(2)**

Column-I	**Column-II**
(i) $\int_0^{\pi/2} \cos x \, dx$ equals	(A) **1**
(ii) $\int_0^{2} (3x^2 + 1) \, dx$ equals	(B) **10**

(A) (i)→(A), (ii)→(B)

(B) (i)→(B), (ii)→(A)

Q22. Fill in the blanks: **(2)**

(i) If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, then $\vec{a} \cdot \vec{a} =$ ________.

(ii) The plane $z = 5$ is parallel to the ________ plane.

Q23. Write TRUE or FALSE: **(2)**

(i) $\tan^{-1}(1) + \tan^{-1}(-1) = 0$.

(ii) The function $f(x) = \frac{1}{x}$ is continuous at $x = 0$.

Q24. Answer as directed: **(2)**

(i) Check whether the statement "If $3 + 4 = 8$, then 5 is odd" is true or false.

(ii) Write the negation of: "For every real number x, $x^2 \geq 0$."

Q25. Fill in the blanks (Applications of Derivatives): **(4)**

(i) The function $f(x) = e^x$ is ________ (increasing/decreasing) on $\mathbb{R}$.

(ii) The slope of the normal to $y = x^2$ at the point $(1, 1)$ is ________.

(iii) If the radius of a sphere is r, then $\frac{dV}{dr} =$ ________.

(iv) The function $f(x) = x^2 - 2x + 5$ has a local minimum at $x =$ ________.

Q26. Fill in the blanks (Matrices and Determinants): **(4)**

(i) A square matrix A is symmetric if ________.

(ii) If A is a 3×4 matrix and B is a 4×2 matrix, then the order of AB is ________.

(iii) The value of $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ is ________.

(iv) If every element of a row of a determinant is multiplied by k, the determinant is multiplied by ________.

Q27. Write TRUE or FALSE: **(4)**

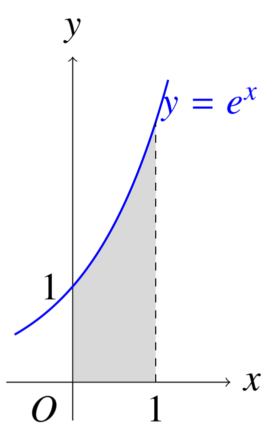
(i) The lines $\frac{x-1}{2} = \frac{y}{3} = \frac{z+1}{4}$ and $\frac{x}{4} = \frac{y-2}{6} = \frac{z}{8}$ are parallel.

(ii) The vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$ are perpendicular.

(iii) The distance of the point $(1, 2, 3)$ from the origin is 14.

(iv) A line makes angles $90°, 60°, 30°$ with the coordinate axes; then $\cos^2 90° + \cos^2 60° + \cos^2 30° = 1$.

Q28. Study the figure showing the curve $y = e^x$, the x-axis and the ordinates $x = 0$ and $x = 1$: **(4)**



(i) Write the value of y at $x = 0$ and at $x = 1$.

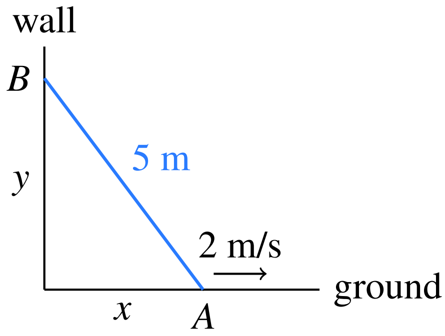
(ii) Write the definite integral representing the shaded area.

(iii) Evaluate the integral.

(iv) Is the shaded area greater than or less than 2 square units? Justify briefly.

Q29. Read and answer (i)–(vi):

A ladder AB of length 5 m leans against a vertical wall. Its foot A is pulled away from the wall along the ground at a constant rate of 2 m/s. Let x be the distance of the foot from the wall and y the height of the top of the ladder above the ground at time t. **(6)**



(i) The relation between x and y is:

(A) $x + y = 5$

(B) $x^2 + y^2 = 25$

(C) $x^2 - y^2 = 25$

(D) $xy = 25$

(ii) Differentiating the relation with respect to t gives:

(A) $x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

(B) $\frac{dx}{dt} + \frac{dy}{dt} = 0$

(C) $x\frac{dx}{dt} - y\frac{dy}{dt} = 0$

(D) $y\frac{dx}{dt} + x\frac{dy}{dt} = 0$

(iii) When $x = 3$ m, the height y is:

(A) 3 m

(B) 5 m

(C) 4 m

(D) 2 m

(iv) Given $\frac{dx}{dt} = 2$ m/s, when $x = 3$ the value of $\frac{dy}{dt}$ is:

(A) $-\frac{3}{2}$ m/s

(B) $\frac{3}{2}$ m/s

(C) $-\frac{2}{3}$ m/s

(D) -2 m/s

(v) The negative sign of $\frac{dy}{dt}$ indicates that the top of the ladder is:

(A) Sliding down the wall

(B) Rising up the wall

(C) Stationary

(D) Moving horizontally

(vi) When $x = 4$ m, the rate at which y decreases is:

(A) $\frac{8}{3}$ m/s

(B) $\frac{3}{8}$ m/s

(C) 2 m/s

(D) $\frac{4}{3}$ m/s

Section: B

Q30. Evaluate $\lim_{x \to 0} \dfrac{\sin 4x + \tan 2x}{3x}$. **(2)**

Q31. Express $A = \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$ as the sum of a symmetric and a skew-symmetric matrix. **(2)**

Q32. Find the principal value of $\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\dfrac{1}{2}\right)$. **(2)**

Q33. Evaluate $\int \tan^2 x \, dx$. **(2)**

Q34. Find the equation of the hyperbola with foci $(\pm 6, 0)$ and eccentricity $\dfrac{3}{2}$. **OR** Find the length of the major and minor axes of the ellipse $9x^2 + 4y^2 = 36$. **(2)**

Q35. Find the intercepts cut off by the plane $2x + y - z = 5$ on the coordinate axes. **(2)**

Q36. Evaluate $\int_0^{\pi/4} \frac{\sin x + \cos x}{\cos^2 x - \sin^2 x + 1}\, dx$ (Hint: put $\cos x - \sin x = t$ after simplifying, or simplify the denominator as $2\cos^2 x$). **(2)**

Q37. Verify that $y = e^{-3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$. **(2)**

Q38. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$, find a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. **(2)**

Q39. If $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$, show that $A^2 - 4A + 3I = 0$ and hence find A^{-1}. **(4)**

Q40. Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$. **(4)**

Q41. Find the vector and Cartesian equations of the line passing through the points $(3, -2, 5)$ and $(3, 1, -4)$. Also state which coordinate remains constant along the line. **(4)**

Q42. Evaluate $\int x^2 e^x\, dx$. **OR** Evaluate $\int \frac{dx}{\sqrt{9 - 4x^2}}$. **(4)**

Q43. A furniture workshop makes chairs and tables. Each chair requires 2 hours of carpentry and 1 hour of polishing; each table requires 3 hours of carpentry and 1 hour of polishing. In a week, at most 36 hours of carpentry and at most 14 hours of polishing are available. The profit is ₹ 80 per chair and ₹ 110 per table. Formulate and solve the LPP graphically to maximise the weekly profit. **(4)**

Q44. Using integration, find the area of the region bounded by the curves $y = x^2$ and $y = x$.

OR

Using integration, find the area of the region in the first quadrant enclosed by the circle $x^2 + y^2 = 32$, the line $y = x$ and the x-axis. **(6)**

Q45. Show that of all rectangles inscribed in a given fixed circle, the square has the maximum area.

OR

Solve the differential equation $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$. **(6)**

Detailed Solutions

Q1.

Solution

Concept: A substitution that recentres the limit at zero converts an unfamiliar form into the standard limit $\lim_{h\to 0}\frac{\sin h}{h} = 1$. The co-function identity $\cos\left(\frac{\pi}{2} - h\right) = \sin h$ performs the conversion.

Solution:

(a) Substitute $h = \frac{\pi}{2} - x$, so that as $x \to \frac{\pi}{2}$, $h \to 0$:

$$\lim_{x\to\pi/2} \frac{\cos x}{\frac{\pi}{2} - x} = \lim_{h\to 0} \frac{\cos\left(\frac{\pi}{2} - h\right)}{h}$$

(b) Apply the co-function identity:

$$= \lim_{h\to 0} \frac{\sin h}{h}$$

(c) Quote the standard limit:

$$= 1$$

Why other options are wrong:

- **Option A:** 0 substitutes $x = \frac{\pi}{2}$ into the numerator only, ignoring the vanishing denominator.
- **Option B:** -1 mishandles the sign: $\frac{d}{dx}\cos x = -\sin x$, but the denominator's derivative is also -1, so the signs cancel.
- **Option D:** both one-sided limits equal 1, so the limit exists.

Final Answer: 1

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept: The determinant of a diagonal matrix is the product of its diagonal entries, and determinants respect inverses: $|A^{-1}| = \frac{1}{|A|}$.

Solution:

(a) Compute the determinant of the diagonal matrix:

$$|A| = 2 \times 3 \times 5 = 30$$

(b) Apply the inverse rule:

$$|A^{-1}| = \frac{1}{|A|} = \frac{1}{30}$$

(c) (Consistency check: $A^{-1} = \text{diag}\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{5}\right)$, whose product is $\frac{1}{30}$. ✓)

Why other options are wrong:

- **Option A:** 30 is $|A|$ itself, not the determinant of the inverse.
- **Option C:** $\frac{1}{10}$ omits the factor 3 from the product.
- **Option D:** 10 is the product 2×5, again dropping an entry and forgetting the reciprocal.

Final Answer: $\frac{1}{30}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: Two lines are parallel exactly when their slopes are equal. For $ax + by = c$ the slope is $-\frac{a}{b}$; equate the two slopes and solve for the parameter.

Solution:

(a) Compute the slope of the first line $kx + 2y = 5$:

$$m_1 = -\frac{k}{2}$$

(b) Compute the slope of the second line $3x + y = 1$:

$$m_2 = -3$$

(c) Impose parallelism $m_1 = m_2$:

$$-\frac{k}{2} = -3$$

(d) Solve:

$$k = 6$$

Why other options are wrong:

- **Option A:** $\frac{3}{2}$ comes from equating $\frac{k}{2} = \frac{3}{1}$ upside down as $\frac{2}{k} = \frac{1}{3}$ wrongly rearranged.
- **Option B:** $\frac{2}{3}$ equates the reciprocal ratios of coefficients.
- **Option C:** -6 carries a sign error — both slopes are already negative.

Final Answer: 6

Answer: (D)

Go Back to Question 3

Q4.

Solution

Concept: For parametric equations $x = x(t)$, $y = y(t)$, the derivative is the ratio of parametric rates: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, provided $\frac{dx}{dt} \neq 0$.

Solution:

(a) Differentiate each coordinate with respect to t:

$$\frac{dx}{dt} = 2at, \qquad \frac{dy}{dt} = 2a$$

(b) Form the ratio:

$$\frac{dy}{dx} = \frac{2a}{2at}$$

(c) Cancel the common factor $2a$:

$$\frac{dy}{dx} = \frac{1}{t}$$

Why other options are wrong:

- **Option A:** t inverts the ratio, dividing $\frac{dx}{dt}$ by $\frac{dy}{dt}$.
- **Option C:** $2t$ is $\frac{dx}{dt}$ with $a = 1$, not the required ratio.
- **Option D:** $\frac{1}{2t}$ forgets that the factor $2a$ cancels completely.

Final Answer: $\frac{1}{t}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: A point on the y-axis has the form $(0, y, 0)$. Equidistance from two given points is imposed by equating the squared distances, which removes the radicals.

Solution:

(a) Let the required point be $P(0, y, 0)$ and equate squared distances to $(2, 3, 1)$ and $(0, -1, 3)$:

$$4 + (y - 3)^2 + 1 = 0 + (y + 1)^2 + 9$$

(b) Expand both sides:

$$5 + y^2 - 6y + 9 = y^2 + 2y + 1 + 9$$

(c) Simplify:

$$14 - 6y = 10 + 2y$$

(d) Solve:

$$4 = 8y \implies y = \frac{1}{2}$$

Why other options are wrong:

- **Option B:** 2 results from dropping the constant 4 (the x-distance) on the left.
- **Option C:** $-\frac{1}{2}$ carries a sign slip when moving $-6y$ across.
- **Option D:** 1 solves $4 = 4y$, a mis-collection of the y terms.

Final Answer: $y = \frac{1}{2}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: The Pythagorean identity $\sec^2\theta = 1 + \tan^2\theta$ evaluates $\sec^2$ of an inverse tangent without finding the angle itself: if $\theta = \tan^{-1} x$ then $\tan\theta = x$ directly.

Solution:

(a) Let $\theta = \tan^{-1} 2$, so that:

$$\tan\theta = 2$$

(b) Apply the identity:

$$\sec^2\theta = 1 + \tan^2\theta = 1 + 4$$

(c) Conclude:

$$\sec^2\left(\tan^{-1} 2\right) = 5$$

Why other options are wrong:

- **Option A:** 2 is $\tan\theta$ itself, not $\sec^2\theta$.
- **Option B:** 3 adds $1 + 2$, using $\tan\theta$ instead of $\tan^2\theta$.
- **Option C:** 4 is $\tan^2\theta$ alone, forgetting the $+1$.

Final Answer: 5

Answer: (D)

Go Back to Question 6

Q7.

Solution

Concept: The diameter-form equation of a circle with diameter endpoints (x_1, y_1) and (x_2, y_2) is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ — built on the fact that an angle in a semicircle is a right angle.

Solution:

(a) Substitute the endpoints $(1, 2)$ and $(3, -4)$ into the diameter form:

$$(x - 1)(x - 3) + (y - 2)(y + 4) = 0$$

(b) Expand the two products:

$$x^2 - 4x + 3 + y^2 + 2y - 8 = 0$$

(c) Collect terms:

$$x^2 + y^2 - 4x + 2y - 5 = 0$$

(d) (Check: centre = midpoint = $(2, -1)$, which matches $-g = 2,\ -f = -1$. ✓)

Why other options are wrong:

- **Option A:** reverses the signs of the linear terms, placing the centre at $(-2, 1)$.
- **Option B:** $+5$ as constant gives radius$^2 = 4 + 1 - 5 = 0$, a point circle.
- **Option C:** swaps the coefficients of x and y, i.e. a centre of $(1, -2)$.

Final Answer: $x^2 + y^2 - 4x + 2y - 5 = 0$

Answer: (D)

Go Back to Question 7

Q8.

Solution

Concept: Simplifying the integrand by term-wise division turns a rational integrand into a sum of power functions, each integrable by the power rule.

Solution:

(a) Divide each term of the numerator by x^2:

$$\frac{x^3 - 1}{x^2} = x - x^{-2}$$

(b) Integrate term by term:

$$\int_1^2 \left(x - x^{-2}\right) dx = \left[\frac{x^2}{2} + \frac{1}{x}\right]_1^2$$

(c) Substitute the limits:

$$= \left(2 + \frac{1}{2}\right) - \left(\frac{1}{2} + 1\right)$$

(d) Simplify:

$$= \frac{5}{2} - \frac{3}{2} = 1$$

Why other options are wrong:

- **Option B:** 2 forgets the $\frac{1}{x}$ term entirely, integrating only x incorrectly.
- **Option C:** $\frac{5}{2}$ is only the upper-limit value, with the lower limit never subtracted.
- **Option D:** $\frac{3}{2}$ is only the lower-limit value.

Final Answer: 1

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: Matrix–vector multiplication produces linear equations: each row of the matrix dotted with the column vector must equal the corresponding entry of the result.

Solution:

(a) Write out the product $A\begin{pmatrix}1\\2\end{pmatrix}$:

$$\begin{pmatrix}1 & a\\2 & b\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}1+2a\\2+2b\end{pmatrix}$$

(b) Equate to the given result $\begin{pmatrix}5\\4\end{pmatrix}$:

$$1+2a=5,\qquad 2+2b=4$$

(c) Solve each equation:

$$2a=4\implies a=2,\qquad 2b=2\implies b=1$$

(d) Hence $(a,b)=(2,1)$.

Why other options are wrong:

- **Option A:** $(1,2)$ swaps the two values.
- **Option B:** $(2,-1)$ solves $2+2b=0$ instead of $2+2b=4$.
- **Option C:** $(-2,1)$ solves $1-2a=5$, a sign error in the first row.

Final Answer: $(2,1)$

Answer: (D)

Go Back to Question 9

Q10.

Solution

Concept: When three vectors sum to zero, isolating one of them — $\vec{c} = -(\vec{a} + \vec{b})$ — and squaring magnitudes exposes the dot product: $|\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}\cdot\vec{b}$.

Solution:

(a) Rearrange the zero-sum condition and square:

$$\vec{c} = -(\vec{a} + \vec{b}) \implies |\vec{c}|^2 = |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}\cdot\vec{b}$$

(b) Substitute the magnitudes:

$$49 = 9 + 25 + 2\,\vec{a}\cdot\vec{b}$$

(c) Solve for the dot product:

$$2\,\vec{a}\cdot\vec{b} = 15 \implies \vec{a}\cdot\vec{b} = \frac{15}{2}$$

(d) Extract the cosine:

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{15/2}{15} = \frac{1}{2}$$

(e) Identify the angle:

$$\theta = 60^\circ$$

Why other options are wrong:

- **Option A:** 30° needs $\cos\theta = \frac{\sqrt{3}}{2}$, not $\frac{1}{2}$.
- **Option B:** 45° needs $\cos\theta = \frac{1}{\sqrt{2}}$.
- **Option D:** 90° needs $\vec{a}\cdot\vec{b} = 0$, but it equals $\frac{15}{2}$.

Final Answer: 60°

Answer: (C)

Go Back to Question 10

Q11.

Solution

Concept: A first order differential equation $\frac{dy}{dx} = F(x, y)$ is homogeneous when F can be written as a function of the single ratio $\frac{y}{x}$ — equivalently $F(\lambda x, \lambda y) = F(x, y)$ for every $\lambda \neq 0$.

Solution:

(a) Split the right-hand side:

$$F(x, y) = \frac{x + y}{x} = 1 + \frac{y}{x}$$

(b) The expression depends on x and y only through the ratio $\frac{y}{x}$, so it is of the form $g\left(\frac{y}{x}\right)$.

(c) Scaling check: $F(\lambda x, \lambda y) = \frac{\lambda x+\lambda y}{\lambda x} = \frac{x+y}{x} = F(x, y)$. ✓

(d) Hence the equation is homogeneous (solvable by $y = vx$).

Why other options are wrong:

- **Option B:** the scaling test above succeeds, so "not homogeneous" is false.
- **Option C:** only the first derivative appears, so the order is 1.
- **Option D:** the derivative appears to the first power; the degree is 1.

Final Answer: Homogeneous

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: The implication $p \rightarrow q$ is false in exactly one situation: a true hypothesis leading to a false conclusion. In every other combination it is true.

Solution:

(a) Record the given truth values: p is T and q is F.

(b) Consult the definition of the conditional: $p \rightarrow q$ is F precisely when p is T and q is F.

(c) This is exactly the given combination.

(d) Hence the truth value of $p \rightarrow q$ is F.

Why other options are wrong:

- **Option A:** T would require either p false or q true; neither holds.
- **Option C:** a statement has exactly one truth value at a time.
- **Option D:** the table of the conditional determines the value completely.

Final Answer: F

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: Composition applies the inner function first: $(f \circ g)(x) = f(g(x))$. Evaluate g at the point, then feed the output into f.

Solution:

(a) Evaluate the inner function at 7:

$$g(7) = 7 - 7 = 0$$

(b) Feed the result into f:

$$f(0) = 0 + 7 = 7$$

(c) Hence:

$$(f \circ g)(7) = 7$$

(d) (In fact $(f \circ g)(x) = (x - 7) + 7 = x$ for every x — the functions are mutually inverse.)

Why other options are wrong:

- **Option A:** 14 evaluates $f(7) = 14$, skipping the inner function g.
- **Option B:** 0 stops at $g(7) = 0$ without applying f.
- **Option D:** -7 applies g twice instead of g then f.

Final Answer: 7

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept: The cofactor of the entry in row i, column j is $C_{ij} = (-1)^{i+j} M_{ij}$, where the minor M_{ij} is the determinant left after deleting row i and column j.

Solution:

(a) Locate the element 6: it sits in row 2, column 3.

(b) Delete row 2 and column 3 to form the minor:

$$M_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = 8 - 14 = -6$$

(c) Attach the sign factor $(-1)^{2+3} = -1$:

$$C_{23} = (-1)(-6) = 6$$

Why other options are wrong:

- **Option B:** -6 is the raw minor without the sign factor $(-1)^{i+j}$.
- **Option C:** 22 deletes the wrong row/column pair.
- **Option D:** -22 combines both errors.

Final Answer: 6

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: The equation $\frac{x^2}{A} + \frac{y^2}{B} = 1$ represents an ellipse only when both denominators are positive and unequal (equality would give a circle, treated separately from a genuine ellipse).

Solution:

(a) Impose positivity of the first denominator:

$$9 - k > 0 \implies k < 9$$

(b) Impose positivity of the second denominator:

$$k - 4 > 0 \implies k > 4$$

(c) Intersect the conditions:

$$4 < k < 9$$

(d) Exclude the circle case $9 - k = k - 4$, i.e. $k = \frac{13}{2}$. The final condition is $4 < k < 9$ with $k \neq \frac{13}{2}$.

Why other options are wrong:

- **Option A:** $k < 4$ makes $k - 4 < 0$, so the equation is a hyperbola, not an ellipse.
- **Option B:** $k > 9$ makes $9 - k < 0$, again a hyperbola.
- **Option D:** $k = \frac{13}{2}$ gives equal denominators — a circle, the one excluded value.

Final Answer: $4 < k < 9$ and $k \neq \frac{13}{2}$

Answer: (C)

Go Back to Question 15

Q16.

Solution

Concept: Any expression $a\sin x + b\cos x$ can be rewritten as $\sqrt{a^2+b^2}\,\sin(x+\varphi)$; since the sine factor never exceeds 1, the maximum value is the amplitude $\sqrt{a^2+b^2}$.

Solution:

(a) Identify the coefficients: $a = 1, b = 1$.

(b) Compute the amplitude:

$$\sqrt{a^2+b^2} = \sqrt{1+1} = \sqrt{2}$$

(c) Rewrite:

$$\sin x + \cos x = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$$

(d) The sine factor attains 1 at $x = \frac{\pi}{4}$, so the maximum value is $\sqrt{2}$.

(e) (Calculus check: $f'(x) = \cos x - \sin x = 0$ at $x = \frac{\pi}{4}$, and $f\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$. ✓)

Why other options are wrong:

- **Option A:** 1 is the maximum of $\sin x$ or $\cos x$ alone, not of their sum.
- **Option B:** 2 adds the two individual maxima, which occur at different points.
- **Option D:** $\frac{1}{\sqrt{2}}$ is the value of each term at $x = \frac{\pi}{4}$, not their sum.

Final Answer: $\sqrt{2}$

Answer: (C)

Go Back to Question 16

Q17.

Solution

Concept: Each inequality selects a half-plane: $x \geq 0$ keeps points on or right of the y-axis, $y \geq 0$ keeps points on or above the x-axis. A system of inequalities selects the intersection of its half-planes.

Solution:

(a) The condition $x \geq 0$ describes the closed right half-plane.

(b) The condition $y \geq 0$ describes the closed upper half-plane.

(c) Their intersection contains exactly the points with both coordinates non-negative.

(d) That region is the first quadrant (including its boundary axes) — the natural domain of every standard LPP.

Why other options are wrong:

- **Option B:** the second quadrant has $x \leq 0$, violating $x \geq 0$.
- **Option C:** the third quadrant violates both inequalities.
- **Option D:** the entire plane ignores both constraints.

Final Answer: First quadrant

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: An exponential of a difference splits into a quotient, $2^{y-x} = \frac{2^y}{2^x}$, allowing separation of variables; the antiderivative of a^u is $\frac{a^u}{\log a}$.

Solution:

(a) Split the exponential and separate:

$$\frac{dy}{dx} = \frac{2^y}{2^x} \implies 2^{-y}\,dy = 2^{-x}\,dx$$

(b) Integrate both sides:

$$\int 2^{-y}dy = \int 2^{-x}dx \implies -\frac{2^{-y}}{\log 2} = -\frac{2^{-x}}{\log 2} + c$$

(c) Multiply by $-\log 2$ and absorb constants:

$$2^{-y} = 2^{-x} + c' \implies 2^{-x} - 2^{-y} = C$$

Why other options are wrong:

- **Option A:** $2^x + 2^y = C$ has positive exponents — the separation produces negative exponents.
- **Option C:** $2^x - 2^y = C$ likewise; differentiating it does not return the given equation.
- **Option D:** $2^{-x} + 2^{-y} = C$ has the wrong relative sign between the two terms.

Final Answer: $2^{-x} - 2^{-y} = C$

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept: For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the foci sit at $(\pm c, 0)$ with $c^2 = a^2 + b^2$; the distance between the two foci is $2c$.

Solution:

(a) Read the parameters:

$$a^2 = 25, \qquad b^2 = 11$$

(b) Apply the focal relation:

$$c^2 = 25 + 11 = 36 \implies c = 6$$

(c) The distance between the foci is:

$$2c = 12$$

Why other options are wrong:

- **Option A:** 6 is c, the distance of *one* focus from the centre.
- **Option C:** 10 is $2a$, the length of the transverse axis.
- **Option D:** $2\sqrt{11}$ is $2b$, the conjugate axis length.

Final Answer: 12

Answer: (B)

Go Back to Question 19

Q20.

Solution

Concept: Two vectors are parallel when their corresponding components are in a common ratio: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Solution:

(a) Set up the proportionality between $(2, 3, -6)$ and $(4, m, -12)$:

$$\frac{2}{4} = \frac{3}{m} = \frac{-6}{-12}$$

(b) The first and third ratios both equal $\frac{1}{2}$, confirming consistency.

(c) Equate the middle ratio to $\frac{1}{2}$:

$$\frac{3}{m} = \frac{1}{2} \implies m = 6$$

Why other options are wrong:

- **Option A:** 3 copies the second component unchanged, ignoring the doubling of the other components.
- **Option B:** 4 copies the first component of the second vector.
- **Option C:** 12 applies a factor of 4 instead of 2.

Final Answer: 6

Answer: (D)

Go Back to Question 20

Q21.

Solution

Concept: Part (i) evaluates a basic trigonometric definite integral with antiderivative $\sin x$. Part (ii) integrates a polynomial term by term with the power rule.

Solution:

(a) For sub-question (i):

$$\int_0^{\pi/2} \cos x \, dx = [\sin x]_0^{\pi/2} = 1 - 0 = 1$$

This matches option (A).

(b) For sub-question (ii):

$$\int_0^{2} (3x^2 + 1) \, dx = \left[x^3 + x\right]_0^2 = 8 + 2 = 10$$

This matches option (B).

Final Answer: (i)→(A), (ii)→(B)

Go Back to Question 21

Q22.

Solution

Concept: Part (i): the dot product of a vector with itself equals the square of its magnitude. Part (ii): the plane $z = c$ is a horizontal plane at height c, parallel to the plane containing the x- and y-axes.

Solution:

(a) For sub-question (i), square the components of $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$ and add:

$$\vec{a} \cdot \vec{a} = |\vec{a}|^2 = 2^2 + (-3)^2 + 1^2 = 4 + 9 + 1 = 14$$

(b) For sub-question (ii), every point of the plane $z = 5$ has the same height 5 above the plane $z = 0$.

(c) The plane $z = 0$ is by definition the xy-plane.

(d) Hence $z = 5$ is parallel to the xy-plane.

Final Answer: (i) 14, (ii) xy-plane

Go Back to Question 22

Q23.

Solution

Concept: Part (i) uses the odd-function property of the inverse tangent, $\tan^{-1}(-x) = -\tan^{-1} x$. Part (ii) applies the definition of continuity: a function must at least be *defined* at a point to be continuous there.

Solution:

(a) For statement (i), evaluate each term:

$$\tan^{-1}(1) = \frac{\pi}{4}, \qquad \tan^{-1}(-1) = -\frac{\pi}{4}$$

(b) Add them:

$$\frac{\pi}{4} + \left(-\frac{\pi}{4}\right) = 0$$

The statement is TRUE.

(c) For statement (ii), the function $f(x) = \frac{1}{x}$ is not defined at $x = 0$ (division by zero).

(d) A function undefined at a point cannot be continuous there; moreover the one-sided limits diverge to $\pm\infty$. FALSE.

Final Answer: (i) TRUE, (ii) FALSE

Go Back to Question 23

Q24.

Solution

Concept: A conditional with a false hypothesis is vacuously true, whatever the conclusion. The negation of a universal statement "for every x, $P(x)$" is the existential "there exists an x such that not $P(x)$".

Solution:

(a) For sub-question (i), examine the hypothesis: $3 + 4 = 8$ is false.

(b) By the truth table of the conditional, $\text{F} \rightarrow$ (anything) is T.

(c) Hence the statement "If $3 + 4 = 8$, then 5 is odd" is **true** (vacuously — and here the conclusion happens to be true as well).

(d) For sub-question (ii), negate the universal quantifier: "There exists a real number x such that $x^2 < 0$."

Final Answer: (i) True (false hypothesis makes the conditional true), (ii) There exists a real number x such that $x^2 < 0$.

Go Back to Question 24

Q25.

Solution

Concept: Four applications of derivatives: monotonicity from the sign of f'; normal slope as negative reciprocal of tangent slope; the derivative of the sphere volume $V = \frac{4}{3}\pi r^3$; and the vertex of an upward parabola as its minimiser.

Solution:

(a) For sub-question (i), $f'(x) = e^x > 0$ for all real x, so e^x is **increasing** on $\mathbb{R}$.

(b) For sub-question (ii), the tangent slope to $y = x^2$ at $(1, 1)$ is $y' = 2x = 2$; the normal slope is:

$$-\frac{1}{2}$$

(c) For sub-question (iii), differentiate the sphere volume:

$$V = \frac{4}{3}\pi r^3 \implies \frac{dV}{dr} = 4\pi r^2$$

(d) For sub-question (iv), set $f'(x) = 2x - 2 = 0$, giving $x = 1$ (and $f'' = 2 > 0$ confirms a minimum).

Final Answer: (i) increasing, (ii) $-\frac{1}{2}$, (iii) $4\pi r^2$, (iv) 1

Go Back to Question 25

Q26.

Solution

Concept: Four matrix–determinant facts: the definition of symmetry $A^T = A$; the order rule for products (rows of the first, columns of the second); the rotation determinant via $\cos^2\theta + \sin^2\theta = 1$; and the row-scaling property of determinants.

Solution:

(a) For sub-question (i), a square matrix is symmetric precisely when it equals its own transpose:

$$A^T = A \quad (\text{i.e. } a_{ij} = a_{ji} \text{ for all } i, j)$$

(b) For sub-question (ii), a 3×4 matrix times a 4×2 matrix yields a matrix of order:

$$3 \times 2$$

(c) For sub-question (iii), expand the determinant:

$$\cos\theta \cdot \cos\theta - (-\sin\theta) \cdot \sin\theta = \cos^2\theta + \sin^2\theta = 1$$

(d) For sub-question (iv), scaling one row by k scales the determinant by exactly:

$$k$$

Final Answer: (i) $A^T = A$, (ii) 3×2, (iii) 1, (iv) k

Go Back to Question 26

Q27.

Solution

Concept: This set tests parallelism of 3D lines via proportional direction ratios, perpendicularity via a zero dot product, the origin-distance formula $\sqrt{x^2 + y^2 + z^2}$, and the direction cosine identity $l^2 + m^2 + n^2 = 1$.

Solution:

(a) For statement (i), compare direction ratios $\langle 2, 3, 4\rangle$ and $\langle 4, 6, 8\rangle$: since $\langle 4, 6, 8\rangle = 2\langle 2, 3, 4\rangle$, the lines are parallel. TRUE.

(b) For statement (ii), compute the dot product:

$$(\hat{i} + \hat{j}) \cdot (\hat{i} - \hat{j}) = 1 - 1 = 0$$

Zero dot product means perpendicular. TRUE.

(c) For statement (iii), compute the distance from the origin:

$$\sqrt{1^2 + 2^2 + 3^2} = \sqrt{14} \neq 14$$

The claim confuses $\sqrt{14}$ with 14. FALSE.

(d) For statement (iv), evaluate the sum:

$$\cos^2 90^\circ + \cos^2 60^\circ + \cos^2 30^\circ = 0 + \frac{1}{4} + \frac{3}{4} = 1$$

The direction cosine identity is satisfied. TRUE.

Final Answer: (i) TRUE, (ii) TRUE, (iii) FALSE, (iv) TRUE

Go Back to Question 27

Q28.

Solution

Concept: The area under $y = e^x$ follows from the self-antiderivative property of the exponential function: $\int e^x dx = e^x$. The bound comparison uses the estimate $e < 3$.

Solution:

(a) For sub-question (i), evaluate the function at the ordinates:

$$y(0) = e^0 = 1, \qquad y(1) = e^1 = e \approx 2.718$$

(b) For sub-question (ii), the shaded area is:

$$\text{Area} = \int_0^1 e^x \, dx$$

(c) For sub-question (iii), integrate:

$$\int_0^1 e^x dx = [e^x]_0^1 = e - 1$$

(d) For sub-question (iv), compare with 2: since $e \approx 2.718 < 3$, we get $e - 1 \approx 1.718 < 2$. The shaded area is **less than** 2 square units.

Final Answer: (i) 1 and e, (ii) $\int_0^1 e^x dx$, (iii) $e - 1$, (iv) less than 2 (since $e < 3$)

Go Back to Question 28

Q29.

Solution

Concept: The sliding ladder is the classic related-rates model: the Pythagorean constraint links x and y; implicit differentiation with respect to time links their rates; substituting instantaneous values yields the required rate.

Solution:

(a) For sub-question (i), the ladder is the hypotenuse of the right triangle formed with the wall and ground:

$$x^2 + y^2 = 25 \implies \text{Option (B)}$$

(b) For sub-question (ii), differentiate implicitly with respect to t (the constant 25 vanishes and the factor 2 cancels):

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0 \implies \text{Option (A)}$$

(c) For sub-question (iii), substitute $x = 3$ into the constraint:

$$y = \sqrt{25 - 9} = 4 \text{ m} \implies \text{Option (C)}$$

(d) For sub-question (iv), insert $x = 3$, $y = 4$, $\frac{dx}{dt} = 2$:

$$3(2) + 4\frac{dy}{dt} = 0 \implies \frac{dy}{dt} = -\frac{6}{4} = -\frac{3}{2} \text{ m/s} \implies \text{Option (A)}$$

(e) For sub-question (v), $\frac{dy}{dt} < 0$ means the height y is decreasing — the top slides *down* the wall: Option (A).

(f) For sub-question (vi), at $x = 4$: $y = \sqrt{25 - 16} = 3$, so:

$$4(2) + 3\frac{dy}{dt} = 0 \implies \frac{dy}{dt} = -\frac{8}{3}$$

The rate of decrease is $\frac{8}{3}$ m/s: Option (A).

Final Answer: (i) B, (ii) A, (iii) C, (iv) A, (v) A, (vi) A

Go Back to Question 29

Q30.

Solution

Concept: Both standard limits $\lim_{x\to 0}\frac{\sin kx}{x} = k$ and $\lim_{x\to 0}\frac{\tan kx}{x} = k$ apply after splitting the fraction across the sum.

Solution:

(a) Split the limit across the two terms:

$$L = \lim_{x\to 0}\frac{\sin 4x}{3x} + \lim_{x\to 0}\frac{\tan 2x}{3x}$$

(b) Evaluate the first term:

$$\lim_{x\to 0}\frac{\sin 4x}{3x} = \frac{4}{3}$$

(c) Evaluate the second term:

$$\lim_{x\to 0}\frac{\tan 2x}{3x} = \frac{2}{3}$$

(d) Add:

$$L = \frac{4}{3} + \frac{2}{3} = 2$$

Final Answer: 2

Go Back to Question 30

Q31.

Solution

Concept: Every square matrix decomposes uniquely as $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$: the first bracket is symmetric, the second skew-symmetric.

Solution:

(a) Write the transpose of $A = \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$:

$$A^T = \begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}$$

(b) Form the symmetric part:

$$P = \frac{1}{2}\left(A + A^T\right) = \frac{1}{2}\begin{pmatrix} 6 & 6 \\ 6 & -2 \end{pmatrix} = \begin{pmatrix} 3 & 3 \\ 3 & -1 \end{pmatrix}$$

(c) Form the skew-symmetric part:

$$Q = \frac{1}{2}\left(A - A^T\right) = \frac{1}{2}\begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$$

(d) Verify: $P^T = P$, $Q^T = -Q$, and

$$P + Q = \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix} = A\checkmark$$

Final Answer: $A = \begin{pmatrix} 3 & 3 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$

Go Back to Question 31

Q32.

Solution

Concept: Principal values: $\sin^{-1}$ takes values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and is odd; $\cos^{-1}$ takes values in $[0, \pi]$. Evaluate each term separately, then add.

Solution:

(a) Evaluate the first term using oddness:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\frac{\sqrt{3}}{2} = -\frac{\pi}{3}$$

(b) Evaluate the second term:

$$\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$$

(c) Add:

$$-\frac{\pi}{3} + \frac{\pi}{3} = 0$$

Final Answer: 0

Go Back to Question 32

Q33.

Solution

Concept: The identity $\tan^2 x = \sec^2 x - 1$ converts the integrand into terms whose antiderivatives are known: $\int \sec^2 x\, dx = \tan x$ and $\int 1\, dx = x$.

Solution:

(a) Substitute the identity:

$$\int \tan^2 x\, dx = \int \left(\sec^2 x - 1\right) dx$$

(b) Split and integrate:

$$= \int \sec^2 x\, dx - \int 1\, dx$$

(c) Write the antiderivatives:

$$= \tan x - x + C$$

Final Answer: $\tan x - x + C$

Go Back to Question 33

Q34.

Solution

Concept: For a horizontal hyperbola, foci $(\pm c, 0)$ give c; the eccentricity gives $a = \frac{c}{e}$; then $b^2 = c^2 - a^2$. For an ellipse in non-standard form, divide by the constant to read a^2 and b^2; the axes have lengths $2a$ (major) and $2b$ (minor).

Solution:

(a) From the foci $(\pm 6, 0)$: $c = 6$. From $e = \frac{3}{2}$:

$$a = \frac{c}{e} = \frac{6}{3/2} = 4 \implies a^2 = 16$$

(b) Compute b^2:

$$b^2 = c^2 - a^2 = 36 - 16 = 20$$

(c) Write the hyperbola:

$$\frac{x^2}{16} - \frac{y^2}{20} = 1$$

(d) **(OR variant)** Divide $9x^2 + 4y^2 = 36$ by 36:

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

Here $9 > 4$, so the major axis is vertical with $a^2 = 9$, $b^2 = 4$:

$$\text{Major axis} = 2a = 6, \qquad \text{Minor axis} = 2b = 4$$

Final Answer: $\frac{x^2}{16} - \frac{y^2}{20} = 1$ (OR: major = 6, minor = 4)

Go Back to Question 34

Q35.

Solution

Concept: Dividing a plane's equation by its constant term brings it to intercept form $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, where a, b, c are the intercepts on the three axes.

Solution:

(a) Divide $2x + y - z = 5$ throughout by 5:

$$\frac{2x}{5} + \frac{y}{5} - \frac{z}{5} = 1$$

(b) Rewrite each term in intercept form:

$$\frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1$$

(c) Read the intercepts:

$$x\text{-intercept} = \frac{5}{2}, \qquad y\text{-intercept} = 5, \qquad z\text{-intercept} = -5$$

(d) (Check: the points $\left(\frac{5}{2}, 0, 0\right)$, $(0, 5, 0)$, $(0, 0, -5)$ all satisfy the original equation. ✓)

Final Answer: $\frac{5}{2}$, 5, −5 on the x-, y-, z-axes

Go Back to Question 35

Q36.

Solution

Concept: Simplify before integrating: the denominator $\cos^2 x - \sin^2 x + 1$ collapses via $1 - \sin^2 x = \cos^2 x$ into $2\cos^2 x$, splitting the integrand into $\sec x \tan x$ and $\sec^2 x$ terms with known antiderivatives.

Solution:

(a) Simplify the denominator:

$$\cos^2 x - \sin^2 x + 1 = \cos^2 x + (1 - \sin^2 x) = 2\cos^2 x$$

(b) Split the integrand:

$$\frac{\sin x + \cos x}{2\cos^2 x} = \frac{1}{2}\left(\sec x \tan x + \sec^2 x\right)$$

(c) Integrate with standard antiderivatives:

$$I = \frac{1}{2}\left[\sec x + \tan x\right]_0^{\pi/4}$$

(d) Substitute the limits ($\sec\frac{\pi}{4} = \sqrt{2}$, $\tan\frac{\pi}{4} = 1$, $\sec 0 = 1$, $\tan 0 = 0$):

$$I = \frac{1}{2}\left[(\sqrt{2} + 1) - (1 + 0)\right] = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

Final Answer: $\frac{1}{\sqrt{2}}$

Go Back to Question 36

Q37.

Solution

Concept: Verifying a solution means substituting the proposed function and its derivatives into the differential equation and confirming the identity holds for all x.

Solution:

(a) Compute the derivatives of $y = e^{-3x}$:

$$\frac{dy}{dx} = -3e^{-3x}, \qquad \frac{d^2y}{dx^2} = 9e^{-3x}$$

(b) Substitute into the left side of the equation:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 9e^{-3x} - 3e^{-3x} - 6e^{-3x}$$

(c) Combine the coefficients:

$$= (9 - 3 - 6)e^{-3x} = 0$$

(d) The equation is satisfied identically, so $y = e^{-3x}$ is a solution. ■

Final Answer: Verified

Go Back to Question 37

Q38.

Solution

Concept: The cross product $\vec{a}\times\vec{b}$ is perpendicular to both factors; dividing it by its own magnitude produces the required unit vector $\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$.

Solution:

(a) Compute the cross product:

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix}$$

(b) Expand along the first row:

$$= \hat{i}(3-(-1)) - \hat{j}(3-2) + \hat{k}(-1-2) = 4\hat{i} - \hat{j} - 3\hat{k}$$

(c) Compute its magnitude:

$$|\vec{a}\times\vec{b}| = \sqrt{16+1+9} = \sqrt{26}$$

(d) Divide to obtain the unit vector:

$$\hat{n} = \frac{4\hat{i} - \hat{j} - 3\hat{k}}{\sqrt{26}}$$

(The negative of this vector is equally valid.)

Final Answer: $\pm\frac{4\hat{i} - \hat{j} - 3\hat{k}}{\sqrt{26}}$

Go Back to Question 38

Q39.

Solution

Concept: Once a matrix satisfies a polynomial identity such as $A^2 - 4A + 3I = 0$, multiplying through by A^{-1} converts the identity into an explicit formula for the inverse — no adjugate computation needed.

Solution:

(a) Compute A^2 for $A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$:

$$A^2 = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{pmatrix} = \begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix}$$

(b) Compute $4A$ and assemble the polynomial:

$$A^2 - 4A + 3I = \begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix} - \begin{pmatrix} 8 & -4 \\ -4 & 8 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

(c) Combine entries:

$$= \begin{pmatrix} 5-8+3 & -4+4+0 \\ -4+4+0 & 5-8+3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \checkmark$$

(d) Multiply the identity by A^{-1}:

$$A - 4I + 3A^{-1} = 0 \implies A^{-1} = \frac{1}{3}(4I - A)$$

(e) Evaluate:

$$A^{-1} = \frac{1}{3}\begin{pmatrix} 4-2 & 0+1 \\ 0+1 & 4-2 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

(f) Verify: $AA^{-1} = \frac{1}{3}\begin{pmatrix} 4-1 & 2-2 \\ -2+2 & -1+4 \end{pmatrix} = I.$ ✓

Final Answer: $A^{-1} = \frac{1}{3}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

Go Back to Question 39

Q40.

Solution

Concept: The doubling formula $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$ (valid for $|x| < 1$) collapses the doubled term; a second application of the addition formula $\tan^{-1}u + \tan^{-1}v = \tan^{-1}\frac{u+v}{1-uv}$ $(uv < 1)$ completes the sum.

Solution:

(a) Apply the doubling formula with $x = \frac{1}{2}$ (note $|x| < 1$):

$$2\tan^{-1}\frac{1}{2} = \tan^{-1}\frac{2\cdot\frac{1}{2}}{1-\frac{1}{4}} = \tan^{-1}\frac{1}{3/4} = \tan^{-1}\frac{4}{3}$$

(b) Add the remaining term via the addition formula with $u = \frac{4}{3}$, $v = \frac{1}{7}$ (here $uv = \frac{4}{21} < 1$):

$$\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}}$$

(c) Simplify the numerator and denominator:

$$\frac{4}{3}+\frac{1}{7} = \frac{28+3}{21} = \frac{31}{21}, \qquad 1-\frac{4}{21} = \frac{17}{21}$$

(d) Form the ratio:

$$\tan^{-1}\frac{31/21}{17/21} = \tan^{-1}\frac{31}{17}$$

(e) Hence $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$. ■

Final Answer: Proved

Go Back to Question 40

Q41.

Solution

Concept: The line through points A and B has vector equation $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$. When one direction ratio is zero, the corresponding coordinate stays constant and the Cartesian form records that coordinate separately.

Solution:

(a) Compute the direction vector from $A(3, -2, 5)$ to $B(3, 1, -4)$:

$$\vec{b} - \vec{a} = (3 - 3,\ 1 - (-2),\ -4 - 5) = (0,\ 3,\ -9)$$

(b) Write the vector equation:

$$\vec{r} = (3\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda\,(3\hat{j} - 9\hat{k})$$

(c) For the Cartesian form, the zero x-direction ratio forces x to stay fixed:

$$x = 3, \qquad \frac{y+2}{3} = \frac{z-5}{-9} \quad \left(\text{equivalently } \frac{y+2}{1} = \frac{z-5}{-3}\right)$$

(d) The x-coordinate remains constant (every point of the line has $x = 3$), because the line is parallel to the yz-plane.

Final Answer: $\vec{r} = (3\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(3\hat{j} - 9\hat{k})$; $x = 3$, $\frac{y+2}{1} = \frac{z-5}{-3}$; x stays constant

Go Back to Question 41

Q42.

Solution

Concept: Repeated integration by parts reduces the power of x one step at a time in $\int x^2 e^x\,dx$; the alternative integral is a standard form $\int \frac{dx}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$ after factoring the coefficient of x^2.

Solution:

(a) First application with $u = x^2$, $dv = e^x\,dx$:

$$\int x^2 e^x\,dx = x^2 e^x - 2\int x e^x\,dx$$

(b) Second application with $u = x$, $dv = e^x\,dx$:

$$\int x e^x\,dx = x e^x - \int e^x\,dx = x e^x - e^x$$

(c) Substitute back:

$$\int x^2 e^x\,dx = x^2 e^x - 2\left(x e^x - e^x\right)$$

(d) Factor:

$$= e^x\left(x^2 - 2x + 2\right) + C$$

(e) Check by differentiating: $e^x(x^2 - 2x + 2) + e^x(2x - 2) = e^x x^2$. ✓

(f) **(OR variant)** Factor the coefficient inside the root:

$$\int \frac{dx}{\sqrt{9-4x^2}} = \int \frac{dx}{2\sqrt{\frac{9}{4}-x^2}} = \frac{1}{2}\sin^{-1}\left(\frac{x}{3/2}\right) + C = \frac{1}{2}\sin^{-1}\frac{2x}{3} + C$$

Final Answer: $e^x(x^2 - 2x + 2) + C$ (OR: $\frac{1}{2}\sin^{-1}\frac{2x}{3} + C$)

Go Back to Question 42

Q43.

Solution

Concept: Translate each weekly resource into a linear constraint, then apply the corner point method on the bounded feasible region: the maximum profit occurs at one of its vertices.

Solution:

(a) Let x = chairs and y = tables per week. The constraints are:

$$\text{Carpentry: } 2x + 3y \le 36, \qquad \text{Polishing: } x + y \le 14, \qquad x, y \ge 0$$

$$\text{Maximise } Z = 80x + 110y$$

(b) Draw the boundary lines: $2x + 3y = 36$ through $(18, 0), (0, 12)$; $x + y = 14$ through $(14, 0), (0, 14)$.

(c) Find their intersection by elimination:

$$2x + 3y = 36, \quad 2x + 2y = 28 \implies y = 8, \; x = 6$$

(d) List the corner points of the feasible region:

$$(0, 0), \quad (14, 0), \quad (6, 8), \quad (0, 12)$$

(e) Evaluate Z at each vertex:

$$Z(0, 0) = 0, \quad Z(14, 0) = 1120, \quad Z(6, 8) = 480 + 880 = 1360, \quad Z(0, 12) = 1320$$

(f) The maximum is at $(6, 8)$: make 6 chairs and 8 tables for a weekly profit of ₹ 1360.

Final Answer: Max profit = | 1360 at $x = 6$ chairs, $y = 8$ tables

Go Back to Question 43

Q44.

Solution

Concept: The area between two intersecting curves is $\int_a^b (\text{upper} - \text{lower})\,dx$ between their intersection points. In the circle–line problem, the first-quadrant region splits naturally at the intersection point into a triangle part (under $y = x$) and a circular part (under the arc).

Solution:

(a) Find the intersections of $y = x^2$ and $y = x$:

$$x^2 = x \implies x(x-1) = 0 \implies x = 0,\ 1$$

(b) Determine the upper curve on $(0, 1)$: at $x = \frac{1}{2}$, the line gives $\frac{1}{2}$ and the parabola $\frac{1}{4}$, so $y = x$ is on top.

(c) Set up and evaluate the integral:

$$\text{Area} = \int_0^1 \left(x - x^2\right) dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \text{ sq. units}$$

Solution

(OR variant)

(a) Find where the line $y = x$ meets the circle $x^2 + y^2 = 32$ in the first quadrant:

$$2x^2 = 32 \implies x = 4 \implies \text{intersection } (4, 4)$$

(b) Split the region at $x = 4$: from 0 to 4 the boundary above is the line $y = x$; from 4 to $\sqrt{32} = 4\sqrt{2}$ it is the circular arc $y = \sqrt{32 - x^2}$:

$$\text{Area} = \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$$

(c) Evaluate the first piece:

$$\int_0^4 x\,dx = \left[\frac{x^2}{2}\right]_0^4 = 8$$

(d) Evaluate the second piece with $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$, $a = 4\sqrt{2}$:

$$\left[\frac{x}{2}\sqrt{32 - x^2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}} = \left(0 + 16 \cdot \frac{\pi}{2}\right) - \left(8 + 16 \cdot \frac{\pi}{4}\right) = 8\pi - 8 - 4\pi = 4\pi - 8$$

(e) Add the two pieces:

$$\text{Area} = 8 + 4\pi - 8 = 4\pi \text{ sq. units}$$

Final Answer: $\frac{1}{6}$ sq. units (OR: 4π sq. units)

Go Back to Question 44

Q45.

Solution

Concept: Constrained optimisation: for a rectangle inscribed in a circle of radius r, the diagonal equals the diameter, linking the two sides. Maximising the area (or equivalently its square) with the first and second derivative tests identifies the optimal shape.

Solution:

(a) Let the rectangle have sides x and y, inscribed in a circle of fixed radius r. The diagonal is the diameter:

$$x^2 + y^2 = 4r^2 \implies y = \sqrt{4r^2 - x^2}$$

(b) Express the area as a single-variable function:

$$A(x) = x\sqrt{4r^2 - x^2}, \qquad 0 < x < 2r$$

(c) Work with the square for cleaner algebra: $S(x) = A^2 = x^2\left(4r^2 - x^2\right) = 4r^2x^2 - x^4$.

(d) Differentiate and set to zero:

$$S'(x) = 8r^2x - 4x^3 = 4x\left(2r^2 - x^2\right) = 0 \implies x = r\sqrt{2}$$

(e) Confirm the maximum:

$$S''(x) = 8r^2 - 12x^2 \implies S''\left(r\sqrt{2}\right) = 8r^2 - 24r^2 = -16r^2 < 0 \checkmark$$

(f) Compute the other side:

$$y = \sqrt{4r^2 - 2r^2} = r\sqrt{2} = x$$

Since $x = y$, the rectangle of maximum area is a square (of area $2r^2$). ■

Solution

(OR variant)

(a) The equation $x\frac{dy}{dx} = y - x\tan\frac{y}{x}$ is homogeneous (right side depends on $\frac{y}{x}$ only). Substitute $y = vx$, so $\frac{dy}{dx} = v + x\frac{dv}{dx}$:

$$x\left(v + x\frac{dv}{dx}\right) = vx - x\tan v$$

(b) Simplify:

$$vx + x^2\frac{dv}{dx} = vx - x\tan v \implies x\frac{dv}{dx} = -\tan v$$

(c) Separate the variables:

$$\cot v\, dv = -\frac{dx}{x}$$

(d) Integrate both sides:

$$\log|\sin v| = -\log|x| + \log C$$

(e) Combine the logarithms:

$$x\sin v = C$$

(f) Revert to the original variables with $v = \frac{y}{x}$:

$$x\sin\frac{y}{x} = C$$

Final Answer: Square of side $r\sqrt{2}$ maximises the area (shown) (OR: $x\sin\frac{y}{x} = C$)

Go Back to Question 45

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	B	3	D	4	B	5	A
6	D	7	D	8	A	9	D	10	C
11	A	12	B	13	C	14	A	15	C
16	C	17	A	18	B	19	B	20	D