

NIOS Class 12 Mathematics Sample Paper – 6

Duration: 180 Minutes

Maximum Marks: 100

Instructions

- This paper contains **45** Questions. The paper is divided into two sections: **Section A – 50** marks, **Section B – 50** marks.
- **Section A** consists of
 - Q.No. 1 to 20** – Multiple Choice type questions (MCQs) carrying **+1** mark each. Select and write the most appropriate option out of the four options given in each of these questions.
 - Q.No. 21 to 29** – **Objective type questions.**
 - Q.No. 21 to 24** carry **02** marks each (with 2 sub-parts of 1 mark each).
 - Q.No. 25 to 28** carry **04** marks each (with 4 sub-parts of 1 mark each).
 - Q.No. 29** carries **06** marks (with 6 sub-parts of 1 mark each). Attempt these questions as per the instructions given for each of the questions 21–29.
- **Section B** consists of
 - Q.No. 30 to 38**– Very Short questions carrying **02** marks each.
 - Q.No. 39 to 43** – Short Answer type questions carrying **04** marks each.
 - Q.No. 44 to 45** – Long Answer type questions carrying **06** marks each. (An internal choice has been provided in some of the questions in Section B. You have to attempt only one of the given choices in such questions.)
- There is **No Negative marking**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Section: A

Q1. If A is a square matrix of order 3 such that $|A| = 5$, then the value of $|2\text{adj}(A)|$ is: **(1)**

(A) 40

(B) 200



(C) 100

(D) 50

Q2. The domain of the function $f(x) = \sin^{-1} \sqrt{x-1}$ is: (1)

(A) $[0, 1]$

(B) $[1, 2]$

(C) $[-1, 1]$

(D) $[1, \infty)$

Q3. The value of $\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{x}$ is equal to: (1)

(A) 0

(B) 1

(C) e

(D) Does not exist

Q4. The value of $\int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx$ is: (1)

(A) π

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) 0

Q5. The direction cosines of a line passing through the origin and equally inclined to the positive direction of coordinate axes are: (1)

(A) $\langle 1, 1, 1 \rangle$

(B) $\langle \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \rangle$

(C) $\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

(D) $\langle 0, 0, 0 \rangle$

Q6. If the equations of two lines are $\vec{r} = \vec{a}_1 + \lambda \vec{b}$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}$, then these lines are always: (1)



- (A) Perpendicular
- (B) Intersecting
- (C) Parallel
- (D) Skew

Q7. The distance of the point $(2, 3, 4)$ from the x -axis is: **(1)**

- (A) 2
- (B) 5
- (C) $\sqrt{13}$
- (D) 4

Q8. The slope of the tangent to the curve $y = x^3 - x$ at $x = 2$ is: **(1)**

- (A) 11
- (B) 12
- (C) 10
- (D) 6

Q9. The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is: **(1)**

- (A) 5
- (B) $\sqrt{13}$
- (C) 12
- (D) 25

Q10. The focus of the parabola $y^2 = -16x$ is located at the point: **(1)**

- (A) $(4, 0)$
- (B) $(-4, 0)$
- (C) $(0, 4)$
- (D) $(0, -4)$



- Q11.** The integrating factor (I.F.) for the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is: **(1)**
- (A) e^x
 (B) $\log x$
 (C) x
 (D) $\frac{1}{x}$
- Q12.** If \vec{a} and \vec{b} are two perpendicular vectors such that $|\vec{a}| = 3$ and $|\vec{b}| = 4$, then $|\vec{a} + \vec{b}|$ equals: **(1)**
- (A) 7
 (B) 1
 (C) 5
 (D) 12
- Q13.** Let R be a relation on the set of natural numbers \mathbb{N} defined by $xRy \iff x + 2y = 8$. The domain of R is: **(1)**
- (A) $\{2, 4, 6\}$
 (B) $\{1, 2, 3, 4\}$
 (C) $\{2, 4, 6, 8\}$
 (D) $\{1, 3, 5\}$
- Q14.** The angle between the lines $x - 2y + 5 = 0$ and $2x + y - 3 = 0$ is: **(1)**
- (A) 0°
 (B) 45°
 (C) 90°
 (D) 60°
- Q15.** If A and B are symmetric matrices of the same order, then $AB - BA$ is always a: **(1)**
- (A) Symmetric matrix



- (B) Skew-symmetric matrix
- (C) Identity matrix
- (D) Zero matrix

Q16. The corner points of the feasible region for an LPP are $(0, 2), (3, 0), (6, 0), (0, 5)$.
The maximum value of $Z = 4x + 3y$ occurs at: **(1)**

- (A) $(0, 5)$
- (B) $(6, 0)$
- (C) $(3, 0)$
- (D) $(0, 2)$

Q17. Which of the following is the contrapositive of the statement: "If it rains, then the game is cancelled"? **(1)**

- (A) If it does not rain, then the game is not cancelled.
- (B) If the game is cancelled, then it rains.
- (C) If the game is not cancelled, then it does not rain.
- (D) It rains if and only if the game is cancelled.

Q18. The value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$ is: **(1)**

- (A) $\frac{\pi}{12}$
- (B) $\frac{11\pi}{12}$
- (C) $\frac{5\pi}{12}$
- (D) $\frac{3\pi}{4}$

Q19. The eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is: **(1)**

- (A) $\frac{3}{5}$
- (B) $\frac{4}{5}$
- (C) $\frac{9}{25}$
- (D) $\frac{5}{3}$



Q20. If $y = \log(\sec x + \tan x)$, then $\frac{dy}{dx}$ is equal to: (1)

- (A) $\sec x$
- (B) $\tan x$
- (C) $\sec x + \tan x$
- (D) $\frac{1}{\sec x + \tan x}$

Q21. Match the properties in Column-I with their correct mathematical values in Column-II: (2)

Column-I	Column-II
(i) For a 3×3 skew-symmetric matrix M , the value of $\det(M)$ is	(A) 0
(ii) If vectors $\vec{p} = \hat{i} + \alpha\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$ are orthogonal, then the value of α is	(B) 2

- (A) (i) \rightarrow (A), (ii) \rightarrow (B)
- (B) (i) \rightarrow (B), (ii) \rightarrow (A)

Q22. Answer the following fill-in-the-blank items: (2)

- (i) The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y = 0$ is _____.
- (ii) The distance between the parallel planes $2x + y + 2z = 8$ and $2x + y + 2z + 4 = 0$ is _____ units.

Q23. Determine the truth value of the statements below: (2)

- (i) Every identity relation on a non-empty set is also an equivalence relation.
- (ii) The function $f(x) = |x|$ is differentiable at $x = 0$.

Q24. Write down the logical negation for the following sentences: (2)

- (i) $\sqrt{5}$ is a rational number.



(ii) All prime numbers are odd integers.

Q25. Fill in the blanks with appropriate answers for the following calculus limits properties: (4)

- (i) $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} = \underline{\hspace{2cm}}$.
- (ii) $\lim_{x \rightarrow 0} (1 + x)^{1/x} = \underline{\hspace{2cm}}$.
- (iii) $\lim_{x \rightarrow \infty} \frac{2x+1}{3x-2} = \underline{\hspace{2cm}}$.
- (iv) $\lim_{x \rightarrow 1} \frac{x^5-1}{x-1} = \underline{\hspace{2cm}}$.

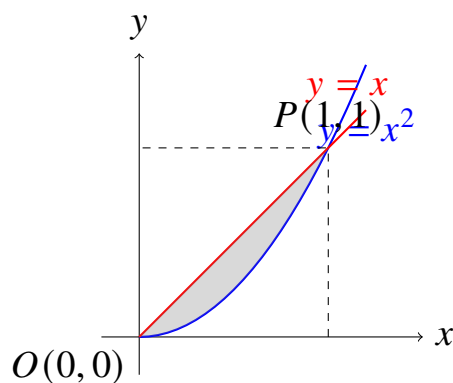
Q26. Fill in the blanks with coordinates relating to the Conic Sections properties: (4)

- (i) The coordinates of the center of the circle $x^2 + y^2 + 4x - 8y = 0$ are $\underline{\hspace{2cm}}$.
- (ii) The length of the latus rectum of the parabola $x^2 = 12y$ is $\underline{\hspace{2cm}}$.
- (iii) The length of the major axis of the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ is $\underline{\hspace{2cm}}$.
- (iv) The equation of the directrix of the parabola $y^2 = 8x$ is $\underline{\hspace{2cm}}$.

Q27. Write TRUE or FALSE for the following properties of Vectors and Matrices: (4)

- (i) If A is an invertible matrix, then $(A^{-1})^{-1} = A$.
- (ii) The scalar triple product $[\vec{a}, \vec{b}, \vec{c}]$ is zero if two vectors are parallel.
- (iii) Matrix multiplication is always commutative for any two square matrices.
- (iv) Projection of vector \hat{i} on \hat{j} is equal to 1.

Q28. Study the geometric representation below showing the line $y = x$ bounding an area with the curve $y = x^2$ in the first quadrant, then answer the following 4 questions: (4)



- (i) Find the x -coordinate of the non-zero point of intersection P .
- (ii) Write down the expression for the upper bounding function $f(x)$ within the interval $[0, 1]$.
- (iii) Set up the definite integral required to evaluate the shaded area.
- (iv) Calculate the final numeric value of the shaded area in square units.

Q29. Read the passage given below and answer the six sub-questions (i to vi) that follow:

An optimization analysis is conducted for a manufacturing plant producing two products under a linear programming model. Let x be the units of Product A and y be the units of Product B. The constraints governing production resources are given by:

$$x + 2y \leq 10, \quad 3x + y \leq 15, \quad x \geq 0, \quad y \geq 0$$

The objective profit function to maximize is designed as $Z = 3x + 2y$. **(6)**

- (i) Which of the following represents a non-negativity constraint in this LPP system?
 - (A) $x + 2y \leq 10$
 - (B) $3x + y \leq 15$
 - (C) $x \geq 0$
 - (D) $Z = 3x + 2y$
- (ii) The boundary corner point produced by the intersection of the two main resource lines $x + 2y = 10$ and $3x + y = 15$ is:
 - (A) (4, 3)
 - (B) (3, 4)
 - (C) (2, 4)
 - (D) (1, 2)
- (iii) The corner point lying on the positive y -axis bounding the feasible region is:
 - (A) (0, 15)
 - (B) (0, 5)
 - (C) (10, 0)



(D) (5, 0)

(iv) The value of the objective function Z evaluated at the origin (0, 0) is:

(A) 15

(B) 10

(C) 0

(D) 18

(v) Evaluate Z at the intersection point found in sub-question (ii). Its value is:

(A) 17

(B) 18

(C) 20

(D) 14

(vi) The maximum value achieved by the profit objective function Z over the entire valid region is:

(A) 15

(B) 17

(C) 18

(D) 10

Section: B

Q30. Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3x + 5$ is a bijective function. (2)

Q31. Find the value of k if the line $\frac{x-1}{2} = \frac{y-2}{k} = \frac{z+1}{-3}$ is perpendicular to the line $\frac{x+2}{3} = \frac{y-1}{2} = \frac{z}{2}$. (2)

Q32. Differentiate $\log(\sin x^2)$ with respect to x . (2)

Q33. Form the differential equation of the family of concentric circles centered at the origin, $x^2 + y^2 = r^2$, where r is an arbitrary constant. (2)



Q34. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

OR

Find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 5\hat{k}$. (2)

Q35. Evaluate the integral: $\int \frac{1}{x(1+\log x)} dx$. (2)

Q36. Find the equation of the normal to the curve $y = 2x^2 + 3 \sin x$ at $x = 0$. (2)

Q37. Find the area of a triangle whose two adjacent sides are represented by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$. (2)

Q38. Find the Cartesian equation of the plane passing through the point (1, 2, 3) and perpendicular to the vector $2\hat{i} + 3\hat{j} - \hat{k}$. (2)

Q39. Solve the following system of linear equations using Cramer’s Rule or Matrix Method:

$$2x + 3y = 8$$

$$3x - 2y = -1$$

(4)

Q40. Prove that $\tan^{-1} \left(\frac{1}{2} \right) + \tan^{-1} \left(\frac{1}{3} \right) = \frac{\pi}{4}$. (4)

Q41. Find the equations of the straight line passing through the point (1, -1, 2) and parallel to the planes $3x + 2y - z = 5$ and $x - y + 2z = 3$. (4)

Q42. Evaluate the indefinite integral: $\int \frac{2x+3}{x^2+3x+2} dx$.

OR

Evaluate the definite integral using properties: $\int_0^\pi \frac{x \sin x}{1+\cos^2 x} dx$ *Visibility*. (4)

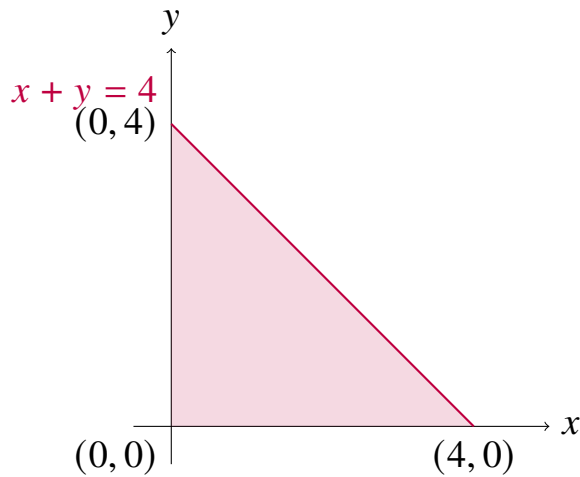
Q43. Carefully construct a graphical framework showing the feasible region bounded



by the system of constraints given below, then shade the area:

$$x + y \leq 4, \quad x \geq 0, \quad y \geq 0$$

Using your graph, compute the minimum value of $Z = 2x + 5y$. (4)



Q44. Find the shortest distance between the two skew lines whose vector equations are: (6)

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

OR

Find the vector and Cartesian equations of the plane passing through the three non-collinear points $A(2, 5, -3)$, $B(-2, -3, 5)$, and $C(5, 3, -3)$.

Q45. Find the intervals in which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is:

- (a) Strictly increasing
- (b) Strictly decreasing

Also, locate its points of local maxima and local minima. (6)



Detailed Solutions

Q1.

Solution

Concept: The problem evaluates the determinant of a scalar product involving the adjoint of a matrix. For an $n \times n$ matrix A and scalar k , the property states $|kA| = k^n|A|$. Additionally, the determinant of the adjoint matrix satisfies the identity $|\text{adj}(A)| = |A|^{n-1}$. Combining these rules allows us to compute the expanded value.

Solution:

- (a) The given matrix A has an order $n = 3$ and its determinant value is $|A| = 5$.
- (b) We need to determine the value of the determinant expression $|2\text{adj}(A)|$.
- (c) Applying the scalar property $|kM| = k^n|M|$ where matrix $M = \text{adj}(A)$ and scalar $k = 2$, we factor out the scalar parameter:

$$|2\text{adj}(A)| = 2^3|\text{adj}(A)| = 8|\text{adj}(A)|$$

- (d) Next, we substitute the standard structural formula for the determinant of an adjoint matrix, which is $|\text{adj}(A)| = |A|^{3-1} = |A|^2$:

$$|2\text{adj}(A)| = 8 \cdot |A|^2$$

- (e) Substituting the numerical parameter $|A| = 5$ into the derived equation yields:

$$8 \cdot (5)^2 = 8 \cdot 25 = 200$$

Final Answer: 200

Answer: (B)

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Q2.

Solution

Concept: The domain of a real-valued composite function involving an inverse trigonometric mapping requires evaluating structural restrictions. For the expression $\sin^{-1}(u)$ to be mathematically defined, its argument must satisfy $-1 \leq u \leq 1$. Additionally, any expression under a square root term must remain non-negative.

Solution:

(a) Consider the given function $f(x) = \sin^{-1} \sqrt{x-1}$.

(b) First, analyze the condition required for the square root operator $\sqrt{x-1}$. The radicand expression must be non-negative:

$$x - 1 \geq 0 \implies x \geq 1$$

(c) Second, evaluate the domain constraints for the arcsine function argument. The input must fall within the closed interval $[-1, 1]$:

$$-1 \leq \sqrt{x-1} \leq 1$$

(d) Since a principal square root expression naturally yields non-negative outputs, the lower bound is satisfied for all valid real values, leaving:

$$0 \leq \sqrt{x-1} \leq 1$$

(e) Squaring all parts of the inequality preserves the relationship structure:

$$0 \leq x - 1 \leq 1$$

(f) Adding 1 across the inequality bounds isolates the independent variable x :

$$1 \leq x \leq 2$$

(g) Intersection of the radical domain restriction $x \geq 1$ and the arcsine constraint gives the final closed interval $[1, 2]$.

Final Answer: $[1, 2]$

Answer: (B)

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Q3.

Solution

Concept: Evaluating limits involving composite transcendental functions often involves standard fundamental limit theorems. The primary exponential fundamental identity state that $\lim_{u \rightarrow 0} \frac{e^u - 1}{u} = 1$, and the standard trigonometric identity states $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.

Solution:

- (a) The problem requires evaluating the expression: $\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{x}$.
- (b) Substituting $x = 0$ directly into the expression results in an indeterminate configuration of the type $\frac{0}{0}$.
- (c) To resolve this indeterminate structure, we algebraic rewrite the expression by multiplying and dividing the denominator by $\sin x$:

$$\lim_{x \rightarrow 0} \left(\frac{e^{\sin x} - 1}{\sin x} \cdot \frac{\sin x}{x} \right)$$

- (d) Using the product rule properties of limits, we split the composite structure into two distinct limit evaluations:

$$\left(\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{\sin x} \right) \cdot \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right)$$

- (e) Let $u = \sin x$. As the variable $x \rightarrow 0$, the mapped variable $u \rightarrow \sin(0) = 0$. This transforms the first component into the standard exponential format:

$$\lim_{u \rightarrow 0} \frac{e^u - 1}{u} = 1$$

- (f) The second component matches the standard fundamental trigonometric limit structure exactly, which evaluates to 1.
- (g) Multiplying these independent values gives: $1 \cdot 1 = 1$.

Final Answer: 1

Answer: (B)

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Q4.

Solution

Concept: Definite integration involving symmetric bounded intervals can be simplified using King’s Property. The property states that $\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$. This technique is highly effective for cyclic trigonometric rational integrands.

Solution:

(a) Let the target integral expression be denoted as:

$$I = \int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx \quad \text{--- (Equation 1)}$$

(b) Apply the definite integral property by replacing the variable x with $(0 + \frac{\pi}{2} - x) = \frac{\pi}{2} - x$:

$$I = \int_0^{\pi/2} \frac{\sin^{100} (\frac{\pi}{2} - x)}{\sin^{100} (\frac{\pi}{2} - x) + \cos^{100} (\frac{\pi}{2} - x)} dx$$

(c) Using standard trigonometric co-function reduction relations, $\sin (\frac{\pi}{2} - x) = \cos x$ and $\cos (\frac{\pi}{2} - x) = \sin x$:

$$I = \int_0^{\pi/2} \frac{\cos^{100} x}{\cos^{100} x + \sin^{100} x} dx \quad \text{--- (Equation 2)}$$

(d) Add Equation 1 and Equation 2 together. Since their integration limits and denominators match, their numerators combine linearly:

$$2I = \int_0^{\pi/2} \frac{\sin^{100} x + \cos^{100} x}{\sin^{100} x + \cos^{100} x} dx$$

(e) The integrand simplifies to unity:

$$2I = \int_0^{\pi/2} 1 dx = [x]_0^{\pi/2} = \frac{\pi}{2}$$

(f) Solving for I yields: $I = \frac{\pi}{4}$.

Final Answer: $\frac{\pi}{4}$

Answer: (C)

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Q5.

Solution

Concept: Direction cosines $\langle l, m, n \rangle$ represent the cosines of the angles a vector makes with the coordinate axes. They must satisfy the identity $l^2 + m^2 + n^2 = 1$. Equal inclination implies that these angles, and thus their cosine values, are identical.

Solution:

- (a) Let the line be inclined at equal angles α, β, γ relative to the positive x, y, z axes.
- (b) Given that the line is equally inclined to the axes, we establish that $\alpha = \beta = \gamma$.
- (c) This condition implies that the direction cosines are equal: $l = \cos \alpha, m = \cos \beta = \cos \alpha$, and $n = \cos \gamma = \cos \alpha$.
- (d) We substitute these identical parameters into the standard direction cosine quadratic identity:

$$l^2 + m^2 + n^2 = 1 \implies \cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$$

- (e) Combine the terms to solve for the quadratic value:

$$3 \cos^2 \alpha = 1 \implies \cos^2 \alpha = \frac{1}{3}$$

- (f) Taking the square root for the specified positive directions yields:

$$\cos \alpha = \frac{1}{\sqrt{3}}$$

- (g) Therefore, each direction cosine component equals $\frac{1}{\sqrt{3}}$, producing the coordinate vector sequence $\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$.

Final Answer: $\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

Answer: (C)

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Q6.

Solution

Concept: The structural equation of a straight line in 3D space is given in vector form by $\vec{r} = \vec{a} + k\vec{b}$, where vector \vec{a} denotes a specific position coordinate point on the line, and vector \vec{b} represents the directional vector parallel to that line.

Solution:

- (a) Analyze the vector equation representing the first line: $\vec{r} = \vec{a}_1 + \lambda\vec{b}$. This line passes through point \vec{a}_1 and is oriented parallel to direction vector \vec{b} .
- (b) Analyze the vector equation representing the second line: $\vec{r} = \vec{a}_2 + \mu\vec{b}$. This line passes through point \vec{a}_2 and is also oriented parallel to direction vector \vec{b} .
- (c) Compare the directional vectors of both spatial lines. Both equations utilize the exact same direction vector parameter, \vec{b} .
- (d) Since both straight lines share an identical directional vector component, they are oriented along the same direction in three-dimensional space.
- (e) In solid geometry, lines that share the same directional vector or have proportional direction numbers are classified as parallel lines.
- (f) Unless the points \vec{a}_1 and \vec{a}_2 lie on the same path (making them coincident), these distinct equations will always define a pair of parallel lines.

Final Answer: Parallel

Answer: (C)

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Q7.

Solution

Concept: The shortest perpendicular distance of any spatial point $P(x, y, z)$ from a coordinate axis is calculated by considering its coordinate values. For the x -axis, the perpendicular projection drops to $(x, 0, 0)$, meaning the distance depends purely on the remaining orthogonal components y and z .

Solution:

- (a) Let the given target spatial point be denoted as $P(2, 3, 4)$. Here, $x = 2$, $y = 3$, and $z = 4$.
- (b) We need to determine the perpendicular distance of this point from the x -axis.
- (c) The perpendicular projection of point $P(2, 3, 4)$ onto the x -axis matches its x -coordinate exactly while zeroing out other components, yielding point $A(2, 0, 0)$.
- (d) Apply the standard Euclidean 3D distance formula between points $P(2, 3, 4)$ and $A(2, 0, 0)$:

$$d = \sqrt{(2 - 2)^2 + (3 - 0)^2 + (4 - 0)^2}$$

- (e) Simplify the terms inside the radical expression:

$$d = \sqrt{0^2 + 3^2 + 4^2} = \sqrt{0 + 9 + 16}$$

- (f) Add the values together to obtain the final radicand:

$$d = \sqrt{25} = 5$$

- (g) Thus, the geometric distance of the point from the x -axis is exactly 5 units.

Final Answer: 5

Answer: (B)

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Q8.

Solution

Concept: The geometric interpretation of the first derivative of a function $y = f(x)$ represents the instantaneous rate of change, which corresponds to the slope of the tangent line drawn to the curve at any given point. Evaluating $\frac{dy}{dx}$ at $x = x_0$ gives the specific slope value.

Solution:

(a) The given equation modeling the continuous curve is: $y = x^3 - x$.

(b) To determine the general slope function of the tangent line, differentiate y with respect to the independent variable x :

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(x)$$

(c) Applying the power rule of differentiation ($\frac{d}{dx}(x^n) = nx^{n-1}$) yields the derivative expression:

$$\frac{dy}{dx} = 3x^2 - 1$$

(d) We are tasked with evaluating this slope specifically at the coordinate boundary value $x = 2$.

(e) Substitute $x = 2$ directly into the derived derivative expression:

$$\text{Slope } (m) = \left. \frac{dy}{dx} \right|_{x=2} = 3(2)^2 - 1$$

(f) Perform the arithmetic operations sequentially by computing the exponent first:

$$m = 3(4) - 1 = 12 - 1 = 11$$

(g) Thus, the numeric slope value of the tangent line at $x = 2$ is exactly 11.

Final Answer: 11

Answer: (A)

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Q9.

Solution

Concept: The general second-degree equation representing a circle in a two-dimensional Cartesian coordinate plane is structured as $x^2 + y^2 + 2gx + 2fy + c = 0$. The radius r of this circle is determined using the coefficients via the standard formula $r = \sqrt{g^2 + f^2 - c}$.

Solution:

- (a) Write down the given specific equation of the circle:

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

- (b) Compare this expression directly with the standard equation coefficients ($2g = -4$, $2f = 6$, $c = -12$).

- (c) Solve for the center coordinate components g and f :

$$g = \frac{-4}{2} = -2, \quad f = \frac{6}{2} = 3$$

- (d) Identify the constant scalar term from the expression, which is $c = -12$.

- (e) Write out the radius equation and substitute these calculated values into it:

$$r = \sqrt{(-2)^2 + (3)^2 - (-12)}$$

- (f) Simplify the arithmetic squares and signs inside the radical operator:

$$r = \sqrt{4 + 9 + 12}$$

- (g) Sum the numbers inside the radical to find the final value:

$$r = \sqrt{25} = 5$$

- (h) The calculated radius of the given circle equation is exactly 5 units.

Final Answer: 5

Answer: (A)

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Q10.

Solution

Concept: A standard parabola opening along the negative direction of the horizontal axis has the structural equation $y^2 = -4ax$, where $a > 0$ represents the distance from the vertex to the focus. For this configuration, the focus point is located on the axis of symmetry at coordinates $(-a, 0)$.

Solution:

- (a) The equation of the parabola provided in the problem is: $y^2 = -16x$.
- (b) Identify the direction of the conic axis. Since the variable x is linear and has a negative coefficient, this parabola opens symmetrically to the left.
- (c) Compare this formula with the standard geometric model equation:

$$-4ax = -16x \implies 4a = 16$$

- (d) Solve this linear equation to find the value of the focal length parameter a :

$$a = \frac{16}{4} = 4$$

- (e) For a parabola of the form $y^2 = -4ax$, the vertex is at the origin $(0, 0)$, and its focus lies on the negative x -axis.
- (f) The standard structural coordinate format for the focus of this parabola type is given by $(-a, 0)$.
- (g) Substituting our calculated parameter $a = 4$ into this coordinate form yields the point $(-4, 0)$.

Final Answer: $(-4, 0)$

Answer: (B)

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Q11.

Solution

Concept: A first-order linear differential equation is represented in the standard form $\frac{dy}{dx} + P(x)y = Q(x)$. The Integrating Factor (I.F.) is a crucial mathematical expression used to transform the left side of the equation into a perfect derivative, and it is calculated using the exponential formula $I.F. = e^{\int P(x) dx}$.

Solution:

- (a) Write down the given first-order differential equation:

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

- (b) Rearrange the terms to explicitly match the standard linear profile:

$$\frac{dy}{dx} + \left(\frac{1}{x}\right)y = x^2$$

- (c) Identify the corresponding functional coefficient functions. By direct comparison, we find $P(x) = \frac{1}{x}$ and $Q(x) = x^2$.

- (d) Substitute the expression for $P(x)$ into the integrating factor integral equation:

$$I.F. = e^{\int \frac{1}{x} dx}$$

- (e) Evaluate the standard indefinite integral in the exponent:

$$\int \frac{1}{x} dx = \log x$$

- (f) Substitute this result back into the exponential base structure:

$$I.F. = e^{\log x}$$

- (g) Using the fundamental inverse property of natural logarithms and exponential functions ($e^{\log u} = u$), the expression simplifies directly to x .

Final Answer: x

Answer: (C)

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Q12.

Solution

Concept: The magnitude of the sum of two vectors is found by utilizing vector dot product identities. For any vectors \vec{a} and \vec{b} , the square of their vector sum is given by $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a} \cdot \vec{b})$. Orthogonal vectors produce a dot product equal to zero.

Solution:

- (a) The problem provides two perpendicular vectors, \vec{a} and \vec{b} .
- (b) Since the vectors are explicitly stated to be perpendicular, their scalar dot product must satisfy the standard orthogonality condition:

$$\vec{a} \cdot \vec{b} = 0$$

- (c) We are given the absolute magnitudes of each individual vector: $|\vec{a}| = 3$ and $|\vec{b}| = 4$.
- (d) Expand the required algebraic vector expression using the distributive property of dot products:

$$|\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a} \cdot \vec{b})$$

- (e) Substitute the known scalar parameters and the orthogonality dot product condition into the expanded equation:

$$|\vec{a} + \vec{b}|^2 = (3)^2 + (4)^2 + 2(0)$$

- (f) Compute the numeric values of the squares and sum them:

$$|\vec{a} + \vec{b}|^2 = 9 + 16 + 0 = 25$$

- (g) Extract the positive square root to find the linear vector magnitude:

$$|\vec{a} + \vec{b}| = \sqrt{25} = 5$$

Final Answer: 5

Answer: (C)

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Q13.

Solution

Concept: The domain of a mathematical relation R defined on a set is the collection of all first elements (x) from the ordered pairs (x, y) that satisfy the given condition. Since the relation is bounded within the set of natural numbers $\mathbb{N} = \{1, 2, 3, \dots\}$, both variables must be positive integers.

Solution:

- (a) The relation R is defined on the natural numbers set \mathbb{N} by the linear constraint equation:

$$x + 2y = 8$$

- (b) Express the independent variable x explicitly in terms of the variable y :

$$x = 8 - 2y$$

- (c) Since the relation elements must belong to \mathbb{N} , the value of x must be a positive integer ($x > 0$). This establishes an upper bound inequality for y :

$$8 - 2y > 0 \implies 2y < 8 \implies y < 4$$

- (d) List the valid positive integer values for y within this open interval: $y \in \{1, 2, 3\}$.
- (e) Substitute each valid value of y back into the isolated equation to compute the corresponding values of x :
- For $y = 1$: $x = 8 - 2(1) = 6$
 - For $y = 2$: $x = 8 - 2(2) = 4$
 - For $y = 3$: $x = 8 - 2(3) = 2$

- (f) Collect the computed inputs to form the complete set of ordered pairs: $R = \{(6, 1), (4, 2), (2, 3)\}$.

- (g) The domain comprises the first components of these points, which gives the set $\{2, 4, 6\}$.

Final Answer: $\{2, 4, 6\}$

Answer: (A)

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Q14.

Solution

Concept: The angle θ between two intersecting lines represented by the standard linear forms $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ can be evaluated via their slope properties. If the product of their slopes satisfies $m_1 \cdot m_2 = -1$, the two lines are mutually perpendicular.

Solution:

(a) Consider the equation of the first straight line: $x - 2y + 5 = 0$.

(b) Rearrange this equation into slope-intercept form ($y = m_1x + c_1$) to isolate the slope:

$$2y = x + 5 \implies y = \frac{1}{2}x + \frac{5}{2} \implies m_1 = \frac{1}{2}$$

(c) Consider the equation of the second straight line: $2x + y - 3 = 0$.

(d) Rearrange this equation into slope-intercept form ($y = m_2x + c_2$) to isolate the slope:

$$y = -2x + 3 \implies m_2 = -2$$

(e) Calculate the product of the two computed line slopes:

$$\text{Product} = m_1 \cdot m_2 = \left(\frac{1}{2}\right) \cdot (-2)$$

(f) Simplifying the fraction yields:

$$m_1 \cdot m_2 = -1$$

(g) According to analytical geometry principles, a slope product of -1 proves that the lines intersect at a right angle. Therefore, the angle between them is exactly 90° .

Final Answer: 90°

Answer: (C)

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Q15.

Solution

Concept: A square matrix M is classified as symmetric if it equals its own transpose ($M^T = M$), and skew-symmetric if it equals its negative transpose ($M^T = -M$). Matrix transposition follows distributive rules: $(A \pm B)^T = A^T \pm B^T$ and $(AB)^T = B^T A^T$.

Solution:

- (a) We are given that A and B are symmetric matrices of the same order. This provides the mathematical identities:

$$A^T = A \quad \text{and} \quad B^T = B$$

- (b) Let us define a new composite matrix variable X to represent the matrix expression:

$$X = AB - BA$$

- (c) To classify this matrix, apply the transpose operator to both sides of the expression:

$$X^T = (AB - BA)^T$$

- (d) Use the distributive property of transposition across matrix subtraction:

$$X^T = (AB)^T - (BA)^T$$

- (e) Apply the reversal law of transposition to each product term:

$$X^T = (B^T A^T) - (A^T B^T)$$

- (f) Substitute the initial conditions ($A^T = A$ and $B^T = B$) into the expression:

$$X^T = BA - AB$$

- (g) Factor out a negative sign to compare it with the original matrix definition:

$$X^T = -(AB - BA) = -X$$

- (h) Since $X^T = -X$, the composite matrix $AB - BA$ is skew-symmetric.

Final Answer: Skew-symmetric matrix

Answer: (B)

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Q16.

Solution

Concept: According to the Fundamental Theorem of Linear Programming, the optimal value (maximum or minimum) of a linear objective function $Z = ax + by$ over a bounded convex feasible region always occurs at one of the corner points (vertices) of that region.

Solution:

- (a) The given profit objective function to maximize is:

$$Z = 4x + 3y$$

- (b) The coordinates of the boundary corner points of the feasible region are listed as:

$$(0, 2), \quad (3, 0), \quad (6, 0), \quad (0, 5)$$

- (c) Evaluate the objective function value at the first corner point (0, 2):

$$Z_{(0,2)} = 4(0) + 3(2) = 0 + 6 = 6$$

- (d) Evaluate the objective function value at the second corner point (3, 0):

$$Z_{(3,0)} = 4(3) + 3(0) = 12 + 0 = 12$$

- (e) Evaluate the objective function value at the third corner point (6, 0):

$$Z_{(6,0)} = 4(6) + 3(0) = 24 + 0 = 24$$

- (f) Evaluate the objective function value at the fourth corner point (0, 5):

$$Z_{(0,5)} = 4(0) + 3(5) = 0 + 15 = 15$$

- (g) Compare the computed values: 6, 12, 24, and 15. The largest value among these is 24.

- (h) This maximum profit output occurs exactly at the coordinates (6, 0).

Final Answer: (6, 0)

Answer: (B)

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Q17.

Solution

Concept: In mathematical logic, a conditional statement is represented symbolically as $P \implies Q$, where P is the hypothesis and Q is the conclusion. The contrapositive of this conditional structure is defined as $\neg Q \implies \neg P$. A statement and its contrapositive are logically equivalent.

Solution:

- (a) Identify the component propositions within the given conditional sentence. Let:
 - P : It rains.
 - Q : The game is cancelled.
- (b) The original sentence format is: "If P , then Q " ($P \implies Q$).
- (c) To formulate the contrapositive statement, we must find the logical negation of both individual components:
 - $\neg Q$: The game is not cancelled.
 - $\neg P$: It does not rain.
- (d) Arrange these negated elements into the contrapositive conditional template: "If $\neg Q$, then $\neg P$ ".
- (e) Translate this symbolic logical format back into a standard English sentence:

"If the game is not cancelled, then it does not rain."
- (f) This matched statement retains the exact logical truth values of the original proposition.

Final Answer: If the game is not cancelled, then it does not rain.

Answer: (C)

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Q18.

Solution

Concept: Evaluating expressions containing inverse trigonometric terms requires finding principal values. The principal value branch for $\tan^{-1} x$ is $(-\frac{\pi}{2}, \frac{\pi}{2})$, and for $\cos^{-1} x$ it is $[0, \pi]$. Negative inputs for arccosine use the identity $\cos^{-1}(-u) = \pi - \cos^{-1} u$.

Solution:

(a) The problem requires evaluating the expression: $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$.

(b) Find the principal value for the first component, $\tan^{-1}(1)$. Since $\tan\left(\frac{\pi}{4}\right) = 1$:

$$\tan^{-1}(1) = \frac{\pi}{4}$$

(c) Analyze the second component, $\cos^{-1}\left(-\frac{1}{2}\right)$. Apply the negative angle identity for arccosine:

$$\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$$

(d) Determine the standard angle where cosine outputs a value of $\frac{1}{2}$, which is $\frac{\pi}{3}$:

$$\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

(e) Combine the two calculated principal angles to evaluate the total sum:

$$\text{Sum} = \frac{\pi}{4} + \frac{2\pi}{3}$$

(f) Find a common denominator, which is 12, to add the fractions:

$$\text{Sum} = \frac{3\pi}{12} + \frac{8\pi}{12} = \frac{11\pi}{12}$$

Final Answer: $\frac{11\pi}{12}$

Answer: (B)

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Q19.

Solution

Concept: The standard equation of a horizontally oriented ellipse centered at the origin is written as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$. The eccentricity e measures the degree of deviation from a perfect circle and is given by the formula $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution:

- (a) Write down the given conic equation of the ellipse:

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

- (b) Identify the parameters by comparing it with the standard equation form. This gives:

$$a^2 = 25 \implies a = 5$$

$$b^2 = 16 \implies b = 4$$

- (c) Since $a^2 > b^2$, the major axis of this ellipse lies along the horizontal x -axis.

- (d) Write down the standard formula used to compute eccentricity:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

- (e) Substitute the known values of a^2 and b^2 into the radical formula:

$$e = \sqrt{1 - \frac{16}{25}}$$

- (f) Perform the fractional subtraction inside the square root operator:

$$e = \sqrt{\frac{25 - 16}{25}} = \sqrt{\frac{9}{25}}$$

- (g) Extract the square root of the fraction to obtain the final linear scalar ratio:

$$e = \frac{3}{5}$$

Final Answer: $\frac{3}{5}$

Answer: (A)

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Q20.

Solution

Concept: Differentiating a composite logarithmic function requires applying the chain rule. For a function $y = \log(u)$, where $u = g(x)$, the derivative is given by $\frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$. Standard derivatives state that $\frac{d}{dx}(\sec x) = \sec x \tan x$ and $\frac{d}{dx}(\tan x) = \sec^2 x$.

Solution:

(a) Let the target equation be: $y = \log(\sec x + \tan x)$.

(b) Apply the chain rule by differentiating the outer logarithmic function with respect to its inner arguments:

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \cdot \frac{d}{dx}(\sec x + \tan x)$$

(c) Differentiate each inner trigonometric function term individually:

$$\frac{d}{dx}(\sec x) = \sec x \tan x, \quad \frac{d}{dx}(\tan x) = \sec^2 x$$

(d) Substitute these individual derivatives back into the chain rule product structure:

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \cdot (\sec x \tan x + \sec^2 x)$$

(e) Factor out the common term $\sec x$ from the numerator expression:

$$\frac{dy}{dx} = \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$$

(f) Cancel out the identical binomial terms $(\sec x + \tan x)$ from both the numerator and denominator:

$$\frac{dy}{dx} = \sec x$$

Final Answer: $\sec x$

Answer: (A)

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Q21.

Solution

Concept: A square matrix M is defined as skew-symmetric if $M^T = -M$. For any odd-order skew-symmetric matrix, taking the determinant shows that $\det(M) = 0$. For perpendicular vectors, their scalar dot product must equal zero, meaning $\vec{p} \cdot \vec{q} = 0$.

Solution:

- (a) Consider statement (i) concerning a 3×3 skew-symmetric matrix M . Using determinant properties, we know $\det(M^T) = \det(-M) = (-1)^3 \det(M) = -\det(M)$. Since $\det(M^T) = \det(M)$, this simplifies to $\det(M) = -\det(M)$, which forces $2 \det(M) = 0$, or $\det(M) = 0$. Thus, property (i) matches column value (A).
- (b) Consider statement (ii) where vectors $\vec{p} = \hat{i} + \alpha\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$ are given as orthogonal.
- (c) Set their scalar dot product equal to zero to fulfill the geometric condition:

$$\vec{p} \cdot \vec{q} = (1)(1) + (\alpha)(-1) + (1)(1) = 0$$

- (d) Simplify the resulting linear algebraic equation to determine the missing variable:

$$1 - \alpha + 1 = 0 \implies 2 - \alpha = 0 \implies \alpha = 2$$

- (e) This matches sub-question (ii) with column value (B). Comparing option sets, option (A) represents the perfect layout.

Final Answer: (i) \rightarrow (A), (ii) \rightarrow (B)

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Q22.

Solution

Concept: The degree of a differential equation is the highest power of the highest-order derivative present, provided the equation is expressed as a polynomial in its derivatives. The shortest distance between two parallel planes $Ax + By + Cz + D_1 = 0$ and $Ax + By + Cz + D_2 = 0$ is found using the formula $d = \frac{|D_1 - D_2|}{\sqrt{A^2 + B^2 + C^2}}$.

Solution:

- (a) For sub-question (i), inspect the given differential equation:

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y = 0$$

- (b) Identify the highest-order derivative present in the expression, which is $\frac{d^2y}{dx^2}$ (order 2). The exponent raised over this specific highest derivative term is 3. Since it forms a valid polynomial sequence, the degree of the equation is 3.
- (c) For sub-question (ii), identify the coefficients of the parallel planes. Rewrite the equations into standard structural format:

$$2x + y + 2z - 8 = 0 \implies D_1 = -8$$

$$2x + y + 2z + 4 = 0 \implies D_2 = 4$$

- (d) Substitute coefficients $A = 2, B = 1, C = 2$ along with constants into the parallel distance formula:

$$d = \frac{|-8 - 4|}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{|-12|}{\sqrt{4 + 1 + 4}} = \frac{12}{\sqrt{9}} = \frac{12}{3} = 4$$

Final Answer: (i) 3, (ii) 4

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Q23.

Solution

Concept: An identity relation I_A maps every element of a set exclusively to itself. An equivalence relation must be reflexive, symmetric, and transitive. Differentiability requires that the function has a well-defined unique tangent slope at that point, meaning left-hand and right-hand derivatives must match.

Solution:

(a) Analyze statement (i) regarding identity relations. Let $I_A = \{(a, a) : a \in A\}$ be defined on a non-empty set A .

(b) Check the conditions for an equivalence relation:

- It is reflexive since $(a, a) \in I_A$ for all $a \in A$.
- It is symmetric because if $(a, b) \in I_A$, then $a = b$, which implies $(b, a) = (a, a) \in I_A$.
- It is transitive because if $(a, b) \in I_A$ and $(b, c) \in I_A$, then $a = b = c$, meaning $(a, c) \in I_A$.

Hence, statement (i) is TRUE.

(c) Analyze statement (ii) concerning the absolute value function $f(x) = |x|$ at the origin $x = 0$.

(d) Calculate the left-hand derivative and right-hand derivative at $x = 0$:

$$LHD = \lim_{h \rightarrow 0^-} \frac{|0 + h| - 0}{h} = -1, \quad RHD = \lim_{h \rightarrow 0^+} \frac{|0 + h| - 0}{h} = 1$$

(e) Since $LHD \neq RHD$, the graph has a sharp corner at the origin and is not differentiable. Hence, statement (ii) is FALSE.

Final Answer: (i) TRUE, (ii) FALSE

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Q24.

Solution

Concept: The logical negation of a proposition P is denoted as $\neg P$ and represents a statement with an opposite truth value. For existential or universal quantifiers, negating an absolute statement like "All X are Y " produces the logically equivalent format "There exists at least one X that is not Y ".

Solution:

- (a) Analyze sentence (i): " $\sqrt{5}$ is a rational number."
- (b) To negate this basic declarative assertion, insert a negation modifier directly into the predicate structure. The resulting logical sentence becomes: " $\sqrt{5}$ is not a rational number" (or equivalently, " $\sqrt{5}$ is an irrational number").
- (c) Analyze sentence (ii): "All prime numbers are odd integers."
- (d) This contains a universal quantifier ("All"). The negation of a universally quantified statement asserts that there is at least one counterexample to the rule.
- (e) Apply the logical negation rule $\neg(\forall x, P(x)) \equiv \exists x, \neg P(x)$ to construct the new proposition.
- (f) The proper phrasing states: "Some prime numbers are not odd integers" or alternatively, "There exists a prime number that is not an odd integer."

Final Answer: (i) $\sqrt{5}$ is not a rational number, (ii) Some prime numbers are not odd integers.

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Q25.

Solution

Concept: Evaluating calculus limits depends on fundamental analytical limits theorems. The basic identity states that $\lim_{u \rightarrow 0} \frac{\sin u}{u} = 1$. The compound exponential theorem specifies $\lim_{x \rightarrow 0} (1 + x)^{1/x} = e$. Algebraic rational limits are solved by evaluating leading coefficients or employing the algebraic power identity $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$.

Solution:

- (a) For sub-question (i), adjust the denominator to evaluate the limit:

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{x} = \lim_{x \rightarrow 0} \left(3 \cdot \frac{\sin 3x}{3x} \right) = 3 \cdot 1 = 3$$

- (b) For sub-question (ii), the expression matches the definition of the base of natural logarithms exactly:

$$\lim_{x \rightarrow 0} (1 + x)^{1/x} = e$$

- (c) For sub-question (iii), evaluate the behavior as $x \rightarrow \infty$. Divide the numerator and denominator by the highest power of x , which is x^1 :

$$\lim_{x \rightarrow \infty} \frac{2 + \frac{1}{x}}{3 - \frac{2}{x}} = \frac{2 + 0}{3 - 0} = \frac{2}{3}$$

- (d) For sub-question (iv), apply the standard polynomial limit theorem where $a = 1$ and power exponent $n = 5$:

$$\lim_{x \rightarrow 1} \frac{x^5 - 1^5}{x - 1} = 5 \cdot (1)^{5-1} = 5 \cdot 1 = 5$$

Final Answer: (i) 3, (ii) e , (iii) $\frac{2}{3}$, (iv) 5

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Q26.

Solution

Concept: Conic sections are defined by standard structural algebraic formulas. A circle $x^2 + y^2 + 2gx + 2fy + c = 0$ has its center at $(-g, -f)$. A parabola $x^2 = 4ay$ has a latus rectum of length $4a$, while $y^2 = 4ax$ has a vertical directrix at $x = -a$. An ellipse $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ with $a > b$ has a vertical major axis of length $2a$.

Solution:

- (a) For item (i), look at the circle equation: $x^2 + y^2 + 4x - 8y = 0$. Compare coefficients to find $2g = 4 \implies g = 2$ and $2f = -8 \implies f = -4$. The center is $(-g, -f) = (-2, 4)$.
- (b) For item (ii), the parabola expression is $x^2 = 12y$. Comparing this directly to $x^2 = 4ay$ yields a latus rectum length of $4a = 12$.
- (c) For item (iii), the ellipse is $\frac{x^2}{9} + \frac{y^2}{25} = 1$. Since $25 > 9$, we have $a^2 = 25 \implies a = 5$. The total length of the major axis along the y -axis is $2a = 2(5) = 10$.
- (d) For item (iv), the parabola equation is $y^2 = 8x$. Comparing this to $y^2 = 4ax$ gives $4a = 8 \implies a = 2$. The equation of the vertical directrix line on the opposite side is $x = -a \implies x = -2$.

Final Answer: (i) $(-2, 4)$, (ii) 12, (iii) 10, (iv) $x = -2$

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Q27.

Solution

Concept: The double inverse matrix property states that the inverse of an inverse matrix returns the original matrix, $(A^{-1})^{-1} = A$. A scalar triple product $[\vec{a}, \vec{b}, \vec{c}]$ computes the volume of a parallelepiped; it is zero if any two vectors are parallel. Matrix multiplication is generally non-commutative ($AB \neq BA$). The scalar vector projection of \vec{u} on \vec{v} is given by $\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|}$.

Solution:

- (a) Statement (i) asserts $(A^{-1})^{-1} = A$. By the mathematical definition of an inverse matrix, multiplying a matrix by its inverse results in the identity matrix. Inverting a second time reverses the operation, making this statement TRUE.
- (b) Statement (ii) states that the scalar triple product is zero if two vectors are parallel. A parallel vector condition means one vector is a scalar multiple of another, causing two rows of the corresponding determinant to be proportional. Thus, the determinant evaluates to zero, making this statement TRUE.
- (c) Statement (iii) claims matrix multiplication is always commutative. This is false because, in general matrix algebra, $AB \neq BA$. Thus, this statement is FALSE.
- (d) Statement (iv) mentions the projection of unit vector \hat{i} on unit vector \hat{j} . Compute using the dot product formula: $\frac{\hat{i} \cdot \hat{j}}{|\hat{j}|} = \frac{0}{1} = 0$. Since the statement claims it equals 1, it is FALSE.

Final Answer: (i) TRUE, (ii) TRUE, (iii) FALSE, (iv) FALSE

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Q28.

Solution

Concept: The area bounded between two intersecting curves $y = f(x)$ and $y = g(x)$ from $x = a$ to $x = b$ is calculated using a definite integral. The bounded area formula is structured as $\int_a^b [f(x) - g(x)] dx$, where $f(x)$ is the upper bounding function and $g(x)$ is the lower bounding function across that interval.

Solution:

(a) For sub-question (i), find the intersection points by setting the two functions equal:
 $x^2 = x \implies x^2 - x = 0 \implies x(x - 1) = 0$. The non-zero solution gives the coordinate position $x = 1$.

(b) For sub-question (ii), examine the bounded region over the domain interval $[0, 1]$. The linear graph $y = x$ sits vertically above the parabolic curve $y = x^2$. Therefore, the upper bounding function expression is $f(x) = x$.

(c) For sub-question (iii), set up the standard definite integral required to calculate the shaded region:

$$\text{Area} = \int_0^1 (x - x^2) dx$$

(d) For sub-question (iv), evaluate this definite integral by applying the fundamental power integration rules:

$$\text{Area} = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \left(\frac{1}{2} - \frac{1}{3} \right) - (0 - 0) = \frac{3 - 2}{6} = \frac{1}{6}$$

Thus, the shaded area is $\frac{1}{6}$ square units.

Final Answer: (i) 1, (ii) x , (iii) $\int_0^1 (x - x^2) dx$, (iv) $\frac{1}{6}$

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Q29.

Solution

Concept: Linear Programming Problems (LPP) optimize a linear objective function subject to a system of linear constraints. Non-negativity constraints restrict decision variables to the first quadrant ($x \geq 0, y \geq 0$). The corner point method evaluates the objective function Z at each boundary vertex to determine the absolute maximum value.

Solution:

- (a) For item (i), the non-negativity conditions prevent negative production quantities. Looking at the options, $x \geq 0$ matches this description (Option C).
- (b) For item (ii), solve the system of boundary lines: $x + 2y = 10$ and $3x + y = 15$. Multiply the second equation by 2 to get $6x + 2y = 30$. Subtracting the first equation gives $5x = 20 \implies x = 4$. Substitute back to find $4 + 2y = 10 \implies y = 3$. The intersection vertex is $(4, 3)$ (Option A).
- (c) For item (iii), find the corner point on the positive y -axis by setting $x = 0$ in the constraint $x + 2y \leq 10 \implies 2y = 10 \implies y = 5$. This yields point $(0, 5)$ (Option B).
- (d) For item (iv), evaluate $Z = 3x + 2y$ at the origin $(0, 0)$: $Z = 3(0) + 2(0) = 0$ *Target* (Option C).
- (e) For item (v), substitute the intersection coordinates $(4, 3)$ into the objective function: $Z = 3(4) + 2(3) = 12 + 6 = 18$ (Option B).
- (f) For item (vi), compare all corner point values: $Z_{(0,0)} = 0, Z_{(5,0)} = 15, Z_{(0,5)} = 10, Z_{(4,3)} = 18$. The absolute maximum value is 18 (Option C).

Final Answer: (i) C, (ii) A, (iii) B, (iv) C, (v) B, (vi) C

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Q30.

Solution

Concept: A function $f : A \rightarrow B$ is classified as a bijective function if it satisfies two conditions: it must be injective (one-to-one) and surjective (onto). Injectivity requires that if $f(x_1) = f(x_2)$, then $x_1 = x_2$. Surjectivity requires that for every output element $y \in B$, there exists an input element $x \in A$ such that $f(x) = y$.

Solution:

(a) We are given the linear function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by the equation $f(x) = 3x + 5$.

(b) First, let us test for the injective (one-to-one) property. Assume there are two domain elements $x_1, x_2 \in \mathbb{R}$ such that their function maps are identical:

$$f(x_1) = f(x_2) \implies 3x_1 + 5 = 3x_2 + 5$$

(c) Subtract the constant value 5 from both sides of the equation:

$$3x_1 = 3x_2$$

(d) Divide by the scalar multiplier 3 to isolate the variables:

$$x_1 = x_2$$

Since $f(x_1) = f(x_2)$ implies $x_1 = x_2$, the function is injective.

(e) Next, test for the surjective (onto) property. Let y be any arbitrary real number in the codomain ($y \in \mathbb{R}$). Set up the equation $f(x) = y$:

$$3x + 5 = y \implies 3x = y - 5 \implies x = \frac{y - 5}{3}$$

(f) Since $y \in \mathbb{R}$, the value $x = \frac{y-5}{3}$ will always evaluate to a valid real number. This means x belongs to the domain \mathbb{R} .

(g) Verify the mapping: $f\left(\frac{y-5}{3}\right) = 3\left(\frac{y-5}{3}\right) + 5 = (y - 5) + 5 = y$. Thus, the function is surjective. Since it is both injective and surjective, it is bijective.

Final Answer: The function is proved to be bijective.

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Q31.

Solution

Concept: Two straight lines in three-dimensional space with direction vectors $\vec{v}_1 = \langle a_1, b_1, c_1 \rangle$ and $\vec{v}_2 = \langle a_2, b_2, c_2 \rangle$ are mutually perpendicular if and only if the scalar dot product of their direction vectors equals zero, which corresponds to the algebraic condition $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

Solution:

- (a) Identify the direction ratios of the first straight line $\frac{x-1}{2} = \frac{y-2}{k} = \frac{z+1}{-3}$ from its denominators:

$$\vec{v}_1 = \langle 2, k, -3 \rangle$$

- (b) Identify the direction ratios of the second straight line $\frac{x+2}{3} = \frac{y-1}{2} = \frac{z}{2}$ from its denominators:

$$\vec{v}_2 = \langle 3, 2, 2 \rangle$$

- (c) Since the lines are perpendicular, set the dot product of their direction vectors equal to zero:

$$\vec{v}_1 \cdot \vec{v}_2 = (2)(3) + (k)(2) + (-3)(2) = 0$$

- (d) Expand the scalar multiplication terms:

$$6 + 2k - 6 = 0$$

- (e) Cancel the constant terms to solve for k :

$$2k = 0 \implies k = 0$$

Final Answer: $k = 0$

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Q32.

Solution

Concept: Differentiating a nested function requires the systematic application of the calculus chain rule: $\frac{d}{dx}[f(g(h(x)))] = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$. The standard derivative formulas are $\frac{d}{du}(\log u) = \frac{1}{u}$, $\frac{d}{dv}(\sin v) = \cos v$, and $\frac{d}{dx}(x^2) = 2x$.

Solution:

(a) Let the composite target function be $y = \log(\sin x^2)$.

(b) Apply the chain rule by differentiating the outermost logarithmic function with respect to its inner argument:

$$\frac{dy}{dx} = \frac{1}{\sin x^2} \cdot \frac{d}{dx}(\sin x^2)$$

(c) Differentiate the inner trigonometric sine function with respect to its polynomial argument x^2 :

$$\frac{d}{dx}(\sin x^2) = \cos x^2 \cdot \frac{d}{dx}(x^2)$$

(d) Evaluate the remaining inner polynomial derivative using the power rule:

$$\frac{d}{dx}(x^2) = 2x$$

(e) Substitute these individual step results back into the full derivative expression:

$$\frac{dy}{dx} = \frac{1}{\sin x^2} \cdot \cos x^2 \cdot 2x$$

(f) Combine terms and utilize the cotangent trigonometric identity ($\cot u = \frac{\cos u}{\sin u}$):

$$\frac{dy}{dx} = 2x \cdot \left(\frac{\cos x^2}{\sin x^2}\right) = 2x \cot x^2$$

Final Answer: $2x \cot x^2$

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Q33.

Solution

Concept: To formulate the differential equation of a family of curves, differentiate the given primitive equation with respect to the independent variable x a number of times equal to the number of independent arbitrary constants, then eliminate those constants from the equations.

Solution:

- (a) Write down the given algebraic equation representing the family of concentric circles:

$$x^2 + y^2 = r^2$$

where r is the single arbitrary parameter representing the radius.

- (b) Differentiate both sides of this equation implicitly with respect to the independent variable x :

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(r^2)$$

- (c) Apply the power rule and the chain rule to the variables:

$$2x + 2y \frac{dy}{dx} = 0$$

- (d) Factor out the common numerical coefficient 2 from the simplified expression:

$$2 \left(x + y \frac{dy}{dx} \right) = 0$$

- (e) Divide by 2 to write the final first-order differential equation:

$$x + y \frac{dy}{dx} = 0$$

This eliminates the constant r^2 completely.

Final Answer: $x + y \frac{dy}{dx} = 0$

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Q34.

Solution

Concept: The scalar projection of a vector \vec{a} onto another vector \vec{b} is defined by the formula $\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$. Alternatively, a unit vector in the direction of any non-zero vector \vec{c} is calculated by dividing the vector by its structural magnitude: $\hat{c} = \frac{\vec{c}}{|\vec{c}|}$.

Solution:

- (a) For the first question part, compute the dot product of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$:

$$\vec{a} \cdot \vec{b} = (2)(1) + (3)(2) + (2)(1) = 2 + 6 + 2 = 10$$

- (b) Calculate the total scalar length magnitude of target vector \vec{b} :

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$$

- (c) Substitute these components into the projection formula:

$$\text{Proj}_{\vec{b}}\vec{a} = \frac{10}{\sqrt{6}} = \frac{5\sqrt{6}}{3}$$

- (d) For the alternative **OR** choice, calculate the vector sum $\vec{c} = \vec{a} + \vec{b}$:

$$\vec{c} = (2 + 1)\hat{i} + (1 + 2)\hat{j} + (-3 + 5)\hat{k} = 3\hat{i} + 3\hat{j} + 2\hat{k}$$

- (e) Find the magnitude of this combined vector:

$$|\vec{c}| = \sqrt{3^2 + 3^2 + 2^2} = \sqrt{9 + 9 + 4} = \sqrt{22}$$

- (f) Construct the required unit direction vector:

$$\hat{c} = \frac{3\hat{i} + 3\hat{j} + 2\hat{k}}{\sqrt{22}}$$

Final Answer: $\frac{10}{\sqrt{6}}$

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Q35.

Solution

Concept: Evaluating an algebraic integral containing logarithmic terms can be simplified using the method of integration by substitution. If we substitute a composite expression with $u = g(x)$, its corresponding differential change is given by $du = g'(x) dx$.

Solution:

- (a) Write down the indefinite integral expression:

$$I = \int \frac{1}{x(1 + \log x)} dx$$

- (b) Choose an appropriate substitution variable to simplify the denominator. Let:

$$u = 1 + \log x$$

- (c) Differentiate this substitution equation with respect to x to find the differential element:

$$dx \cdot \frac{du}{dx} = \frac{1}{x} \implies du = \frac{1}{x} dx$$

- (d) Replace the corresponding elements in the original integral using these u -variables:

$$I = \int \frac{1}{1 + \log x} \cdot \left(\frac{1}{x} dx\right) = \int \frac{1}{u} du$$

- (e) Integrate this standard rational reciprocal form:

$$I = \log |u| + C$$

where C represents the arbitrary constant of integration.

- (f) Substitute the original expression for u back into the logarithmic solution:

$$I = \log |1 + \log x| + C$$

Final Answer: $\log |1 + \log x| + C$

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Q36.

Solution

Concept: The slope of the tangent line to a curve $y = f(x)$ at a specific point is given by its derivative $m_t = \left. \frac{dy}{dx} \right|_{x=x_0}$. Since a normal line is perpendicular to the tangent line, its slope m_n satisfies the orthogonal relationship $m_n = -\frac{1}{m_t}$. The equation of a straight line is $y - y_0 = m_n(x - x_0)$.

Solution:

- (a) Determine the y-coordinate of the curve $y = 2x^2 + 3 \sin x$ at the evaluation point $x = 0$:

$$y_0 = 2(0)^2 + 3 \sin(0) = 0 \implies \text{Point} = (0, 0)$$

- (b) Find the general derivative expression by differentiating the curve function:

$$\frac{dy}{dx} = \frac{d}{dx}(2x^2 + 3 \sin x) = 4x + 3 \cos x$$

- (c) Substitute $x = 0$ into this derivative to find the tangent slope m_t :

$$m_t = 4(0) + 3 \cos(0) = 0 + 3(1) = 3$$

- (d) Calculate the perpendicular normal line slope m_n :

$$m_n = -\frac{1}{m_t} = -\frac{1}{3}$$

- (e) Set up the equation of the normal line passing through the point $(0, 0)$ with slope $-\frac{1}{3}$:

$$y - 0 = -\frac{1}{3}(x - 0) \implies y = -\frac{1}{3}x$$

- (f) Convert this into standard linear form:

$$3y = -x \implies x + 3y = 0$$

Final Answer: $x + 3y = 0$

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Q37.

Solution

Concept: The area of a triangle with adjacent vector sides \vec{a} and \vec{b} equals half the magnitude of their vector cross product: $\text{Area} = \frac{1}{2}|\vec{a} \times \vec{b}|$. The cross product is computed by expanding a 3×3 determinant containing the standard basis unit vectors.

Solution:

- (a) Write down the given adjacent side vectors:

$$\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}, \quad \vec{b} = \hat{i} - \hat{j} + \hat{k}$$

- (b) Set up the determinant structure to compute the vector cross product $\vec{a} \times \vec{b}$:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

- (c) Expand the matrix along its first row:

$$\vec{a} \times \vec{b} = \hat{i}(1 \cdot 1 - 4 \cdot (-1)) - \hat{j}(3 \cdot 1 - 4 \cdot 1) + \hat{k}(3 \cdot (-1) - 1 \cdot 1)$$

- (d) Simplify the components inside the parentheses:

$$\vec{a} \times \vec{b} = \hat{i}(1 + 4) - \hat{j}(3 - 4) + \hat{k}(-3 - 1) = 5\hat{i} + \hat{j} - 4\hat{k}$$

- (e) Calculate the magnitude of this cross product vector:

$$|\vec{a} \times \vec{b}| = \sqrt{5^2 + 1^2 + (-4)^2} = \sqrt{25 + 1 + 16} = \sqrt{42}$$

- (f) Divide this result by 2 to find the triangle area:

$$\text{Area} = \frac{1}{2}\sqrt{42} \text{ square units}$$

Final Answer: $\frac{1}{2}\sqrt{42}$

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Q38.

Solution

Concept: The Cartesian equation of a flat plane passing through a point (x_0, y_0, z_0) with a perpendicular normal vector $\vec{n} = A\hat{i} + B\hat{j} + C\hat{k}$ is expressed by the linear dot product relationship $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$.

Solution:

- (a) Identify the coordinates of the given point through which the plane passes:

$$(x_0, y_0, z_0) = (1, 2, 3)$$

- (b) Extract the direction components from the given perpendicular normal vector $\vec{n} = 2\hat{i} + 3\hat{j} - \hat{k}$:

$$A = 2, \quad B = 3, \quad C = -1$$

- (c) Substitute these coordinate and vector values into the standard linear plane equation:

$$2(x - 1) + 3(y - 2) + (-1)(z - 3) = 0$$

- (d) Distribute the multiplication coefficients across each binomial expression:

$$2x - 2 + 3y - 6 - z + 3 = 0$$

- (e) Collect and combine the constant scalar values together:

$$2x + 3y - z + (-2 - 6 + 3) = 0$$

- (f) Simplify to find the final equation of the plane:

$$2x + 3y - z - 5 = 0 \implies 2x + 3y - z = 5$$

Final Answer: $2x + 3y - z = 5$

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Q39.

Solution

Concept: A system of linear equations can be solved using Cramer’s Rule, which uses determinants. For a two-variable system, the variable solutions are found using the ratios $x = \frac{\Delta_x}{\Delta}$ and $y = \frac{\Delta_y}{\Delta}$, provided that the main coefficient determinant satisfies $\Delta \neq 0$.

Solution:

- (a) Write down the given linear system of equations:

$$2x + 3y = 8$$

$$3x - 2y = -1$$

- (b) Construct and evaluate the principal coefficient matrix determinant (Δ):

$$\Delta = \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = (2)(-2) - (3)(3) = -4 - 9 = -13$$

- (c) Construct the x -determinant (Δ_x) by replacing the first column with the constant vector values:

$$\Delta_x = \begin{vmatrix} 8 & 3 \\ -1 & -2 \end{vmatrix} = (8)(-2) - (3)(-1) = -16 + 3 = -13$$

- (d) Construct the y -determinant (Δ_y) by replacing the second column with the constant vector values:

$$\Delta_y = \begin{vmatrix} 2 & 8 \\ 3 & -1 \end{vmatrix} = (2)(-1) - (8)(3) = -2 - 24 = -26$$

- (e) Compute the final variable values using the Cramer’s Rule ratios:

$$x = \frac{\Delta_x}{\Delta} = \frac{-13}{-13} = 1$$

$$y = \frac{\Delta_y}{\Delta} = \frac{-26}{-13} = 2$$

Final Answer: $x = 1, y = 2$

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Q40.

Solution

Concept: To prove inverse trigonometric identities, use the standard angle addition identity for the arctangent function: $\tan^{-1} u + \tan^{-1} v = \tan^{-1} \left(\frac{u+v}{1-uv} \right)$, which holds true provided that the product of the arguments satisfies the inequality $uv < 1$.

Solution:

- (a) Write down the left-hand side (LHS) of the identity that needs to be verified:

$$\text{LHS} = \tan^{-1} \left(\frac{1}{2} \right) + \tan^{-1} \left(\frac{1}{3} \right)$$

- (b) Check the condition for the arctangent addition formula by multiplying the arguments:

$$u \cdot v = \left(\frac{1}{2} \right) \cdot \left(\frac{1}{3} \right) = \frac{1}{6}$$

Since $\frac{1}{6} < 1$, the standard addition formula can be applied directly.

- (c) Substitute these values into the arctangent addition identity:

$$\text{LHS} = \tan^{-1} \left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2} \cdot \frac{1}{3} \right)} \right)$$

- (d) Simplify the fractional expressions in both the numerator and the denominator:

$$\text{Numerator} = \frac{3 + 2}{6} = \frac{5}{6}, \quad \text{Denominator} = 1 - \frac{1}{6} = \frac{5}{6}$$

- (e) Substitute these values back into the arctangent expression:

$$\text{LHS} = \tan^{-1} \left(\frac{\frac{5}{6}}{\frac{5}{6}} \right) = \tan^{-1}(1)$$

- (f) Since $\tan \left(\frac{\pi}{4} \right) = 1$, this evaluates to $\frac{\pi}{4}$, which matches the right-hand side (RHS).

Final Answer: The identity is proved to be equal to $\frac{\pi}{4}$.

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Q41.

Solution

Concept: A line parallel to two intersecting planes is parallel to the line of intersection of those planes. Its direction vector \vec{b} is perpendicular to both normal vectors \vec{n}_1 and \vec{n}_2 , meaning $\vec{b} = \vec{n}_1 \times \vec{n}_2$. The symmetric equation of a line passing through (x_0, y_0, z_0) with direction $\langle a, b, c \rangle$ is $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$.

Solution:

- (a) Extract the normal vectors from the planes $3x + 2y - z = 5$ and $x - y + 2z = 3$:

$$\vec{n}_1 = 3\hat{i} + 2\hat{j} - \hat{k}, \quad \vec{n}_2 = \hat{i} - \hat{j} + 2\hat{k}$$

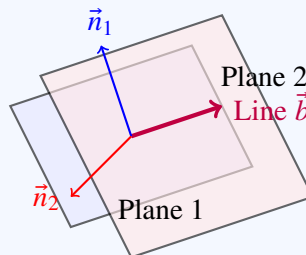
- (b) Find the direction vector \vec{b} using the cross product:

$$\vec{b} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$\vec{b} = \hat{i}(4 - 1) - \hat{j}(6 + 1) + \hat{k}(-3 - 2) = 3\hat{i} - 7\hat{j} - 5\hat{k}$$

- (c) Use the given point $(1, -1, 2)$ to construct the symmetric line equations:

$$\frac{x - 1}{3} = \frac{y + 1}{-7} = \frac{z - 2}{-5}$$



Final Answer: $\frac{x-1}{3} = \frac{y+1}{-7} = \frac{z-2}{-5}$

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Q42.

Solution

Concept: For the indefinite option, use substitution since the numerator is the exact derivative of the denominator quadratic polynomial. For the definite integral option, apply King’s property:
 $\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$.

Solution:

(a) For the **first option:** Let $u = x^2 + 3x + 2$. Then $du = (2x + 3) dx$.

$$I = \int \frac{1}{u} du = \log |u| + C = \log |x^2 + 3x + 2| + C$$

(b) For the **OR option:** Let $I = \int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$. Applying King’s property:

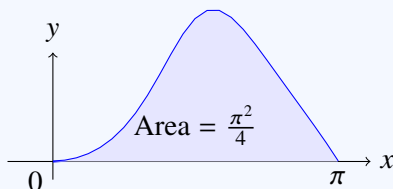
$$I = \int_0^\pi \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx = \int_0^\pi \frac{(\pi - x) \sin x}{1 + \cos^2 x} dx$$

(c) Add both integral representations:

$$2I = \pi \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx$$

(d) Substitute $t = \cos x \implies dt = -\sin x dx$. The limits transform from $[0, \pi]$ to $[1, -1]$:

$$2I = \pi \int_{-1}^1 \frac{1}{1 + t^2} dt = 2\pi \int_0^1 \frac{1}{1 + t^2} dt = 2\pi [\tan^{-1} t]_0^1 = 2\pi \left(\frac{\pi}{4}\right) \implies I = \frac{\pi^2}{4}$$



Final Answer: $\log |x^2 + 3x + 2| + C$

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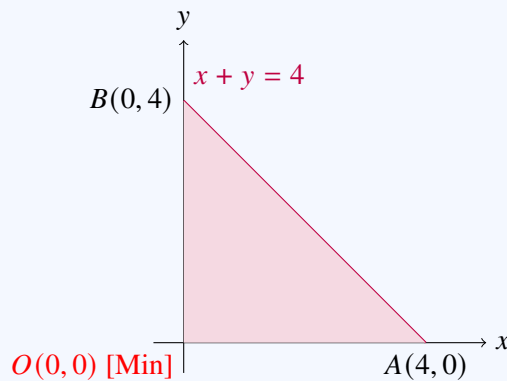
Q43.

Solution

Concept: In Linear Programming, the feasible region is bounded by linear inequalities. By the Corner Point Theorem, the minimum value of a linear objective function occurs at one of the vertices of this bounded polygon.

Solution:

- (a) Graph the boundary line $x + y = 4$. The intercepts on the axes are $(4, 0)$ and $(0, 4)$.
- (b) The non-negativity constraints restrict the region to the first quadrant. Since $(0, 0)$ satisfies $0 + 0 \leq 4$, the feasible area is the right triangle with corners at $O(0, 0)$, $A(4, 0)$, and $B(0, 4)$.
- (c) Evaluate the objective function $Z = 2x + 5y$ at each vertex:
 - At $O(0, 0)$: $Z = 2(0) + 5(0) = 0$
 - At $A(4, 0)$: $Z = 2(4) + 5(0) = 8$
 - At $B(0, 4)$: $Z = 2(0) + 5(4) = 20$
- (d) The absolute minimum value is 0, which occurs at the origin $(0, 0)$.



Final Answer: Minimum value is 0 at $(0, 0)$.

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Q44.

Solution

Concept: The shortest distance d between two skew lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is calculated using the formula $d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$.

Solution:

(a) Extract vector components from the lines: $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$, $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$.

(b) Calculate displacement vector: $\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$.

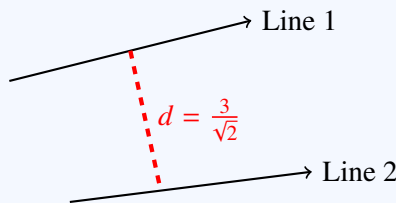
(c) Compute the cross product of the direction vectors:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}(-2 - 1) - \hat{j}(2 - 2) + \hat{k}(1 + 2) = -3\hat{i} + 3\hat{k}$$

(d) Calculate its magnitude: $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 0^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$.

(e) Compute the scalar dot product: $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (1)(-3) + (-3)(0) + (-2)(3) = -9$.

(f) Evaluate the final shortest distance: $d = \frac{|-9|}{3\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$ units.



Final Answer: $\frac{3}{\sqrt{2}}$

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Q45.

Solution

Concept: A function is strictly increasing where its first derivative $f'(x) > 0$ and strictly decreasing where $f'(x) < 0$. Local extrema occur at critical points where $f'(x) = 0$, and are analyzed using the First or Second Derivative Test.

Solution:

(a) Compute the derivative of $f(x) = 2x^3 - 3x^2 - 36x + 7$:

$$f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x - 3)(x + 2)$$

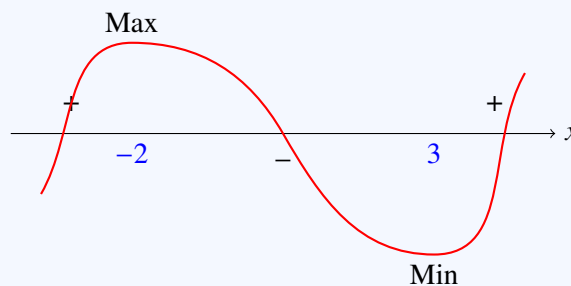
(b) Set $f'(x) = 0$ to get the critical points: $x = 3$ and $x = -2$.

(c) Determine intervals based on the sign of $f'(x)$:

- $(-\infty, -2) \cup (3, \infty)$: $f'(x) > 0 \implies$ Strictly Increasing
- $(-2, 3)$: $f'(x) < 0 \implies$ Strictly Decreasing

(d) Apply the First Derivative Test:

- At $x = -2$, $f'(x)$ changes from positive to negative \implies Local Maxima.
- At $x = 3$, $f'(x)$ changes from negative to positive \implies Local Minima.



Final Answer: Strictly increasing in $(-\infty, -2) \cup (3, \infty)$ and strictly decreasing in $(-2, 3)$. Local maximum at $x = -2$ and local minimum at $x = 3$.

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	B	4	C	5	C
6	C	7	B	8	A	9	A	10	B
11	C	12	C	13	A	14	C	15	B
16	B	17	C	18	B	19	A	20	A

