NIOS Class 12 Mathematics Sample Paper – 9

Duration: 180 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **45** Questions. The paper is divided into two sections: **Section A – 50** marks, **Section B – 50** marks.
- **Section A** consists of
  -**Q.No. 1 to 20** – Multiple Choice type questions (MCQs) carrying **+1 mark** each. Select and write the most appropriate option out of the four options given in each of these questions.
  -**Q.No. 21 to 29** – **Objective type questions**.
  -**Q.No. 21 to 24** carry **02 marks** each (with 2 sub-parts of 1 mark each).
  -**Q.No. 25 to 28** carry **04 marks** each (with 4 sub-parts of 1 mark each).
  -**Q.No. 29** carries **06 marks** (with 6 sub-parts of 1 mark each). Attempt these questions as per the instructions given for each of the questions 21–29.
- **Section B** consists of
  -**Q.No. 30 to 38**– Very Short questions carrying **02 marks** each.
  -**Q.No. 39 to 43** – Short Answer type questions carrying **04 marks** each.
  -**Q.No. 44 to 45** – Long Answer type questions carrying **06 marks** each.
  (An internal choice has been provided in some of the questions in Section B. You have to attempt only one of the given choices in such questions.)
- There is **No Negative marking**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Section: A

Q1. The value of $\lim_{x\to 0} \dfrac{e^{3x}-1}{\sin 2x}$ is:

(1)

(A) $\dfrac{3}{2}$

(B) $\frac{2}{3}$

(C) 3

(D) 1

Q2. If A is a 3×3 matrix and $|A| = -2$, then $|-2A|$ equals:

(1)

(A) 16

(B) -16

(C) 4

(D) -4

Q3. The radius of the circle $x^2 + y^2 + 6x - 8y + 9 = 0$ is:

(1)

(A) 4

(B) 5

(C) $\sqrt{34}$

(D) 2

Q4. If $f(x) = \frac{2x-1}{x+3}, x \neq -3$, then $f^{-1}(x)$ is:

(1)

(A) $\frac{x+3}{2-x}$

(B) $\frac{2x+1}{3-x}$

(C) $\frac{3x+1}{2-x}$

(D) $\frac{x-1}{x+3}$

Q5. The scalar projection of $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ is:

(1)

(A) $\frac{4}{3}$
(B) 4
(C) $\frac{2}{3}$
(D) $\frac{3}{4}$

Q6. If $y = x^x$ for $x > 0$, then $\left.\frac{dy}{dx}\right|_{x=1}$ equals:

(1)

(A) 1
(B) e
(C) 0
(D) 2

Q7. The matrix $\begin{pmatrix} k & 3 \\ 12 & k \end{pmatrix}$ is singular when:

(1)

(A) $k = \pm 6$
(B) $k = 6$ only
(C) $k = -6$ only
(D) $k = \pm 3$

Q8. The distance between the parallel lines $5x - 12y + 13 = 0$ and $5x - 12y - 26 = 0$ is:

(1)

(A) 3
(B) $\frac{13}{3}$
(C) 39
(D) 1

Q9. $\int_0^1 \frac{3x^2}{1+x^3}\,dx$ equals:

(1)

(A) $\log 2$

(B) $2\log 2$

(C) 1

(D) $\frac{1}{2}$

Q10. The converse of the implication $p \to q$ is:

(1)

(A) $\neg q \to \neg p$

(B) $q \to p$

(C) $\neg p \to \neg q$

(D) $p \wedge q$

Q11. The length of the latus rectum of the parabola $y^2 = 8x$ is:

(1)

(A) 8

(B) 4

(C) 2

(D) 16

Q12. The domain of $\sin^{-1}(3x-2)$ is:

(1)

(A) $[-1, 1]$

(B) $\left[\frac{1}{3}, 1\right]$

(C) $\left[0, \frac{2}{3}\right]$

(D) $\left[\frac{2}{3}, 1\right]$

Q13. The function $f(x) = x^3 - 3x^2 - 9x + 2$ is increasing on:

(1)

(A) $(-1, 3)$

(B) $(-\infty, 3)$

(C) $(-\infty, -1) \cup (3, \infty)$

(D) $\mathbb{R}$

Q14. If A is a 3×3 matrix with $|A| = 2$, then $|\operatorname{adj} A|$ is:

(1)

(A) 4

(B) 2

(C) 8

(D) $\frac{1}{4}$

Q15. The equation of the plane through the origin and perpendicular to $\hat{i} - 2\hat{j} + 4\hat{k}$ is:

(1)

(A) $x + 2y + 4z = 0$

(B) $x - 2y + 4z = 0$

(C) $2x - y + 4z = 0$

(D) $x - 2y - 4z = 0$

Q16. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x$ is:

(1)

(A) 1

(B) 2

(C) 3

(D) Not defined

Q17. The eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is:

(1)

(A) $\frac{4}{5}$

(B) $\frac{3}{5}$

(C) $\frac{5}{4}$

(D) $\frac{2}{5}$

Q18. $\int_{-\pi}^{\pi} x \cos x \, dx$ equals:

(1)

(A) 0

(B) 2π

(C) -2π

(D) π^2

Q19. If $A = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$, then A^{-1} is:

(1)

(A) $\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$

(B) $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix}$

(C) $\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$

(D) $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$

Q20. The angle between the planes $x + y + z = 0$ and $x - y + z = 0$ is:

(1)

(A) 0

(B) $\frac{\pi}{2}$

(C) $\cos^{-1}\left(\frac{1}{3}\right)$

(D) $\cos^{-1}\left(-\frac{1}{3}\right)$

Q21. Match Column-I with Column-II:

(2)

Column-I	**Column-II**
(i) The function $f(x) = x^3$ on $\mathbb{R}$	(A) Principal range of $\cos^{-1} x$ is $[0, \pi]$
(ii) The inverse trigonometric function $\cos^{-1} x$	(B) One-one function

(A) (i)→(A), (ii)→(B)

(B) (i)→(B), (ii)→(A)

Q22. Fill in the blanks:

(2)

(i) If A is of order 3×2 and B is of order 2×4, then the order of AB is ________.

(ii) $\det \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} =$ ________.

Q23. Write TRUE for correct statement and FALSE for incorrect statement:

(2)

(i) In a bounded feasible region of a linear programming problem, the maximum or minimum value of a linear objective function occurs at a corner point.

(ii) The negation of $p \lor q$ is $\neg p \lor \neg q$.

Q24. Write the negation of each of the following statements:

(2)

(i) All square matrices are invertible.

(ii) There exists a line parallel to both coordinate axes.

Q25. Fill in the blanks:

(4)

(i) $\lim_{x \to 0} \frac{\sqrt{1+4x}-1}{x} =$ ________.

(ii) $\frac{d}{dx} \log(1+x^2) =$ ________.

(iii) $\int 5x^4 \, dx =$ ________.

(iv) $\int_0^2 x \, dx =$ ________.

Q26. Fill in the blanks:

(4)

(i) The midpoint of $(-2, 5)$ and $(4, -1)$ is ________.

(ii) The slope of $2x + 3y - 7 = 0$ is ________.

(iii) The centre of $x^2 + y^2 - 10x + 6y + 18 = 0$ is ________.

(iv) The eccentricity of the parabola $y^2 = 16x$ is ________.

Q27. Write TRUE for correct statement and FALSE for incorrect statement:

(4)

(i) For square matrices A and B of the same order, $AB = BA$ always.

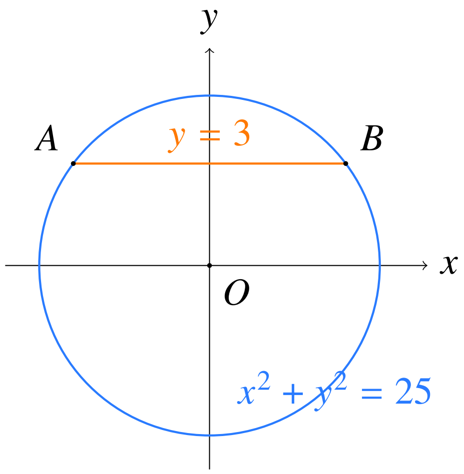
(ii) A square matrix with zero determinant is singular.

(iii) If A is invertible, then $(A^{-1})^{-1} = A$.

(iv) For a 3×3 matrix A, $\det(2A) = 2 \det A$.

Q28. Study the figure of the circle $x^2 + y^2 = 25$ cut by the line $y = 3$, and answer the following:

(4)



(i) Write the radius of the circle.

(ii) Find the coordinates of A and B.

(iii) Find the length of chord AB.

(iv) Find the distance of chord AB from the centre.

Q29. Read the passage and answer (i)–(vi):

A workshop makes two items X and Y. One unit of X requires 3 hours on Machine I and 1 hour on Machine II. One unit of Y requires 1 hour on Machine I and 2 hours on Machine II. Machine I is available for 60 hours and Machine II is available for 50 hours. Profit per unit of X is Rs. 40 and profit per unit of Y is Rs. 30. Let x and y denote the number of units of X and Y respectively.

(6)

(i) The Machine I constraint is:

(A) $x + 2y \leq 50$

(B) $3x + y \leq 60$

(C) $x + y \leq 60$

(D) $40x + 30y \leq 60$

(ii) The Machine II constraint is:

(A) $3x + y \leq 50$

(B) $2x + y \leq 50$

(C) $x + 2y \leq 50$

(D) $x + y \leq 50$

(iii) The objective function is:

(A) Maximize $Z = 40x + 30y$

(B) Maximize $Z = 3x + y$

(C) Minimize $Z = 40x + 30y$

(D) Maximize $Z = x + 2y$

(iv) The point of intersection of $3x + y = 60$ and $x + 2y = 50$ is:

(A) $(18, 14)$

(B) $(15, 15)$

(C) $(20, 0)$

(D) $(14, 18)$

(v) The profit at $(20, 0)$ is:

(A) Rs. 600

(B) Rs. 800

(C) Rs. 1000

(D) Rs. 1100

(vi) The maximum profit is:

(A) Rs. 1100

(B) Rs. 800

(C) Rs. 750

(D) Rs. 1250

Section: B

Q30. Show that $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 2x - 5$ is one-one.

(2)

Q31. Find the inverse of $A = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$.

(2)

Q32. Evaluate $\lim_{x \to 0} \frac{\log(1 + 5x)}{x}$.

(2)

Q33. Differentiate $y = e^{2x} \sin 3x$ with respect to x.

(2)

Q34. Find a unit vector in the direction of $3\hat{i} - 4\hat{j} + 12\hat{k}$.

OR

Find the angle between $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$.

(2)

Q35. Find the Cartesian equation of the line passing through $(1, -2, 3)$ and having direction ratios $2, 3, -1$.

(2)

Q36. Evaluate $\int_0^1 e^{2x} \, dx$.

(2)

Q37. Find the principal value of $\cos^{-1}\left(\cos \frac{5\pi}{3}\right)$.

(2)

Q38. Find the area of the triangle with vertices $A(1, 0, 0)$, $B(0, 2, 0)$ and $C(0, 0, 2)$.

(2)

Q39. For $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, verify the Cayley-Hamilton relation $A^2 - 4A + 3I = 0$. Hence find A^{-1}.

(4)

Q40. Evaluate $\int \frac{x+5}{x^2+4x+3}\, dx$.

(4)

Q41. Find the distance of the point $P(1, 2, 3)$ from the plane $2x - y + 2z - 5 = 0$. Also find the foot of the perpendicular from P to the plane.

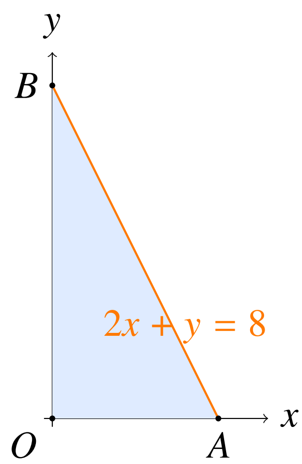
(4)

Q42. Solve the differential equation $\frac{dy}{dx} + 2y = e^{-x}$, given that $y(0) = 1$.

(4)

Q43. In the figure, the line $2x + y = 8$ meets the coordinate axes at A and B. Find the intercepts and the area of triangle OAB.

(4)



Q44. Find the area bounded by the parabola $y = 4 - x^2$ and the x-axis, using integration.

(6)

Q45. Find the equation of the plane through $A(1,0,0)$, $B(0,2,0)$ and $C(0,0,3)$. Also find the volume of the tetrahedron formed by this plane and the three coordinate planes.

(6)

Detailed Solutions

Q1.

Solution

Concept: For small values of x, the standard limits $\lim_{u\to 0}\frac{e^u-1}{u}=1$ and $\lim_{u\to 0}\frac{\sin u}{u}=1$ are used. When the argument is multiplied by a constant, that constant must be adjusted carefully.

Solution:

Step 1: Write the expression as

$$\lim_{x\to 0}\frac{e^{3x}-1}{\sin 2x}.$$

Step 2: Multiply and divide by the natural arguments $3x$ and $2x$:

$$\frac{e^{3x}-1}{\sin 2x}=\frac{e^{3x}-1}{3x}\cdot\frac{3x}{2x}\cdot\frac{2x}{\sin 2x}.$$

Step 3: Apply the standard limits. As $x\to 0$, $3x\to 0$ and $2x\to 0$.

$$\lim_{x\to 0}\frac{e^{3x}-1}{3x}=1, \qquad \lim_{x\to 0}\frac{2x}{\sin 2x}=1.$$

Step 4: Therefore,

$$\lim_{x\to 0}\frac{e^{3x}-1}{\sin 2x}=1\cdot\frac{3}{2}\cdot 1=\frac{3}{2}.$$

Final Answer: $\boxed{\frac{3}{2}}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: If A is an $n \times n$ matrix, then $|kA| = k^n|A|$. The scalar multiplier affects every row, so for a 3×3 matrix the determinant is multiplied by the cube of the scalar.

Solution:

Step 1: Here A is a 3×3 matrix, so $n = 3$.

Step 2: Use the determinant property:

$$|-2A| = (-2)^3|A|.$$

Step 3: Substitute $|A| = -2$:

$$|-2A| = (-8)(-2) = 16.$$

Step 4: The positive sign appears because both factors are negative.

Final Answer: $\boxed{16}$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept: The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$. Its centre is $(-g, -f)$ and radius is $\sqrt{g^2 + f^2 - c}$. Completing squares gives the same result and also checks the radius clearly.

Solution:

Step 1: Compare

$$x^2 + y^2 + 6x - 8y + 9 = 0$$

with $x^2 + y^2 + 2gx + 2fy + c = 0$.

Step 2: We get $2g = 6$, so $g = 3$, and $2f = -8$, so $f = -4$. Also, $c = 9$.

Step 3: Radius is

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{3^2 + (-4)^2 - 9}.$$

Step 4: Simplify:

$$r = \sqrt{9 + 16 - 9} = \sqrt{16} = 4.$$

Final Answer: $\boxed{4}$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept: To find the inverse of a function, write $y = f(x)$, interchange the roles algebraically by solving for x in terms of y, and then replace y by x in the final expression. The denominator should also be checked because division by zero is not allowed.

Solution:

Step 1: Let

$$y = \frac{2x - 1}{x + 3}.$$

Step 2: Cross multiply:

$$y(x + 3) = 2x - 1.$$

Step 3: Expand and collect the terms containing x on one side:

$$xy + 3y = 2x - 1 \quad \Rightarrow \quad xy - 2x = -1 - 3y.$$

Step 4: Factor out x:

$$x(y - 2) = -(1 + 3y).$$

Step 5: Divide by $y - 2$:

$$x = \frac{1 + 3y}{2 - y}.$$

Step 6: Hence

$$f^{-1}(x) = \frac{3x + 1}{2 - x}.$$

Final Answer: $\boxed{\frac{3x + 1}{2 - x}}$

Answer: (C)

Go Back to Question 4

Q5.

Solution

Concept: The scalar projection of $\vec{a}$ on $\vec{b}$ is $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$. It measures the signed component of $\vec{a}$ in the direction of $\vec{b}$.

Solution:

Step 1: Compute the dot product:

$$\vec{a}\cdot\vec{b} = (2)(1) + (-1)(2) + (2)(2) = 2 - 2 + 4 = 4.$$

Step 2: Compute the magnitude of $\vec{b}$:

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3.$$

Step 3: Scalar projection is therefore

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{4}{3}.$$

Final Answer: $\boxed{\frac{4}{3}}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: For a variable power expression x^x, logarithmic differentiation is convenient. Write $y = x^x$, take logarithm on both sides, and then differentiate implicitly.

Solution:

Step 1: Let $y = x^x$. Taking logarithm,

$$\log y = x \log x.$$

Step 2: Differentiate both sides with respect to x:

$$\frac{1}{y}\frac{dy}{dx} = \log x + 1.$$

Step 3: Hence

$$\frac{dy}{dx} = x^x(\log x + 1).$$

Step 4: At $x = 1$,

$$\left.\frac{dy}{dx}\right|_{x=1} = 1^1(\log 1 + 1) = 1(0 + 1) = 1.$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: A square matrix is singular when its determinant is zero. For a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the determinant is $ad - bc$.

Solution:

Step 1: The determinant is

$$\begin{vmatrix} k & 3 \\ 12 & k \end{vmatrix} = k \cdot k - 3 \cdot 12 = k^2 - 36.$$

Step 2: For singularity, set the determinant equal to zero:

$$k^2 - 36 = 0.$$

Step 3: Factor the difference of squares:

$$(k - 6)(k + 6) = 0.$$

Step 4: Therefore,

$$k = 6 \quad \text{or} \quad k = -6.$$

Final Answer: $\boxed{k = \pm 6}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: The distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$. The coefficients of x and y must be exactly the same in both equations.

Solution:

Step 1: The two lines are

$$5x - 12y + 13 = 0, \qquad 5x - 12y - 26 = 0.$$

Step 2: Here $a = 5$, $b = -12$, $c_1 = 13$, and $c_2 = -26$.

Step 3: Distance is

$$d = \frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}} = \frac{|-26 - 13|}{\sqrt{5^2 + (-12)^2}}.$$

Step 4: Simplify:

$$d = \frac{39}{\sqrt{25 + 144}} = \frac{39}{13} = 3.$$

Final Answer: $\boxed{3}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: When the numerator is the derivative of the denominator, the integral becomes logarithmic. The formula is $\int \frac{f'(x)}{f(x)}\, dx = \log|f(x)| + C$.

Solution:

Step 1: Let the denominator be

$$u = 1 + x^3.$$

Step 2: Differentiate:

$$du = 3x^2\, dx.$$

Step 3: The integral becomes

$$\int_0^1 \frac{3x^2}{1 + x^3}\, dx = \int_{u=1}^{u=2} \frac{du}{u}.$$

Step 4: Evaluate:

$$\int_1^2 \frac{du}{u} = [\log u]_1^2 = \log 2 - \log 1 = \log 2.$$

Final Answer: $\boxed{\log 2}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: For an implication $p \to q$, the converse is obtained by interchanging the hypothesis and conclusion. Thus the converse reads "if q, then p".

Solution:

Step 1: Original implication:

$$p \to q.$$

Step 2: Interchange the position of p and q.

Step 3: The converse becomes

$$q \to p.$$

Step 4: Note that the contrapositive would be $\neg q \to \neg p$, which is a different statement.

Final Answer: $\boxed{q \to p}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: For the parabola $y^2 = 4ax$, the length of the latus rectum is $4a$. Therefore, when the equation is already in the form $y^2 = 4ax$, the coefficient of x directly gives the latus rectum length.

Solution:

Step 1: Compare

$$y^2 = 8x$$

with the standard form

$$y^2 = 4ax.$$

Step 2: Therefore,

$$4a = 8.$$

Step 3: The length of the latus rectum is $4a$, so it is

$$8.$$

Final Answer: $\boxed{8}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: For $\sin^{-1} u$ to be defined as a real number, the input u must lie in the interval $[-1, 1]$. This gives an inequality involving x.

Solution:

Step 1: Here the input of the inverse sine function is $3x - 2$.

Step 2: Write the required condition:

$$-1 \leq 3x - 2 \leq 1.$$

Step 3: Add 2 throughout:

$$1 \leq 3x \leq 3.$$

Step 4: Divide by 3:

$$\frac{1}{3} \leq x \leq 1.$$

Final Answer: $\boxed{\left[\frac{1}{3}, 1\right]}$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: A differentiable function is increasing where its derivative is positive. For polynomial functions, the sign of the derivative is studied by factoring it and making a sign chart across its critical points.

Solution:

Step 1: Differentiate the function:

$$f(x) = x^3 - 3x^2 - 9x + 2,$$

$$f'(x) = 3x^2 - 6x - 9.$$

Step 2: Factor the derivative:

$$f'(x) = 3(x^2 - 2x - 3) = 3(x - 3)(x + 1).$$

Step 3: Critical points are $x = -1$ and $x = 3$.

Step 4: Test the sign of $f'(x)$: it is positive for $x < -1$, negative for $-1 < x < 3$, and positive for $x > 3$.

Step 5: Hence the function is increasing on

$$(-\infty, -1) \cup (3, \infty).$$

Final Answer: $\boxed{(-\infty, -1) \cup (3, \infty)}$

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept: For an $n \times n$ matrix A, the determinant of its adjoint is $|\operatorname{adj} A| = |A|^{n-1}$. Here the order is 3, so the exponent is 2.

Solution:

Step 1: Given $|A| = 2$ and A is 3×3.

Step 2: Apply the formula:

$$|\operatorname{adj} A| = |A|^{3-1} = |A|^2.$$

Step 3: Substitute $|A| = 2$:

$$|\operatorname{adj} A| = 2^2 = 4.$$

Final Answer: $\boxed{4}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: A plane through the origin with normal vector $a\hat{i} + b\hat{j} + c\hat{k}$ has equation $ax + by + cz = 0$. The coefficients of x, y, z are the components of the normal vector.

Solution:

Step 1: The given normal vector is

$$\hat{i} - 2\hat{j} + 4\hat{k}.$$

Step 2: Therefore $a = 1$, $b = -2$, and $c = 4$.

Step 3: Since the plane passes through the origin, the constant term is zero.

Step 4: Hence the equation is

$$x - 2y + 4z = 0.$$

Final Answer: $\boxed{x - 2y + 4z = 0}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: The order of a differential equation is the order of the highest derivative present. The degree is the power of the highest-order derivative after the equation is polynomial in derivatives.
Solution:

Step 1: The given differential equation is

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x.$$

Step 2: The highest derivative present is $\frac{d^2y}{dx^2}$, so the order is 2.

Step 3: This highest derivative occurs with power 2.

Step 4: The equation is polynomial in derivatives, so the degree is defined and equals 2.

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$, eccentricity is $e = \sqrt{1 - \frac{b^2}{a^2}}$.
Solution:

Step 1: From

$$\frac{x^2}{25} + \frac{y^2}{9} = 1,$$

we have $a^2 = 25$ and $b^2 = 9$.

Step 2: Use the formula:

$$e = \sqrt{1 - \frac{b^2}{a^2}}.$$

Step 3: Substitute:

$$e = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}.$$

Final Answer: $\boxed{\frac{4}{5}}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: If $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)\,dx = 0$. A product of an odd function and an even function is odd.

Solution:

Step 1: Here x is an odd function and $\cos x$ is an even function.

Step 2: Therefore $x\cos x$ is odd because

$$(-x)\cos(-x) = -x\cos x.$$

Step 3: The limits are symmetric from $-\pi$ to π.

Step 4: Hence

$$\int_{-\pi}^{\pi} x\cos x\,dx = 0.$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: The inverse of a diagonal matrix is obtained by replacing each non-zero diagonal entry by its reciprocal. Off-diagonal zero entries remain zero.

Solution:

Step 1: The given matrix is

$$A = \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}.$$

Step 2: Since both diagonal entries are non-zero, the inverse exists.

Step 3: Take reciprocals of the diagonal entries:

$$2 \mapsto \frac{1}{2}, \qquad -3 \mapsto -\frac{1}{3}.$$

Step 4: Hence

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix}.$$

Final Answer: $\boxed{\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix}}$

Answer: (B)

Go Back to Question 19

Q20.

Solution

Concept: The angle between two planes is the angle between their normal vectors. If the normal vectors are $\vec{n}_1$ and $\vec{n}_2$, then $\cos\theta = \dfrac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$ for the acute angle.

Solution:

Step 1: Normal vectors of the planes are

$$\vec{n}_1 = (1, 1, 1), \qquad \vec{n}_2 = (1, -1, 1).$$

Step 2: Dot product:

$$\vec{n}_1 \cdot \vec{n}_2 = 1 - 1 + 1 = 1.$$

Step 3: Magnitudes:

$$|\vec{n}_1| = \sqrt{3}, \qquad |\vec{n}_2| = \sqrt{3}.$$

Step 4: Therefore

$$\cos\theta = \frac{1}{3}.$$

Step 5: Hence

$$\theta = \cos^{-1}\left(\frac{1}{3}\right).$$

Final Answer: $\boxed{\cos^{-1}\left(\frac{1}{3}\right)}$

Answer: (C)

Go Back to Question 20

Q21.

Solution

Concept: A function is one-one if distinct inputs give distinct outputs. The function x^3 is strictly increasing on $\mathbb{R}$, so it is one-one. The principal value range of $\cos^{-1} x$ is fixed as $[0, \pi]$.

Solution:

Step 1: The function $f(x) = x^3$ is strictly increasing on the real line, so it is one-one. Thus (i) matches (B).

Step 2: For $\cos^{-1} x$, the principal value always lies between 0 and π. Thus (ii) matches (A).

Step 3: Therefore the correct matching is

$$(i) \longrightarrow (B), \qquad (ii) \longrightarrow (A).$$

Final Answer: $\boxed{(i) \longrightarrow (B),\ (ii) \longrightarrow (A)}$

Go Back to Question 21

Q22.

Solution

Concept: Matrix multiplication AB is possible when the number of columns of A equals the number of rows of B. The resulting matrix has order equal to rows of A by columns of B. A 2×2 determinant is evaluated as $ad - bc$.

Solution:

Step 1: For part (i), A is 3×2 and B is 2×4. The inner numbers match, so AB exists.

Step 2: The order of AB is outer numbers, that is 3×4.

Step 3: For part (ii),

$$\det \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} = 5 \cdot 3 - 2 \cdot 7 = 15 - 14 = 1.$$

Final Answer: $\boxed{\text{(i) } 3 \times 4, \quad \text{(ii) } 1}$

Go Back to Question 22

Q23.

Solution

Concept: In linear programming, the corner point theorem states that the optimum of a linear objective function over a bounded feasible region occurs at a vertex. In logic, De Morgan's law says $\neg(p \vee q) = \neg p \wedge \neg q$.

Solution:

Step 1: Statement (i) is correct by the corner point theorem of linear programming. Hence it is TRUE.

Step 2: Statement (ii) says that the negation of $p \vee q$ is $\neg p \vee \neg q$.

Step 3: By De Morgan's law, the correct negation is

$$\neg(p \vee q) = \neg p \wedge \neg q.$$

Step 4: Therefore statement (ii) is FALSE.

Final Answer: $\boxed{\text{(i) TRUE, (ii) FALSE}}$

Go Back to Question 23

Q24.

Solution

Concept: The negation of a universal statement beginning with "All" is an existential statement beginning with "There exists at least one". The negation of an existential statement beginning with "There exists" is a universal denial such as "No" or "There does not exist".

Solution:

Step 1: Statement (i) is: "All square matrices are invertible."

Step 2: Its negation is: "There exists at least one square matrix which is not invertible."

Step 3: Statement (ii) is: "There exists a line parallel to both coordinate axes."

Step 4: Its negation is: "There does not exist any line parallel to both coordinate axes."

Final Answer:

$$\boxed{\text{(i) Some square matrix is not invertible.}}$$

$$\boxed{\text{(ii) No line is parallel to both coordinate axes.}}$$

Go Back to Question 24

Q25.

Solution

Concept: This question uses basic limits, logarithmic differentiation, elementary integration, and definite integration. For radicals, rationalization is useful; for $\log(1 + x^2)$, apply the chain rule; for power integration, increase the exponent by one and divide by the new exponent.

Solution:

Step 1: For part (i), rationalize:

$$\frac{\sqrt{1+4x}-1}{x} \cdot \frac{\sqrt{1+4x}+1}{\sqrt{1+4x}+1} = \frac{4}{\sqrt{1+4x}+1}.$$

Putting $x = 0$ gives $\frac{4}{2} = 2$.

Step 2: For part (ii), by chain rule,

$$\frac{d}{dx}\log(1+x^2) = \frac{2x}{1+x^2}.$$

Step 3: For part (iii),

$$\int 5x^4\,dx = 5 \cdot \frac{x^5}{5} + C = x^5 + C.$$

Step 4: For part (iv),

$$\int_0^2 x\,dx = \left[\frac{x^2}{2}\right]_0^2 = \frac{4}{2} = 2.$$

Final Answer: $\boxed{\text{(i) } 2,\ \text{(ii) } \frac{2x}{1+x^2},\ \text{(iii) } x^5 + C,\ \text{(iv) } 2}$

Go Back to Question 25

Q26.

Solution

Concept: Coordinate geometry uses standard formulas: midpoint formula, slope formula from a line, circle centre from the general equation, and eccentricity of a parabola. A parabola always has eccentricity 1.

Solution:

Step 1: Midpoint of $(-2, 5)$ and $(4, -1)$ is

$$\left(\frac{-2+4}{2}, \frac{5+(-1)}{2}\right) = (1, 2).$$

Step 2: For $2x + 3y - 7 = 0$, slope is $-\frac{a}{b} = -\frac{2}{3}$.

Step 3: For $x^2 + y^2 - 10x + 6y + 18 = 0$, compare with $x^2 + y^2 + 2gx + 2fy + c = 0$.

$$2g = -10 \Rightarrow g = -5, \quad 2f = 6 \Rightarrow f = 3.$$

Centre is $(-g, -f) = (5, -3)$.

Step 4: A parabola has eccentricity 1.

Final Answer: $\boxed{\text{(i) } (1, 2),\ \text{(ii) } -\frac{2}{3},\ \text{(iii) } (5, -3),\ \text{(iv) } 1}$

Go Back to Question 26

Q27.

Solution

Concept: Matrix multiplication is generally not commutative. A square matrix is singular exactly when its determinant is zero. The inverse of an inverse returns the original matrix. For an $n \times n$ matrix, $\det(kA) = k^n \det A$.

Solution:

Step 1: Statement (i) is FALSE because, in general, $AB \neq BA$.

Step 2: Statement (ii) is TRUE because determinant zero is the defining test for singularity.

Step 3: Statement (iii) is TRUE because reversing the inverse operation gives the original matrix.

Step 4: Statement (iv) is FALSE because for a 3×3 matrix,

$$\det(2A) = 2^3 \det A = 8 \det A,$$

not $2 \det A$.

Final Answer: $\boxed{\text{(i) FALSE, (ii) TRUE, (iii) TRUE, (iv) FALSE}}$

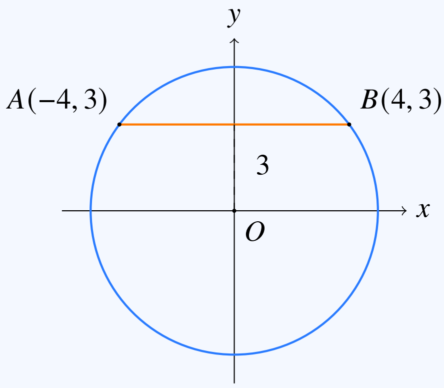
Go Back to Question 27

Q28.

Solution

Concept: Points of intersection of a line and a circle are found by substituting the line equation into the circle equation. The chord length is the distance between the two intersection points.

Solution:



Step 1: The circle is $x^2 + y^2 = 25$, so its radius is $\sqrt{25} = 5$.

Step 2: On the line $y = 3$, substitute into the circle:

$$x^2 + 3^2 = 25 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4.$$

Thus $A(-4, 3)$ and $B(4, 3)$.

Step 3: The chord length is

$$AB = 4 - (-4) = 8.$$

Step 4: The perpendicular distance from the centre $O(0, 0)$ to the line $y = 3$ is 3 units.

Final Answer: $\boxed{\text{(i) } 5,\ \text{(ii) } (-4, 3), (4, 3),\ \text{(iii) } 8,\ \text{(iv) } 3}$

Go Back to Question 28

Q29.

Solution

Concept: In a linear programming word problem, resource limitations form inequalities, and profit forms the objective function. The maximum of a linear objective function over the feasible region occurs at a corner point.

Solution:

Step 1: Machine I usage is 3 hours for X and 1 hour for Y, so

$$3x + y \leq 60.$$

Thus part (i) is option B.

Step 2: Machine II usage is 1 hour for X and 2 hours for Y, so

$$x + 2y \leq 50.$$

Thus part (ii) is option C.

Step 3: Profit is Rs. 40 per unit of X and Rs. 30 per unit of Y, so the objective is

$$\text{Maximize } Z = 40x + 30y.$$

Thus part (iii) is option A.

Step 4: Solve the two boundary equations:

$$3x + y = 60, \qquad x + 2y = 50.$$

From the first equation, $y = 60 - 3x$. Substitute in the second:

$$x + 2(60 - 3x) = 50 \Rightarrow x + 120 - 6x = 50 \Rightarrow -5x = -70 \Rightarrow x = 14.$$

Then $y = 60 - 42 = 18$. Thus part (iv) is option D.

Step 5: At $(20, 0)$,

$$Z = 40(20) + 30(0) = 800.$$

Thus part (v) is option B.

Step 6: Check corner profits: at $(20, 0)$, $Z = 800$; at $(0, 25)$, $Z = 750$; at $(14, 18)$,

$$Z = 40(14) + 30(18) = 560 + 540 = 1100.$$

The maximum profit is Rs. 1100, so part (vi) is option A.

Final Answer: (i) B, (ii) C, (iii) A, (iv) D, (v) B, (vi) A

Go Back to Question 29

Q30.

Solution

Concept: A function f is one-one if $f(a) = f(b)$ implies $a = b$. For a linear function with non-zero coefficient of x, two different inputs cannot give the same output.

Solution:

Step 1: Let $a, b \in \mathbb{R}$ and assume

$$f(a) = f(b).$$

Step 2: Using $f(x) = 2x - 5$, we get

$$2a - 5 = 2b - 5.$$

Step 3: Add 5 to both sides:

$$2a = 2b.$$

Step 4: Divide by 2:

$$a = b.$$

Step 5: Therefore f is one-one.

Final Answer: $\boxed{f \text{ is one-one}}$

Go Back to Question 30

Q31.

Solution

Concept: For a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, its inverse is $A^{-1} = \dfrac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, provided $ad - bc \neq 0$.

Solution:

Step 1: For

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix},$$

the determinant is

$$|A| = 1 \cdot 7 - 2 \cdot 3 = 7 - 6 = 1.$$

Step 2: Since $|A| \neq 0$, the inverse exists.

Step 3: Apply the inverse formula:

$$A^{-1} = \frac{1}{1}\begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}.$$

Final Answer: $\boxed{\begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}}$

Go Back to Question 31

Q32.

Solution

Concept: The standard logarithmic limit is $\lim_{u\to 0} \frac{\log(1+u)}{u} = 1$. If the expression contains $5x$, we multiply and divide by 5 to form the standard structure.

Solution:

Step 1: Write

$$\frac{\log(1+5x)}{x} = 5 \cdot \frac{\log(1+5x)}{5x}.$$

Step 2: Let $u = 5x$. As $x \to 0$, $u \to 0$.

Step 3: Therefore

$$\lim_{x\to 0} \frac{\log(1+5x)}{x} = 5 \cdot 1 = 5.$$

Final Answer: $\boxed{5}$

Go Back to Question 32

Q33.

Solution

Concept: The derivative of a product is found by the product rule: $(uv)' = u'v + uv'$. Here one factor is exponential and the other is trigonometric, so both must be differentiated.

Solution:

Step 1: Let $u = e^{2x}$ and $v = \sin 3x$.

Step 2: Then

$$u' = 2e^{2x}, \qquad v' = 3\cos 3x.$$

Step 3: Apply product rule:

$$\frac{dy}{dx} = 2e^{2x}\sin 3x + e^{2x} \cdot 3\cos 3x.$$

Step 4: Factor e^{2x}:

$$\frac{dy}{dx} = e^{2x}(2\sin 3x + 3\cos 3x).$$

Final Answer: $\boxed{e^{2x}(2\sin 3x + 3\cos 3x)}$

Go Back to Question 33

Q34.

Solution

Concept: A unit vector in the direction of a vector $\vec{v}$ is obtained by dividing $\vec{v}$ by its magnitude. For the alternative part, the angle between vectors is found from $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$.

Solution:

Step 1: For the first part,

$$\vec{v} = 3\hat{i} - 4\hat{j} + 12\hat{k}.$$

Step 2: Its magnitude is

$$|\vec{v}| = \sqrt{3^2 + (-4)^2 + 12^2} = \sqrt{9 + 16 + 144} = 13.$$

Step 3: Hence the required unit vector is

$$\frac{\vec{v}}{|\vec{v}|} = \frac{3\hat{i} - 4\hat{j} + 12\hat{k}}{13}.$$

Step 4: For the OR part,

$$\vec{a} \cdot \vec{b} = (1)(2) + (2)(1) + (2)(-2) = 2 + 2 - 4 = 0.$$

Since the dot product is zero, the angle is 90°.

Final Answer: $\boxed{\frac{3\hat{i} - 4\hat{j} + 12\hat{k}}{13}}$ OR $\boxed{90^\circ}$

Go Back to Question 34

Q35.

Solution

Concept: A line through (x_1, y_1, z_1) with direction ratios a, b, c has Cartesian equation $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$.

Solution:

Step 1: The point is $(1, -2, 3)$.

Step 2: Direction ratios are $2, 3, -1$.

Step 3: Substitute into the standard form:

$$\frac{x-1}{2} = \frac{y-(-2)}{3} = \frac{z-3}{-1}.$$

Step 4: Hence

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-1}.$$

Final Answer: $\boxed{\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-1}}$

Go Back to Question 35

Q36.

Solution

Concept: The integral of e^{ax} is $\frac{e^{ax}}{a} + C$. For a definite integral, evaluate the antiderivative at the upper limit and subtract the value at the lower limit.

Solution:

Step 1: Find the antiderivative:

$$\int e^{2x}\,dx = \frac{e^{2x}}{2}.$$

Step 2: Apply the limits:

$$\int_0^1 e^{2x}\,dx = \left[\frac{e^{2x}}{2}\right]_0^1.$$

Step 3: Substitute:

$$\frac{e^2}{2} - \frac{1}{2} = \frac{e^2-1}{2}.$$

Final Answer: $\boxed{\frac{e^2-1}{2}}$

Go Back to Question 36

Q37.

Solution

Concept: The principal range of $\cos^{-1} x$ is $[0, \pi]$. Even if the original angle lies outside this interval, the inverse cosine returns the angle in $[0, \pi]$ having the same cosine value.

Solution:

Step 1: Evaluate the inner cosine:

$$\cos \frac{5\pi}{3} = \frac{1}{2}.$$

Step 2: Therefore

$$\cos^{-1}\left(\cos \frac{5\pi}{3}\right) = \cos^{-1}\left(\frac{1}{2}\right).$$

Step 3: In the principal range $[0, \pi]$, the angle whose cosine is $\frac{1}{2}$ is $\frac{\pi}{3}$.

Final Answer: $\boxed{\frac{\pi}{3}}$

Go Back to Question 37

Q38.

Solution

Concept: The area of a triangle formed by two vectors $\vec{u}$ and $\vec{v}$ from a common vertex is $\frac{1}{2}|\vec{u} \times \vec{v}|$.

Solution:

Step 1: Take $A(1, 0, 0)$ as the common vertex.

Step 2: Form vectors:

$$\overrightarrow{AB} = B - A = (-1, 2, 0), \qquad \overrightarrow{AC} = C - A = (-1, 0, 2).$$

Step 3: Compute cross product:

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{vmatrix} = 4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}.$$

Step 4: Magnitude is

$$\sqrt{4^2 + 2^2 + 2^2} = \sqrt{24} = 2\sqrt{6}.$$

Step 5: Area of the triangle is

$$\frac{1}{2}(2\sqrt{6}) = \sqrt{6}.$$

Final Answer: $\boxed{\sqrt{6} \text{ square units}}$

Go Back to Question 38

Q39.

Solution

Concept: The Cayley-Hamilton theorem states that every square matrix satisfies its own characteristic equation. For this question, the relation is already provided, so we verify it directly by computing A^2 and simplifying $A^2 - 4A + 3I$.

Solution:

Step 1: Given

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

Step 2: Compute

$$A^2 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}.$$

Step 3: Now calculate

$$A^2 - 4A + 3I = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} - \begin{pmatrix} 8 & 4 \\ 4 & 8 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}.$$

Step 4: Simplifying entries gives

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

So the relation is verified.

Step 5: From

$$A^2 - 4A + 3I = 0,$$

multiply by A^{-1}:

$$A - 4I + 3A^{-1} = 0.$$

Step 6: Hence

$$3A^{-1} = 4I - A.$$

Step 7: Therefore

$$A^{-1} = \frac{1}{3}\left(\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\right) = \frac{1}{3}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}.$$

Final Answer: $\boxed{A^{-1} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix}}$

Go Back to Question 39

Q40.

Solution

Concept: A rational function with a factorable quadratic denominator can be integrated by partial fractions. Since $x^2 + 4x + 3 = (x + 1)(x + 3)$, we express the integrand as a sum of two simple fractions.

Solution:

Step 1: Factor the denominator:

$$x^2 + 4x + 3 = (x + 1)(x + 3).$$

Step 2: Write

$$\frac{x + 5}{(x + 1)(x + 3)} = \frac{A}{x + 1} + \frac{B}{x + 3}.$$

Step 3: Multiply by $(x + 1)(x + 3)$:

$$x + 5 = A(x + 3) + B(x + 1).$$

Step 4: Substitute $x = -1$:

$$4 = 2A \Rightarrow A = 2.$$

Step 5: Substitute $x = -3$:

$$2 = -2B \Rightarrow B = -1.$$

Step 6: Therefore

$$\int \frac{x + 5}{x^2 + 4x + 3}\,dx = \int \left(\frac{2}{x + 1} - \frac{1}{x + 3}\right) dx.$$

Step 7: Integrate term by term:

$$2\log|x + 1| - \log|x + 3| + C.$$

Final Answer: $\boxed{2\log|x + 1| - \log|x + 3| + C}$

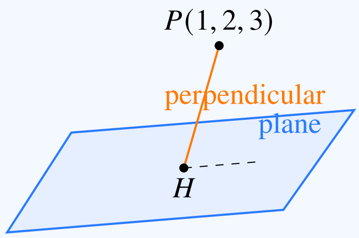
Go Back to Question 40

Q41.

Solution

Concept: The distance from a point (x_1, y_1, z_1) to a plane $ax + by + cz + d = 0$ is $\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$. The foot of the perpendicular is found by moving from the point in the direction of the plane normal.

TikZ Diagram Explanation: The diagram shows a plane, the point P, the foot H, and the perpendicular segment PH.



Solution:

Step 1: Plane is $2x - y + 2z - 5 = 0$, so $a = 2$, $b = -1$, $c = 2$, $d = -5$.

Step 2: For $P(1, 2, 3)$,

$$ax_1 + by_1 + cz_1 + d = 2(1) - 2 + 2(3) - 5 = 1.$$

Step 3: Denominator is

$$\sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{9} = 3.$$

Step 4: Hence distance is

$$d = \frac{|1|}{3} = \frac{1}{3}.$$

Step 5: The normal vector is $\vec{n} = (2, -1, 2)$ and $|\vec{n}|^2 = 9$.

Step 6: Foot of perpendicular is

$$H = P - \frac{2(1) - 2 + 2(3) - 5}{9}\vec{n}.$$

Step 7: Therefore

$$H = (1, 2, 3) - \frac{1}{9}(2, -1, 2) = \left(\frac{7}{9}, \frac{19}{9}, \frac{25}{9}\right).$$

Final Answer: $\boxed{\text{Distance} = \frac{1}{3}, \quad H\left(\frac{7}{9}, \frac{19}{9}, \frac{25}{9}\right)}$

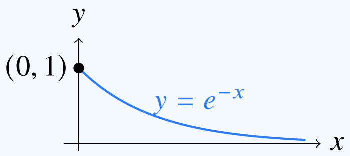
Go Back to Question 41

Q42.

Solution

Concept: A first-order linear differential equation $\frac{dy}{dx} + Py = Q$ is solved using integrating factor $e^{\int P\,dx}$. After multiplying by the integrating factor, the left side becomes the derivative of a product.

TikZ Diagram Explanation: The solution obtained is $y = e^{-x}$, a decreasing exponential curve passing through $(0, 1)$.



Solution:

Step 1: The differential equation is

$$\frac{dy}{dx} + 2y = e^{-x}.$$

Step 2: Here $P = 2$, so the integrating factor is

$$I.F. = e^{\int 2\,dx} = e^{2x}.$$

Step 3: Multiply the equation by e^{2x}:

$$e^{2x}\frac{dy}{dx} + 2e^{2x}y = e^{x}.$$

Step 4: The left side is

$$\frac{d}{dx}\left(ye^{2x}\right) = e^{x}.$$

Step 5: Integrate both sides:

$$ye^{2x} = e^{x} + C.$$

Step 6: Therefore

$$y = e^{-x} + Ce^{-2x}.$$

Step 7: Use $y(0) = 1$:

$$1 = 1 + C \Rightarrow C = 0.$$

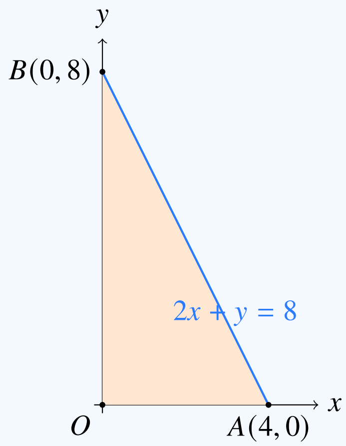
Final Answer: $\boxed{y = e^{-x}}$

Go Back to Question 42

Q43.

Solution

Concept: The intercepts of a line with the coordinate axes are found by setting the other variable equal to zero. The area of a right triangle formed with the axes is $\frac{1}{2} \times$ base $\times$ height.
TikZ Diagram Explanation: The shaded triangle has base OA on the x-axis and height OB on the y-axis.



Solution:

Step 1: For the x-intercept, put $y = 0$ in $2x + y = 8$:

$$2x = 8 \Rightarrow x = 4.$$

So $A = (4, 0)$.

Step 2: For the y-intercept, put $x = 0$:

$$y = 8.$$

So $B = (0, 8)$.

Step 3: The base of triangle OAB is $OA = 4$ and height is $OB = 8$.

Step 4: Area is

$$\frac{1}{2} \times 4 \times 8 = 16.$$

Final Answer: $\boxed{A(4, 0),\ B(0, 8),\ \text{Area} = 16 \text{ square units}}$

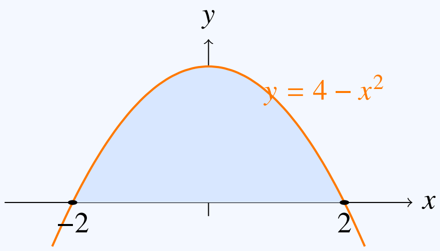
Go Back to Question 43

Q44.

Solution

Concept: The area bounded by a curve and the x-axis is obtained by integrating the upper curve over the interval where it meets the axis. First find the points where $y = 0$, then integrate y between those limits.

TikZ Diagram Explanation: The shaded region lies under $y = 4 - x^2$ and above the x-axis from $x = -2$ to $x = 2$.



Solution:

Step 1: Find the points where the parabola meets the x-axis:

$$4 - x^2 = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2.$$

Step 2: On $[-2, 2]$, the curve $y = 4 - x^2$ is above the x-axis.

Step 3: Area is

$$A = \int_{-2}^{2} (4 - x^2)\, dx.$$

Step 4: Integrate:

$$A = \left[4x - \frac{x^3}{3}\right]_{-2}^{2}.$$

Step 5: Substitute upper and lower limits:

$$A = \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right).$$

Step 6: Simplify:

$$A = \frac{16}{3} + \frac{16}{3} = \frac{32}{3}.$$

Final Answer: $\boxed{\frac{32}{3} \text{ square units}}$

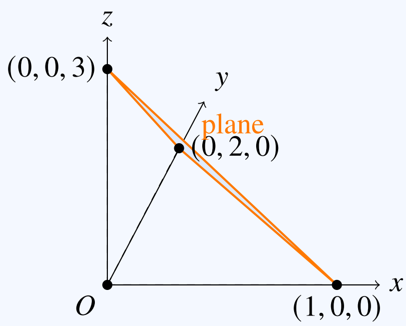
Go Back to Question 44

Q45.

Solution

Concept: A plane cutting the coordinate axes at a, b, c has intercept form $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. The volume of the tetrahedron formed by this plane and the coordinate planes is $\frac{abc}{6}$.

TikZ Diagram Explanation: The triangle with intercepts on the three coordinate axes forms one face of the tetrahedron, while the coordinate planes form the other faces.



Solution:

Step 1: The plane passes through intercept points

$$A(1, 0, 0), \quad B(0, 2, 0), \quad C(0, 0, 3).$$

Step 2: Therefore the intercepts are $a = 1$, $b = 2$, and $c = 3$.

Step 3: Use intercept form:

$$\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1.$$

Step 4: Multiply by 6 to write the Cartesian equation:

$$6x + 3y + 2z = 6.$$

Step 5: The tetrahedron formed with coordinate planes has intercept lengths 1, 2, 3 on the three axes.

Step 6: Its volume is

$$V = \frac{abc}{6} = \frac{1 \cdot 2 \cdot 3}{6} = 1.$$

Final Answer: $\boxed{6x + 3y + 2z = 6, \quad V = 1 \text{ cubic unit}}$

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	A	3	A	4	C	5	A
6	A	7	A	8	A	9	A	10	B
11	A	12	B	13	C	14	A	15	B
16	B	17	A	18	A	19	B	20	C