N 819– MATHEMATICS (71) ALGEBRA - PART I - 2025 Question Paper with Solutions

Time Allowed : 2 Hours	**Maximum Marks :** 40

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 2 hours duration.
2. All questions are compuslory.
3. Use of calculator is not allowed.
4. This question paper is divided into 2 sections. This question paper contains total 11 pages.
5. The maximum marks are 40.

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1. (A) Choose the correct alternative from given:

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

(i) Write the degree of the given determinant.

(A) 1
(B) 2
(C) 3
(D) 4

Correct Answer: (B) 2

Solution: The degree of a determinant is directly related to the size of the matrix it represents. The degree is simply the number of rows or columns in the matrix (which are equal for square matrices). For the given determinant:

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

This is a 2×2 matrix, which means it has two rows and two columns. The degree of a determinant is equal to the number of rows (or columns) in a square matrix. Thus, the degree of the determinant in this case is 2. Therefore, the correct answer is (B) 2.

Quick Tip

For a square matrix of size $n \times n$, the degree of its determinant is n.

(ii) From the following equations, which one is the quadratic equation?

(A) $\frac{5}{x} - 3 = x^2$
(B) $x(x + 5) = 2$
(C) $n - 1 = 2n$
(D) $\frac{1}{x^2}(x + 2) = x$

Correct Answer: (B)

Solution: A quadratic equation is defined as an equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants, and the highest power of the variable (in this case, x) is 2.
Let's evaluate each option:
- (A) $\frac{5}{x} - 3 = x^2$: This equation contains $\frac{1}{x}$ and x^2, so it is not a quadratic equation. - (B) $x(x + 5) = 2$: This simplifies to:

$$x^2 + 5x = 2$$

Rearranging gives:

$$x^2 + 5x - 2 = 0$$

This is a quadratic equation because the highest power of x is 2. - (C) $n - 1 = 2n$: This is a linear equation (the highest power of n is 1). - (D) $\frac{1}{x^2}(x + 2) = x$: This is not a quadratic equation because it involves $\frac{1}{x^2}$ and not just x^2.
Thus, the correct answer is (B), because $x(x + 5) = 2$ simplifies to a quadratic equation.

> **Quick Tip**
>
> A quadratic equation always has the highest power of the variable as 2, and it is in the form $ax^2 + bx + c = 0$.

(iii) Find the common difference of the following A.P.: 4, 4, 4, ...

(A) 1
(B) 8
(C) 4
(D) 0

Correct Answer: (D) 0

Solution: In an arithmetic progression (A.P.), the common difference (d) is the difference between any two consecutive terms. The general form of an A.P. is:

$$a, a + d, a + 2d, a + 3d, \ldots$$

For the given sequence: 4, 4, 4, ..., the terms are all the same. The difference between any two consecutive terms is:

$$d = 4 - 4 = 0$$

Thus, the common difference of this A.P. is 0.

Quick Tip

In an arithmetic progression (A.P.), the common difference is calculated as $d = a_{n+1} - a_n$, where a_n and a_{n+1} are two consecutive terms. If all terms are the same, the common difference is 0.

(iv) Which number cannot represent a probability?

(A) $\frac{2}{3}$
(B) $\frac{15}{10}$
(C) 15%
(D) 0.7

Correct Answer: (B) $\frac{15}{10}$

Solution: A probability is a measure of the likelihood of an event occurring and it always lies between 0 and 1, inclusive. Therefore, any value greater than 1 or less than 0 cannot represent a valid probability. Let's evaluate each of the given options:

- (A) $\frac{2}{3} \approx 0.667$, which is between 0 and 1, so it can represent a probability.
- (B) $\frac{15}{10} = 1.5$, which is greater than 1, so it cannot represent a probability.
- (C) 15% is equal to 0.15, which is a valid probability because it lies between 0 and 1.
- (D) 0.7 is between 0 and 1, so it is a valid probability.

Thus, the number $\frac{15}{10}$ is the only one that cannot represent a probability because it exceeds 1.

Quick Tip

A probability value must always be between 0 and 1, inclusive. Values greater than 1 or less than 0 cannot represent a probability.

(B) Solve the following subquestions:

(i) If $2x + y = 7$ and $x + 2y = 11$, then find the value of $x + y$.

Solution: We are given the following system of equations:

$$2x + y = 7 \quad \text{(Equation 1)}$$

$$x + 2y = 11 \quad \text{(Equation 2)}$$

We need to solve for $x + y$. Let's use the method of substitution or elimination. We will first solve for y from Equation 1.
From Equation 1:

$$2x + y = 7 \quad \Rightarrow \quad y = 7 - 2x \quad \cdots (3)$$

Now, substitute Equation (3) for y in Equation 2:

$$x + 2(7 - 2x) = 11$$

Simplifying:

$$x + 14 - 4x = 11$$
$$-3x + 14 = 11$$
$$-3x = -3 \quad \Rightarrow \quad x = 1$$

Now substitute $x = 1$ into Equation (3) to find y:

$$y = 7 - 2(1) = 7 - 2 = 5$$

Thus, $x = 1$ and $y = 5$, so:

$$x + y = 1 + 5 = 6$$

Therefore, the value of $x + y$ is 6.

Quick Tip

When solving simultaneous equations, substitute one equation into the other to eliminate one variable and simplify the system.

(ii) Find the first term of the given sequence: $t_n = 3n - 4$.

Solution: The given general term of the sequence is:

$$t_n = 3n - 4$$

To find the first term, we substitute $n = 1$ into the formula:

$$t_1 = 3(1) - 4 = 3 - 4 = -1$$

Thus, the first term of the sequence is -1.

Quick Tip

To find the first term of a sequence, substitute $n = 1$ into the general formula for the sequence.

(iii) How many alpha numerals are there in the format of GSTIN?

Solution: GSTIN (Goods and Services Tax Identification Number) is a unique identifier given to taxpayers in India. The GSTIN is in the format:

AAAPL1234C1Z5

The format consists of 15 characters in total: - The first 2 characters are letters (alpha), - The next 10 characters are alphanumeric (alpha-numeric), - The 13th character is a letter (alpha), - The 14th character is a number (numeric), - The 15th character is a letter (alpha).
Thus, there are 14 alphanumeric characters in the GSTIN format (13 alphabetic and 1 numeric).

Quick Tip

GSTIN always consists of 15 characters, with a specific format containing both alphabetic and numeric characters.

(iv) Two coins are tossed simultaneously. Write the sample space S.

Solution: When two coins are tossed simultaneously, there are four possible outcomes: - The first coin can be heads (H) or tails (T), - The second coin can be heads (H) or tails (T).
Thus, the sample space S is:

$$S = \{HH, HT, TH, TT\}$$

There are 4 possible outcomes in the sample space.

Quick Tip

The sample space for two coin tosses contains all the possible combinations of heads and tails. The sample space size is 4.

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2. (A) Complete and write any two activities from the following:

(i) Complete the following table to draw the graph of $x + 2y = 4$.

x	-2	☐
y	☐	1
(x, y)	☐	☐

Solution: We are given the equation of a straight line $x + 2y = 4$, and we need to complete the table of values for the equation in order to plot the graph.

Step 1: Substituting $x = -2$ Substitute $x = -2$ into the equation:

$$x + 2y = 4 \quad \Rightarrow \quad -2 + 2y = 4 \quad \Rightarrow \quad 2y = 6 \quad \Rightarrow \quad y = 3.$$

So, when $x = -2$, the corresponding value of y is $y = 3$, and the point is $(-2, 3)$.

Step 2: Substituting $x = 2$ Substitute $x = 2$ into the equation:

$$x + 2y = 4 \quad \Rightarrow \quad 2 + 2y = 4 \quad \Rightarrow \quad 2y = 2 \quad \Rightarrow \quad y = 1.$$

So, when $x = 2$, the corresponding value of y is $y = 1$, and the point is $(2, 1)$.
Step 3: Table of Values Now, we have the completed table of values:

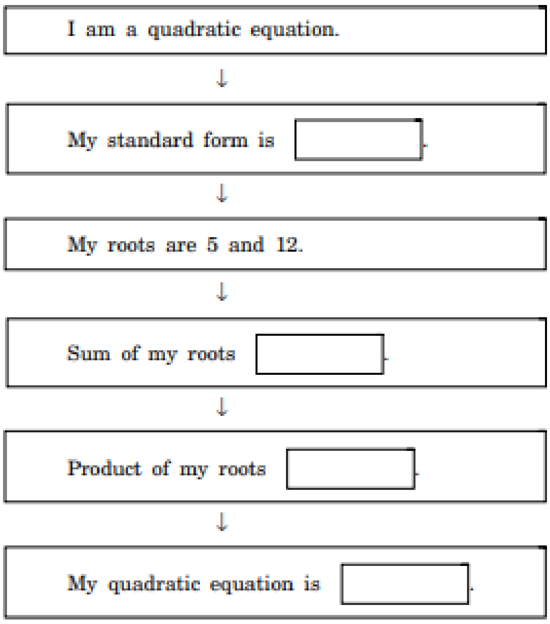
x	-2	2
y	3	1
(x, y)	$(-2, 3)$	$(2, 1)$

These are two points on the graph. By plotting these points and drawing a straight line through them, we will have the graph of the equation $x + 2y = 4$.

Quick Tip

To plot the graph of a linear equation, choose values for x, solve for y, and plot the corresponding points. Connecting at least two points will give you a straight line.

(ii) Complete the following activity to form a quadratic equation.



Solution : I am a quadratic equation.

My standard form is:

$$ax^2 + bx + c = 0$$

The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b, and c are constants, and the highest power of x is 2.

My roots are 5 and 12.

The roots of the quadratic equation are the values of x that satisfy the equation. In this case, the roots are given as 5 and 12.

The sum of the roots is:

$$\text{Sum of the roots} = 5 + 12 = 17$$

The product of the roots is:

$$\text{Product of the roots} = 5 \times 12 = 60$$

Sum of my roots: 17

Product of my roots: 60

We know from Vieta's formulas that for a quadratic equation $ax^2 + bx + c = 0$: - The sum of the roots r_1 and r_2 is given by $r_1 + r_2 = -\frac{b}{a}$, - The product of the roots $r_1 \times r_2$ is given by $r_1 \times r_2 = \frac{c}{a}$.
Using the sum and product of the roots, we can write the quadratic equation in the form:

$$x^2 - (\text{Sum of the roots})x + \text{Product of the roots} = 0$$

Substitute the sum and product of the roots:

$$x^2 - 17x + 60 = 0$$

My quadratic equation is:

$$x^2 - 17x + 60 = 0$$

This is the quadratic equation whose roots are 5 and 12.

Quick Tip

To form a quadratic equation from the given roots, use the sum and product of the roots to write the equation as $(x - r_1)(x - r_2) = 0$, where r_1 and r_2 are the roots. Alternatively, use Vieta's formulas to directly derive the equation.

(iii). Pushpmala has invested 24,000 and purchased share of FV 20 at a premium of 4. Complete the following activity to find the number of shares she purchased.
Activity :
FV = 20
Premium = 4
MV = FV + $\boxed{}$
= 20 + $\boxed{}$
= 24
Number of shares = $\frac{\textbf{Total investment}}{\textbf{MV}}$
$= \frac{24{,}000}{\boxed{}}$

$= \boxed{}$ **shares.**

Correct Answer: The completed activity is provided in the solution. The final answer is 1000 shares.

Solution:

Step 1: Understanding the Concept:
This problem involves basic concepts of share market investments.

- **Face Value (FV):** The nominal value of a share printed on the share certificate.
- **Premium:** The amount paid for a share that is above its Face Value.
- **Market Value (MV):** The price at which a share is actually bought or sold in the market.
- **Total Investment:** The total amount of money used to purchase the shares.

Step 2: Key Formula or Approach:
The formulas needed to solve this activity are:
1. To calculate the Market Value (MV) of a share when it's at a premium:

$$\text{MV} = \text{Face Value (FV)} + \text{Premium}$$

2. To calculate the number of shares purchased:

$$\text{Number of shares} = \frac{\text{Total Investment}}{\text{Market Value (MV) per share}}$$

Step 3: Detailed Explanation:
We will complete the activity step-by-step using the given information and formulas.
Part 1: Calculate the Market Value (MV)
Given:
Face Value (FV) = 20
Premium = 4
Using the formula for MV:

$$\text{MV} = \text{FV} + \text{Premium}$$
$$\text{MV} = 20 + 4$$
$$\text{MV} = 24$$

The first two blanks are 'Premium' and '4'.

Part 2: Calculate the Number of Shares
Given:
Total Investment = 24,000
Market Value (MV) = 24 (calculated above)
Using the formula for the number of shares:

$$\text{Number of shares} = \frac{\text{Total Investment}}{\text{MV}}$$

$$\text{Number of shares} = \frac{24,000}{24}$$
$$\text{Number of shares} = 1000$$

The next two blanks are '24' and '1000'.

Step 4: Final Answer:
The completed activity is as follows:
MV = FV + **Premium**
= 20 + **4**
= 24
Number of shares = $\frac{\text{Total investment}}{\text{MV}}$
= $\frac{24{,}000}{\mathbf{24}}$
= **1000** shares.
Pushpmala purchased 1000 shares.

> **Quick Tip**
>
> Always distinguish between Face Value (FV) and Market Value (MV). All calculations for the number of shares, investment, and returns are based on the Market Value, which is the actual transaction price. The dividend, however, is always calculated on the Face Value.

(B) Solve any four subquestions from the following :

(i) Solve the simultaneous equations

$$x + y = 3 \ ; \ 3x - 2y = 4$$

Correct Answer: $x = 2, \ y = 1$

Solution: Step 1: Write the system and isolate one variable from the simpler equation:

$$\begin{aligned} x + y &= 3 \\ 3x - 2y &= 4 \end{aligned} \qquad \Longrightarrow \qquad y = 3 - x \text{ (from (1))}.$$

Step 2: Substitute $y = 3 - x$ into (2) and solve for x:

$$3x - 2(3 - x) = 4 \ \Rightarrow \ 3x - 6 + 2x = 4 \ \Rightarrow \ 5x = 10 \ \Rightarrow \ x = 2.$$

Step 3: Back-substitute into (1) to get y:

$$2 + y = 3 \ \Rightarrow \ y = 1.$$

Check: In (2), $3(2) - 2(1) = 6 - 2 = 4$, so the solution is correct.

Quick Tip

For two linear equations, use *substitution* when a variable has coefficient 1 or -1, and *elimination* when you can quickly cancel a variable by scaling and adding the equations. Always do a quick check in both equations.

(ii) Solve the quadratic equation $m^2 + 14m + 13 = 0$.

Correct Answer: $m = -1,\ m = -13$

Solution: Step 1: Factor by splitting the middle term. We want two numbers whose sum is 14 and product is 13: they are 1 and 13.

$$m^2 + 14m + 13 = m^2 + m + 13m + 13 = m(m+1) + 13(m+1) = (m+1)(m+13).$$

Step 2: Set each factor to zero:

$$m + 1 = 0 \Rightarrow m = -1, \qquad m + 13 = 0 \Rightarrow m = -13.$$

Verification: Sum of roots $= -1 + (-13) = -14 = -\frac{14}{1}$ and product $= (-1)(-13) = 13 = \frac{13}{1}$, matching the coefficients.

Quick Tip

For $ax^2 + bx + c = 0$: if it factors neatly, use factoring; otherwise use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. A quick check: sum of roots $= -\frac{b}{a}$ and product $= \frac{c}{a}$.

(iii) Find the 19th term of the A.P. 7, 13, 19, 25, ...

Correct Answer: $T_{19} = 115$

Solution: Step 1: Identify the first term and common difference:

$$a = 7, \qquad d = 13 - 7 = 6.$$

Step 2: Use the n-th term formula $T_n = a + (n-1)d$:

$$T_{19} = 7 + (19 - 1) \cdot 6 = 7 + 18 \cdot 6 = 7 + 108 = 115.$$

Sanity check: Terms grow by 6 each step; from $T_1 = 7$ to T_{19} we add 18 steps of size 6, i.e., 108.

Quick Tip

In any A.P., $T_n = a + (n-1)d$ and $S_n = \dfrac{n}{2}\big(2a + (n-1)d\big)$. Memorize both; they cover most A.P. questions quickly.

(iv) A share with market value 2000 is sold through a broker who charges 0.5% brokerage. Find the amount received by the seller.

Correct Answer: 1990

Solution: Step 1: Compute brokerage as 0.5% of the sale (market) value:

$$\text{Brokerage} = \frac{0.5}{100} \times 2000 = 0.005 \times 2000 = 10.$$

Step 2: Net amount received = Sale value − Brokerage:

$$\text{Amount received} = 2000 - 10 = 1990.$$

Interpretation: Brokerage is a fee deducted from the sale proceeds, so the seller receives the market value minus this fee.

Quick Tip

For percentage deductions like brokerage, commission, or discount, use Net = Gross $\times \left(1 - \frac{r}{100}\right)$. Here, $2000 \times (1 - 0.5\%) = 2000 \times 0.995 = 1990$.

(v) The following table shows the number of students and the time they utilized daily for their studies. Find the mean time spent by the students for their studies.

Class Time (In hours)	Class Marks (x_i)	No. of Students (f_i)	$f_i x_i$
0–2	1	8	08
2–4	3	14	42
4–6	5	18	90
6–8	7	10	70
8–10	9	10	90

Correct Answer: $\bar{x} = 5$

Solution:
Step 1: Compute $\sum f_i$:

$$\sum f_i = 8 + 14 + 18 + 10 + 10 = 60.$$

Step 2: Compute $\sum f_i x_i$:

$$\sum f_i x_i = 0 \cdot 8 + 3 \cdot 14 + 5 \cdot 18 + 7 \cdot 10 + 9 \cdot 10 = 0 + 42 + 90 + 70 + 90 = 300.$$

Step 3: Apply the mean formula:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{300}{60} = 5.$$

Quick Tip

When dealing with frequency distributions:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Always check totals: $\sum f_i$ (total frequency) and $\sum f_i x_i$ (sum of products) before dividing.

3(A) Complete and write any one activity from the following :
(i) Shri Maniklal has purchased 300 shares of F.V. 100, for M.V. 120. Company has paid dividend at 7%. Complete the following activity to find the rate of return on his investment.
Activity:
F.V. = 100
Number of shares = 300
Market value = 120

(a) Sum invested = M.V. × No. of shares

$\therefore \quad = \square \times \square$

$= 36{,}000$

(b) Dividend per share = F.V. × rate of dividend

$= \square \times \frac{\square}{100}$

$= 7$

$\therefore$ Total dividend received = 300 × 7

$= \square$

(c) Rate of return $= \frac{\text{Dividend income}}{\text{Sum invested}} \times 100$

$= \frac{2{,}100}{36{,}000} \times 100$

$= \square$ %.

Correct Answer: The completed activity is shown in the solution.

Solution:

Step 1: Understanding the Concept:
This activity requires calculating the rate of return on an investment in shares. The rate of return is the percentage of the total dividend income relative to the total amount invested

(based on Market Value).

Step 2: Key Formula or Approach:
1. **Sum Invested** = Market Value (M.V.) × Number of shares.
2. **Dividend Income** = (Face Value (F.V.) × Dividend Rate) × Number of shares.
3. **Rate of Return** = $\left(\frac{\text{Total Dividend Income}}{\text{Sum Invested}}\right) \times 100$.

Step 3: Detailed Explanation:
Here is the completed activity with the blanks filled in.

F.V. = 100
Number of shares = 300
Market value = 120

(a) Sum invested = M.V. × No. of shares
∴ = $\boxed{\mathbf{120}} \times \boxed{\mathbf{300}}$
= 36,000

(b) Dividend per share = F.V. × rate of dividend
$= \boxed{\mathbf{100}} \times \frac{\boxed{\mathbf{7}}}{100}$
= 7
∴ Total dividend received = 300 × 7
$= \boxed{\mathbf{2{,}100}}$

(c) Rate of return = $\frac{\text{Dividend income}}{\text{Sum invested}} \times 100$
$= \frac{2{,}100}{36{,}000} \times 100$
$= \frac{2{,}100}{360}$
$= \frac{210}{36} = \frac{35}{6} \approx 5.83$
$= \boxed{\mathbf{5.83}}\,\%$.

Step 4: Final Answer:
The rate of return on the investment is 5.83%.

> **Quick Tip**
>
> Always remember that the dividend is calculated on the Face Value (F.V.) of a share, but the investment amount and the rate of return are calculated based on the Market Value (M.V.).

(ii) A two digit number is to be formed from the digits 2, 3, 5 without repetition of the digits. Complete the following activity to find the probability that the number so formed is an odd number.
Activity:

Let S be the sample space.
$\therefore$ S = {23, 25, 32, $\boxed{}$, 52, 53}
$\therefore$ n(S) = $\boxed{}$
Event A: The number so formed is an odd number.
$\therefore$ A = {23, 25, $\boxed{}$, 53}
$\therefore$ n(A) = 4

$\therefore$ P(A) = $\frac{\boxed{}}{n(S)}$ ......(Formula)

$\therefore$ P(A) = $\frac{\boxed{}}{6}$

$\therefore$ P(A) = $\frac{\boxed{}}{3}$

Correct Answer: The completed activity is shown in the solution.

Solution:

Step 1: Understanding the Concept:
This activity involves finding the probability of an event. We first need to list all possible outcomes (the sample space) when forming two-digit numbers from the digits 2, 3, 5 without repetition. Then, we identify the favorable outcomes (the odd numbers) and use the probability formula.

Step 2: Key Formula or Approach:
Probability of an event A, P(A) = $\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{n(A)}{n(S)}$.

Step 3: Detailed Explanation:
Here is the completed activity with the blanks filled in.

Let S be the sample space. The two-digit numbers formed from 2, 3, 5 without repetition are:

- Using 2 in tens place: 23, 25
- Using 3 in tens place: 32, 35
- Using 5 in tens place: 52, 53

$\therefore$ S = {23, 25, 32, $\boxed{\mathbf{35}}$, 52, 53}
$\therefore$ n(S) = $\boxed{\mathbf{6}}$
Event A: The number so formed is an odd number. An odd number has an odd digit (3 or 5) in the unit's place. The odd numbers from the sample space are 23, 25, 35, 53.
$\therefore$ A = {23, 25, $\boxed{\mathbf{35}}$, 53}
$\therefore$ n(A) = 4

$\therefore$ P(A) = $\frac{\boxed{\mathbf{n(A)}}}{n(S)}$ ......(Formula)

$\therefore$ P(A) = $\frac{\boxed{\mathbf{4}}}{6}$

$\therefore$ P(A) = $\frac{\boxed{\mathbf{2}}}{3}$

Step 4: Final Answer:
The probability that the number formed is an odd number is $\frac{2}{3}$.

> **Quick Tip**
>
> For probability questions, always start by systematically listing all possible outcomes in the sample space (S). This prevents errors in counting n(S). Then, carefully list the outcomes that satisfy the event's condition to find n(A).

(B) Solve any two subquestions from the following :

(i) Solve the following simultaneous equations by Cramer's rule:

$$4x + 3y = 18;\ 3x - 2y = 5$$

Correct Answer: (x, y) = (3, 2)

Solution:

Step 1: Understanding the Concept:
Cramer's rule is a method for solving a system of linear equations using determinants. For a system of two equations:
$a_1x + b_1y = c_1$
$a_2x + b_2y = c_2$
The solution is given by $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$.

Step 2: Key Formula or Approach:
The determinants are calculated as follows:

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1b_2 - c_2b_1$$

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = a_1c_2 - a_2c_1$$

Step 3: Detailed Explanation:
The given equations are:
$4x + 3y = 18 \ldots (1)$
$3x - 2y = 5 \ldots (2)$
Here, $a_1 = 4, b_1 = 3, c_1 = 18$ and $a_2 = 3, b_2 = -2, c_2 = 5$.

First, we calculate the determinant D:

$$D = \begin{vmatrix} 4 & 3 \\ 3 & -2 \end{vmatrix} = (4 \times -2) - (3 \times 3) = -8 - 9 = -17$$

Next, we calculate the determinant D_x:

$$D_x = \begin{vmatrix} 18 & 3 \\ 5 & -2 \end{vmatrix} = (18 \times -2) - (5 \times 3) = -36 - 15 = -51$$

Now, we calculate the determinant D_y:

$$D_y = \begin{vmatrix} 4 & 18 \\ 3 & 5 \end{vmatrix} = (4 \times 5) - (3 \times 18) = 20 - 54 = -34$$

Using Cramer's rule:

$$x = \frac{D_x}{D} = \frac{-51}{-17} = 3$$
$$y = \frac{D_y}{D} = \frac{-34}{-17} = 2$$

Step 4: Final Answer:
The solution to the simultaneous equations is $x = 3$ and $y = 2$.

> **Quick Tip**
>
> Be very careful with signs when calculating determinants, especially when coefficients are negative. A small sign error will lead to an incorrect final answer. It's a good practice to double-check your determinant calculations.

(ii) Solve the following quadratic equation by using formula method:

x^2 - 2x - 3 = 0

Correct Answer: x = 3 or x = -1

Solution:

Step 1: Understanding the Concept:
The formula method is used to find the roots of a quadratic equation of the form $ax^2+bx+c = 0$.

Step 2: Key Formula or Approach:
The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The term $b^2 - 4ac$ is called the discriminant (Δ).

Step 3: Detailed Explanation:
The given quadratic equation is $x^2 - 2x - 3 = 0$.
Comparing this with the standard form $ax^2 + bx + c = 0$, we get:

$$a = 1, \quad b = -2, \quad c = -3$$

First, we calculate the discriminant, $\Delta = b^2 - 4ac$:

$$\Delta = (-2)^2 - 4(1)(-3)$$

$$\Delta = 4 - (-12)$$

$$\Delta = 4 + 12 = 16$$

Now, we substitute the values of a, b, and Δ into the quadratic formula:

$$x = \frac{-(-2) \pm \sqrt{16}}{2(1)}$$

$$x = \frac{2 \pm 4}{2}$$

The two roots are:

$$x_1 = \frac{2+4}{2} = \frac{6}{2} = 3$$

$$x_2 = \frac{2-4}{2} = \frac{-2}{2} = -1$$

Step 4: Final Answer:
The roots of the quadratic equation are 3 and -1.

> **Quick Tip**
>
> Always calculate the discriminant ($b^2 - 4ac$) separately first. This simplifies the main formula and helps avoid calculation errors. If the discriminant is a perfect square, you know the roots will be rational.

(iii) A committee of two members is to be formed from three boys and two girls. Find the probability of the following events:
Event A: At least one girl must be a member of the committee.
Event B: Committee must be of one boy and one girl.

Correct Answer: P(A) = 7/10, P(B) = 6/10 or 3/5

Solution:

Step 1: Understanding the Concept:
This problem involves calculating probabilities of events based on combinations. First, we need to find the total number of ways to form the committee (sample space), and then find the number of ways for each specific event.
Let the three boys be B_1, B_2, B_3 and the two girls be G_1, G_2. Total persons = 5.

Step 2: Key Formula or Approach:
1. The number of combinations of choosing r items from a set of n is given by ${}^nC_r = \frac{n!}{r!(n-r)!}$.

2. Probability of an event $= \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$.

Step 3: Detailed Explanation:
Sample Space (S):
We need to form a committee of 2 members from a total of 5 people (3 boys + 2 girls).

$$n(S) = {}^5C_2 = \frac{5!}{2!(5-2)!} = \frac{5 \times 4}{2 \times 1} = 10$$

So, there are 10 possible committees.

Event A: At least one girl must be a member.
"At least one girl" means the committee can have (one girl and one boy) OR (two girls).
Case 1: One girl and one boy Number of ways = (Choose 1 girl from 2) × (Choose 1 boy from 3)

$$= {}^2C_1 \times {}^3C_1 = 2 \times 3 = 6$$

Case 2: Two girls Number of ways = (Choose 2 girls from 2)

$$= {}^2C_2 = 1$$

Total number of ways for Event A, n(A) = 6 + 1 = 7.
Probability of Event A:

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{10}$$

Alternatively, for Event A: The complement of "at least one girl" is "no girls" (i.e., both are boys). Number of ways to choose 2 boys from 3 = ${}^3C_2 = 3$. P(no girl) = $\frac{3}{10}$. P(at least one girl) = 1 - P(no girl) = $1 - \frac{3}{10} = \frac{7}{10}$.

Event B: Committee must be of one boy and one girl.
This is the same as Case 1 calculated for Event A. Number of ways for Event B, n(B) = (Choose 1 boy from 3) × (Choose 1 girl from 2)

$$n(B) = {}^3C_1 \times {}^2C_1 = 3 \times 2 = 6$$

Probability of Event B:

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{10} = \frac{3}{5}$$

Step 4: Final Answer:
The probability of Event A (at least one girl) is $\frac{7}{10}$.
The probability of Event B (one boy and one girl) is $\frac{6}{10}$ or $\frac{3}{5}$.

Quick Tip

For "at least one" probability questions, it is often much easier to calculate the probability of the complementary event ("none") and subtract it from 1.

(iv) In a general store the prices of different articles and its demand is shown in the following frequency distribution table. Find the Median of the prices.

Price in Rupees	**No. of Articles**
Less than 20	140
20–40	100
40–60	80
60–80	60
80–100	20

Correct Answer: Median = 32

Solution:

Step 1: Understanding the Concept:
The median is the middle value in a dataset. For grouped data, we find the median class and then use a formula to estimate the median value. The given table has a 'less than' type entry for the first row, which we interpret as the first class interval (0-20).

Step 2: Key Formula or Approach:
The formula for the median of grouped data is:

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f}\right] \times h$$

where:

- L = Lower class boundary of the median class.
- N = Total frequency (Σf).
- cf = Cumulative frequency of the class preceding the median class.
- f = Frequency of the median class.
- h = Class width of the median class.

Step 3: Detailed Explanation:
First, we construct the frequency distribution table with class intervals, frequencies (f), and cumulative frequencies (cf). The class "Less than 20" is taken as 0-20.

Class Interval	**Frequency (f)**	**Cumulative Frequency (cf)**
0–20	140	140
20–40	100	140 + 100 = 240
40–60	80	240 + 80 = 320
60–80	60	320 + 60 = 380
80–100	20	380 + 20 = 400
Total	**N = 400**	

1. **Find N/2**:

$$N = \Sigma f = 400$$

$$\frac{N}{2} = \frac{400}{2} = 200$$

2. **Identify the Median Class**: The cumulative frequency just greater than 200 is 240. The corresponding class is **20–40**. This is the median class.

3. **Identify the values for the formula**:

- L (Lower limit of median class) = 20
- cf (Cumulative frequency of the preceding class) = 140
- f (Frequency of the median class) = 100
- h (Class width) = 40 - 20 = 20

4. **Calculate the Median**:

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f}\right] \times h$$

$$\text{Median} = 20 + \left[\frac{200 - 140}{100}\right] \times 20$$

$$\text{Median} = 20 + \left[\frac{60}{100}\right] \times 20$$

$$\text{Median} = 20 + (0.6) \times 20$$

$$\text{Median} = 20 + 12$$

$$\text{Median} = 32$$

Step 4: Final Answer:
The Median of the prices is 32.

Quick Tip

When a frequency table contains a mix of 'less than' and class intervals, carefully construct a standard class interval table first. The key is to correctly create the cumulative frequency (cf) column, as all values in the median formula depend on it.

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4. Solve any two subquestions from the following :
(i) Find the value of 'm' if the quadratic equation

$$(m - 12)\ x^2 + 2(m - 12)x + 2 = 0$$

has real and equal roots.

Correct Answer: m = 14

Solution:

Step 1: Understanding the Concept:
For a quadratic equation of the form $ax^2 + bx + c = 0$, the roots are real and equal if its discriminant (Δ) is equal to zero.

Step 2: Key Formula or Approach:
The discriminant is given by the formula:

$$\Delta = b^2 - 4ac$$

For real and equal roots, we must have:

$$b^2 - 4ac = 0$$

Step 3: Detailed Explanation:
The given quadratic equation is $(m - 12)x^2 + 2(m - 12)x + 2 = 0$.
Comparing this with the standard form $ax^2 + bx + c = 0$, we get:

$$a = m - 12$$

$$b = 2(m - 12)$$

$$c = 2$$

For the equation to be quadratic, $a \neq 0$, which means $m - 12 \neq 0$, so $m \neq 12$.
Now, we set the discriminant to zero:

$$\Delta = [2(m - 12)]^2 - 4(m - 12)(2) = 0$$

$$4(m - 12)^2 - 8(m - 12) = 0$$

We can factor out $4(m - 12)$ from the expression:

$$4(m - 12)[(m - 12) - 2] = 0$$

$$4(m - 12)(m - 14) = 0$$

This gives two possible solutions: $m - 12 = 0$ or $m - 14 = 0$.
So, $m = 12$ or $m = 14$.
However, as we established earlier, for the equation to be quadratic, $m \neq 12$.
Therefore, we must discard the solution $m = 12$.
The only valid solution is $m = 14$.

Step 4: Final Answer:
The value of 'm' for which the quadratic equation has real and equal roots is 14.

Quick Tip

When solving for a variable in the coefficient of the x^2 term, always check if your solution makes that coefficient zero. If it does, that value must be rejected because the equation would no longer be quadratic.

(ii) A farmer borrows 1,000 and agrees to repay with a total interest of 140, in 12 instalments. Each instalment being less than the preceding instalment by 10. What should be the amount of his first and last instalment?

Correct Answer: First instalment = 150, Last instalment = 40

Solution:

Step 1: Understanding the Concept:
The instalments form an Arithmetic Progression (A.P.) because the difference between consecutive instalments is constant. We are given the total amount to be repaid (sum of the A.P.), the number of terms, and the common difference. We need to find the first and last terms.

Step 2: Key Formula or Approach:
1. Sum of n terms of an A.P.: $S_n = \frac{n}{2}[2a+(n-1)d]$ 2. The n-th term of an A.P.: $t_n = a+(n-1)d$ where a is the first term, n is the number of terms, and d is the common difference.

Step 3: Detailed Explanation:
Total amount to be repaid = Principal + Interest

$$S_n = 1000 + 140 = 1140$$

Number of instalments, $n = 12$.
Each instalment is 10 less than the preceding one, so the common difference, $d = -10$.
Let the first instalment be a.
Using the sum formula to find a:

$$S_{12} = \frac{12}{2}[2a + (12-1)(-10)]$$

$$1140 = 6[2a + (11)(-10)]$$

$$1140 = 6[2a - 110]$$

Divide both sides by 6:

$$\frac{1140}{6} = 2a - 110$$

$$190 = 2a - 110$$

$$190 + 110 = 2a$$

$$300 = 2a$$

$$a = 150$$

So, the first instalment is 150.
Now, we find the last instalment, which is the 12th term (t_{12}).
Using the n-th term formula:

$$t_{12} = a + (12-1)d$$

$$t_{12} = 150 + (11)(-10)$$

$$t_{12} = 150 - 110$$

$$t_{12} = 40$$

So, the last instalment is 40.

Step 4: Final Answer:
The amount of the first instalment is 150 and the amount of the last instalment is 40.

Quick Tip

Pay close attention to the wording. "Less than the preceding" implies a negative common difference. Also, remember that the total amount repaid (S_n) is the sum of the principal and the interest.

(iii) The following table shows the marks of 180 students in Mathematics.

Marks	No. of Students
0–10	25
10–20	x
20–30	30
30–40	2x
40–50	65

Find the value of 'x' and draw histogram.

Correct Answer: x = 20. The histogram is to be drawn based on the calculated frequencies.

Solution:

Step 1: Understanding the Concept:
The total number of students is the sum of the frequencies of all class intervals. We can set up an equation to find the value of 'x'. A histogram is a graphical representation of this frequency distribution where class intervals are on the x-axis and frequencies are on the y-axis.

Step 2: Key Formula or Approach:
Total Frequency (N) = Sum of all individual frequencies (Σf_i).

Step 3: Detailed Explanation:
Part 1: Finding the value of 'x'
The total number of students is given as 180.

$$N = \Sigma f_i = 180$$

Summing the frequencies from the table:

$$25 + x + 30 + 2x + 65 = 180$$

Combine the constant terms and the 'x' terms:

$$(25 + 30 + 65) + (x + 2x) = 180$$

$$120 + 3x = 180$$

$$3x = 180 - 120$$

$$3x = 60$$

$$x = \frac{60}{3} = 20$$

Now we can find the frequencies for the classes 10-20 and 30-40:

- Frequency for 10–20 $= x = 20$
- Frequency for 30–40 $= 2x = 2(20) = 40$

The complete frequency table is:

Marks	**No. of Students**
0–10	25
10–20	20
20–30	30
30–40	40
40–50	65
Total	**180**

Part 2: Drawing the Histogram

To draw the histogram:

1. Draw the X-axis and Y-axis on graph paper.
2. On the X-axis, represent the Marks (class intervals). Choose an appropriate scale, for example, 1 cm = 10 marks. The intervals are continuous (0-10, 10-20, etc.).
3. On the Y-axis, represent the Number of Students (frequency). Choose an appropriate scale, for example, 1 cm = 10 students.
4. For each class interval, draw a rectangle (bar) with the width equal to the class interval and height equal to the corresponding frequency. The bars should be adjacent to each other without any gaps.
   - For 0–10, the bar height is 25.
   - For 10–20, the bar height is 20.
   - For 20–30, the bar height is 30.
   - For 30–40, the bar height is 40.
   - For 40–50, the bar height is 65.
5. Label the axes and provide a title for the histogram, e.g., "Marks Distribution of 180 Students".

Step 4: Final Answer:
The value of x is 20. The histogram should be drawn on graph paper as described above using the calculated frequencies.

> **Quick Tip**
>
> For histograms, ensure that the class intervals are continuous. If they are not, you must adjust the boundaries first. In this case, the classes are already continuous, so you can draw the adjacent bars directly. Always choose a clear and consistent scale for both axes.

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5. Solve any one of the following subquestions :

(i) Draw the graphs representing the equation 2x = y + 2 and 4x + 3y = 24 on the same graph paper. Find the area of the triangle formed by these lines and the X-axis.

Correct Answer: Area = 10 sq. units

Solution:

Step 1: Understanding the Concept:
We need to plot two linear equations on a graph. The two lines and the X-axis will form a triangle. The vertices of this triangle will be the x-intercept of the first line, the x-intercept of the second line, and the point of intersection of the two lines. We can then find the area of this triangle.

Step 2: Key Formula or Approach:
Area of a triangle = $\frac{1}{2} \times$ base $\times$ height.

Step 3: Detailed Explanation:
Part 1: Plotting the graphs
First, we find at least two points for each equation to draw the lines.
Line 1: 2x = y + 2 $\implies$ y = 2x - 2

x	0	1	2
y	-2	0	2

Points are (0, -2), (1, 0), (2, 2). The x-intercept is (1, 0).
Line 2: 4x + 3y = 24

x	0	6	3
y	8	0	4

Points are (0, 8), (6, 0), (3, 4). The x-intercept is (6, 0).
The graphs should be plotted on graph paper using these points.

Part 2: Finding the vertices of the triangle
The vertices of the triangle are:

- Vertex A: The x-intercept of Line 1, which is **(1, 0)**.
- Vertex B: The x-intercept of Line 2, which is **(6, 0)**.
- Vertex C: The intersection point of the two lines.

To find Vertex C, we solve the two equations simultaneously:

1. $y = 2x - 2$
2. $4x + 3y = 24$

Substitute (1) into (2):

$$4x + 3(2x - 2) = 24$$
$$4x + 6x - 6 = 24$$
$$10x = 30$$
$$x = 3$$

Now find y by substituting x=3 into equation (1):

$$y = 2(3) - 2 = 6 - 2 = 4$$

So, Vertex C is **(3, 4)**.

Part 3: Finding the area of the triangle
The triangle has vertices A(1, 0), B(6, 0), and C(3, 4). The base of the triangle is the segment AB on the X-axis.

$$\text{Base} = \text{distance between } (1, 0) \text{ and } (6, 0) = 6 - 1 = 5 \text{ units.}$$

The height of the triangle is the perpendicular distance from vertex C to the base on the X-axis. This is the y-coordinate of point C.

$$\text{Height} = 4 \text{ units.}$$

Now, we calculate the area:

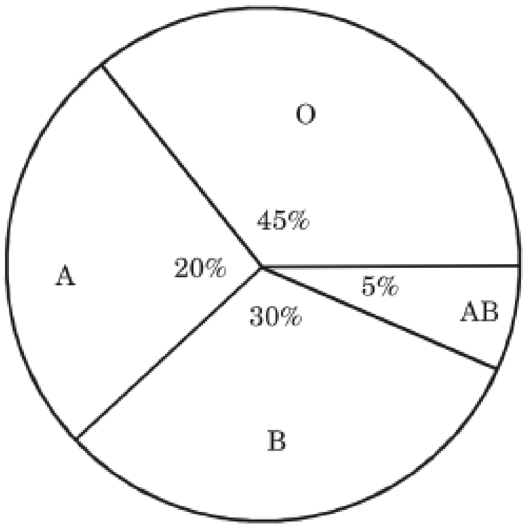
$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$\text{Area} = \frac{1}{2} \times 5 \times 4 = 10 \text{ sq. units.}$$

Step 4: Final Answer:
The area of the triangle formed by the lines and the X-axis is 10 square units.

Quick Tip

To easily find the vertices of a triangle formed by two lines and an axis, find the intercepts of the lines with that axis and the intersection point of the two lines. The base will be the distance between the intercepts, and the height will be the perpendicular coordinate of the intersection point.

(ii) The following pie-diagram shows percentage of persons according to blood group in a blood group checking camp. Answer the following questions:



(a) Find the measure of central angle for each blood group.
(b) Find the total number of persons, if there are 600 persons of blood group B.

Correct Answer: (a) O=162°, A=72°, B=108°, AB=18°. (b) Total persons = 2000.

Solution:

Step 1: Understanding the Concept:
A pie chart represents a whole dataset, where the total is equivalent to 100% or 360°. The central angle for each sector is proportional to the percentage it represents.

Step 2: Key Formula or Approach:

(a) Central Angle $= \frac{\text{Percentage of component}}{100} \times 360°$

(b) Total number $= \frac{\text{Given number for a component}}{\text{Percentage of that component}} \times 100$

Step 3: Detailed Explanation:
(a) Find the measure of central angle for each blood group.

- **Blood Group O (45%):**
  Central Angle $= \frac{45}{100} \times 360° = 0.45 \times 360° = 162°$
- **Blood Group A (20%):**
  Central Angle $= \frac{20}{100} \times 360° = 0.20 \times 360° = 72°$

- **Blood Group B (30%):**
  Central Angle $= \frac{30}{100} \times 360^\circ = 0.30 \times 360^\circ = 108^\circ$
- **Blood Group AB (5%):**
  Central Angle $= \frac{5}{100} \times 360^\circ = 0.05 \times 360^\circ = 18^\circ$

(Check: $162 + 72 + 108 + 18 = 360^\circ$)

(b) Find the total number of persons, if there are 600 persons of blood group B.
From the pie chart, the percentage of persons with blood group B is 30%.
Let the total number of persons be 'T'.
We are given that 30% of T is 600.

$$\frac{30}{100} \times T = 600$$

To find T, we rearrange the formula:

$$T = \frac{600 \times 100}{30}$$

$$T = 20 \times 100$$

$$T = 2000$$

So, the total number of persons in the camp was 2000.

Step 4: Final Answer:
(a) The central angles are: O = 162°, A = 72°, B = 108°, AB = 18°.
(b) The total number of persons is 2000.

Quick Tip

When converting percentages to degrees for a pie chart, you can remember that 1% is equal to 3.6° (360/100). You can then simply multiply each percentage by 3.6 to find the central angle.