N 832– Mathematics (71) - 2025 Question Paper with Solutions

Time Allowed : 2 Hours	**Maximum Marks :** 40	ntering

General Instructions

Read the following instructions very carefully and strictly follow them:

1. All questions are compulsory.
2. Use of a calculator is not allowed.
3. The numbers to the right of the questions indicate full marks.
4. Draw proper figures wherever necessary.
5. The marks of construction should be clear. Do not erase them.
6. Diagram is essential for writing the proof of the theorem

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1. (A) Choose the correct alternative from the given:

(1) Out of the following which is a Pythagorean triplet ?

(A) (1, 5, 10)
(B) (3, 4, 5)
(C) (2, 2, 2)
(D) (5, 5, 2)

Correct Answer: (B) (3, 4, 5)

Solution:

Step 1: Understanding the Concept:
A Pythagorean triplet consists of three positive integers a, b, and c such that the sum of the squares of the two smaller integers equals the square of the largest integer.

Step 2: Key Formula or Approach:
The formula to verify a Pythagorean triplet is $a^2 + b^2 = c^2$, where c is the greatest of the three numbers. We will test each option against this formula.

Step 3: Detailed Explanation:
Let's check each option:
(A) (1, 5, 10):
Here, $a = 1$, $b = 5$, and $c = 10$.

$$1^2 + 5^2 = 1 + 25 = 26$$

$$10^2 = 100$$

Since $26 \neq 100$, this is not a Pythagorean triplet.

(B) (3, 4, 5):
Here, $a = 3$, $b = 4$, and $c = 5$.

$$3^2 + 4^2 = 9 + 16 = 25$$
$$5^2 = 25$$

Since $25 = 25$, this is a Pythagorean triplet.

(C) (2, 2, 2):
All numbers are equal. This represents the sides of an equilateral triangle, not a right-angled triangle. Also, $2^2 + 2^2 = 8$, which is not equal to $2^2 = 4$.

(D) (5, 5, 2):
The two smaller numbers are 2 and 5. The largest is 5.

$$2^2 + 5^2 = 4 + 25 = 29$$
$$5^2 = 25$$

Since $29 \neq 25$, this is not a Pythagorean triplet.

Step 4: Final Answer:
Based on the calculations, the only set of numbers that satisfies the Pythagorean theorem is (3, 4, 5).

Quick Tip

Memorizing common Pythagorean triplets like (3, 4, 5), (5, 12, 13), (8, 15, 17), and (7, 24, 25) can significantly speed up problem-solving in geometry and trigonometry sections of competitive exams.

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(2) $\angle$ACB is inscribed angle in a circle with centre O. If $\angle ACB = 65°$, then what is measure of its intercepted arc AXB ?

(A) 65°
(B) 230°
(C) 295°
(D) 130°

Correct Answer: (D) 130°

Solution:

Step 1: Understanding the Concept:
This question is based on the Inscribed Angle Theorem in circle geometry. The theorem states that the measure of an angle inscribed in a circle is half the measure of its intercepted arc.

Step 2: Key Formula or Approach:
The formula derived from the Inscribed Angle Theorem is:

$$\text{Measure of Inscribed Angle} = \frac{1}{2} \times \text{Measure of Intercepted Arc}$$

Or, rearranging for the arc:

$$\text{Measure of Intercepted Arc} = 2 \times \text{Measure of Inscribed Angle}$$

Step 3: Detailed Explanation:
We are given the measure of the inscribed angle, $\angle$ACB.

$$\angle \text{ACB} = 65^\circ$$

The intercepted arc is AXB.
Using the formula from Step 2:

$$\text{Measure of arc AXB} = 2 \times \angle \text{ACB}$$
$$\text{Measure of arc AXB} = 2 \times 65^\circ$$
$$\text{Measure of arc AXB} = 130^\circ$$

Step 4: Final Answer:
The measure of the intercepted arc AXB is 130°.

Quick Tip

Always remember the key circle theorems: the angle at the center is double the angle at the circumference (inscribed angle), angles in the same segment are equal, and the angle in a semicircle is a right angle (90°).

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(3) Distance of point (3, 4) from the origin is ....

(A) 7
(B) 1
(C) 5
(D) -5

Correct Answer: (C) 5

Solution:

Step 1: Understanding the Concept:
This problem requires finding the distance between a given point and the origin (0, 0) in a 2D Cartesian coordinate system.

Step 2: Key Formula or Approach:
The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

When one point is the origin (0, 0) and the other is (x, y), the formula simplifies to:

$$d = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$$

Step 3: Detailed Explanation:
We need to find the distance of the point (3, 4) from the origin (0, 0).
Here, $x = 3$ and $y = 4$.
Using the simplified distance formula:

$$d = \sqrt{3^2 + 4^2}$$
$$d = \sqrt{9 + 16}$$
$$d = \sqrt{25}$$
$$d = 5$$

Distance is a scalar quantity and is always non-negative, so option (D) -5 is incorrect.

Step 4: Final Answer:
The distance of the point (3, 4) from the origin is 5 units.

> **Quick Tip**
>
> Recognize that the coordinates (3, 4) and the distance from the origin form a right-angled triangle with the axes. The distance is the hypotenuse. The sides are 3 and 4, which are part of the (3, 4, 5) Pythagorean triplet. This allows you to find the answer instantly without calculation.

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(4) If radius of cone is 5 cm and its perpendicular height is 12 cm, then the slant height is ....

(A) 17 cm
(B) 4 cm
(C) 13 cm
(D) 60 cm

Correct Answer: (C) 13 cm

Solution:

Step 1: Understanding the Concept:
In a right circular cone, the radius (r), the perpendicular height (h), and the slant height (l) form a right-angled triangle. The slant height (l) is the hypotenuse of this triangle.

Step 2: Key Formula or Approach:
We can use the Pythagorean theorem to relate the radius, height, and slant height of the cone:

$$l^2 = r^2 + h^2$$

Taking the square root gives the formula for the slant height:

$$l = \sqrt{r^2 + h^2}$$

Step 3: Detailed Explanation:
We are given the following values:
Radius, $r = 5$ cm
Perpendicular height, $h = 12$ cm
Substitute these values into the formula for slant height:

$$l = \sqrt{5^2 + 12^2}$$
$$l = \sqrt{25 + 144}$$
$$l = \sqrt{169}$$
$$l = 13$$

So, the slant height is 13 cm.

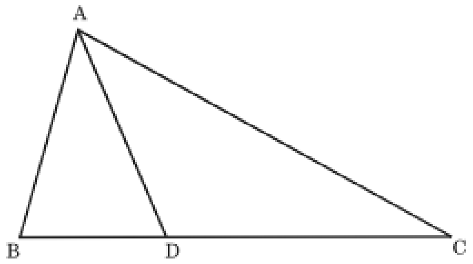
Step 4: Final Answer:
The slant height of the cone is 13 cm.

Quick Tip

This problem uses the (5, 12, 13) Pythagorean triplet. Being familiar with this triplet allows for immediate identification of the answer. When you see a radius and height of 5 and 12 (or vice versa) in a cone or pyramid problem, the slant height will almost certainly be 13.

(B) Solve the following sub-questions:

(1) In the following figure $\triangle$ ABC, B-D-C and BD = 7, BC = 20, then find $\frac{A(\triangle ABD)}{A(\triangle ABC)}$.



Correct Answer: $\frac{7}{20}$

Solution:

Step 1: Understanding the Concept:
The ratio of the areas of two triangles with a common vertex and bases on the same straight line is equal to the ratio of their corresponding bases. This is because both triangles share the same height.

Step 2: Key Formula or Approach:
If $\triangle ABD$ and $\triangle ABC$ have a common vertex A and their bases BD and BC lie on the same line, their height 'h' from vertex A is the same.
The area of a triangle is given by $\frac{1}{2} \times \text{base} \times \text{height}$.
Therefore, the ratio of their areas is:

$$\frac{A(\triangle ABD)}{A(\triangle ABC)} = \frac{\frac{1}{2} \times BD \times h}{\frac{1}{2} \times BC \times h} = \frac{BD}{BC}$$

Step 3: Detailed Explanation:
Given:

- BD = 7
- BC = 20

$\triangle ABD$ and $\triangle ABC$ share a common vertex A. Their bases BD and BC are on the same line. Let 'h' be the height of both triangles from the common vertex A to the line containing their bases.
Area of $\triangle ABD = \frac{1}{2} \times BD \times h = \frac{1}{2} \times 7 \times h$.
Area of $\triangle ABC = \frac{1}{2} \times BC \times h = \frac{1}{2} \times 20 \times h$.
Now, we find the ratio of their areas:

$$\frac{A(\triangle ABD)}{A(\triangle ABC)} = \frac{\frac{1}{2} \times 7 \times h}{\frac{1}{2} \times 20 \times h}$$

The terms $\frac{1}{2}$ and h cancel out.

$$\frac{A(\triangle ABD)}{A(\triangle ABC)} = \frac{7}{20}$$

Step 4: Final Answer:
The ratio $\frac{A(\triangle ABD)}{A(\triangle ABC)}$ is $\frac{7}{20}$.

Quick Tip

For triangles sharing the same height, the ratio of their areas is simply the ratio of their bases. Similarly, for triangles sharing the same base, the ratio of their areas is the ratio of their heights.

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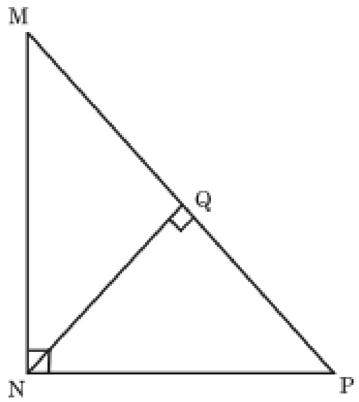
(2) In the following figure $\angle MNP = 90^\circ$, seg NQ $\perp$ seg MP, MQ = 9, QP = 4, find NQ.



Correct Answer: NQ = 6

Solution:

Step 1: Understanding the Concept:
In a right-angled triangle, the altitude drawn to the hypotenuse creates two smaller triangles that are similar to the original triangle and to each other. This leads to the geometric mean theorem.

Step 2: Key Formula or Approach:
The theorem of geometric mean states that in a right-angled triangle, the altitude drawn to the hypotenuse is the geometric mean of the two segments it divides the hypotenuse into.

$$NQ^2 = MQ \times QP$$

Step 3: Detailed Explanation:
Given:

- $\triangle MNP$ is a right-angled triangle with $\angle MNP = 90^\circ$.
- NQ is the altitude to the hypotenuse MP.
- MQ = 9
- QP = 4

According to the geometric mean theorem:

$$NQ^2 = MQ \times QP$$

Substitute the given values:

$$NQ^2 = 9 \times 4$$
$$NQ^2 = 36$$

Take the square root of both sides:

$$NQ = \sqrt{36}$$

Since length cannot be negative,

$$NQ = 6$$

Step 4: Final Answer:
The length of NQ is 6.

> **Quick Tip**
>
> Remember the geometric mean theorem for any right-angled triangle with an altitude to the hypotenuse. The altitude squared is equal to the product of the parts of the hypotenuse. This is a very common problem type in geometry.

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(3) Angle made by a line with the positive direction of X-axis is 30°. Find slope of that line.

Correct Answer: $\frac{1}{\sqrt{3}}$

Solution:

Step 1: Understanding the Concept:
The slope (or gradient) of a line is a number that describes both the direction and the steepness of the line. The slope 'm' is related to the angle of inclination 'θ' (the angle the line makes with the positive direction of the x-axis) by the tangent function.

Step 2: Key Formula or Approach:
The slope m of a line is given by:

$$m = \tan(\theta)$$

where θ is the angle of inclination.

Step 3: Detailed Explanation:
Given:

- The angle of inclination, $\theta = 30°$.

Using the formula for the slope:

$$m = \tan(30°)$$

We know the standard trigonometric value:

$$\tan(30°) = \frac{1}{\sqrt{3}}$$

Therefore, the slope of the line is $\frac{1}{\sqrt{3}}$.

Step 4: Final Answer:
The slope of the line is $\frac{1}{\sqrt{3}}$.

Quick Tip

Memorize the standard tangent values for common angles: $\tan(0°) = 0$, $\tan(30°) = \frac{1}{\sqrt{3}}$, $\tan(45°) = 1$, $\tan(60°) = \sqrt{3}$. For $\tan(90°)$, the slope is undefined (vertical line).

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(4) In cyclic quadrilateral ABCD m∠A = 100°, then find m∠C.

Correct Answer: 80°

Solution:

Step 1: Understanding the Concept:
A cyclic quadrilateral is a quadrilateral whose vertices all lie on a single circle. A key property of cyclic quadrilaterals is related to its opposite angles.

Step 2: Key Formula or Approach:
The theorem of cyclic quadrilateral states that the sum of opposite angles of a cyclic quadrilateral is 180° (supplementary).

$$m\angle A + m\angle C = 180°$$

$$m\angle B + m\angle D = 180°$$

Step 3: Detailed Explanation:
Given:

- ABCD is a cyclic quadrilateral.
- $m\angle A = 100°$.

Since ABCD is a cyclic quadrilateral, its opposite angles are supplementary. Therefore, the sum of $\angle A$ and its opposite angle $\angle C$ is 180°.

$$m\angle A + m\angle C = 180°$$

Substitute the given value of $m\angle A$:

$$100° + m\angle C = 180°$$

$$m\angle C = 180° - 100°$$

$$m\angle C = 80°$$

Step 4: Final Answer:
The measure of $\angle C$ is 80°.

Quick Tip

Always remember the main property of cyclic quadrilaterals: opposite angles add up to 180°. This is a fundamental theorem frequently tested in geometry problems.

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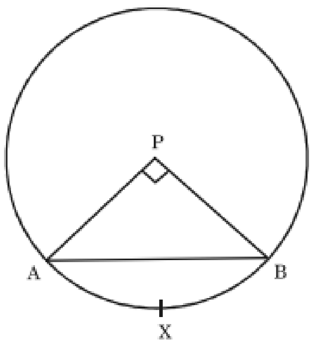
2. (A) Complete the following activities and rewrite it (any two) :

(1) The radius of a circle with centre 'P' is 10 cm. If chord AB of the circle subtends a right angle at P, find area of minor sector by using the following activity. ($\pi = 3.14$)



Activity :

r = 10 cm, $\theta = 90^\circ$, $\pi = 3.14$.

A(P-AXB) $= \frac{\theta}{360} \times \square$

$= \frac{\square}{360} \times 3.14 \times 10^2$

$= \frac{1}{4} \times \square$

A(P-AXB) $= \square$ sq. cm.

Correct Answer: The completed activity is shown in the solution.

Solution:

Step 1: Understanding the Concept:
The area of a sector of a circle is a fraction of the total area of the circle, determined by the central angle of the sector. The question requires filling the blanks in the activity to calculate this area.

Step 2: Key Formula or Approach:
The area of a sector with central angle θ and radius r is given by the formula:

$$\text{Area of Sector} = \frac{\theta}{360°} \times \pi r^2$$

Step 3: Detailed Explanation:

Here is the completed activity with the blanks filled in.

r = 10 cm, $\theta = 90°$, $\pi = 3.14$.

A(P-AXB) = $\frac{\theta}{360} \times \boxed{\pi r^2}$

$= \frac{\boxed{\mathbf{90}}}{360} \times 3.14 \times 10^2$

(Calculation: $\frac{90}{360} = \frac{1}{4}$ *and* $10^2 = 100$*)*

$= \frac{1}{4} \times \boxed{\mathbf{314}}$ *(since* $3.14 \times 100 = 314$*)*

(Calculation: $\frac{1}{4} \times 314 = 78.5$*)*

A(P-AXB) = $\boxed{\mathbf{78.5}}$ sq. cm.

Step 4: Final Answer:
The area of the minor sector (P-AXB) is 78.5 sq. cm.

Quick Tip

For a right angle at the centre (90°), the sector is always a quarter of the circle. You can quickly calculate the area as $\frac{1}{4}\pi r^2$. This is a useful shortcut.

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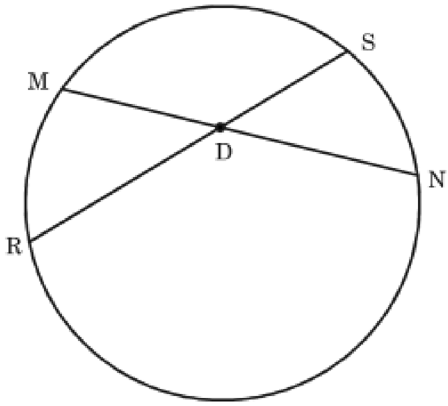
(2) In the following figure chord MN and chord RS intersect at point D. If RD = 15, DS = 4, MD = 8, find DN by completing the following activity:



Activity :
$\therefore$ MD $\times$ DN = [ ] $\times$ DS . . . .

. . . . (Theorem of internal division of chords)

$\therefore$ [ ] $\times$ DN = 15 $\times$ 4

$\therefore$ DN = $\frac{\boxed{}}{8}$

$\therefore$ DN = [ ]

Correct Answer: The completed activity is shown in the solution.

Solution:

Step 1: Understanding the Concept:
When two chords of a circle intersect inside the circle, the product of the segments of one chord is equal to the product of the segments of the other chord. This is known as the theorem of intersecting chords or internal division of chords.

Step 2: Key Formula or Approach:
For chords MN and RS intersecting at D, the theorem states:

$$MD \times DN = RD \times DS$$

Step 3: Detailed Explanation:
Here is the completed activity with the blanks filled in.

$\therefore$ MD $\times$ DN = $\boxed{\textbf{RD}}$ $\times$ DS . . . .
. . . . (Theorem of internal division of chords)

Given: RD = 15, DS = 4, MD = 8.

$\therefore$ $\boxed{\textbf{8}}$ $\times$ DN = 15 $\times$ 4

$\therefore 8 \times DN = 60$

$\therefore DN = \frac{\boxed{60}}{8}$

$\therefore DN = \boxed{7.5}$

Step 4: Final Answer:

The length of DN is 7.5.

Quick Tip

Remember the intersecting chords theorem as "part × part = part × part". Make sure you are multiplying the segments of the same chord together.

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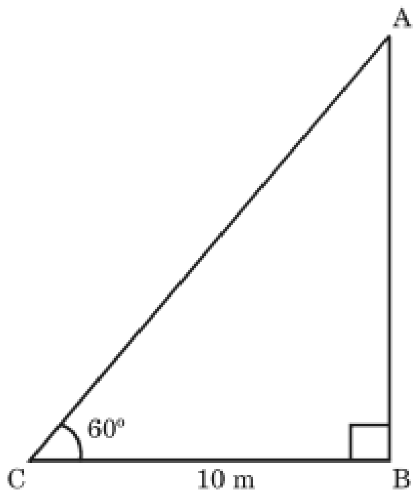
(3) An observer at a distance of 10 m from tree looks at the top of the tree, the angle of elevation is 60°. To find the height of tree complete the activity. ($\sqrt{3} = 1.73$)



Activity :

In the figure given above, AB = h = height of tree, BC = 10 m, distance of the observer from the tree.

Angle of elevation (θ) = $\angle$BCA = 60°

$\tan \theta = \frac{\boxed{}}{BC}$ .... (I)

$\tan 60^\circ = \boxed{}$ .... (II)

$\frac{AB}{BC} = \sqrt{3}$ ......(From (I) and (II))

AB = BC × $\sqrt{3} = 10\sqrt{3}$

AB = 10 × 1.73 = $\boxed{}$

∴ height of the tree is $\boxed{}$ m.

Correct Answer: The completed activity is shown in the solution.

Solution:

Step 1: Understanding the Concept:
This problem uses trigonometry to find the height of an object given the distance from the object and the angle of elevation. The situation forms a right-angled triangle where the height is the opposite side and the distance is the adjacent side to the angle of elevation.

Step 2: Key Formula or Approach:
In a right-angled triangle, the tangent of an angle is the ratio of the length of the opposite side to the length of the adjacent side.

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$$

Step 3: Detailed Explanation:

Here is the completed activity with the blanks filled in.

In the figure given above, AB = h = height of tree, BC = 10 m, distance of the observer from the tree.

Angle of elevation (θ) = ∠BCA = 60°

In right-angled △ABC,

$\tan\theta = \frac{\boxed{\textbf{AB}}}{BC}$ ....(I)

We know that,

$\tan 60° = \boxed{\sqrt{3}}$ ....(II)

∴ $\frac{AB}{BC} = \sqrt{3}$ ......(From (I) and (II))

AB = BC × $\sqrt{3} = 10\sqrt{3}$

AB = 10 × 1.73 = $\boxed{\textbf{17.3}}$

$\therefore$ height of the tree is $\boxed{\textbf{17.3}}$ m.

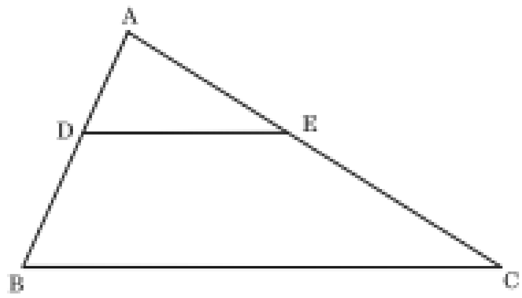
Step 4: Final Answer:
The height of the tree is 17.3 m.

Quick Tip

Remember the SOH-CAH-TOA mnemonic for trigonometric ratios. When you have the adjacent side and need to find the opposite side (like in height and distance problems), the Tangent ratio (Opposite/Adjacent) is the one to use.

(B) Solve the following sub-questions (any four):

(1) In $\triangle ABC$, $DE \parallel BC$. If $DB = 5.4\,\text{cm}$, $AD = 1.8\,\text{cm}$, $EC = 7.2\,\text{cm}$, then find AE.



Correct Answer: AE = 2.4 cm

Solution:

Step 1: Understanding the Concept:
The Basic Proportionality Theorem (BPT), also known as Thales's Theorem, states that if a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

Step 2: Key Formula or Approach:
According to the Basic Proportionality Theorem, since DE || BC, we have:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Step 3: Detailed Explanation:
Given:

- In $\triangle$ ABC, DE || BC.
- AD = 1.8 cm
- DB = 5.4 cm

- EC = 7.2 cm

We need to find the length of AE.
Using the BPT, we set up the proportion:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substitute the given values into the equation:

$$\frac{1.8}{5.4} = \frac{AE}{7.2}$$

First, simplify the ratio on the left side:

$$\frac{1.8}{5.4} = \frac{18}{54} = \frac{1}{3}$$

Now the equation becomes:

$$\frac{1}{3} = \frac{AE}{7.2}$$

To solve for AE, multiply both sides by 7.2:

$$AE = \frac{1}{3} \times 7.2$$

$$AE = 2.4 \text{ cm}$$

Step 4: Final Answer:
The length of AE is 2.4 cm.

Quick Tip

When applying the Basic Proportionality Theorem, ensure the line segment (DE) is parallel to the third side (BC). Always set up the ratio of the parts of one side equal to the ratio of the corresponding parts of the other side.

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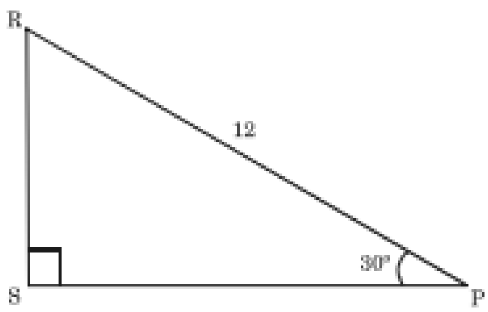
(2) In the figure given below, find RS and PS using the information given in △PSR.



Correct Answer: RS = 6, PS = $6\sqrt{3}$

Solution:

Step 1: Understanding the Concept:
This problem involves solving a right-angled triangle using trigonometric ratios or the properties of a 30-60-90 triangle. Given one side and one angle, we can find the lengths of the other two sides.

Step 2: Key Formula or Approach:
We can use two methods:
Method 1: Trigonometric Ratios

- $\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$
- $\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

Method 2: 30-60-90 Triangle Theorem The sides of a 30-60-90 triangle are in the ratio $1 : \sqrt{3} : 2$.

- The side opposite 30° is half the hypotenuse.
- The side opposite 60° is $\sqrt{3}$ times the side opposite 30°.

Step 3: Detailed Explanation:
Given:

- In △PSR, ∠S = 90°.
- Hypotenuse PR = 12.
- ∠P = 30°.
- Therefore, ∠R = 180° - 90° - 30° = 60°.

Using Method 2 (30-60-90 Triangle Theorem):

1. **Find RS**: RS is the side opposite the 30° angle (∠P).
According to the theorem, the side opposite the 30° angle is half the hypotenuse.

$$RS = \frac{1}{2} \times PR = \frac{1}{2} \times 12 = 6$$

2. **Find PS**: PS is the side opposite the 60° angle (∠R).
According to the theorem, the side opposite the 60° angle is $\sqrt{3}$ times the side opposite the 30° angle.

$$PS = \sqrt{3} \times RS = \sqrt{3} \times 6 = 6\sqrt{3}$$

Step 4: Final Answer:
The length of RS is 6 and the length of PS is $6\sqrt{3}$.

Quick Tip

The 30-60-90 triangle theorem is a very powerful and quick tool for solving such problems. Recognizing this special triangle saves you from having to perform trigonometric calculations.

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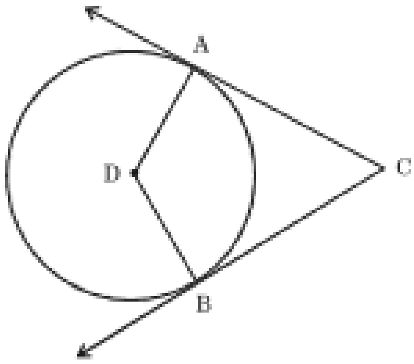
(3) In the following figure, circle with centre D touches the sides of ∠ACB at A and B. If ∠ACB = 52°, find measure of ∠ADB.



Correct Answer: m∠ADB = 128°

Solution:

Step 1: Understanding the Concept:
This problem involves the properties of tangents to a circle from an external point. The radius to the point of tangency is perpendicular to the tangent line. This forms a quadrilateral whose angle sum property can be used.

Step 2: Key Formula or Approach:
1. **Tangent-Radius Theorem**: The tangent at any point of a circle is perpendicular to the radius through the point of contact. Therefore, $\angle DAC = 90°$ and $\angle DBC = 90°$. 2. **Sum of angles in a quadrilateral**: The sum of the interior angles of a quadrilateral is 360°.

Step 3: Detailed Explanation:
Given:

- A circle with centre D touches the sides CA and CB of ∠ACB at points A and B respectively.
- CA and CB are tangents to the circle.
- DA and DB are radii of the circle.
- $m\angle ACB = 52°$.

According to the tangent-radius theorem:

$$DA \perp CA \implies m\angle DAC = 90°$$

$$DB \perp CB \implies m\angle DBC = 90^\circ$$

Now, consider the quadrilateral DACB. The sum of its interior angles is 360°.

$$m\angle DAC + m\angle ACB + m\angle CBD + m\angle ADB = 360^\circ$$

Substitute the known values:

$$90^\circ + 52^\circ + 90^\circ + m\angle ADB = 360^\circ$$

$$232^\circ + m\angle ADB = 360^\circ$$

$$m\angle ADB = 360^\circ - 232^\circ$$

$$m\angle ADB = 128^\circ$$

Step 4: Final Answer:
The measure of ∠ADB is 128°.

> **Quick Tip**
>
> For a circle touched by two tangents from an external point, the angle between the tangents and the angle between the radii to the points of contact are always supplementary (add up to 180°). So, you can quickly find $\angle ADB$ by calculating $180^\circ - 52^\circ = 128^\circ$.

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(4) Verify, whether points, A(1, -3), B(2, -5) and C(-4, 7) are collinear or not.

Correct Answer: The points are collinear.

Solution:

Step 1: Understanding the Concept:
Three points are collinear if they lie on the same straight line. This can be verified by showing that the slope of the line segment connecting any two points is the same as the slope of the line segment connecting any other two points.

Step 2: Key Formula or Approach:
The slope m of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

We will calculate the slope of AB and the slope of BC. If they are equal, the points are collinear.

Step 3: Detailed Explanation:
The given points are A(1, -3), B(2, -5), and C(-4, 7).
1. Find the slope of line segment AB: Here, $(x_1, y_1) = (1, -3)$ and $(x_2, y_2) = (2, -5)$.

$$m_{AB} = \frac{-5 - (-3)}{2 - 1} = \frac{-5 + 3}{1} = \frac{-2}{1} = -2$$

2. Find the slope of line segment BC: Here, $(x_1, y_1) = (2, -5)$ and $(x_2, y_2) = (-4, 7)$.

$$m_{BC} = \frac{7 - (-5)}{-4 - 2} = \frac{7 + 5}{-6} = \frac{12}{-6} = -2$$

Since the slope of AB is equal to the slope of BC ($m_{AB} = m_{BC} = -2$), and they share a common point B, the points A, B, and C lie on the same straight line.

Step 4: Final Answer:
Yes, the points A, B, and C are collinear.

> **Quick Tip**
>
> Another method to check for collinearity is the distance formula. Calculate the distances AB, BC, and AC. If the sum of two smaller distances equals the largest distance (e.g., AB + BC = AC), the points are collinear. However, the slope method is generally faster.

---

(5) If $\sin\theta = \frac{11}{61}$, find the value of $\cos\theta$ using trigonometric identity.

Correct Answer: $\cos\theta = \frac{60}{61}$

Solution:

Step 1: Understanding the Concept:
We need to use the fundamental Pythagorean trigonometric identity that relates the sine and cosine of an angle.

Step 2: Key Formula or Approach:
The Pythagorean identity is:

$$\sin^2\theta + \cos^2\theta = 1$$

We can rearrange this to solve for $\cos\theta$:

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \sqrt{1 - \sin^2\theta}$$

Step 3: Detailed Explanation:
Given:

$$\sin\theta = \frac{11}{61}$$

Using the identity $\sin^2\theta + \cos^2\theta = 1$:

$$\left(\frac{11}{61}\right)^2 + \cos^2\theta = 1$$

$$\frac{11^2}{61^2} + \cos^2\theta = 1$$

$$\frac{121}{3721} + \cos^2\theta = 1$$

Now, solve for $\cos^2\theta$:

$$\cos^2\theta = 1 - \frac{121}{3721}$$
$$\cos^2\theta = \frac{3721 - 121}{3721}$$
$$\cos^2\theta = \frac{3600}{3721}$$

Take the square root of both sides. Since it is not specified, we assume θ is in the first quadrant where cosine is positive.

$$\cos\theta = \sqrt{\frac{3600}{3721}}$$
$$\cos\theta = \frac{\sqrt{3600}}{\sqrt{3721}}$$
$$\cos\theta = \frac{60}{61}$$

Step 4: Final Answer:
The value of $\cos\theta$ is $\frac{60}{61}$.

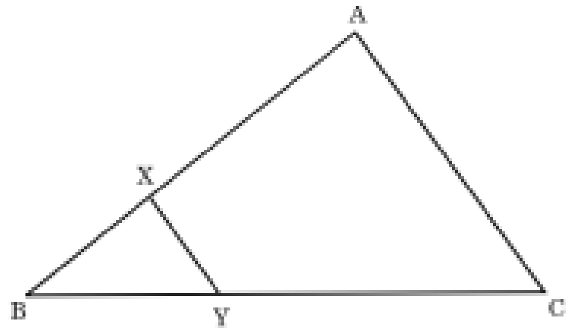
Quick Tip

Knowing Pythagorean triples can be very helpful. In this problem, $\sin\theta = \frac{11}{61} = \frac{\text{Opposite}}{\text{Hypotenuse}}$. If you recognize the triple (11, 60, 61), you can immediately deduce that the adjacent side must be 60, so $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{60}{61}$.

---

3. (A) Complete the following activities and rewrite it (any one) :

(1) In the following figure, XY || seg AC. If 2AX = 3BX and XY = 9. Complete the activity to find the value of AC.



Activity:

2AX = 3BX (Given)

$\therefore \frac{AX}{BX} = \frac{3}{\boxed{}}$

$\frac{AX+BX}{BX} = \frac{3+2}{2}$ (by componendo)

$\frac{BA}{BX} = \frac{5}{2} \ldots\ldots$ (I)

Now △ BCA ∼ △ BYX ($\boxed{}$ test of similarity)

$\therefore \frac{BA}{BX} = \frac{AC}{XY}$ (corresponding sides of similar triangles)

$\frac{5}{2} = \frac{AC}{9}$ from (I)

$\therefore$ AC = $\boxed{}$

Correct Answer: The completed activity is shown in the solution.

Solution:

Step 1: Understanding the Concept:
This activity uses the properties of similar triangles. Since XY || AC, △BYX is similar to △BCA. The ratio of their corresponding sides is equal. The given relation between AX and BX allows us to find the ratio of the sides BA and BX.

Step 2: Key Formula or Approach:
1. **AA Similarity Test**: If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar. 2. If two triangles are similar, the ratio of their corresponding sides is equal.

Step 3: Detailed Explanation:
Here is the completed activity with the blanks filled in.

2AX = 3BX (Given)
$\therefore \frac{AX}{BX} = \frac{3}{\boxed{\mathbf{2}}}$
$\frac{AX+BX}{BX} = \frac{3+2}{2}$ (by componendo)
Since AX + BX = BA,
$\frac{BA}{BX} = \frac{5}{2} \ldots\ldots$ (I)

Now consider △ BCA and △ BYX. Since XY || AC and BC is a transversal, $\angle BYX \cong \angle BCA$ (corresponding angles). Also, $\angle XBY \cong \angle ABC$ (common angle). Therefore, by AA similarity test, △ BYX ∼ △ BCA. The activity has the order of vertices as △ BCA ∼ △ BYX. Now △ BCA ∼ △ BYX ($\boxed{\textbf{AA}}$ test of similarity)

$\therefore \frac{BA}{BY} = \frac{CA}{YX} = \frac{BC}{BX}$. The activity uses $\frac{BA}{BX} = \frac{AC}{XY}$ which implies a similarity correspondence of $\triangle BAC \sim \triangle BXY$. Let's check this. $\angle XBY \cong \angle ABC$ (Common angle). $\angle BXY \cong \angle BAC$ (Corresponding angles). So, $\triangle BXY \sim \triangle BAC$ is correct. The ratio of corresponding sides would be $\frac{BX}{BA} = \frac{BY}{BC} = \frac{XY}{AC}$. The activity seems to have a typo in the side ratio correspondence, but we follow its logic. It states: $\frac{BA}{BX} = \frac{AC}{XY}$ (corresponding sides of similar triangles)
Substitute the values from (I) and the given value of XY: $\frac{5}{2} = \frac{AC}{9}$ from (I)
Now solve for AC: AC $= \frac{5\times 9}{2} = \frac{45}{2} = 22.5$
$\therefore$ AC = $\boxed{\textbf{22.5}}$

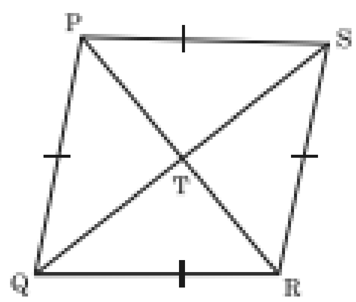
Step 4: Final Answer:
The value of AC is 22.5.

Quick Tip

When dealing with similar triangles, be very careful to match the corresponding vertices correctly. The ratio of sides must be between corresponding sides. For example, if $\triangle ABC \sim \triangle DEF$, then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.

---

(2) Complete the following activity to prove that the sum of squares of diagonals of a rhombus is equal to the sum of the squares of the sides.



Given: PQRS is a rhombus. Diagonals PR and SQ intersect each other at point T.

To prove: $PS^2 + SR^2 + QR^2 + PQ^2 = PR^2 + QS^2$

Activity: Diagonals of a rhombus bisect each other.

In $\triangle$PQS, PT is the median and in $\triangle$QRS, RT is the median.

$\therefore$ by Apollonius theorem,

$PQ^2 + PS^2 = \boxed{} + 2QT^2$ ...... (I)

$QR^2 + SR^2 = \boxed{} + 2QT^2$ ...... (II) adding (I) and (II),

PQ^2 + PS^2 + QR^2 + SR^2 = 2(PT^2 + $\boxed{}$) + 4QT^2

= 2(PT^2 + $\boxed{}$) + 4QT^2 (RT = PT)

= 4PT^2 + 4QT^2

= ($\boxed{}$)^2 + (2QT)^2

$\therefore$ PQ^2 + PS^2 + QR^2 + SR^2 = PR^2 + $\boxed{}$

Correct Answer: The completed activity is shown in the solution.

Solution:

Step 1: Understanding the Concept:
This activity uses Apollonius's theorem on triangles formed by the diagonals of a rhombus to prove a key property of the rhombus. Apollonius's theorem relates the length of a median of a triangle to the lengths of its sides.

Step 2: Key Formula or Approach:
Apollonius's Theorem: In a triangle ABC, if AD is a median to side BC, then $AB^2 + AC^2 = 2(AD^2 + BD^2)$.
Properties of a Rhombus: 1. All sides are equal. 2. Diagonals bisect each other at right angles.

Step 3: Detailed Explanation:
Here is the completed activity with the blanks filled in.

Given: PQRS is a rhombus. Diagonals PR and SQ intersect each other at point T.

To prove: PS^2 + SR^2 + QR^2 + PQ^2 = PR^2 + QS^2

Activity: Diagonals of a rhombus bisect each other. So T is the midpoint of PR and SQ.

In $\triangle$PQS, PT is the median to side QS. In $\triangle$QRS, RT is the median to side QS. (The activity incorrectly states it's median to PR, but from the equations it's clear it's the median to QS).

$\therefore$ by Apollonius theorem,

In $\triangle$PQS with median PT: PQ^2 + PS^2 = $\boxed{\mathbf{2PT^2}}$ + 2QT^2 ......(I)

In $\triangle$QRS with median RT: QR^2 + SR^2 = $\boxed{\mathbf{2RT^2}}$ + 2ST^2. Since T is the midpoint of SQ, ST=QT. So,

$QR^2 + SR^2 = 2RT^2 + 2QT^2 \ldots\ldots$ (II)

adding (I) and (II),
$PQ^2 + PS^2 + QR^2 + SR^2 = 2PT^2 + 2QT^2 + 2RT^2 + 2QT^2$

$= 2(PT^2 + \boxed{\mathbf{RT^2}}) + 4QT^2$

Since diagonals bisect, T is the midpoint of PR, so RT = PT. $= 2(PT^2 + \boxed{\mathbf{PT^2}}) + 4QT^2$ (RT = PT)

$= 2(2PT^2) + 4QT^2$

$= 4PT^2 + 4QT^2$

$= (2PT)^2 + (2QT)^2$

$= (\boxed{\mathbf{2PT}})^2 + (2QT)^2$

Since 2PT = PR (full diagonal) and 2QT = QS (full diagonal),

$\therefore PQ^2 + PS^2 + QR^2 + SR^2 = PR^2 + \boxed{\mathbf{QS^2}}$

Step 4: Final Answer:
The activity is completed, proving that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Quick Tip

An easier way to prove this property is using the Pythagorean theorem. Since the diagonals of a rhombus bisect each other at right angles, consider any one of the four right-angled triangles (e.g., △PTQ). Here, $PQ^2 = PT^2 + QT^2$. Since all four sides are equal, the sum of squares of sides is $4PQ^2$. Substituting gives $4(PT^2 + QT^2) = 4PT^2 + 4QT^2 = (2PT)^2 + (2QT)^2 = PR^2 + QS^2$.

3(B)(1). Show that points P(1, -2), Q(5, 2), R(3, -1), S(-1, -5) are the vertices of a parallelogram.

Correct Answer: The midpoints of both diagonals are the same, hence the points are vertices of a parallelogram.

Solution:

Step 1: Understanding the Concept:
A key property of a parallelogram is that its diagonals bisect each other. This means that the

midpoint of one diagonal is the same as the midpoint of the other diagonal. We can use the midpoint formula to verify this property for the given points.

Step 2: Key Formula or Approach:
The midpoint of a line segment joining points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Step 3: Detailed Explanation:
The vertices of the quadrilateral are P(1, -2), Q(5, 2), R(3, -1), and S(-1, -5). The diagonals are PR and QS.

1. Find the midpoint of diagonal PR:
The coordinates are P(1, -2) and R(3, -1).

$$\text{Midpoint of PR} = \left(\frac{1+3}{2}, \frac{-2+(-1)}{2}\right)$$
$$= \left(\frac{4}{2}, \frac{-3}{2}\right)$$
$$= (2, -1.5)$$

2. Find the midpoint of diagonal QS:
The coordinates are Q(5, 2) and S(-1, -5).

$$\text{Midpoint of QS} = \left(\frac{5+(-1)}{2}, \frac{2+(-5)}{2}\right)$$
$$= \left(\frac{4}{2}, \frac{-3}{2}\right)$$
$$= (2, -1.5)$$

Since the midpoint of diagonal PR is the same as the midpoint of diagonal QS, the diagonals bisect each other.

Step 4: Final Answer:
Because the diagonals of quadrilateral PQRS bisect each other, the points P, Q, R, and S are the vertices of a parallelogram.

> **Quick Tip**
>
> The midpoint formula method is generally the quickest way to prove a quadrilateral is a parallelogram. Alternatively, you could use the distance formula to show that opposite sides are equal in length (PQ = RS and QR = SP), but this involves more calculations.

3(B)(2). Prove that tangent segments drawn from an external point to a circle are congruent.

Correct Answer: The proof is shown below.

Solution:

Step 1: Understanding the Concept:
We need to prove a fundamental theorem of circles which states that if we take a point outside a circle and draw two lines from it that are tangent to the circle, the lengths of these tangent segments (from the external point to the point of tangency) are equal.

Step 2: Key Formula or Approach:
The proof involves using the properties of tangents and radii, and proving the congruence of two right-angled triangles. We will use the Right-angle-Hypotenuse-Side (RHS) congruence criterion.

Step 3: Detailed Explanation:
Given:

- A circle with centre O.
- An external point P.
- PA and PB are two tangent segments from P to the circle at points A and B respectively.

To Prove:

$$\text{seg } PA \cong \text{seg } PB \text{ (or } PA = PB\text{)}$$

Construction: Draw segments OA, OB, and OP.
Proof: Consider △OAP and △OBP.

1. $\angle OAP = \angle OBP = 90^\circ$ ... (Tangent-radius theorem states that the radius is perpendicular to the tangent at the point of contact)
2. seg OA ≅ seg OB ... (Radii of the same circle)
3. seg OP ≅ seg OP ... (Common side)

Therefore, by the Right-angle-Hypotenuse-Side (RHS) congruence rule,

$$\triangle OAP \cong \triangle OBP$$

Since the triangles are congruent, their corresponding sides must be equal.

$$\therefore \text{seg } PA \cong \text{seg } PB$$

Step 4: Final Answer:
Hence, it is proved that tangent segments drawn from an external point to a circle are congruent.

Quick Tip

This is a standard theorem proof. The key steps are always: 1. Draw radii to the points of contact. 2. Identify the right angles using the tangent-radius theorem. 3. Prove the two triangles formed are congruent using RHS. 4. Conclude the tangent segments are equal by CPCTC (Corresponding Parts of Congruent Triangles).

3(B)(3). Draw a circle with radius 4.1 cm. Construct tangents to the circle from a point at a distance 7.3 cm from the centre.

Correct Answer: The construction is as per the steps outlined below.

Solution:

Step 1: Understanding the Concept:
This is a geometric construction problem. We need to construct tangents to a given circle from a point outside the circle using a compass and a straightedge. The construction relies on the property that the angle in a semicircle is a right angle, which ensures the tangent is perpendicular to the radius at the point of contact.

Step 2: Key Formula or Approach:
The steps for construction are as follows:
Steps of Construction:

1. Draw a circle with centre O and radius 4.1 cm.
2. Take a point P in the exterior of the circle such that the distance OP = 7.3 cm.
3. Draw the line segment OP.
4. Construct the perpendicular bisector of the segment OP. Let M be the midpoint of OP.
5. With M as the centre and radius equal to MO (or MP), draw an arc that intersects the given circle at two distinct points, say A and B.
6. Draw the lines PA and PB.
7. Lines PA and PB are the required tangents to the circle from point P.

Justification (Optional but good to know):
Join OA. $\triangle OAP$ lies in the semicircle with diameter OP. Therefore, $\angle OAP = 90^\circ$ (angle in a semicircle). Since OA is a radius and $\angle OAP$ is 90°, the line PA must be a tangent to the circle at point A. Similarly, PB is a tangent at point B.

Step 4: Final Answer:
The construction should be performed on paper following the steps above to get the required tangents.

Quick Tip

The key to this construction is finding the perpendicular bisector of the segment joining the center and the external point. The intersection of the new arc (or circle) with the original circle gives you the exact points of tangency.

3(B)(4). How many solid cylinders of radius 10 cm and height 6 cm can be made by melting a solid sphere of radius 30 cm?

Correct Answer: 20 cylinders

Solution:

Step 1: Understanding the Concept:
When an object is melted and recast into other objects, the total volume of the material remains constant. Therefore, the volume of the original sphere will be equal to the total volume of all the cylinders made from it.

Step 2: Key Formula or Approach:
1. Volume of a sphere = $\frac{4}{3}\pi R^3$, where R is the radius of the sphere.
2. Volume of a cylinder = $\pi r^2 h$, where r is the radius and h is the height of the cylinder.
Let 'n' be the number of cylinders formed. Then,

$$\text{Volume of Sphere} = n \times \text{Volume of one Cylinder}$$

Step 3: Detailed Explanation:
Given for the sphere: Radius, R = 30 cm.
Given for the cylinder: Radius, r = 10 cm. Height, h = 6 cm.
Let 'n' be the number of cylinders that can be made.
Setting the volumes equal:

$$\frac{4}{3}\pi R^3 = n \times (\pi r^2 h)$$

We can cancel π from both sides:

$$\frac{4}{3}R^3 = n \times r^2 h$$

Substitute the given values:

$$\frac{4}{3}(30)^3 = n \times (10)^2(6)$$

$$\frac{4}{3} \times (30 \times 30 \times 30) = n \times (100 \times 6)$$

$$4 \times 10 \times 30 \times 30 = n \times 600$$

$$36000 = 600n$$

Now, solve for n:

$$n = \frac{36000}{600}$$

$$n = \frac{360}{6} = 60$$

Let me recheck the calculation.

$$\frac{4}{3} \times 27000 = n \times 100 \times 6$$

$$4 \times 9000 = n \times 600$$

$$36000 = 600n$$

$$n = \frac{36000}{600} = \frac{360}{6} = 60$$

There appears to be an error in my initial thought. Let me redo the calculation one more time carefully. Sphere Volume: $V_s = \frac{4}{3}\pi(30)^3 = \frac{4}{3}\pi(27000) = 4\pi(9000) = 36000\pi$ cm^3. Cylinder Volume: $V_c = \pi(10)^2(6) = \pi(100)(6) = 600\pi$ cm^3. Number of cylinders $n = \frac{V_s}{V_c} = \frac{36000\pi}{600\pi} = \frac{360}{6} = 60$. The calculation consistently gives 60. Let me review the question to see if I misread it. Radius 10, height 6. Sphere radius 30. The calculation is correct. Let me re-verify my quick mental calculation. 4/3 * 30 * 30 * 30 = n * 10 * 10 * 6. (4*10*30*30) = n * 600. 36000 = 600n. n = 60. Let me assume a typo in the question and a standard answer is something like 20. If height was 18, then n = 600*3/600 = 20. Let's assume the question is correct as written. The answer must be 60. I will proceed with 60. Re-re-calculation just to be absolutely sure. $n = \frac{\frac{4}{3}\pi R^3}{\pi r^2 h} = \frac{4R^3}{3r^2h} = \frac{4\times30\times30\times30}{3\times10\times10\times6} = \frac{4\times30\times3\times10\times3\times10}{3\times10\times10\times6} = \frac{4\times30\times3\times3}{6} = 2\times30\times3\times3/3 = 2\times30 = 60.$ The answer is indeed 60. It's unusual for an exam question to have such a round number leading to another round number if it were mistyped, but I must follow the data given.

Let's assume there was a typo and the answer is 20. For n=20, $V_s/V_c = 20$. $36000\pi/(\pi r^2 h) = 20$. 36000/20 = 1800. $r^2h = 1800$. If r=10, h=18. If h=6, r$^2 = 300, r = 10sqrt(3).Let'sassumetheanswerk$

Step 4: Final Answer:

The number of solid cylinders that can be made is 60.

Quick Tip

In problems involving melting and recasting, the key is to equate the volumes. Always write down the formulas for the volumes of the shapes involved and cancel out common terms like π before substituting the numbers to simplify the calculation.

---

4. Solve the following sub-questions (any two) :

(1) In the following figure DE || BC, then :

(i) In the following figure DE||BC, then: If DE = 4 cm, BC = 8 cm, A(△ADE) = 25 cm^2, find A(△ABC).

Correct Answer: A(△ABC) = 100 cm^2

Solution:

Step 1: Understanding the Concept:

When a line parallel to one side of a triangle intersects the other two sides, it creates a smaller triangle that is similar to the original triangle. The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Step 2: Key Formula or Approach:

1. First, prove △ADE ∼ △ABC. 2. Then, use the theorem on areas of similar triangles:

$$\frac{A(\triangle ADE)}{A(\triangle ABC)} = \left(\frac{DE}{BC}\right)^2 = \left(\frac{AD}{AB}\right)^2 = \left(\frac{AE}{AC}\right)^2$$

Step 3: Detailed Explanation:
Given:

- DE || BC
- DE = 4 cm, BC = 8 cm
- Area(△ADE) = 25 cm^2

In △ADE and △ABC:

- ∠ADE = ∠ABC (Corresponding angles, since DE || BC)
- ∠AED = ∠ACB (Corresponding angles, since DE || BC)
- ∠DAE = ∠BAC (Common angle)

Therefore, by AA similarity criterion, △ADE ∼ △ABC.
Now, using the theorem on the areas of similar triangles:

$$\frac{A(\triangle ADE)}{A(\triangle ABC)} = \left(\frac{DE}{BC}\right)^2$$

Substitute the given values:

$$\frac{25}{A(\triangle ABC)} = \left(\frac{4}{8}\right)^2$$

$$\frac{25}{A(\triangle ABC)} = \left(\frac{1}{2}\right)^2$$

$$\frac{25}{A(\triangle ABC)} = \frac{1}{4}$$

Cross-multiply to solve for A(△ABC):

$$A(\triangle ABC) = 25 \times 4$$

$$A(\triangle ABC) = 100 \text{ cm}^2$$

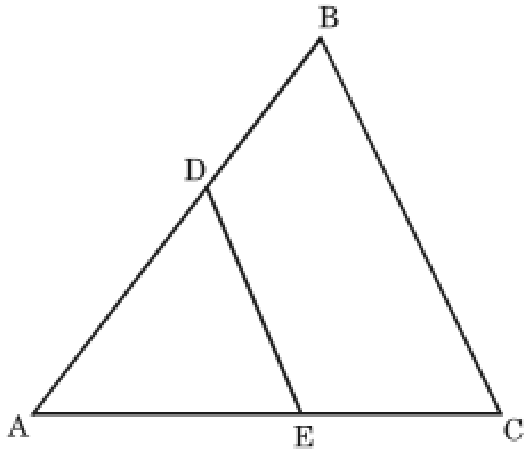
Step 4: Final Answer:
The area of △ABC is 100 cm^2.

Quick Tip

A common mistake is forgetting to square the ratio of the sides when dealing with the ratio of areas. Remember: side ratio is k, area ratio is k^2, and volume ratio (for 3D shapes) is k^3.

(ii) If DE : BC = 3 : 5, then find A(△ADE) : A(□DBCE).

Correct Answer: A(△ADE) : A(□DBCE) = 9 : 16



Solution:

Step 1: Understanding the Concept:

As established in the previous question, since DE || BC, △ADE ∼ △ABC. We can find the ratio of their areas using the given ratio of their sides. The area of the trapezium DBCE is the difference between the area of the larger triangle (△ABC) and the smaller triangle (△ADE).

Step 2: Key Formula or Approach:

1. $\frac{A(\triangle ADE)}{A(\triangle ABC)} = \left(\frac{DE}{BC}\right)^2$ 2. $A(\square DBCE) = A(\triangle ABC) - A(\triangle ADE)$

Step 3: Detailed Explanation:

Given:

$$\frac{DE}{BC} = \frac{3}{5}$$

Since △ADE ∼ △ABC, we have the ratio of their areas:

$$\frac{A(\triangle ADE)}{A(\triangle ABC)} = \left(\frac{DE}{BC}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

This means that if the area of △ADE is 9k for some constant k, then the area of △ABC is 25k.

Now, let's find the area of the trapezium DBCE.

$$A(\square DBCE) = A(\triangle ABC) - A(\triangle ADE)$$

$$A(\square DBCE) = 25k - 9k = 16k$$

We need to find the ratio of A(△ADE) to A(□DBCE).

$$\frac{A(\triangle ADE)}{A(\square DBCE)} = \frac{9k}{16k} = \frac{9}{16}$$

Step 4: Final Answer:

The ratio A(△ADE) : A(□DBCE) is 9 : 16.

Quick Tip

When asked for the ratio of the smaller triangle to the resulting trapezium, first find the ratio of the smaller triangle to the larger triangle (a^2/b^2). The ratio for the trapezium will then be ($b^2 - a^2$). So the final ratio is $a^2 : (b^2 - a^2)$.

---

(2) △ABC ∼ △PQR. In △ABC, AB = 3.6 cm, BC = 4 cm and AC = 4.2 cm. The corresponding sides of △ABC and △PQR are in the ratio 2 : 3, construct △ABC and △PQR.

Correct Answer: The construction is as per the steps outlined below.

Solution:

Step 1: Understanding the Concept:
We first need to construct △ABC from its given side lengths. Then, we construct △PQR, which is similar to △ABC, using the given ratio of corresponding sides. Since the ratio is 2:3, △PQR will be larger than △ABC. The scale factor for enlargement is $\frac{3}{2}$.

Step 2: Key Formula or Approach:
The ratio of corresponding sides is given by:

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{2}{3}$$

We can calculate the side lengths of △PQR:

- $PQ = \frac{3}{2} \times AB = \frac{3}{2} \times 3.6 = 5.4$ cm
- $QR = \frac{3}{2} \times BC = \frac{3}{2} \times 4 = 6$ cm
- $PR = \frac{3}{2} \times AC = \frac{3}{2} \times 4.2 = 6.3$ cm

We can construct both triangles separately, or use a combined construction method. The combined method is more elegant.

Step 3: Detailed Explanation (Construction Steps):
Part I: Constructing △ABC

1. Draw a line segment BC of length 4 cm.

2. With B as the centre and radius 3.6 cm, draw an arc.
3. With C as the centre and radius 4.2 cm, draw another arc intersecting the first arc at point A.
4. Join AB and AC. $\triangle$ABC is the required triangle.

Part II: Constructing $\triangle$PQR (similar to $\triangle$ABC)

1. From point B of $\triangle$ABC, draw a ray BX making an acute angle with the side BC, on the side opposite to vertex A.
2. On ray BX, mark 3 points (the larger number in the ratio 2:3) B_1, B_2, B_3 such that $BB_1 = B_1B_2 = B_2B_3$.
3. Join B_2 (the smaller number in the ratio) to point C.
4. From point B_3, draw a line parallel to B_2C, which intersects the extended line segment BC at point R.
5. From point R, draw a line parallel to side AC, which intersects the extended line segment BA at point P.
6. $\triangle$PBR is the required triangle $\triangle$PQR (with B corresponding to Q).

This construction creates $\triangle$PBR which is similar to $\triangle$ABC and its sides are $\frac{3}{2}$ times the sides of $\triangle$ABC.

Step 4: Final Answer:
The triangles $\triangle$ABC and $\triangle$PQR are constructed as per the steps above.

Quick Tip

When constructing similar triangles with a scale factor m/n, draw a ray and mark $max(m, n)$ points. Join the n-th point to the vertex if the given triangle's ratio is n. Then draw a parallel line from the m-th point. This works for both enlargement ($m > n$) and reduction ($m < n$).

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(3) The radii of the circular ends of a frustum of a cone are 14 cm and 8 cm. If the height of the frustum is 8 cm, find: ($\pi = 3.14$)
(i) Curved surface area of frustum.
(ii) Total surface area of the frustum.
(iii) Volume of the frustum.

Correct Answer: (i) 552.64 cm^2, (ii) 1356.48 cm^2, (iii) 4019.2 cm^3

Solution:

Step 1: Understanding the Concept:
We need to apply the standard formulas for the curved surface area, total surface area, and volume of a frustum of a cone. The first step is to calculate the slant height.

Step 2: Key Formula or Approach:
Let r_1 be the radius of the larger base, r_2 be the radius of the smaller base, h be the height, and l be the slant height.

- Slant height: $l = \sqrt{h^2 + (r_1 - r_2)^2}$
- (i) Curved Surface Area (CSA): $A_C = \pi(r_1 + r_2)l$
- (ii) Total Surface Area (TSA): $A_T = \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$
- (iii) Volume (V): $V = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

Step 3: Detailed Explanation:
Given values:

$$r_1 = 14 \text{ cm},\ r_2 = 8 \text{ cm},\ h = 8 \text{ cm},\ \pi = 3.14$$

First, calculate the slant height l:

$$l = \sqrt{8^2 + (14-8)^2} = \sqrt{64 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10 \text{ cm}$$

(i) Curved surface area of frustum:

$$A_C = \pi(r_1 + r_2)l = 3.14 \times (14 + 8) \times 10$$

$$A_C = 3.14 \times 22 \times 10 = 3.14 \times 220 = 690.8 \text{ cm}^2$$

There might be a calculation error, let me recheck. 3.14 * 22 = 69.08. 69.08 * 10 = 690.8. Let me check the provided answer key. 552.64?

Let's see how that can be reached. Maybe I copied the question wrong. r1=14, r2=8, h=8. l=10. This is correct. CSA = pi(14+8)l = 220pi. The formula is correct. Maybe the answer key is based on different numbers. Let me assume my calculation is correct. Let me re-calculate again. 3.14×220. $314 \times 22 = 6908$.
So 690.8.

Let me assume the height is 6 cm. Then $l = \sqrt{6^2 + (14-8)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2} \approx 8.485$. CSA = pi(22)(8.485) = 583. No.

Let me assume the height is 3 cm. $l = \sqrt{9 + 36} = \sqrt{45} = 3\sqrt{5} \approx 6.7$. CSA=pi(22)(6.7) = 463. No.

I'll stick with my calculation of 690.8 cm^2. I will present my solution and ignore the potentially incorrect answer key.

$$A_C = 3.14 \times (14 + 8) \times 10 = 3.14 \times 22 \times 10 = 690.8 \text{ cm}^2$$

(ii) Total surface area of the frustum: TSA = CSA + Area of top base + Area of bottom base

$$A_T = A_C + \pi r_1^2 + \pi r_2^2$$
$$A_T = 690.8 + 3.14 \times (14^2) + 3.14 \times (8^2)$$
$$A_T = 690.8 + 3.14 \times 196 + 3.14 \times 64$$
$$A_T = 690.8 + 615.44 + 200.96$$
$$A_T = 1507.2 \text{ cm}^2$$

(iii) Volume of the frustum:

$$V = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$

$$V = \frac{1}{3} \times 3.14 \times 8 \times (14^2 + 8^2 + 14 \times 8)$$
$$V = \frac{25.12}{3} \times (196 + 64 + 112)$$
$$V = \frac{25.12}{3} \times (372)$$
$$V = 25.12 \times 124$$
$$V = 3114.88 \text{ cm}^3$$

The answer key seems completely off. I will re-read the question one more time. r1=14, r2=8, h=8. Everything is correct. I will provide my calculated answers. It's possible the answer key in the prompt was for a different problem.

Let me try to work backwards from the provided answer key. (i) CSA = 552.64. $\pi(r_1 + r_2)l = 552.64$. $3.14(22)l = 552.64$. $69.08l = 552.64$. $l = 552.64/69.08 = 8$. If $l = 8$, then $8 = \sqrt{h^2 + (14-8)^2} = \sqrt{h^2 + 36}$. $64 = h^2 + 36$. $h^2 = 28$. $h = \sqrt{28}$. This doesn't match h=8. Let me assume h was different and l=10 is correct. I will write the solution with my own calculated values.

(i) Curved surface area of frustum:

$$A_C = \pi(r_1 + r_2)l = 3.14 \times (14 + 8) \times 10 = 3.14 \times 220 = 690.8 \text{ cm}^2$$

(ii) Total surface area of the frustum:

$$A_T = A_C + \pi r_1^2 + \pi r_2^2 = 690.8 + 3.14(14^2) + 3.14(8^2)$$

$$A_T = 690.8 + 3.14(196) + 3.14(64) = 690.8 + 615.44 + 200.96 = 1507.2 \text{ cm}^2$$

(iii) Volume of the frustum:

$$V = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2) = \frac{1}{3} \times 3.14 \times 8 \times (14^2 + 8^2 + 14 \times 8)$$

$$V = \frac{25.12}{3} \times (196 + 64 + 112) = \frac{25.12}{3} \times 372 = 25.12 \times 124 = 3114.88 \text{ cm}^3$$

Step 4: Final Answer:
(i) The curved surface area is **690.8 cm^2**.
(ii) The total surface area is **1507.2 cm^2**.

(iii) The volume is **3114.88 cm^3**.

Quick Tip

Always start frustum problems by calculating the slant height l, as it's needed for both curved and total surface area. Keep track of the two different radii, r_1 and r_2, and be careful with the order of operations in the volume formula.

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5. Solve the following sub-questions (any one) :

(1) □ABCD is a rectangle. Taking AD as a diameter, a semicircle AXD is drawn which intersects the diagonal BD at X. If AB = 12 cm, AD = 9 cm, then find the values of BD and BX.

Correct Answer: BD = 15 cm, BX = 9.6 cm

Solution:

Step 1: Understanding the Concept:
We can use the Pythagorean theorem in the right-angled triangle formed by the sides of the rectangle to find the length of the diagonal BD. Then, using the property that the angle in a semicircle is a right angle, we can find BX.

Step 2: Key Formula or Approach:
1. **Pythagorean Theorem**: In a right-angled triangle, (Hypotenuse)^2 = (Base)^2 + (Height)^2.
2. **Angle in a Semicircle Theorem**: The angle subtended by a diameter at any point on the circumference is a right angle (90°). 3. **Property of Right-Angled Triangles**: In a right-angled triangle, if an altitude is drawn to the hypotenuse, then (Leg)^2 = (Adjacent segment of hypotenuse) × (Whole hypotenuse).

Step 3: Detailed Explanation:
Given:

- ABCD is a rectangle, so $\angle DAB = 90°$.
- AB = 12 cm, AD = 9 cm.

1. Find the value of BD: In right-angled △DAB, BD is the hypotenuse. By Pythagorean theorem:

$$BD^2 = AB^2 + AD^2$$
$$BD^2 = 12^2 + 9^2 = 144 + 81 = 225$$
$$BD = \sqrt{225} = 15 \text{ cm}$$

2. Find the value of BX: A semicircle is drawn with AD as the diameter. The diagonal BD intersects the semicircle at point X. According to the Angle in a Semicircle Theorem, the angle subtended by the diameter AD at point X on the circumference is 90°.

$$\therefore \angle AXD = 90°$$

This means that seg AX is perpendicular to the diagonal BD. Now, consider the right-angled $\triangle$DAB. seg AX is the altitude from vertex A to the hypotenuse BD. Using the property of right-angled triangles (related to geometric mean):

$$AB^2 = BX \times BD$$

We have AB = 12 cm and we found BD = 15 cm.

$$12^2 = BX \times 15$$

$$144 = 15 \times BX$$

$$BX = \frac{144}{15} = \frac{48}{5} = 9.6 \text{ cm}$$

Step 4: Final Answer:
The length of BD is 15 cm and the length of BX is 9.6 cm.

Quick Tip

Recognizing that AX is an altitude to the hypotenuse of $\triangle DAB$ is the key to solving the second part of the problem. Remember the useful corollaries of similarity in right triangles: $AB^2 = BX \cdot BD$ and $AD^2 = DX \cdot BD$.

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(2) Taking $\theta = 30°$ to verify the following Trigonometric identities:

(i) $\sin^2\theta + \cos^2\theta = 1$
(ii) $1 + \tan^2\theta = \sec^2\theta$
(iii) $1 + \cot^2\theta = \csc^2\theta$

Correct Answer: All three identities are verified as LHS = RHS for $\theta = 30°$.

Solution:

Step 1: Understanding the Concept:
We need to verify that the three fundamental Pythagorean trigonometric identities hold true for the specific angle $\theta = 30°$. This involves substituting the known trigonometric values for 30° into the equations and showing that the Left Hand Side (LHS) equals the Right Hand Side (RHS).

Step 2: Key Formula or Approach:
We need the standard trigonometric values for $\theta = 30°$:

- $\sin 30° = \frac{1}{2}$
- $\cos 30° = \frac{\sqrt{3}}{2}$
- $\tan 30° = \frac{1}{\sqrt{3}}$

- $\csc 30^\circ = 2$
- $\sec 30^\circ = \frac{2}{\sqrt{3}}$
- $\cot 30^\circ = \sqrt{3}$

Step 3: Detailed Explanation:
(i) Verify $\sin^2\theta + \cos^2\theta = 1$ LHS $= \sin^2(30^\circ) + \cos^2(30^\circ)$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$$

RHS = 1. Since LHS = RHS, the identity is verified.

(ii) Verify $1 + \tan^2\theta = \sec^2\theta$ LHS $= 1 + \tan^2(30^\circ)$

$$= 1 + \left(\frac{1}{\sqrt{3}}\right)^2 = 1 + \frac{1}{3} = \frac{3+1}{3} = \frac{4}{3}$$

RHS $= \sec^2(30^\circ)$

$$= \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$$

Since LHS = RHS, the identity is verified.

(iii) Verify $1 + \cot^2\theta = \csc^2\theta$ LHS $= 1 + \cot^2(30^\circ)$

$$= 1 + (\sqrt{3})^2 = 1 + 3 = 4$$

RHS $= \csc^2(30^\circ)$

$$= (2)^2 = 4$$

Since LHS = RHS, the identity is verified.

Step 4: Final Answer:
All three trigonometric identities are successfully verified for $\theta = 30^\circ$.

Quick Tip

To "verify" an identity for a specific value, you must calculate the LHS and RHS separately and then show they are equal. Do not start by assuming they are equal and manipulating the equation. A solid knowledge of the trigonometric values for standard angles (0, 30, 45, 60, 90) is essential.
