OJEE 2026 Shift-1 LE. Tech (Diploma)

Question Paper with Solutions (Memory-Based)

Conducted by Odisha Joint Entrance Examination Committee (OJEEC))

OJEE
ODISHA JOINT ENTRANCE EXAMINATION

General Instructions

(i) The examination will be conducted in Computer-Based Test (CBT) mode.

(ii) Each question carries +4 marks for correct answer and 1 mark for wrong answer.

(iii) The total number of questions and duration will vary depending on the course.

(iv) Duration of the exam is 2 hours (120 minutes).

1. For a scalar function $\vec{F}(x,y,z) = x^2 + 3y^2 + 2z^2$, the directional derivative at the point P(1, 2, -1) is the direction of a vector $(\hat{i} + \hat{j} + 2\hat{k})$ is

(A) -18
(B) $-3\sqrt{6}$
(C) $3\sqrt{6}$
(D) 18

Correct Answer: (B) $-3\sqrt{6}$

Solution:

Step 1: Understanding the Question:
We need to calculate the directional derivative of a given scalar function $F(x,y,z)$ at a specific point in the direction of a given vector.
Note: To logically arrive at the provided answer key (B) $-3\sqrt{6}$, we must assume a typographical error in the problem statement. The coordinates of point P are likely intended to be $(1,-2,-1)$. We will proceed with this assumption to justify the given answer.

Step 2: Key Formula or Approach:

The directional derivative of a scalar function $F(x,y,z)$ in the direction of vector $\vec{v}$ is given by:

$$D_{\vec{v}}F = \nabla F \cdot \hat{u}$$

where ∇F is the gradient of F evaluated at the given point, and $\hat{u}$ is the unit vector in the direction of $\vec{v}$.

Step 3: Detailed Explanation:

First, find the gradient of the function $F(x,y,z) = x^2 + 3y^2 + 2z^2$:

$$\nabla F = \frac{\partial F}{\partial x}\hat{i} + \frac{\partial F}{\partial y}\hat{j} + \frac{\partial F}{\partial z}\hat{k}$$

$$\nabla F = 2x\hat{i} + 6y\hat{j} + 4z\hat{k}$$

Evaluate the gradient at the assumed point $P(1,-2,-1)$:

$$\nabla F|_{(1,-2,-1)} = 2(1)\hat{i} + 6(-2)\hat{j} + 4(-1)\hat{k} = 2\hat{i} - 12\hat{j} - 4\hat{k}$$

Next, find the unit vector $\hat{u}$ for the direction vector $\vec{v} = \hat{i} + \hat{j} + 2\hat{k}$:

$$|\vec{v}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$\hat{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$$

Now, compute the directional derivative by taking the dot product:

$$D_{\vec{v}}F = (2\hat{i} - 12\hat{j} - 4\hat{k}) \cdot \left[\frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})\right]$$

$$D_{\vec{v}}F = \frac{1}{\sqrt{6}}[(2)(1) + (-12)(1) + (-4)(2)]$$

$$D_{\vec{v}}F = \frac{1}{\sqrt{6}}(2 - 12 - 8)$$

$$D_{\vec{v}}F = \frac{-18}{\sqrt{6}}$$

Rationalizing the denominator gives:

$$D_{\vec{v}}F = \frac{-18 \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} = \frac{-18\sqrt{6}}{6} = -3\sqrt{6}$$

Step 4: Final Answer:

The directional derivative is $-3\sqrt{6}$.

Quick Tip: Always remember to normalize the direction vector to a unit vector before calculating the dot product for a directional derivative.

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2. There are 3 candidate for a Mathematics, 5 for chemistry and 4 for a Physics scholarship. In how many ways can the scholarship be awarded.

(A) 12

(B) 60

(C) 20

(D) none of these

Correct Answer: (B) 60

Solution:

Step 1: Understanding the Question:

We need to find the total number of ways to award scholarships. Based on the options, the question implies awarding one scholarship for each of the three distinct subjects (Mathematics, Chemistry, and Physics).

Step 2: Key Formula or Approach:

We use the Fundamental Principle of Counting (Multiplication Rule). If one event can occur in m ways and a second independent event can occur in n ways, then the two events can occur together in $m \times n$ ways.

Step 3: Detailed Explanation:

Number of ways to award the Mathematics scholarship $= 3$ (since there are 3 candidates)

Number of ways to award the Chemistry scholarship $= 5$ (since there are 5 candidates)

Number of ways to award the Physics scholarship $= 4$ (since there are 4 candidates)

Since the selections for each subject are independent of one another, the total number of ways to award all three scholarships is the product of the individual choices:

$$\text{Total ways} = 3 \times 5 \times 4$$

$$\text{Total ways} = 60$$

Step 4: Final Answer:

The scholarships can be awarded in 60 ways.

Quick Tip: When events are independent and you need them all to occur ("AND" condition), multiply their respective possibilities. If it were a choice of awarding only ONE scholarship in total ("OR" condition), you would add them $(3 + 5 + 4 = 12)$.

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3. Consider an infinite series with first term a and common ratio r. If its sum is 4 and the second term is $\frac{3}{4}$, then

(A) $a = \frac{7}{4}, r = \frac{3}{7}$
(B) $a = 2, r = \frac{3}{8}$
(C) $a = \frac{3}{2}, r = \frac{1}{2}$
(D) $a = 3, r = \frac{1}{4}$

Correct Answer: (D) $a = 3, r = \frac{1}{4}$

Solution:

Step 1: Understanding the Question:

We are dealing with an infinite geometric progression. We are given its sum to infinity and the value of its second term. We need to find the first term (a) and the common ratio (r).

Step 2: Key Formula or Approach:

The sum to infinity of a geometric series is $S_\infty = \frac{a}{1-r}$ (where $|r| < 1$).
The n-th term of a geometric series is $T_n = ar^{n-1}$. Thus, the second term is $T_2 = ar$.

Step 3: Detailed Explanation:

From the given information, we can set up two equations: 1) $S_\infty = \frac{a}{1-r} = 4 \implies a = 4(1-r)$
2) $T_2 = ar = \frac{3}{4} \implies a = \frac{3}{4r}$
Equating the two expressions for a:

$$4(1-r) = \frac{3}{4r}$$

Multiply both sides by $4r$:

$$16r(1-r) = 3$$

$$16r - 16r^2 = 3$$

Rearrange into a standard quadratic equation:

$$16r^2 - 16r + 3 = 0$$

Factorizing the quadratic equation:

$$16r^2 - 12r - 4r + 3 = 0$$

$$4r(4r-3) - 1(4r-3) = 0$$

$$(4r-1)(4r-3) = 0$$

This gives two possible values for r: $r = \frac{1}{4}$ or $r = \frac{3}{4}$
Now, let's find the corresponding values for a using $a = \frac{3}{4r}$: If $r = \frac{1}{4}$:

$$a = \frac{3}{4(\frac{1}{4})} = \frac{3}{1} = 3$$

This corresponds to pair $(a = 3, r = \frac{1}{4})$, which matches option D.
If $r = \frac{3}{4}$:

$$a = \frac{3}{4(\frac{3}{4})} = \frac{3}{3} = 1$$

This gives pair $(a = 1, r = \frac{3}{4})$, which is not among the options.

Step 4: Final Answer:

The correct values are $a = 3, r = \frac{1}{4}$.

Quick Tip: When dealing with simultaneous equations from sequences, substituting a in terms of r usually leads straight to a solvable quadratic equation. Always check both roots against the given options.

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4. If a, b, c are positive numbers then value of $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$, is

(A) ≥ 10
(B) ≥ 9
(C) ≥ 12
(D) None of these

Correct Answer: (B) ≥ 9

Solution:

Step 1: Understanding the Question:

We are asked to find the minimum possible value or the lower bound of an algebraic expression involving three positive numbers a, b, and c.

Step 2: Key Formula or Approach:

This problem can be elegantly solved using the Arithmetic Mean-Geometric Mean (AM-GM) inequality, which states that for non-negative real numbers, the arithmetic mean is greater than or equal to the geometric mean. Alternatively, algebraic expansion paired with the property that $x + \frac{1}{x} \geq 2$ for positive x works perfectly.

Step 3: Detailed Explanation:

Let's expand the given expression:

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

$$= a\left(\frac{1}{a}\right)+a\left(\frac{1}{b}\right)+a\left(\frac{1}{c}\right)+b\left(\frac{1}{a}\right)+b\left(\frac{1}{b}\right)+b\left(\frac{1}{c}\right)+c\left(\frac{1}{a}\right)+c\left(\frac{1}{b}\right)+c\left(\frac{1}{c}\right)$$

$$= 1+\frac{a}{b}+\frac{a}{c}+\frac{b}{a}+1+\frac{b}{c}+\frac{c}{a}+\frac{c}{b}+1$$

Grouping reciprocal terms together:

$$= 3 + \left(\frac{a}{b} + \frac{b}{a}\right) + \left(\frac{b}{c} + \frac{c}{b}\right) + \left(\frac{c}{a} + \frac{a}{c}\right)$$

By the AM-GM inequality, for any two positive numbers x and y:

$$\frac{x}{y} + \frac{y}{x} \geq 2\sqrt{\left(\frac{x}{y}\right)\left(\frac{y}{x}\right)} = 2\sqrt{1} = 2$$

Applying this to each paired term in our expansion:

$$\frac{a}{b} + \frac{b}{a} \geq 2$$

$$\frac{b}{c} + \frac{c}{b} \geq 2$$

$$\frac{c}{a} + \frac{a}{c} \geq 2$$

Substituting these minimum values back into the grouped expression:

$$\text{Value} \geq 3 + 2 + 2 + 2$$

$$\text{Value} \geq 9$$

Equality holds only when $a = b = c$.

Step 4: Final Answer:

The value is ≥ 9.

Quick Tip: This is a direct application of the AM-HM inequality for n variables: $\frac{x_1+x_2+\cdots+x_n}{n} \geq \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}}$. For $n = 3$, moving the denominators yields $(a + b + c)(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}) \geq 3 \times 3 = 9$.

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5. In a mechanical refrigeration system, the highest temperature of refrigerant occurs

(A) Between condenser and evaporator

(B) In evaporator

(C) Before expansion valve

(D) Between compressor and condenser

Correct Answer: (D) Between compressor and condenser

Solution:

Step 1: Understanding the Question:

The question asks to identify the location of the highest refrigerant temperature within a standard vapor-compression refrigeration cycle.

Step 3: Detailed Explanation:

Let's analyze the state of the refrigerant across the standard components of a refrigeration cycle:

- **Compressor:** The refrigerant enters as a low-pressure, low-temperature vapor. The compressor does mechanical work on the fluid, compressing it. This process is ideally isentropic, leading to a significant increase in both pressure and temperature. The refrigerant exits the compressor as a highly superheated vapor.
- **Condenser:** The hot, high-pressure vapor enters the condenser where it rejects heat to the environment. It cools down to its saturation temperature and condenses into a liquid. Thus, temperature decreases here.
- **Expansion Valve:** The high-pressure liquid undergoes an isenthalpic expansion, causing a sharp drop in both pressure and temperature.
- **Evaporator:** The cold, low-pressure mixture enters the evaporator, absorbs heat from the refrigerated space, and evaporates back into a vapor before returning to the compressor. The temperature is lowest here.

Therefore, the highest temperature in the entire cycle is reached immediately after the compression process, right before any heat is rejected. This physical location is exactly between the compressor and the condenser.

Step 4: Final Answer:

The highest temperature occurs between the compressor and condenser.

Quick Tip: In any vapor-compression cycle (refrigeration or heat pump), the compressor's discharge line always carries the hottest fluid, while the expansion valve's exit carries the coldest.

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6. A self-locking screw is one which

(A) Has locking arrangement

(B) Has a hole drilled through for inserting locking pin

(C) Has coefficient of friction equal to or greater than the tangent of the load angle

(D) Has fine pitch screw threads

Correct Answer: (C) Has coefficient of friction equal to or greater than the tangent of the load angle

Solution:

Step 1: Understanding the Question:

We need to identify the correct technical condition that makes a power screw "self-locking".

Step 3: Detailed Explanation:

A self-locking screw is defined mechanically as a screw that will not lower or unwind under the action of the axial load alone. It requires an external applied torque to lower the load.

The condition for self-locking depends on the relationship between the friction angle (ϕ) and the helix angle or load angle (α). For a screw to be self-locking, the friction holding it in place must be greater than or equal to the force component tending to unwind it down the thread incline.

Mathematically, this geometric condition is expressed as:

$$\phi \geq \alpha$$

Taking the tangent of both sides:

$$\tan(\phi) \geq \tan(\alpha)$$

By definition, the coefficient of static friction, μ, is equal to the tangent of the friction angle ($\mu = \tan(\phi)$).

Therefore, the condition for self-locking becomes:

$$\mu \geq \tan(\alpha)$$

This translates precisely to the statement that a screw is self-locking if its coefficient of friction is equal to or greater than the tangent of the load angle.

Step 4: Final Answer:

A self-locking screw has a coefficient of friction equal to or greater than the tangent of the load angle.

Quick Tip: Efficiency of a self-locking screw is always less than 50%. If efficiency exceeds 50%, the screw is "overhauling" and will unwind under its own load.

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7. At 'break-even point'

(A) Constant expenses = Profits
(B) Total sales = variable expenses
(C) Variable expenses - Profits = Total sales
(D) None of the above

Correct Answer: (D) None of the above

Solution:

Step 1: Understanding the Question:

The question asks for the true mathematical relationship that defines the "break-even point" in business or industrial economics.

Step 3: Detailed Explanation:

The break-even point (BEP) is defined as the level of production or sales at which total revenues exactly equal total costs, resulting in a net profit of strictly zero.

The fundamental economic equation is:

$$\text{Total Sales (Revenue)} = \text{Total Costs}$$

Total Costs can be broken down into Fixed Costs (Constant expenses) and Variable Costs (Variable expenses).

$$\text{Total Sales} = \text{Constant expenses} + \text{Variable expenses}$$

Let's evaluate the given options based on this defining equation (where Profit = 0):

- (A) **Constant expenses = Profits:** Incorrect. At BEP, Profit is exactly 0, while constant expenses are typically a positive baseline value.
- (B) **Total sales = variable expenses:** Incorrect. This statement implies that constant expenses are zero (Total Sales−Variable expenses = 0), which ignores fixed costs entirely.
- (C) **Variable expenses - Profits = Total sales:** Since Profit = 0 at BEP, this equation simplifies to Variable expenses = Total sales, which is identical to option (B) and is therefore incorrect.

Since none of the statements A, B, or C correctly describe the break-even condition, the correct choice must be "None of the above".

Step 4: Final Answer:

None of the listed equations correctly define the break-even point.

Quick Tip: At break-even point, Contribution Margin (Total Sales - Variable Expenses) exactly equals Fixed Costs. Always start from the fundamental identity: Profit = Sales - (Fixed Costs + Variable Costs) = 0.

