Permutations And Combinations JEE Main PYQ - 2

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Permutations and Combinations

1. If all the words with or without meaning made using all the letters of the word "KANPUR" are arranged as in a dictionary, then the word at 440th position in this arrangement is: (+4, -1)

 a. PRNAKU

 b. PRKANU

 c. PRKAUN

 d. PRNAUK

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2. The number of 6-letter words, with or without meaning, that can be formed using the letters of the word "MATHS" such that any letter that appears in the word must appear at least twice is: (+4, -1)

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3. Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to: (+4, -1)

 a. 8925

 b. 8750

 c. 9100

 d. 8575

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4. The number of different 5 digit numbers greater than 50000 that can be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, such that the sum of their first and last digits should not be more than 8, is: (+4, -1)

 a. 5719

 b. 4608

 c. 5720

d. 4607

5. Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to: (+4, -1)

a. 8925

b. 8750

c. 9100

d. 8575

6. The number of solutions of the equation (+4, -1)

$$\left(\frac{9}{x}-\frac{9}{\sqrt{x}}+2\right)\left(\frac{2}{x}-\frac{7}{\sqrt{x}}+3\right)=0$$

is:

a. 3

b. 1

c. 2

d. 4

7. The number of different 5 digit numbers greater than 50000 that can be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, such that the sum of their first and last digits should not be more than 8, is: (+4, -1)

a. 5719

b. 4608

c. 5720

d. 4607

8. The number of five-digit numbers, greater than 40000 and divis ible by 5, which can be formed using the digits 0, 1, 3, 5, 7, and 9 without repetition, is equal to: (+4, -1)

a. 120

b. 132

c. 72

d. 96

9. The lines $L_1, L_2, \ldots, L_{20}$ are distinct. For $n = 1, 2, 3, \ldots, 10$, all the lines L_{2n-1} are parallel to each other, and all the lines L_{2n} pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \ldots, L_{20}\}$ is equal to: (+4, -1)

10. If all the words with or without meaning made using all the letters of the word "NAGPUR" are arranged as in a dictionary, then the word at 315th position in this arrangement is (+4, -1)

a. NRAGUP

b. NRAGPU

c. NRAPGU

d. NRAPUG

11. Let the set $S = \{2, 4, 8, 16, ..., 512\}$ be partitioned into 3 sets A, B, C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \phi$. The maximum number of such possible partitions of S is equal to: (+4, -1)

a. 1680

b. 1520

c. 1710

d. 1640

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12. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50^{th} word is : **(+4, -1)**

a. OBBHJ

b. HBBJO

c. OBBJH

d. JBBOH

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13. There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each group, then the number of ways of selecting 4 men and 4 women is **(+4, -1)**

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14. The number of ways five alphabets can be chosen from the alphabets of the word MATHEMATICS, where the chosen alphabets are not necessarily distinct, is equal to : **(+4, -1)**

a. 175

b. 181

c. 177

d. 179

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15. The number of ways of getting a sum 16 on throwing a dice four times is __________. **(+4, -1)**

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16. Let $A = \{1, 3, 7, 9, 11\}$ and $B = \{2, 4, 5, 7, 8, 10, 12\}$. Then the total number of one-one maps $f : A \to B$, such that $f(1) + f(3) = 14$, is: **(+4, -1)**

a. 180

b. 120

c. 480

d. 240

17. There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding A and B, of a triangle $\triangle ABC$. Similarly, there are 6 points $P_6, P_7, \ldots, P_{11}$ on the side BC and 7 points $P_{12}, P_{13}, \ldots, P_{18}$ on the side CA of the triangle.The number of triangles that can be formed using the points $P_1, P_2, \ldots, P_{18}$ as vertices, is: **(+4, -1)**

a. 776

b. 751

c. 796

d. 771

18. If for some m, n; ${}^6C_m + 2\left({}^6C_{m+1}\right) +^6 C_{m+2} >^8 C_3$ and ${}^{n-1}P_3 :^n P_4 = 1 : 8$, then ${}^nP_{m+1} +^{n+1} C_m$ is equal to **(+4, -1)**

a. 380

b. 376

c. 384

d. 372

19. The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is **(+4, -1)**

a. 406

b. 130

c. 142

d. 136

20. The number of integers, between 100 and 1000 having the sum of their digits equals to 14, is______ . **(+4, -1)**

21. The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to _____ **(+4, -1)**

22. If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to: **(+4, -1)**

a. 47

b. 53

c. 51

d. 43

23. In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections : A, B and C . A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is _______ . **(+4, -1)**

24. Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to **(+4, -1)**

a. 18

b. 16

c. 12

d. 15

25. Let $\alpha = \frac{(4!)!}{(4!)^{3!}}$ and $\beta = \frac{(5!)!}{(5!)^{4!}}$. Then: **(+4, -1)**

a. $\alpha \in \mathbb{N}$ and $\beta \notin \mathbb{N}$

b. $\alpha \notin \mathbb{N}$ and $\beta \in \mathbb{N}$

c. $\alpha \in \mathbb{N}$ and $\beta \in \mathbb{N}$

d. $\alpha \notin \mathbb{N}$ and $\beta \notin \mathbb{N}$

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26. Number of 5 letters words made from the word "MATHEMATICS" is equal to **(+4, -1)**

a. 13540

b. 13560

c. 14210

d. 17310

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27. The 315th word in dictionary arranged in order for the word 'NAGPUR' is **(+4, -1)**

a. NRAGPU

b. NRPGUA

c. NPRGUA

d. NRAPGU

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28. If ${}^{n+1}C_{r+1}: {}^{n}C_{r}: {}^{n-1}C_{r-1} = 55: 35: 21$ then the value of n + r is _____. **(+4, -1)**

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29. The 50th word in the dictionary using the letters B, B, H, J, O is: **(+4, -1)**

a. OBBJH

b. OBBHJ

c. JHBBO

d. BBHOJ

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30. If S = {2, 4, 8, 16, ..., 512}. If S is broken in 3 equal subsets A, B and C such that $A \cap B = B \cap C = C \cap A = \varphi$ and $A \cup B \cup C = S$ then maximum number of ways to break is **(+4, -1)**

a. $^9C_3$

b. $\frac{9!}{(3!)3}$

c. $\frac{9!}{(3!)4}$

d. $\frac{9!}{(3!)^2}$

Answers

1. **Answer: c**

Explanation:

Arranging the letters alphabetically: {A, K, N, P, R, U}

Step 1: Words starting with A = 5! = 120

Step 2: Words starting with K = 5! = 120

Step 3: Words starting with N = 5! = 120

Step 4: Words starting with PA = 4! = 24

Step 5: Words starting with PK = 4! = 24

Step 6: Words starting with PN = 4! = 24

Step 7: Words starting with PRKA = 3! = 6

Step 8: PRKAN is the 439th word

Step 9: PRKAUN is the 440th word

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2. **Answer: 15 - 15**

Explanation:

Step 1: Identify the letters in the word "MATHS" and count how many times each can be repeated to form a 6-letter word.
Step 2: Use the counting principle to find the total number of valid words where each letter appears at least twice.
Step 3: The total number of such words is calculated as 15.

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3. **Answer: b**

Explanation:

We need to choose 5 individuals from group A (which has 7 boys and 3 girls) and 3 individuals from group B (which has 6 boys and 5 girls).

For group A, we can select 3 boys and 2 girls, or 4 boys and 1 girl. The number of ways to select these members can be calculated using combinations:

$$\text{Ways for group A} = \binom{7}{4} \times \binom{3}{1} + \binom{7}{3} \times \binom{3}{2}.$$

For group B, we can select the remaining individuals:

$$\text{Ways for group B} = \binom{6}{1} \times \binom{5}{2} + \binom{6}{2} \times \binom{5}{1}.$$

Multiplying the total number of ways for both groups gives the final answer.

Final Answer: 8750.

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4. Answer: b

Explanation:

To solve the problem, we need to count the number of 5-digit numbers $d_1d_2d_3d_4d_5$ where each digit d_i is from the set {0, 1, 2, 3, 4, 5, 6, 7}, the number is greater than 50000, and $d_1 + d_5 \leq 8$.

1. Determine the possible values for d_1:
Since the number must be greater than 50000, d_1 can only be 5, 6, or 7. Thus, $d_1 \in \{5, 6, 7\}$.

2. Analyze the constraint $d_1 + d_5 \leq 8$ for each possible value of d_1:

- If $d_1 = 5$, then $5 + d_5 \leq 8$, which implies $d_5 \leq 3$. Therefore, $d_5 \in \{0, 1, 2, 3\}$. There are 4 possibilities for d_5.
- If $d_1 = 6$, then $6 + d_5 \leq 8$, which implies $d_5 \leq 2$. Therefore, $d_5 \in \{0, 1, 2\}$. There are 3 possibilities for d_5.
- If $d_1 = 7$, then $7 + d_5 \leq 8$, which implies $d_5 \leq 1$. Therefore, $d_5 \in \{0, 1\}$. There are 2 possibilities for d_5.

3. Determine the number of possibilities for d_2, d_3, and d_4:
Since there are no restrictions on d_2, d_3, and d_4 other than belonging to the set {0, 1, 2,

3, 4, 5, 6, 7}, each of them can take 8 possible values. Therefore, there are $8 \times 8 \times 8 = 8^3 = 512$ possibilities for $d_2d_3d_4$.

4. Calculate the total number of such 5-digit numbers:
We consider each case for d_1 separately and sum the results:

- If $d_1 = 5$, there are 4 choices for d_5 and 512 choices for $d_2d_3d_4$. So there are $1 \times 512 \times 4 = 2048$ such numbers.
- If $d_1 = 6$, there are 3 choices for d_5 and 512 choices for $d_2d_3d_4$. So there are $1 \times 512 \times 3 = 1536$ such numbers.
- If $d_1 = 7$, there are 2 choices for d_5 and 512 choices for $d_2d_3d_4$. So there are $1 \times 512 \times 2 = 1024$ such numbers.

Therefore, the total number of such 5-digit numbers is $2048 + 1536 + 1024 = 4608$.

Final Answer:
The total number of such 5 digit numbers is 4608.

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5. **Answer: b**

Explanation:

We need to choose 5 individuals from group A (which has 7 boys and 3 girls) and 3 individuals from group B (which has 6 boys and 5 girls). For group A, we can select 3 boys and 2 girls, or 4 boys and 1 girl. The number of ways to select these members can be calculated using combinations:

$$\text{Ways for group A} = \binom{7}{4} \times \binom{3}{1} + \binom{7}{3} \times \binom{3}{2}.$$

For group B, we can select the remaining individuals:

$$\text{Ways for group B} = \binom{6}{1} \times \binom{5}{2} + \binom{6}{2} \times \binom{5}{1}.$$

Multiplying the total number of ways for both groups gives the final answer. **Final Answer:** 8750.

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6. **Answer: c**

Explanation:

Step 1: The given equation is a product of two factors. For the equation to hold, either one or both factors must be zero.
Step 2: Solve each factor separately: 1. $\frac{9}{x} - \frac{9}{\sqrt{x}} + 2 = 0$ 2. $\frac{2}{x} - \frac{7}{\sqrt{x}} + 3 = 0$
Step 3: Solve each of the resulting equations for x, and ensure that the solutions satisfy the conditions of the problem.
Step 4: After solving both equations, you will find that there are 2 distinct solutions for x. Thus, the correct answer is (3).

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7. Answer: b

Explanation:

Step 1: Determine the first digit restrictions.

The first digit must be ≥ 5 to ensure the number is greater than 50000. This restricts the first digit to 5, 6, or 7.

Step 2: Calculate possible combinations for each first digit.

For each valid first digit d_1 (5, 6, or 7), determine possible last digits d_5 such that their sum $d_1 + d_5 \leq 8$:

$$\begin{aligned} &\text{For } d_1 = 5: \quad \text{Possible } d_5 \text{ are } 0, 1, 2, 3 \quad (4 \text{ choices}) \\ &\text{For } d_1 = 6: \quad \text{Possible } d_5 \text{ are } 0, 1, 2 \quad (3 \text{ choices}) \\ &\text{For } d_1 = 7: \quad \text{Possible } d_5 \text{ are } 0, 1 \quad (2 \text{ choices}) \end{aligned}$$

Step 3: Count total combinations.

Each of the middle three digits (d_2, d_3, d_4) can be any of the 8 digits (0-7). Calculating the combinations for each case:

$$\begin{aligned} &\text{For } d_1 = 5: \quad 4 \times 8^3 = 2048 \\ &\text{For } d_1 = 6: \quad 3 \times 8^3 = 1536 \\ &\text{For } d_1 = 7: \quad 2 \times 8^3 = 1024 \end{aligned}$$

Step 4: Sum over all valid first digits.

$$\text{Total combinations} = 2048 + 1536 + 1024 = 4608.$$

Conclusion:

The total number of such 5-digit numbers greater than 50000, formed under the given constraints, is 4608.

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8. Answer: a

Explanation:

A five-digit number is divisible by 5 if its last digit is either 0 or 5. The number must also be greater than 40000.

Case 1: The last digit is 0 - If the last digit is 0, the first digit can be 5, 7, or 9 (since the number must be greater than 40000). This gives us 3 choices for the first digit.
- For the remaining three digits, we have 4 remaining choices (we've used two digits already), and these can be arranged in:

$$4 \times 3 \times 2 = 4P3 = 24 \, \text{ways}.$$

- So, the number of five-digit numbers ending in 0 is:

$$3 \times 24 = 72.$$

Case 2: The last digit is 5 - If the last digit is 5, the first digit can be 7 or 9 (since the number must be greater than 40000, and we can't use 5 again). This gives us 2 choices for the first digit.
- For the remaining three digits, we have 4 remaining choices, and they can be arranged in:

$$4 \times 3 \times 2 = 4P3 = 24 \, \text{ways}.$$

- So, the number of five-digit numbers ending in 5 is:

$$2 \times 24 = 48.$$

Total Number of Five-Digit Numbers: Adding the counts from both cases, we get:

$$72 + 48 = 120.$$

Final Answer: There are **120 such five-digit numbers**.

9. **Answer: 101 - 101**

Explanation:

Given:
- Lines L_{2n-1} $(n = 1, 2, \ldots, 10)$ are parallel to each other.
- Lines L_{2n} $(n = 1, 2, \ldots, 10)$ pass through a common point P.

Step 1: Points of Intersection between L_{2n-1} and L_{2m}
Since all L_{2n-1} lines are parallel, they do not intersect among themselves. Similarly, all L_{2n} lines pass through the point P, so they intersect at P and do not form additional intersection points among themselves.

However, each line L_{2n-1} intersects each line L_{2m} exactly once (since they are not parallel), leading to:

$$10 \times 10 = 100 \text{ intersection points}$$

Step 2: Points of Intersection among L_{2n} Lines
All L_{2n} lines pass through the common point P. Therefore, there is exactly one intersection point among these lines at P.

Step 3: Total Number of Points of Intersection
The total number of points of intersection is given by:

$$100 + 1 = 101$$

Conclusion: The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \ldots, L_{20}\}$ is 101.

10. **Answer: c**

Explanation:

Arranging the letters in alphabetical order: NAGPUR

Starting with A: $5! = 120$ positions

Starting with G: $5! = 120$ positions, cumulative: 240

Starting with N and A: $4! = 24$ positions, cumulative: 264

Starting with N and G: $4! = 24$ positions, cumulative: 288

Starting with N and P: $4! = 24$ positions, cumulative: 312

Now, starting with N, R, and A:

$$\text{NRAGUP} = 1, \text{ cumulative: } 313$$

$$\text{NRAGPU} = 1, \text{ cumulative: } 314$$

$$\text{NRAPGU} = 1, \text{ cumulative: } 315$$

Thus, the word at the 315^{th} position is **NRAPGU**.

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11. Answer: a

Explanation:

The set $S = \{2, 2^2, 2^3, \ldots, 2^9\}$ contains 9 elements. To partition S into 3 subsets A, B, C of equal size, each subset must have exactly 3 elements.

The number of ways to partition the set can be calculated using the formula:

$$\text{Number of partitions} = \frac{9!}{(3!3!3!)} \times 3!.$$

Expanding this expression:

$$\text{Number of partitions} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4}{6 \times 6} \times 6 = 1680.$$

Therefore, the maximum number of such possible partitions of S is **1680**.

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12. Answer: c

Explanation:

To find the 50th word in the dictionary order of permutations of the letters in **BHBJO**, we proceed as follows:

Step 1: Arrange the letters alphabetically

The letters in the word BHBJO, arranged in alphabetical order, are:

$$B, B, H, J, O.$$

Step 2: Total number of permutations

The total number of permutations of these letters is:

$$\frac{5!}{2!} = \frac{120}{2} = 60 \quad \text{(since there are 2 repeated } B's\text{)}.$$

Step 3: Determine the block sizes

Fix the first letter in alphabetical order and calculate the number of permutations for each block.

Case 1: First letter B

If the first letter is B, the remaining letters are B, H, J, O. The number of permutations of these is:

$$\frac{4!}{2!} = \frac{24}{2} = 12.$$

Thus, the first 12 words start with B.

Case 2: First letter H

If the first letter is H, the remaining letters are B, B, J, O. The number of permutations of these is:

$$\frac{4!}{2!} = \frac{24}{2} = 12.$$

Thus, the next 12 words (from 13 to 24) start with H.

Case 3: First letter J

If the first letter is J, the remaining letters are B, B, H, O. The number of permutations of these is:

$$\frac{4!}{2!} = \frac{24}{2} = 12.$$

Thus, the next 12 words (from 25 to 36) start with J.

Case 4: First letter O

If the first letter is O, the remaining letters are B, B, H, J. The number of permutations of these is:

$$\frac{4!}{2!} = \frac{24}{2} = 12.$$

Thus, the next 12 words (from 37 to 48) start with O.

Step 4: Focus on the 49th to 60th words

The 49th to 60th words start with OB, since the first letter is O and the second letter must now be B. The remaining letters to permute are B, H, J. These permutations are:

$$OBBHJ, OBBJH, OBHBJ, OBJHB, OBJBH, OBJHB.$$

The second word in this list is the 50th word: **OBBJH.**

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13. **Answer: 5626 - 5626**

Explanation:

We need to select 4 men (M) and 4 women (W) from the two groups. Consider the following cases:

From Group A	From Group B	Ways of Selection
4M	4W	$\binom{4}{4} \cdot \binom{4}{4} = 1$
3M1W	1M3W	$\binom{4}{3} \cdot \binom{5}{1} \cdot \binom{5}{3} \cdot \binom{4}{1} = 400$
2M2W	2M2W	$\binom{4}{2} \cdot \binom{5}{2} \cdot \binom{5}{2} \cdot \binom{4}{2} = 3600$
1M3W	3M1W	$\binom{4}{1} \cdot \binom{5}{3} \cdot \binom{5}{1} \cdot \binom{4}{3} = 1600$
4W	4M	$\binom{5}{4} \cdot \binom{5}{4} = 25$
Total		5626

Final Answer: 5626.

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14. Answer: d

Explanation:

AA, MM, TT, H, I, C, S, E

(1) All distinct

8C5   56

(2) 2 same, 3 different

3C1 × 7C3   105

(3) 2 same 1^{st} kind, 2 same 2^{nd} kind, 1 different

3C2 × 6C1   18

Total → 179

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15. Answer: 125 - 125

Explanation:

Given expression:

$$(x^1 + x^2 + \cdots + x^6)^4$$

Rewriting using binomial expansion:

$$x^4 \left(\frac{1 - x^6}{1 - x} \right)^4$$

Expanding:

$$x^4 (1 - x^6)^4 (1 - x)^{-4}$$

Further expanding using binomial theorem:

$$x^4 [1 - 4x^6 + 6x^{12} \dots] \cdot (1 - x)^{-4}$$

Applying binomial expansion to each term:

$$(x^4 - 4x^{10} + 6x^{16} \dots) \cdot (1 + \binom{15}{12} x^{12} + \binom{9}{6} x^6 \dots)$$

Simplifying:

$$(x^4 - 4x^{10} + 6x^{16}) \cdot \left(1 + \binom{15}{12} x^{12} + \binom{9}{6} x^6 \dots \right)$$

Computing coefficients:

$$\binom{15}{3} - 4 \cdot \binom{9}{6} + 6$$

Calculating values:

$$= 35 \times 13 - 6 \times 8 \times 7 + 6$$

Simplifying further:

$$= 455 - 336 + 6$$

Final result:

$$= 125$$

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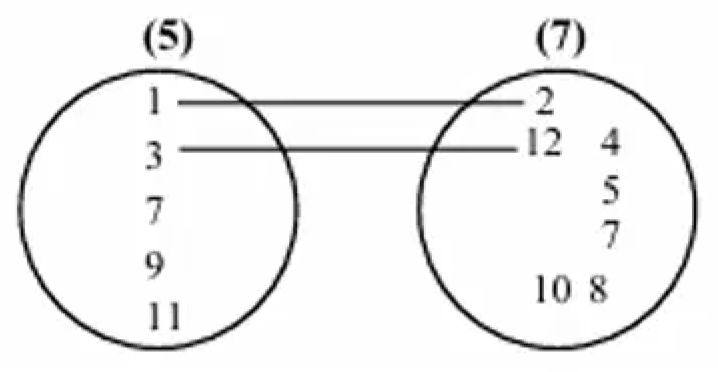
16. **Answer: d**

Explanation:



Given sets:

$$A = \{1, 3, 7, 9, 11\} \quad \text{and} \quad B = \{2, 4, 5, 7, 8, 10, 12\}$$

$$A = \{1, 3, 7, 9, 11\}$$

$$B = \{2, 4, 5, 7, 8, 10, 12\}$$

$$f(1) + f(3) = 14$$

(i) $2 + 12$

(ii) $4 + 10$

$$2 \times (2 \times 5 \times 4 \times 3) = 240$$

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17. **Answer: b**

Explanation:

Total Points on the Triangle: There are 5 points on AB, 6 points on BC, and 7 points on CA, for a total of:

$$5 + 6 + 7 = 18 \text{ points}$$

Selecting 3 Points to Form a Triangle: To form a triangle, we need to select any 3 points out of these 18 points. The total ways to choose 3 points out of 18 is:

$$\binom{18}{3} = \frac{18 \times 17 \times 16}{3 \times 2 \times 1} = 816$$

Subtracting Collinear Points:
We need to subtract cases where the selected 3 points are collinear, as these do not form a triangle:

Points on AB: There are $\binom{5}{3} = 10$ ways to select 3 collinear points from the 5 points on AB.

Points on BC: There are $\binom{6}{3} = 20$ ways to select 3 collinear points from the 6 points on BC.

Points on CA: There are $\binom{7}{3} = 35$ ways to select 3 collinear points from the 7 points on CA.

Therefore, the number of ways to select collinear points is:

$$10 + 20 + 35 = 65$$

Calculating the Number of Triangles: Subtract the collinear cases from the total selections:

$$816 - 65 = 751$$

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18. **Answer: d**

Explanation:

Given:

$${}^6C_m + 2({}^6C_{m+1}) + {}^6C_{m+2} > {}^8C_3$$

$${}^7C_{m+1} + {}^7C_{m+2} > {}^8C_3$$

$${}^8C_{m+2} > {}^8C_3$$

∴ m = 2

Step 2: Using the relation

$${}^{n-1}P_3 : {}^nP_4 = 1 : 8$$

$$\frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{8}$$

∴ n = 8

Step 3:

$$^{n}P_{m+1} + {}^{n+1}C_m = {}^{8}P_3 + {}^{9}C_2$$

$$= 8 \times 7 \times 6 + \frac{9 \times 8}{2}$$

$$= 372$$

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19. Answer: d

Explanation:

To find the number of ways to distribute 21 identical apples among three children such that each child gets at least 2 apples, we can solve the problem using the "stars and bars" theorem. This is a classic example of a combinatorics problem where we need to distribute indistinguishable objects into distinguishable bins with certain restrictions.

First, assign 2 apples to each child to meet the condition that each child gets at least 2 apples. Therefore, we distribute:

- Apples given initially to each child: $3 \times 2 = 6$ apples

Now, we have $21 - 6 = 15$ apples left to distribute among the 3 children with no further restrictions.

According to the stars and bars method, the problem now is equivalent to finding the number of non-negative integer solutions to the equation:

$$x_1 + x_2 + x_3 = 15$$

where x_1, x_2, and x_3 are the number of additional apples given to the first, second, and third child, respectively.

The number of solutions is given by the formula for combinations with repetition, which is:

$\binom{n+k-1}{k-1}$

In our case, $n = 15$ (apples left) and $k = 3$ (children), so:

$\binom{15+3-1}{3-1} = \binom{17}{2}$

Calculate $\binom{17}{2}$ as follows:

$\binom{17}{2} = \frac{17\times16}{2\times1} = \frac{272}{2} = 136$

Thus, the number of ways to distribute the apples under the given conditions is **136**.

Therefore, the correct answer is **136**.

---

20. Answer: 70 - 70

Explanation:

To find the number of integers between 100 and 1000 whose digits sum to 14, consider a number in this range as a three-digit number of the form abc, with a, b, c as digits. The conditions are:

- $100 \leq 100a + 10b + c < 1000$
- $a + b + c = 14$
- 1 ≤ a ≤ 9 (since a three-digit number cannot start with 0)

We can thus deduce:

- $1 \leq a \leq 9$
- $0 \leq b, c \leq 9$

For each valid value of a, calculate the possible combinations of b and c such that they sum with a to 14:

- For $a = 1$, $b + c = 13$; valid combinations: (4,9), (5,8), (6,7), (7,6), (8,5), (9,4)
- For $a = 2$, $b + c = 12$; valid combinations: (3,9), (4,8), (5,7), (6,6), (7,5), (8,4), (9,3)
- For $a = 3$, $b + c = 11$; valid combinations: (2,9), (3,8), (4,7), (5,6), (6,5), (7,4), (8,3), (9,2)
- For $a = 4$, $b + c = 10$; valid combinations: (1,9), (2,8), (3,7), (4,6), (5,5), (6,4), (7,3), (8,2), (9,1)

- For $a = 5$, $b + c = 9$; valid combinations: (0,9), (1,8), (2,7), (3,6), (4,5), (5,4), (6,3), (7,2), (8,1), (9,0)
- For $a = 6$, $b + c = 8$; valid combinations: (0,8), (1,7), (2,6), (3,5), (4,4), (5,3), (6,2), (7,1), (8,0)
- For $a = 7$, $b + c = 7$; valid combinations: (0,7), (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (7,0)
- For $a = 8$, $b + c = 6$; valid combinations: (0,6), (1,5), (2,4), (3,3), (4,2), (5,1), (6,0)
- For $a = 9$, $b + c = 5$; valid combinations: (0,5), (1,4), (2,3), (3,2), (4,1), (5,0)

Adding the number of combinations for each a gives 6 + 7 + 8 + 9 + 10 + 9 + 8 + 7 + 6 = 70.

The total number is 70. Hence, the value is within the range of 70.70, confirming correctness.

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21. **Answer: 3734 - 3734**

Explanation:

The word **DISTRIBUTION** contains the letters: I, I, I, T, T, D, S, R, B, U, O, N.

We calculate the number of distinct 4-letter words that can be formed by considering different cases of letter repetition:

1. **Case 1:** Three letters the same and one different (a, a, a, b)

$$\binom{4}{1} \times \frac{4!}{3!} = 32$$

2. **Case 2:** Two letters each repeated twice (a, a, b, b)

$$\frac{4!}{2! \cdot 2!} = 6$$

3. **Case 3:** Two identical letters and two distinct ones (a, b, c, c)

$$\binom{2}{1} \times \binom{2}{1} \times \frac{4!}{2!} = 672$$

4. **Case 4:** All letters different (a, b, c, d)

$$\binom{4}{1} \times 4! = 3024$$

Total number of possible words:

$$\text{Total} = 3024 + 672 + 6 + 32 = 3734$$

---

22. Answer: c

Explanation:

Total ways to partition 5 into 4 parts are:

- 5, 0, 0, 0 → 1 way
- 4, 1, 0, 0 → $\frac{5!}{4!} = 5 \text{ ways}$
- 3, 2, 0, 0 → $\frac{5!}{3!2!} = 10 \text{ ways}$
- 2, 2, 1, 0 → $\frac{5!}{2!2!1!} = 15 \text{ ways}$
- 2, 1, 1, 1 → $\frac{5!}{2!1!1!1!} = 10 \text{ ways}$
- 3, 1, 1, 0 → $\frac{5!}{3!1!1!} = 10 \text{ ways}$

Total:

$$1 + 5 + 10 + 15 + 10 + 10 = 51 \text{ ways}$$

---

23. Answer: 11376 - 11376

Explanation:

Define the variables: Let x_A be the number of questions selected from section A. Let x_B be the number of questions selected from section B. Let x_C be the number of questions selected from section C. We know that $x_A + x_B + x_C = 15$ with the constraints $x_A \geq 4$, $x_B \geq 4$, and $x_C \geq 4$.

Transform the variables: Introduce new variables:

$$y_A = x_A - 4, \quad y_B = x_B - 4, \quad y_C = x_C - 4$$

The equation becomes:

$$y_A + y_B + y_C = 3$$

Count the non-negative integer solutions: The number of non-negative integer solutions is given by:

$$\text{Number of solutions} = \binom{n+k-1}{k-1}$$

Here, $n = 3$ and $k = 3$:

$$\text{Number of solutions} = \binom{5}{2} = 10$$

Calculate the total combinations: Total selections can be computed as:

$$\text{Total ways} = \sum C(8, x_A) \times C(6, x_B) \times C(6, x_C)$$

$$= 56 \times 6 + 28 \times 6 \times 15 \times 2 + 56 \times 15 \times 2 + 70 \times 6 \times 2 + 8 \times 15 \times 15$$

= 2016 + 5040 + 1680 + 840 + 1800 = 11376

Thus, the answer is: 11376

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24. Answer: d

Explanation:

To solve this problem, we need to find the number of ways to distribute 8 identical books into 4 identical shelves where some shelves can be left empty. This is a combinatorial problem related to the partitioning of numbers.

The number of ways of distributing n identical items into r identical groups is equivalent to finding the number of partitions of n into at most r parts, each of which is a non-negative integer.

In our case, $n = 8$ and $r = 4$. We need to find the partitions of 8 into at most 4 parts.

The possible partitions are as follows:

- $8 = 8$
- $8 = 7 + 1$

- $8 = 6 + 2$
- $8 = 6 + 1 + 1$
- $8 = 5 + 3$
- $8 = 5 + 2 + 1$
- $8 = 5 + 1 + 1 + 1$
- $8 = 4 + 4$
- $8 = 4 + 3 + 1$
- $8 = 4 + 2 + 2$
- $8 = 4 + 2 + 1 + 1$
- $8 = 4 + 1 + 1 + 1 + 1$
- $8 = 3 + 3 + 2$
- $8 = 3 + 3 + 1 + 1$
- $8 = 3 + 2 + 2 + 1$

There are 15 different partitions, and hence the number of ways to arrange the 8 identical books into 4 identical shelves is **15**.

Therefore, the correct answer is **15**.

---

25. Answer: c

Explanation:

To solve this problem, we need to determine whether the expressions α and β belong to the set of natural numbers $\mathbb{N}$.

1. **Expression for α:**
   We have $\alpha = \frac{(4!)!}{(4!)^{3!}}$. First, calculate $4!$: $4! = 4 \times 3 \times 2 \times 1 = 24$.
   Now substitute into the expression for α: $\alpha = \frac{24!}{24^6}$.
   Here, $24!$ represents the factorial of 24, which is the product of all positive integers up to 24.
   The denominator, 24^6, suggests raising 24 to the power 6.
   Since $24!$ is a factorial, it includes 24 as a factor many times, ensuring that 24^6 will divide $24!$ without leaving any decimal or fraction, thus $\alpha \in \mathbb{N}$.
2. **Expression for β:**
   We have $\beta = \frac{(5!)!}{(5!)^{4!}}$. Calculate $5!$: $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.
   Now substitute into the expression for β: $\beta = \frac{120!}{120^{24}}$.

Again, $120!$ includes 120 as a factor repeatedly, more than sufficient for ensuring 120^{24} divides $120!$ completely, so $\beta \in \mathbb{N}$.

3. **Conclusion:**
   Both α and β are in $\mathbb{N}$.

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26. **Answer: b**

Explanation:

The Correct answer is option is (B) :13560

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27. **Answer: d**

Explanation:

The Correct answer is option is (D) : NRAPGU

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28. **Answer: 16 - 16**

Explanation:

The correct answer is 16.

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29. **Answer: a**

Explanation:

The Correct answer is option is (A) : OBBJH

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30. **Answer: b**

Explanation:

The Correct answer is option is (B) : $\frac{9!}{(3!)3}$