Permutations And Combinations JEE Main PYQ - 3

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

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1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Permutations and Combinations

1. Total numbers of 3-digit numbers that are divisible by 6 and can be formed by using the digits 1, 2, 3, 4, 5 with repetition, is___. (+4, -1)

2. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is (+4, -1)

   a. 324

   b. 326

   c. 327

   d. 328

3. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is (+4, -1)

   a. 79

   b. 86

   c. 84

   d. 89

4. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is (+4, -1)

   a. 507

   b. 432

   c. 472

   d. 400

5. The number of ways to distribute the 21 identical apples to three children's so that each child gets at least 2 apples. (+4, -1)

6. Five people are distributed in four identical rooms. A room can also contain zero people. Find the number of ways to distribute them. (+4, -1)

a. 47

b. 53

c. 43

d. 51

7. Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1, 2, 3, 4, 5 and 6 without repetition of digits. Then the total number of such numbers is _______________. (+4, -1)

8. The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to: (+4, -1)

a. 205

b. 615

c. 510

d. 430

9. There are ten boys $B_1, B_2, \ldots, B_{10}$ and five girls $G_1, G_2, \ldots, G_5$ in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is ________ (+4, -1)

10. The total number of four digit numbers such that each of first three digits is divisible by the last digit, is equal to ________. (+4, -1)

11. The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5, 7 and 9 is _____. (+4, -1)

12. If (+4, -1)

$({}^{40}C_0) + ({}^{41}C_1) + ({}^{42}C_2) + ... + ({}^{60}C_{20}) \frac{m}{n} \, {}^{60}C_{20}$

m and n are coprime, then $m + n$ is equal to _____.

13. If (+4, -1)

$\sum_{k=1}^{31} ({}^{31}C_k)({}^{31}C_{k-1}) - \sum_{k=1}^{30} ({}^{30}C_k)({}^{30}C_{k-1}) = \frac{\alpha(60!)}{(30!)(31!)}$

where $\alpha \in R$, then the value of 16α is equal to

a. 1411

b. 1320

c. 1615

d. 1855

14. The letters of the work 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is ______. (+4, -1)

15. **Eight persons are to be transported from city A to city B in three cars of different makes. If each car can accommodate at most three persons, then the number of ways in which they can be transported is:** (+4, -1)

a. 1120

b. 560

c. 1680

d. 3360

16. **If the number of words, with or without meaning, which can be made using all the letters of the word *MATHEMATICS* in which C and S do not come** (+4, -1)

together, is (6!)k , is equal to

a. 5670

b. 1890

c. 595

d. 657

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17. Using the number 1, 2, 3 ... 7, total numbers of 7 digit number which does not contain string 154 or 2367 is (Repetition is not allowed) **(+4, -1)**

a. 4897

b. 4898

c. 4896

d. 4899

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18. Using all the letters of the word MATHS, then rank of the word THAMS is: **(+4, -1)**

a. 101

b. 102

c. 103

d. 104

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19. Find out the rank of MONDAY in English dictionary if all alphabets are arranged in order? **(+4, -1)**

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20. Rank of the word PUBLIC is? **(+4, -1)**

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21. Find all the four letter words with two vowels and 2 consonants from the word UNIVERSE? **(+4, -1)**

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22. If ${}^{2n}C_3 : {}^{n}C_3 = 10$, then $\frac{n^2+3n}{n^2-3n+4}$ is equal to **(+4, -1)**

23. The number of ways to distribute 20 chocolates among three students such that each student gets atleast one chocolate is **(+4, -1)**

a. ${}^{22}C_2$

b. ${}^{19}C_2$

c. ${}^{19}C_3$

d. ${}^{22}C_3$

24. **If** ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$**, then** $n^2 + n + 15$ **is equal to :** **(+4, -1)**

25. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$, then $n^2 + n + 15$ is equal to : **(+4, -1)**

26. The total number of six digit numbers, formed using the digits 4,5,9 only and divisible by 6 , is __ **(+4, -1)**

27. Let $f(x) = 2x^n + \lambda$, where $\lambda \in \mathbb{R}$ and $n \in \mathbb{N}$. Given that $f(4) = 133$ and $f(5) = 255$, What is the sum of all the positive integer divisors of $f(3) - f(2)$? **(+4, -1)**

a. 59

b. 60

c. 61

d. 58

28. There is an unlimited number of identical balls of three different colours. How many arrangements of almost 7 ball in a row can be made by using them? **(+4, -1)**

a. (A) 2180

b. (B) 343

c. (C) 399

d. (D) 3279

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29. Let Tn denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $_{+1}$ - = 21. Then n equals: **(+4, -1)**

a. (A) 5

b. (B) 7

c. (C) 6

d. (D) 4

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30. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the students selected has opted neither for NCC nor for NSS is: **(+4, -1)**

a. (A) $\frac{1}{3}$

b. (B) $\frac{1}{6}$

c. (C) $\frac{2}{3}$

d. (D) $\frac{5}{6}$

Answers

1. Answer: 16 - 16

Explanation:

For a number to be divisible by 6, it must be divisible by both 2 and 3.
Divisibility by 2: The last digit (unit place) must be an even number.
From the available digits 1, 2, 3, 4, 5, the even digits are 2 and 4.
Hence, the last digit must be either 2 or 4.
Divisibility by 3: The sum of the digits must be divisible by 3.
For the last digit being 2:
The first two digits (a, b) must be chosen such that the sum $a + b + 2$ is divisible by 3.
The possible pairs for $a + b = 1, 3, 5, 7, 9$ (mod 3) are (1, 3), (3, 1), (2, 5), (5, 2), (4, 3), and (3, 4).
For the last digit being 4:
Similarly, $a + b + 4$ must be divisible by 3, yielding 8 possible combinations.
Thus, the total number of valid combinations is 8 + 8 = 16.

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2. Answer: c

Explanation:

The word MONDAY has 6 distinct letters. The number of words that can be formed with all the letters of MONDAY is given by $6! = 720$.
Now, let's find the position of the word MONDAY in the dictionary:
The number of words starting with the letter D is $5! = 120$.
The number of words starting with MA is $4! = 24$.
The number of words starting with MD is $4! = 24$.
The number of words starting with MN is $4! = 24$.
The number of words starting with MOA is $3! = 6$.
The number of words starting with MOD is $3! = 6$.
The number of words starting with MONA is $2! = 2$.
The word MONDAY itself is the last one.
Therefore, the rank of MONDAY is:

$$120 + 120 + 24 + 24 + 24 + 6 + 6 + 2 + 1 = 327.$$

3. **Answer: d**

Explanation:

(A)Arrange the letters of OUGHT in alphabetical order: G, H, O, T, U. (B) Count words starting with each letter before ""T"":

$$G \to 4!, \quad H \to 4!, \quad O \to 4!.$$

(C) Count words starting with ""T"" but second letter as:

$$TG \to 3!, \quad TH \to 3!, \quad TOG \to 2!, \quad TOH \to 2!.$$

(D) Add the counts:

$$4! + 4! + 4! + 3! + 3! + 2! + 2! + 1! = 89.$$

Concepts:

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- In the process of rearranging the numbers, subsets of sets are created to determine all possible arrangement sequences of a single data point.
- A permutation is used in many events of daily life. It is used for a list of data where the data order matters.

Combination:

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4. **Answer: b**

Explanation:

(A)Total 3-digit numbers:

$$900 = 999 - 100 + 1.$$

(B)Numbers divisible by 3:

$$\frac{900}{3} = 300.$$

(C)Numbers divisible by 4:

$$\frac{900}{4} = 225.$$

(D)Numbers divisible by both 3 and 4 (i.e., divisible by 12):

$$\frac{900}{12} = 75.$$

(E)Numbers divisible by either 3 or 4:

$$300 + 225 - 75 = 450.$$

(F) Numbers divisible by 48:

$$\frac{900}{48} = 18.$$

(G) Numbers divisible by either 3 or 4 but not 48:

$$450 - 18 = 432.$$

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5. **Answer: 136 - 136**

Explanation:

The answer is 136.

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6. **Answer: d**

Explanation:

The correct option is (D): 51

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7. **Answer: 30 - 30**

Explanation:

The correct answer is 30
Number must start by 1 or 2 and for divisibility by 4 last two digits shall be divisible by 4

∴ _ _ $\underline{12}$ → 0 cases

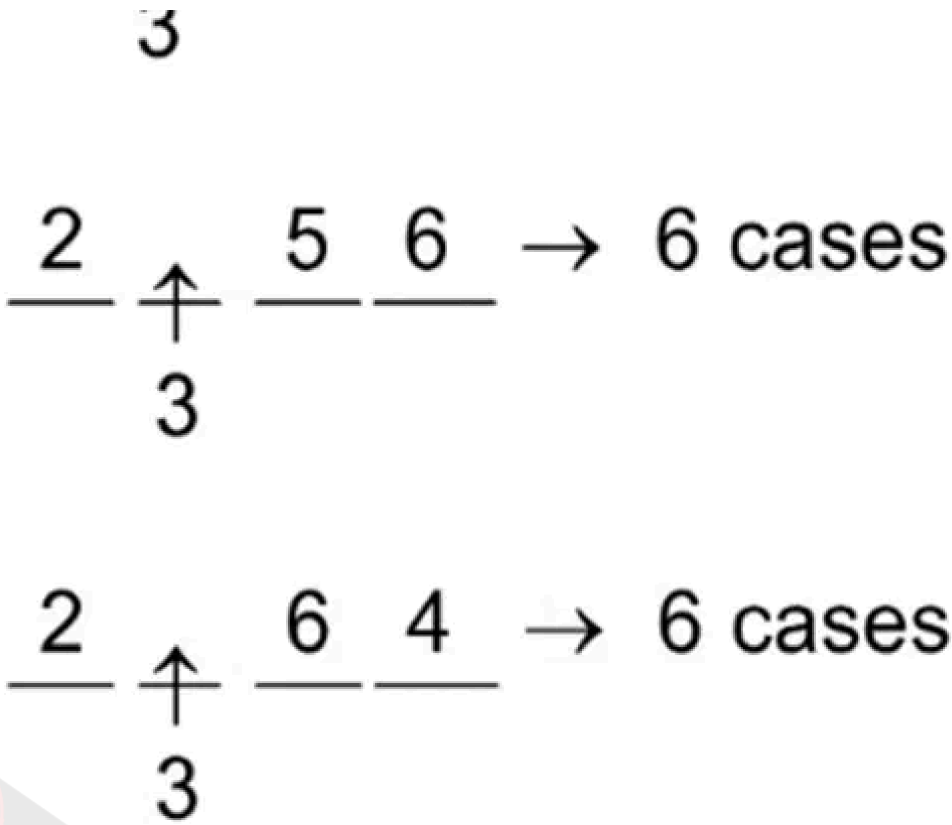
$\underline{2}$ $\underset{3}{\uparrow}$ $\underline{1}$ $\underline{6}$ → 3 cases

$\underline{1}$ $\underset{3}{\uparrow}$ $\underline{2}$ $\underline{4}$ → 3 cases

$\underline{1}$ $\underset{3}{\uparrow}$ $\underline{3}$ $\underline{2}$ → 3 cases

$\underline{2}$ $\underset{3}{\uparrow}$ $\underline{3}$ $\underline{6}$ → 6 cases

$\underline{1}$ $\uparrow$ $\underline{5}$ $\underline{2}$ → 3 cases



⇒ Total 30 numbers

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8. **Answer: d**

Explanation:

The correct answer is (D) : 430
By multinomial theorem, no. of ways to distribute 30 identical candies among four children C_1, C_2 and C_3, C_4
= Coefficient of x^{30} in $(x^4 + x^5 + \ldots + x^7)(x^2 + x^3 + \ldots + x^6)(1 + x + x^2 \ldots)^2$
=Coefficient of x^{24} in $\frac{(1-x^4)(1-x^5)(1-x^{31})^2}{(1-x)(1-x)(1-x)^2}$
= Coefficient of x^{24} in $(1 - x^4 - x^5 + x^9)(1 - x)^{-4}$
= ${}^{27}C_{24} - {}^{23}C_{20} - {}^{22}C_{19} + {}^{18}C_{15} = 430$

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9. **Answer: 1120 - 1120**

Explanation:

The correct answer is 1120
Required number of ways = Total ways of selection – ways in which B_1 and B_2 are present together.
$={}^{10}C_3 \cdot {}^{5}C_3 - {}^{8}C_1 \cdot {}^{5}C_3$
=10(120-8)
= 1120

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10. **Answer: 1086 - 1086**

Explanation:

The correct answer is 1086
Let pqrs is 4-digit number.
So , first 3-digit pqr should be divisible by 's'
If s=1 , then numbers = 9×10×10 = 900
If s=2 , then numbers = 4×5×5 = 100
If s=3, then numbers = 3×4×4 = 48
If s=4, then numbers = 2×3×3 = 18
If s=5, then numbers = 1×2×2 = 4 numbers
If s=6,7,8 and 9 then , numbers = 4×4 = 16
∴ Total 4 digit Number = 900 + 100 + 48 + 18 + 4 + 16 = 1086

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11. **Answer: 576 - 576**

Explanation:

Sum of all given numbers = 31

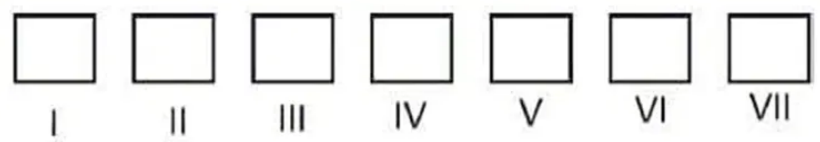

Fig.

Difference between odd and even positions must be 0, 11 or 22, but 0 and 22 are not possible.
Therefore , Only difference 11 is possible
This is possible only when either 1, 2, 3, 4 is filled in odd position in some order and remaining in other
order. Similar arrangements of 2, 3, 5 or 7, 2, 1 or 4, 5, 1 at even positions.
∴ Total possible arrangements = $(4! \times 3!) \times 4$
= 576

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12. **Answer: 102 - 102**

Explanation:

The correct answer is 102

$^{40}C_0 +^{41} C_1 +^{42} C_2 + \ldots\ldots +^{60} C_{20}$

$= {}^{40}C_{40} + {}^{41}C_{40} + {}^{42}C_{40} + \ldots\ldots + {}^{60}C_{40}$

$=^{61} C_{41}$

$= \frac{61}{41} \cdot {}^{60}C_{41}$

∴ m = 61 , n = 41

Therefore , $m + n$ = 102

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13. **Answer: a**

Explanation:

The correct answer is (A) : 1411

$$\sum_{k=1}^{31}({}^{31}C_k)({}^{31}C_{k-1}) - \sum_{k=1}^{30}({}^{30}C_k)({}^{30}C_{k-1})$$
$$= \sum_{k=1}^{31}({}^{31}C_k).({}^{31}C_{32-k}) - \sum_{k=1}^{30}({}^{30}C_k).({}^{30}C_{k-1})$$
$$= {}^{62}C_{32} - {}^{60}C_{31}$$
$$= \frac{60!}{31!29!}\left(\frac{62.61}{32.30} - 1\right) = \frac{60!}{31!29!}\frac{2822}{32.30}$$

$$\alpha = \frac{2822}{32}$$
$$\Rightarrow 16\alpha = 1411$$

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14. **Answer: 1492 - 1492**

Explanation:

Arranging letters in alphabetical order A D I K M N N to find rank of MANKIND and making arrangements of a dictionary we get
A .... $\frac{6!}{2!}$ =360
D 360
I 360
K 360
$M\ A\ D$ .... $\frac{4!}{2!}$ =12
$M\ A\ I$ 12
$M\ A\ K$ 12
$M\ A\ N\ D$ .... 3! = 6
$M\ A\ N\ I$ 6
$M\ A\ N\ K\ D$ 2
$M\ A\ N\ K\ I\ D$ 1
$M\ A\ N\ K\ I\ N\ D$ 1
∴ Rank of MANKIND = 1440 + 36 + 12 + 2 + 2
= 1492

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15. **Answer: c**

Explanation:

Step 1: Understanding the problem There are 8 persons to be transported, and there are 3 cars. Each car can carry at most 3 persons. We need to calculate the number of ways to assign these 8 persons to the cars.
Step 2: Distribute the persons in the cars To distribute the 8 persons into 3 cars, with each car holding a maximum of 3 persons, we first assign 3 persons to two cars and 2 persons to the third car. The number of ways to select 3 persons for the first car from the 8 is given by:

$$\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

Now, 5 persons remain, so the number of ways to select 3 persons for the second car is:

$$\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5 \times 4}{2 \times 1} = 10$$

Finally, 2 persons remain, so the number of ways to assign them to the third car is:

$$\binom{2}{2} = 1$$

Step 3: Consider the arrangements of the cars Since there are 3 cars of different makes, the arrangement of the cars is important. Therefore, the number of ways to assign the selected persons to the cars is multiplied by the number of ways to arrange the cars, which is 3! (since there are 3 cars).

$$\text{Total ways} = \frac{8!}{3!3!2!} = \frac{8 \times 7 \times 6 \times 5 \times 4}{4 \times 6} = 56 \times 30 = 1680$$

Thus, the number of ways in which the persons can be transported is 1680.

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16. Answer: a

Explanation:

1) Letter multiset

MATHEMATICS (11 letters):
$M^2,\ A^2,\ T^2,\ H,\ E,\ I,\ C,\ S.$

2) Total arrangements (no restriction)

$$N_{\text{all}} = \frac{11!}{2!\,2!\,2!} = \frac{11!}{8}.$$

3) Arrangements with C and S together

Treat the block $\boxed{CS}$ (or $\boxed{SC}$) as one item. Then we have 10 items: the block + $M^2, A^2, T^2, H, E, I.$
External permutations:

$$\frac{10!}{2!\,2!\,2!} = \frac{10!}{8}, \quad \text{and internal choices for the block } (CS/SC) = 2!.$$

Hence

$$N_{\text{together}} = 2 \cdot \frac{10!}{8} = \frac{10!}{4}.$$

4) Required count (not together)

$$N = N_{\text{all}} - N_{\text{together}} = \frac{11!}{8} - \frac{10!}{4} = \frac{10!}{8}(11-2) = \frac{9}{8}\,10!.$$

5) Express as $(6!)\,k$

Write $10! = 6! \cdot 7 \cdot 8 \cdot 9 \cdot 10$. Then

$$N = \frac{9}{8}\,10! = \frac{9}{8}(6! \cdot 7 \cdot 8 \cdot 9 \cdot 10) = 6!\left(\tfrac{9}{8} \cdot 7 \cdot 8 \cdot 9 \cdot 10\right) = 6!\,(7 \cdot 9 \cdot 9 \cdot 10) = 6!\,\underline{5670}.$$

Therefore $k = 5670$.

Final: Number of required words $= (6!)\,\mathbf{5670}$. Hence, $k = \boxed{5670}$.

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17. **Answer: b**

Explanation:

The numbers are $1, 2, 3, 4, 5, 6, 7$.

- **Step 1:** Numbers having the string (154):
  - Positions: $(154), 2, 3, 6, 7$
  - Total permutations: $5! = 120$
- **Step 2:** Numbers having the string (2467):
  - Positions: $(2467), 1, 3, 5$
  - Total permutations: $4! = 24$
- **Step 3:** Numbers having both strings (154) and (2467):
  - Positions: $(154), (2467)$
  - Total permutations: $2! = 2$
- **Step 4:** Apply the Inclusion-Exclusion Principle:

$$n((154) \cup (2467)) = 5! + 4! - 2!.$$

- Simplify:

$$n((154) \cup (2467)) = 120 + 24 - 2 = 142.$$

- **Step 5:** Total numbers:

Total permutations: $7! = 5040$.

- **Step 6:** Numbers having neither (154) nor (2467):

Required numbers = $5040 - 142 = 4898$.

Final Answer: The required numbers are 4898.

Concepts:

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18. Answer: c

Explanation:

Given:

We are tasked to determine the rank of the word "THAMS" in alphabetical order using the factorial method.

Step 1: Assign Numerical Positions to Letters

The alphabetical order of the letters T, H, A, M, S is:

$$A = 1,\ H = 2,\ M = 3,\ S = 4,\ T = 5.$$

Thus, the word "THAMS" corresponds to the sequence: $5, 2, 1, 3, 4$.

Step 2: Count Permutations for Each Letter

- For T: There are 4 letters after T in alphabetical order (H, A, M, S). The number of permutations is:

$$4! = 24.$$

- For H: After H, there is 1 letter (A) that comes earlier in the sequence. The number of permutations is:

$$3! \cdot 1 = 6.$$

- For A: There are no letters after A that need to be considered. The number of permutations is:

$$2! \cdot 0 = 0.$$

- For M: Similarly, there are no letters after M to consider. The number of permutations is:

$$1! \cdot 0 = 0.$$

- For S: There are no letters after S. The number of permutations is:

$$0! \cdot 0 = 0.$$

Step 3: Compute Total Permutations Before "THAMS"

Total permutations before "THAMS" is given by:

$$4 \cdot 4! + 3! \cdot 1 + 0 + 0 + 0 = 4 \cdot 24 + 6 = 96 + 6 = 102.$$

Step 4: Add 1 for the Word Itself

Rank of "THAMS" is:

$$102 + 1 = 103.$$

Final Answer:

The rank of the word "THAMS" is 103.

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19. **Answer: 327 - 327**

Explanation:

The correct answer is 327.

3	5	4	2	1	6
M	O	N	D	A	Y
2	3	2	1	0	0
5!	4!	3!	2!	1!	0!

$\therefore$ Rank = $(2 \times 5! + 3 \times 4! + 2 \times 3! + 1 \times 2!) + 1$
= 240+72+12+2+1
= 327

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20. **Answer: 582 - 582**

Explanation:

The rank of the word "PUBLIC" is:
= (4x5! + 4x4! + 0x3! + 2x2! + 1x1! + 0x0!) + 1
= ((4x120) + 96 + 0 + 4 + 1 + 0) + 1
= 480 + 100 + 2
= 582

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21. **Answer: 432 - 432**

Explanation:

The correct answer is 432
Vowels - I, U, E,
Consonants - N, V, R, S
$\Rightarrow {}^3C_2 \times {}^4C_2 \times 4!$
$= 3 \times 6 \times 24$
$= 432$
So, 432 four letters word can be made

Concepts:

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22. **Answer: 2 - 2**

Explanation:

$\frac{^{2n}C_3}{^nC_3} = 10 \Rightarrow \frac{2n.(2n-1).(2n-2)}{n.(n-1)(n-2)}$

$\Rightarrow \frac{(2n-1).2}{n-2}$

$\Rightarrow n = 8$

Therefore, $\frac{n^2+3n}{n^2-3n+4} = \frac{88}{44} = 2$

The answer is 2

Concepts:

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23. **Answer: b**

Explanation:

let x, y, z are number of chocolates three students get
$x + y + z = 20; x, y, z \geq 1$
Therefore, the number of ways is ${}^{19}C_2$

Concepts:

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24. Answer: 45 - 45

Explanation:

$$\frac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!} = \frac{11}{21}$$
$$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{11}{21}$$
$$\Rightarrow \frac{2n+1}{(n+1)(n+2)} = \frac{11}{42}$$
$$\Rightarrow n = 5$$

$\Rightarrow n^2 + n + 15$
$= 25 + 5 + 15 = 45$
So , the correct answer is 45.

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---

25. Answer: 45 - 45

Explanation:

The correct answer is 45.

$\frac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!} = \frac{11}{21}$
$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{11}{21}$
$\Rightarrow \frac{2n+1}{(n+1)(n+2)} = \frac{11}{42}$
$\Rightarrow n = 5$

$\Rightarrow n^2 + n + 15$
$= 25 + 5 + 15 = 45$

Concepts:

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26. **Answer: 81 - 81**

Explanation:

The number must be divisible by 6, so it must be divisible by both 2 and 3. For divisibility by 2, the last digit must be 4. For divisibility by 3, the sum of the digits must be divisible by 3.
Let us consider different cases based on the distribution of digits:
Case 1: All digits are the same
For 44444, there is only 1 number.
Case 2: Two distinct digits

$(4, 5)$: Numbers of the form 44455:

$$\frac{5!}{3!2!} = 10.$$

$(4, 9)$: Numbers of the form 44499:

$$\frac{5!}{3!2!} = 10.$$

Case 3: Three distinct digits
For $4, 5, 9$, let us consider the permutations:
Digits: $4, 5, 9, 4, 4$:

$$\frac{5!}{3!} = 20.$$

Digits: $4, 5, 9, 5, 5$:

$$\frac{5!}{3!2!} = 5.$$

Digits: $4, 5, 9, 9, 9$:

$$\frac{5!}{3!2!} = 5.$$

Digits: $4, 5, 9, 4, 5$:

$$\frac{5!}{2!2!1!} = 30.$$

Total

$$1 + 10 + 10 + 20 + 5 + 5 + 30 = 81.$$

Conclusion
The total number of such numbers is:

$$\boxed{81}.$$

Concepts:

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---

27. Answer: b

Explanation:

We are given:

- $f(4) = 2 \cdot 4^n + \lambda = 133$ (1)
- $f(5) = 2 \cdot 5^n + \lambda = 255$ (2)

Subtract equation (1) from equation (2):

$$2 \cdot 5^n + \lambda - (2 \cdot 4^n + \lambda) = 255 - 133$$

$$2(5^n - 4^n) = 122$$

$$5^n - 4^n = 61$$

Testing integral values of n:

For $n = 3$, we have:

$5^3 - 4^3 = 125 - 64 = 61$

Thus, $n = 3$. Substituting back into equation (1) to find λ:

$2 \cdot 4^3 + \lambda = 133$

$128 + \lambda = 133$

$\lambda = 5$

Now, compute $f(3) - f(2)$:

$f(3) = 2 \cdot 3^3 + 5 = 2 \cdot 27 + 5 = 54 + 5 = 59$

$f(2) = 2 \cdot 2^3 + 5 = 2 \cdot 8 + 5 = 16 + 5 = 21$

$f(3) - f(2) = 59 - 21 = 38$

Find the positive integer divisors of 38:

- 1, 2, 19, 38

Sum of divisors:

$1 + 2 + 19 + 38 = 60$

Thus, the correct answer is option (2).

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---

28. Answer: d

Explanation:

Explanation:
Given, the question we have an unlimited number of identical balls of three different colours.Now,There is an unlimited number of identical three rows.Number of arrangements of 7 balls in a row $= 3^7$ Number of arrangements of 6 balls in a row $= 3^6$ Number of arrangements of 5 balls in a row $= 3^5$ Number of arrangements of 4 balls in a row $= 3^4$ Number of arrangements of 3 balls in a row $= 3^3$ Number of arrangements of 2 balls in a row $= 3^2$ Number of arrangements of 1 ball in a row $= 3$ Therefore, the number of arrangements of almost 7 balls in a row that can be made (n)=$3 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6 + 3^7$ As we can see it is in GP. We will use GP because we can see here that there is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number. $= \frac{3(3^7 - 1)}{3 - 1}$ $\frac{3}{2}(2187 - 1)$ $\frac{3}{2} \times 2186$ 3×1093 $= 3279$Hence, the correct option is (D).

---

29. Answer: b

Explanation:

Explanation:
Given: $_{+1}$ - = 21.......... (i)where = no. of triangles formed by 'n' vertices of polygon = $^nC3$We have to find the value of n.From (i), we have $^{+1}$ $_3$ - $_3$ = 21

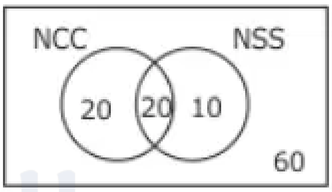
$\frac{(\ +1)\ (\ -1)}{6} - \frac{(\ -1)(\ -2)}{6} = 21$ [Using combination and factorial]
$\frac{-1}{6}[(\ +1) - (\ -2)] = 21$ $\frac{-1\times3}{6} = 21$ $(\ -1) = 42$ $^2 - \ - 42 = 0$
$(\ -7)(\ +6) = 0$ $= 7$ Hence, the correct option is (B).

---

30. Answer: b

Explanation:

Explanation:
Given:Total number of students = 60 ( ) = 40 ( ) = 30 ( ) = 20We have to find the probability that the student selected has opted for neither NCC nor NSS.Consider,



The probability that the student selected has neither for NCC nor NSS. = $\frac{(\ )}{(\ )} = \frac{10}{60} = \frac{1}{6}$
Hence, the correct option is (B).