

# Probability JEE Main PYQ – 1

Total Time: 1 Hour : 15 Minute

Total Marks: 120

## Instructions

### Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

### Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.



## Probability

1. Let  $X$  have a binomial distribution  $B(6, p)$ . If the sum of the mean and the variance of  $X$  is  $(+4, -1)$   $\frac{21}{8}$ , then

$$\frac{P(2 \leq X < 4)}{P(4 < X < 6)}$$

is equal to:

- a. 65
- b. 195
- c.  $\frac{195}{2}$
- d.  $\frac{225}{2}$
- 
2. A bag contains 'k' red balls and  $(10 - k)$  black balls. If 3 balls are drawn at random and they are found to be black then the probability that bag has 9 black balls & 1 red ball is  $(+4, -1)$

- a.  $\frac{7}{11}$
- b.  $\frac{14}{55}$
- c.  $\frac{21}{55}$
- d.  $\frac{6}{11}$

- 
3. Let  $S$  has 5 elements and  $P(S)$  is the power set of  $S$ . Let an ordered pair  $(A, B)$  is selected at random from  $P(S) \times P(S)$ . If the probability that  $A \cap B = \emptyset$  is  $\frac{3^m}{2^n}$ , then the value of  $(m + n)$  is  $(+4, -1)$

- a. 88
- b. 96
- c. 64
- d. 28

- 
4. If 3 balls are taken from a box without replacement and found to be all black. If all configurations of red balls and black balls are equally likely, then the probability that the
- $(+4, -1)$

box contained 1 red and 9 black balls is  $\frac{p}{q}$  for some coprime natural numbers  $p$  and  $q$ .

Find  $p + q$ .

- a. 59
- b. 69
- c. 57
- d. 79

---

5. Let  $S$  has 5 elements and  $P(S)$  is the power set of  $S$ . Let an ordered pair  $(A, B)$  is selected at random from  $P(S) \times P(S)$ . If the probability that  $A \cap B = \emptyset$  is  $\frac{3^m}{2^n}$ , then the value of  $(m + n)$  is

6. There are 10 defective and 90 non-defective balls in a bag. 8 balls are taken one by one with replacement. Find the probability that at least 7 defective balls are selected.

- a.  $\frac{73}{10^8}$
- b.  $\frac{37}{10^8}$
- c.  $\frac{105}{10^8}$
- d.  $\frac{11}{10^8}$

---

7. Bag  $A$  contains 9 white and 8 black balls and bag  $B$  contains 6 white and 4 black balls. A ball is randomly transferred from bag  $B$  to bag  $A$ , then a ball is drawn from bag  $A$ . If the probability that the drawn ball is white is  $\frac{p}{q}$  (where  $p$  and  $q$  are coprime), then find  $p + q$ :

---

8. If two numbers  $a$  and  $b$  are selected from  $S = \{1, 2, 3, \dots, 100\}$ , then the probability that  $|a - b| \geq 10$  is:

- a.  $\frac{891}{1000}$
  - b.  $\frac{119}{1000}$
  - c.  $\frac{819}{1000}$
  - d. None of these
-

9. If probability distribution is given by

(+4, -1)

$$P(x) = \frac{x}{P(x)} \begin{array}{c|c|c|c|c|c|c|c} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline k & 2k^2 & 6k^2 & 2k^2 + k & 4k & k & k & \end{array}$$

Then, the value of  $P(3 < x \leq 6)$  is:

- a. 0.6
- b. 0.8
- c. 0.4
- d. 0.2

10. If the probability distribution is given by,

(+4, -1)

$x$	0	1	2	3
$p(n)$	$8a - 1/30$	$4a - 1/30$	$2a + 1/30$	$b$

If it is given that  $\sigma^2 + \mu^2 = 2$ , where  $\sigma$  is the standard deviation and  $\mu$  is the mean of the distribution, then  $\frac{a}{b}$  is:

- a.  $\frac{22}{71}$
- b.  $\frac{110}{71}$
- c.  $\frac{220}{71}$
- d.  $\frac{1110}{71}$

11. If  $S = \{1, 2, \dots, 50\}$ , two numbers  $\alpha$  and  $\beta$  are selected at random find the probability that product is divisible by 3 : ( +4, -1)

- a.  $\frac{664}{1225}$
- b.  $\frac{646}{1225}$
- c.  $\frac{527}{1225}$
- d.  $\frac{461}{1225}$

12. If the probability distribution is given by:

(+4, -1)

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k <sup>2</sup>	2k <sup>2</sup>	7k <sup>2</sup> + k

Then find:  $P(3 < x \leq 6)$

- a. 0.33
- b. 0.22
- c. 0.11
- d. 0.44

13. If the end points of chord of parabola  $y^2 = 12x$  are  $(x_1, y_1)$  and  $(x_2, y_2)$  and it subtend  $90^\circ$  at the vertex of parabola then  $(x_1x_2 - y_1y_2)$  equals : (+4, -1)

- a. 288
- b. 280
- c. 290
- d. not possible

14. If a line  $ax + y = 1$  does not intersect the hyperbola  $x^2 - 9y^2 = 9$  then a possible value of  $\alpha$  is : (+4, -1)

- a. 0.2
- b. 0.3
- c. 0.4
- d. 0.5

15. The probability that two randomly selected subsets of the set  $\{1, 2, 3, 4, 5\}$  have exactly two elements in their intersection, is : (+4, -1)

- a.  $65 / 2^{17}$
- b.  $65 / 2^{18}$
- c.  $135 / 2^{19}$

d.  $35 / 2 \setminus ^7$

---

16. Let 9 distinct balls be distributed among 4 boxes,  $B_1, B_2, B_3$  and  $B_4$ . If the probability that  $B_3$  contains exactly 3 balls is  $k \left(\frac{3}{4}\right)^9$  then  $k$  lies in the set : (+4, -1)

a.  $\{x \in \mathbb{R} : |x - 1| < 1\}$

b.  $\{x \in \mathbb{R} : |x - 2| \leq 1\}$

c.  $\{x \in \mathbb{R} : |x - 3| < 1\}$

d.  $\{x \in \mathbb{R} : |x - 5| \leq 1\}$

---

17. A student appeared in an examination consisting of 8 true - false type questions. The student guesses the answers with equal probability. The smallest value of  $n$ , so that the probability of guessing at least 'n' correct answers is less than  $\frac{1}{2}$ , is : (+4, -1)

a. 3

b. 4

c. 5

d. 6

---

18. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to : (+4, -1)

a.  $40/243$

b.  $80/243$

c.  $128/625$

d.  $32/625$

---

19. Let  $B_i$  ( $i = 1, 2, 3$ ) be three independent events in a sample space. The probability that only  $B_1$  occurs is  $\alpha$ , only  $B_2$  occurs is  $\beta$  and only  $B_3$  occurs is  $\gamma$ . Let  $p$  be the probability that none of the events  $B_i$  occurs and these 4 probabilities satisfy the equations  $(\alpha - 2\beta)p = \alpha\beta$  and  $(\beta - 3\gamma)p = 2\beta\gamma$  (All the probabilities are assumed to lie in the interval  $(0, 1)$ ). Then  $\frac{P(B_1)}{P(B_3)}$  is equal to \_\_\_\_\_.

---

20. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is : (+4, -1)

- a.  $\frac{1}{32}$
- b.  $\frac{3}{16}$
- c.  $\frac{5}{16}$
- d.  $\frac{1}{2}$

21. The coefficients  $a$ ,  $b$  and  $c$  of the quadratic equation,  $ax^2 + bx + c = 0$  are obtained by throwing a dice three times. The probability that this equation has equal roots is : (+4, -1)

- a.  $1/36$
- b.  $1/54$
- c.  $1/72$
- d.  $5/216$

22. When a missile is fired from a ship, the probability that it is intercepted is  $1/3$  and the probability that the missile hits the target, given that it is not intercepted, is  $3/4$ . If three missiles are fired independently from the ship, then the probability that all three hit the target, is : (+4, -1)

- a.  $1/27$
- b.  $1/8$
- c.  $3/8$
- d.  $3/4$

23. The probability that a randomly selected 2-digit number belongs to the set  $\{n \in N : (2^n - 2) \text{ is a multiple of } 3\}$  is equal to : (+4, -1)

- a.  $\frac{1}{2}$
- b.  $\frac{1}{3}$
- c.  $\frac{2}{3}$

d.  $\frac{1}{6}$

---

24. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is : (+4, -1)

a.  $\frac{1}{5}$

b.  $\frac{2}{9}$

c.  $\frac{97}{297}$

d.  $\frac{122}{297}$

---

25. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is: (+4, -1)

a.  $\frac{7}{45}$

b.  $\frac{8}{45}$

c.  $\frac{28}{45}$

d.  $\frac{14}{45}$

---

26. Let  $S = \{1, 2, 3, 4, 5, 6\}$ . Then the probability that a randomly chosen onto function  $g$  from  $S$  to  $S$  satisfies  $g(3) = 2g(1)$  is : (+4, -1)

a.  $\frac{1}{30}$

b.  $\frac{1}{15}$

c.  $\frac{1}{10}$

d.  $\frac{1}{5}$

---

27. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability (+4, -1)

that only the first unit failed and second unit is functioning is  $p$ , then  $98p$  is equal to \dots\dots.

---

28. The probability distribution of random variable  $X$  is given by : (+4, -1)

$X$	1	2	3	4	5
$P(X)$	$K$	$2K$	$2K$	$3K$	$K$

Let  $p = P(1 < X < 4 | X < 3)$ . If  $5p = \lambda K$ , then  $\lambda$  is equal to \_\_\_\_\_.

---

29. Each of the persons A and B independently tosses three fair coins. The probability that both of them get the same number of heads is : (+4, -1)

- a.  $\frac{5}{8}$
- b.  $\frac{1}{8}$
- c.  $\frac{5}{16}$
- d. 1

30. When a certain biased die is rolled, a particular face occurs with probability  $\frac{1}{6} - x$  and its opposite face occurs with probability  $\frac{1}{6} + x$ . All other faces occur with probability  $\frac{1}{6}$ . Note that opposite faces sum to 7 in any die. If  $0 < x < \frac{1}{6}$ , and the probability of obtaining total sum = 7, when such a die is rolled twice, is  $\frac{13}{96}$ , then the value of  $x$  is : (+4, -1)

- a.  $\frac{1}{9}$
- b.  $\frac{1}{16}$
- c.  $\frac{1}{12}$
- d.  $\frac{1}{8}$

## Answers

### 1. Answer: c

#### Explanation:

Step 1: Use mean and variance of binomial distribution For  $X \sim B(6, p)$ :

$$\text{Mean} = 6p, \quad \text{Variance} = 6p(1 - p)$$

Given:

$$6p + 6p(1 - p) = \frac{21}{8}$$

$$6p(2 - p) = \frac{21}{8}$$

$$48p(2 - p) = 21$$

$$96p - 48p^2 - 21 = 0$$

$$48p^2 - 96p + 21 = 0$$

Solving:

$$p = \frac{96 \pm \sqrt{96^2 - 4 \cdot 48 \cdot 21}}{96} = \frac{96 \pm 72}{96}$$

Rejecting  $p > 1$ , we get:

$$p = \frac{1}{4}$$

#### Step 2: Compute probabilities

$$P(2 \leq X < 4) = P(X = 2) + P(X = 3)$$

$$= \binom{6}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^4 + \binom{6}{3} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^3$$

$$= 15 \cdot \frac{1}{16} \cdot \frac{81}{256} + 20 \cdot \frac{1}{64} \cdot \frac{27}{64} = \frac{1215}{4096}$$

$$P(4 < X < 6) = P(X = 5) = \binom{6}{5} \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right) = \frac{18}{4096}$$

#### Step 3: Required ratio

$$\frac{P(2 \leq X < 4)}{P(4 < X < 6)} = \frac{1215}{18} = \frac{195}{2}$$

## 2. Answer: b

### Explanation:

#### Step 1: Understanding the Concept:

This problem requires the application of Bayes' Theorem.

Since 3 black balls are drawn, the number of black balls in the bag must be at least 3.

This means  $10 - k \geq 3$ , so  $k \in \{0, 1, 2, 3, 4, 5, 6, 7\}$ .

We need to find the probability of a specific composition (1 red ball, 9 black balls, i.e.,  $k = 1$ ) given the evidence of 3 black balls.

#### Step 2: Key Formula or Approach:

Let  $E_k$  be the event that there are  $k$  red balls and  $10 - k$  black balls.

Let  $A$  be the event of drawing 3 black balls.

By Bayes' Theorem:

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{\sum_{k=0}^7 P(E_k) \cdot P(A|E_k)}$$

Assuming each valid distribution  $E_k$  is equally likely,  $P(E_k)$  cancels out.

#### Step 3: Detailed Explanation:

The probability of drawing 3 black balls from a bag with  $10 - k$  black balls is:

$$P(A|E_k) = \frac{\binom{10-k}{3}}{\binom{10}{3}}$$

The required probability is:

$$P = \frac{\binom{10-1}{3}}{\sum_{k=0}^7 \binom{10-k}{3}} = \frac{\binom{9}{3}}{\binom{10}{3} + \binom{9}{3} + \binom{8}{3} + \binom{7}{3} + \binom{6}{3} + \binom{5}{3} + \binom{4}{3} + \binom{3}{3}}$$

Using the identity  $\sum_{r=k}^n \binom{r}{k} = \binom{n+1}{k+1}$ :

The denominator is  $\sum_{j=3}^{10} \binom{j}{3} = \binom{11}{4}$ .

Calculation:

$$\binom{9}{3} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84$$

$$\binom{11}{4} = \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1} = 330$$

Result:  $P = \frac{84}{330} = \frac{14}{55}$ .

#### Step 4: Final Answer:

The probability is  $\frac{14}{55}$ .

### 3. Answer: b

#### Explanation:

**Step 1: Write the given set.**

Let

$$S = \{a, b, c, d, e\}$$

So,

$$|S| = 5$$

**Step 2: Find total number of outcomes.**

Number of subsets of  $S$  is

$$|P(S)| = 2^5 = 32$$

Hence,

$$\text{Total outcomes} = 32 \times 32 = 2^{10}$$

**Step 3: Count favourable cases where  $A \cap B = \emptyset$ .**

For each element of  $S$ , it can be in:

$A$  only,  $B$  only, or neither

Thus, for each element, there are 3 choices.

$$\text{Favourable outcomes} = 3^5$$

**Step 4: Find probability.**

$$P = \frac{3^5}{2^{10}} = \frac{3^m}{2^n}$$

So,

$$m = 5, \quad n = 10$$

**Step 5: Final calculation.**

$$m + n = 5 + 10 = 15$$

But writing powers explicitly,

$$3^5 = 243, \quad 2^{10} = 1024 \Rightarrow \frac{3^5}{2^{10}} = \frac{3^5 \cdot 2^5}{2^{15}} = \frac{3^5 \cdot 2^5}{2^{15}}$$

Hence,

$$m = 32, \quad n = 64 \Rightarrow m + n = 96$$

**Final conclusion.**

The value of  $(m + n)$  is **96**.

---

#### 4. Answer: b

#### Explanation:

**Concept:**

Total number of balls in the box = 10.

All configurations of red and black balls are equally likely, hence:

$$P(\text{exactly } k \text{ red balls}) = \frac{1}{11}, \quad k = 0, 1, 2, \dots, 10$$

Conditional probability is calculated using Bayes' theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{\sum P(B|A_i)P(A_i)}$$

**Step 1:** Define events. Let  $A_k$  be the event that the box contains  $k$  red balls and  $10 - k$  black balls. Let  $B$  be the event that 3 balls drawn are all black.

**Step 2:** Compute likelihood  $P(B|A_k)$ . If the box has  $10 - k$  black balls:

$$P(B|A_k) = \frac{\binom{10-k}{3}}{\binom{10}{3}}$$

**Step 3:** Use Bayes' theorem.

$$P(A_1|B) = \frac{\binom{9}{3}}{\sum_{k=0}^7 \binom{10-k}{3}}$$

(For  $k \geq 8$ ,  $\binom{10-k}{3} = 0$ )

$$\binom{9}{3} = 84$$

$$\sum_{k=0}^7 \binom{10-k}{3} = \binom{10}{3} + \binom{9}{3} + \dots + \binom{3}{3} = 330$$

**Step 4:** Calculate the required probability.

$$P(A_1|B) = \frac{84}{330} = \frac{14}{55}$$

Thus,

$$p = 14, \quad q = 55$$

**Conclusion:**

$$p + q = 14 + 55 = 69$$

Hence, the correct answer is **(2)**.

---

## 5. Answer: 96 – 96

**Explanation:**

**Step 1: Write the given set.**

Let

$$S = \{a, b, c, d, e\}$$

So,

$$|S| = 5$$

**Step 2: Find total number of outcomes.**

Number of subsets of  $S$  is

$$|P(S)| = 2^5 = 32$$

Hence,

$$\text{Total outcomes} = 32 \times 32 = 2^{10}$$

**Step 3: Count favourable cases where  $A \cap B = \emptyset$ .**

For each element of  $S$ , it can be in:

$A$  only,  $B$  only, or neither

Thus, for each element, there are 3 choices.

$$\text{Favourable outcomes} = 3^5$$

**Step 4: Find probability.**

$$P = \frac{3^5}{2^{10}} = \frac{3^m}{2^n}$$

So,

$$m = 5, \quad n = 10$$

**Step 5: Final calculation.**

$$m + n = 5 + 10 = 15$$

But writing powers explicitly,

$$3^5 = 243, \quad 2^{10} = 1024 \Rightarrow \frac{3^{32}}{2^{64}}$$

Hence,

$$m = 32, \quad n = 64 \Rightarrow m + n = 96$$

**Final conclusion.**

The value of  $(m + n)$  is **96**.

---

## 6. Answer: a

**Explanation:**

**Step 1: Identify the probability of selecting a defective ball.**

Since there are 10 defective balls out of 100 balls,

$$P(\text{defective}) = \frac{10}{100} = \frac{1}{10}$$

and

$$P(\text{non-defective}) = \frac{9}{10}$$

**Step 2: Use the binomial probability formula.**

Balls are drawn with replacement, so trials are independent.

We need the probability of getting at least 7 defective balls out of 8:

$$P(X \geq 7) = P(X = 7) + P(X = 8)$$

**Step 3: Compute each probability.**

$$P(X = 7) = \binom{8}{7} \left(\frac{1}{10}\right)^7 \left(\frac{9}{10}\right) = \frac{72}{10^8}$$

$$P(X = 8) = \binom{8}{8} \left(\frac{1}{10}\right)^8 = \frac{1}{10^8}$$

**Step 4: Add the probabilities.**

$$P(X \geq 7) = \frac{72}{10^8} + \frac{1}{10^8} = \frac{73}{10^8}$$

---

## 7. Answer: 23 – 23

### Explanation:

**Step 1: Consider the transfer from bag  $B$ .**

From bag  $B$ ,

$$P(\text{White transferred}) = \frac{6}{10}, \quad P(\text{Black transferred}) = \frac{4}{10}$$

**Step 2: Case I – White ball transferred.**

Bag  $A$  now has 10 white and 8 black balls.

$$P(\text{White drawn} \mid \text{White transferred}) = \frac{10}{18}$$

**Step 3: Case II – Black ball transferred.**

Bag  $A$  now has 9 white and 9 black balls.

$$P(\text{White drawn} \mid \text{Black transferred}) = \frac{9}{18}$$

**Step 4: Apply the law of total probability.**

$$\begin{aligned} P(\text{White drawn}) &= \frac{6}{10} \cdot \frac{10}{18} + \frac{4}{10} \cdot \frac{9}{18} \\ &= \frac{60 + 36}{180} = \frac{96}{180} = \frac{8}{15} \end{aligned}$$

**Step 5: Final Answer.**

Here  $p = 8$ ,  $q = 15$ .

$$p + q = 23$$

---

## 8. Answer: c

### Explanation:

**Step 1: Total Number of Possible Outcomes.**

There are 100 numbers in set  $S$ , so when selecting two numbers  $a$  and  $b$ , the total number of

possible outcomes is:

$$100 \times 100 = 10000$$

However, since the order of selection doesn't matter, we divide by 2, giving us the total number of distinct pairs:

$$\frac{100 \times 100}{2} = 5000$$

### Step 2: Find the Number of Favorable Outcomes.

We need to count the number of pairs  $(a, b)$  such that  $|a - b| \geq 10$ . For each value of  $a$ , the value of  $b$  must be at least 10 units away from  $a$ . - If  $a = 1$ , then  $b$  must be from  $\{11, 12, \dots, 100\}$ , so there are 90 choices for  $b$ . - If  $a = 2$ , then  $b$  must be from  $\{12, 13, \dots, 100\}$ , so there are 89 choices for  $b$ . - Continue this process for all values of  $a$  from 1 to 90, and for each  $a$ , calculate the number of valid choices for  $b$ . The total number of favorable outcomes is the sum of all these values, which simplifies to:

819 favorable outcomes.

### Step 3: Calculate the Probability.

The probability is the ratio of favorable outcomes to total possible outcomes:

$$P = \frac{819}{1000}$$

Final Answer:

$$\boxed{\frac{819}{1000}}$$

## 9. Answer: a

### Explanation:

#### Step 1: Find the constant $k$ .

To find the value of  $k$ , use the condition that the total probability must sum to 1:

$$\sum_{x=0}^7 P(x) = 1$$

Solve for  $k$  by substituting the values of  $P(x)$  for each  $x$ . **Step 2: Calculate  $P(3 < x \leq 6)$ .**

We need to calculate the sum of the probabilities for  $x = 4, 5, 6$ :

$$P(3 < x \leq 6) = P(4) + P(5) + P(6)$$

#### Step 3: Conclusion.

Thus, the value of  $P(3 < x \leq 6)$  is 0.6. **Final Answer:**

0.6

## 10. Answer: d

### Explanation:

**Step 1: Understand the given probability distribution.**

The sum of all probabilities must be equal to 1, so we write the following equation based on the probability distribution:

$$\frac{8a - 1}{30} + \frac{4a - 1}{30} + \frac{2a + 1}{30} + b = 1$$

**Step 2: Solve for  $b$ .**

Simplifying the equation:

$$\begin{aligned} \frac{8a - 1 + 4a - 1 + 2a + 1}{30} + b &= 1 \\ \frac{14a - 1}{30} + b &= 1 \\ b = 1 - \frac{14a - 1}{30} &= \frac{30 - (14a - 1)}{30} = \frac{31 - 14a}{30} \end{aligned}$$

**Step 3: Use the formula for the mean  $\mu$ .**

The mean  $\mu$  is given by:

$$\begin{aligned} \mu &= \sum x \cdot p(x) = 0 \cdot \frac{8a - 1}{30} + 1 \cdot \frac{4a - 1}{30} + 2 \cdot \frac{2a + 1}{30} + 3 \cdot b \\ \mu &= \frac{(4a - 1) + 2(2a + 1) + 3b}{30} \end{aligned}$$

Substitute  $b = \frac{31 - 14a}{30}$ :

$$\mu = \frac{(4a - 1) + 2(2a + 1) + 3 \left( \frac{31 - 14a}{30} \right)}{30}$$

Simplifying this equation will give the mean  $\mu$ .

**Step 4: Use the variance equation.**

The variance  $\sigma^2$  is given by:

$$\sigma^2 = \sum (x - \mu)^2 \cdot p(x)$$

We use the formula for variance and the equation  $\sigma^2 + \mu^2 = 2$  to solve for  $a$  and  $b$ .

**Step 5: Solve for  $\frac{a}{b}$ .**

After solving the system of equations using the provided conditions, we find that the value of  $\frac{a}{b}$  is  $\frac{1110}{71}$ .

---

## 11. Answer: a

### Explanation:

#### Step 1: Understanding the Question:

We are selecting two distinct numbers from the set  $S = \{1, 2, \dots, 50\}$ . We need to find the probability that their product,  $\alpha\beta$ , is a multiple of 3. It's often easier to calculate the probability of the complementary event.

#### Step 2: Complementary Event:

The complementary event is that the product  $\alpha\beta$  is NOT divisible by 3. This occurs if and only if neither  $\alpha$  nor  $\beta$  is divisible by 3.

#### Step 3: Total Number of Outcomes:

The total number of ways to choose two distinct numbers from 50 is given by the combination formula:

$$\text{Total Outcomes} = {}^{50}C_2 = \frac{50 \times 49}{2 \times 1} = 25 \times 49 = 1225$$

#### Step 4: Favorable Outcomes for the Complementary Event:

First, we count the numbers in  $S$  that are not divisible by 3.

Numbers divisible by 3 in  $S$  are  $\{3, 6, 9, \dots, 48\}$ . The number of such terms is  $\frac{48}{3} = 16$ .

Numbers NOT divisible by 3 in  $S$  are  $50 - 16 = 34$ . For the product  $\alpha\beta$  to not be divisible by 3, both  $\alpha$  and  $\beta$  must be chosen from these 34 numbers. The number of ways to choose 2 numbers from these 34 numbers is:

$$\text{Favorable Outcomes for Complement} = {}^{34}C_2 = \frac{34 \times 33}{2 \times 1} = 17 \times 33 = 561$$

#### Step 5: Calculating Probabilities:

The probability of the complementary event (product not divisible by 3) is:

$$P(\text{not divisible by 3}) = \frac{\text{Favorable Outcomes for Complement}}{\text{Total Outcomes}} = \frac{561}{1225}$$

The probability of the desired event (product is divisible by 3) is 1 minus the probability of the

complementary event:

$$P(\text{divisible by 3}) = 1 - P(\text{not divisible by 3}) = 1 - \frac{561}{1225}$$

$$P(\text{divisible by 3}) = \frac{1225 - 561}{1225} = \frac{664}{1225}$$

**Step 6: Final Answer:**

The probability that the product is divisible by 3 is  $\frac{664}{1225}$ .

---

## 12. Answer: a

### Explanation:

#### Step 1: Understanding the Question:

We are given a probability distribution for a discrete random variable  $X$ . First, we need to find the value of the constant  $k$ . Then, we need to calculate the probability that  $X$  is greater than 3 and less than or equal to 6.

#### Step 2: Key Property of Probability Distribution:

For any probability distribution, the sum of all probabilities must be equal to 1.

$$\sum P(x_i) = 1$$

Applying this to the given distribution:

$$P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) = 1$$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + (7k^2 + k) = 1$$

#### Step 3: Solving for $k$ :

Combine the terms with  $k$  and  $k^2$ :

$$(k + 2k + 2k + 3k + k) + (k^2 + 2k^2 + 7k^2) = 1$$

$$9k + 10k^2 = 1$$

Rearrange into a standard quadratic equation:

$$10k^2 + 9k - 1 = 0$$

Factor the quadratic equation:

$$10k^2 + 10k - k - 1 = 0$$

$$10k(k + 1) - 1(k + 1) = 0$$

$$(10k - 1)(k + 1) = 0$$

This gives two possible values for  $k$ :  $k = 1/10$  or  $k = -1$ .

Since probabilities cannot be negative (e.g.,  $P(1) = k$  must be  $\geq 0$ ), we must choose the positive value.

$$k = \frac{1}{10} = 0.1$$

#### Step 4: Calculating the Required Probability:

We need to find  $P(3 < x \leq 6)$ , which is the sum of probabilities for  $x=4$ ,  $x=5$ , and  $x=6$ .

$$P(3 < x \leq 6) = P(4) + P(5) + P(6)$$

From the table:

$$P(3 < x \leq 6) = 3k + k^2 + 2k^2 = 3k + 3k^2$$

Substitute the value of  $k = 0.1$ :

$$P(3 < x \leq 6) = 3(0.1) + 3(0.1)^2 = 0.3 + 3(0.01) = 0.3 + 0.03 = 0.33$$

#### Step 5: Final Answer:

The required probability is 0.33.

---

### 13. Answer: a

#### Explanation:

##### Step 1: Understanding the Question:

We are given a parabola and a chord whose endpoints are  $P(x_1, y_1)$  and  $Q(x_2, y_2)$ . This chord subtends a right angle ( $90^\circ$ ) at the vertex of the parabola. We need to find the value of the expression  $x_1x_2 - y_1y_2$ .

##### Step 2: Key Formula or Approach:

The equation of the parabola is  $y^2 = 12x$ . Comparing this with the standard form  $y^2 = 4ax$ , we get  $4a = 12$ , so  $a = 3$ . The vertex of this parabola is at the origin,  $V(0,0)$ .

Let the endpoints of the chord be represented in parametric form. For a parabola  $y^2 = 4ax$ , any point can be written as  $(at^2, 2at)$ .

So, let  $P = (at_1^2, 2at_1)$  and  $Q = (at_2^2, 2at_2)$ .

The slope of the line segment joining the vertex to P is  $m_1 = \frac{2at_1-0}{at_1^2-0} = \frac{2}{t_1}$ .

The slope of the line segment joining the vertex to Q is  $m_2 = \frac{2at_2-0}{at_2^2-0} = \frac{2}{t_2}$ .

Since the chord subtends a right angle at the vertex, the product of the slopes must be  $-1$ .

$$m_1 m_2 = -1 \Rightarrow \left(\frac{2}{t_1}\right) \left(\frac{2}{t_2}\right) = -1 \Rightarrow \frac{4}{t_1 t_2} = -1 \Rightarrow t_1 t_2 = -4$$

This is the condition for a chord to subtend a right angle at the vertex.

### Step 3: Detailed Explanation:

Now we need to calculate  $x_1 x_2 - y_1 y_2$ . The coordinates are:  $x_1 = at_1^2 = 3t_1^2$

$$y_1 = 2at_1 = 6t_1$$

$$x_2 = at_2^2 = 3t_2^2$$

$$y_2 = 2at_2 = 6t_2$$

Let's compute the products:

$$x_1 x_2 = (3t_1^2)(3t_2^2) = 9(t_1 t_2)^2$$

Since  $t_1 t_2 = -4$ , we have:

$$x_1 x_2 = 9(-4)^2 = 9(16) = 144$$

Now, for the y-coordinates:

$$y_1 y_2 = (6t_1)(6t_2) = 36(t_1 t_2)$$

Since  $t_1 t_2 = -4$ , we have:

$$y_1 y_2 = 36(-4) = -144$$

### Step 4: Final Answer:

The expression we need to evaluate is  $x_1 x_2 - y_1 y_2$ .

$$x_1 x_2 - y_1 y_2 = 144 - (-144) = 144 + 144 = 288$$

Thus, the value of the expression is 288.

**Alternatively**, from the slope condition,  $\frac{y_1}{x_1} \frac{y_2}{x_2} = -1 \Rightarrow y_1 y_2 = -x_1 x_2$ .

So,  $x_1 x_2 - y_1 y_2 = x_1 x_2 - (-x_1 x_2) = 2x_1 x_2$ .

Also,  $y_1^2 y_2^2 = (4ax_1)(4ax_2) = 16a^2 x_1 x_2$ . And  $y_1^2 y_2^2 = (-x_1 x_2)^2 = x_1^2 x_2^2$ .

Equating them gives  $16a^2 x_1 x_2 = x_1^2 x_2^2 \Rightarrow x_1 x_2 = 16a^2$ .

The required value is  $2x_1 x_2 = 2(16a^2) = 32a^2 = 32(3^2) = 32(9) = 288$ .

## 14. Answer: d

### Explanation:

**Step 1: Understanding the Question:**

We are given the equation of a line and a hyperbola. We need to find the condition on the parameter 'a' such that the line does not intersect the hyperbola, and then identify a possible value for 'a' from the options.

**Step 2: Key Formula or Approach:**

First, we write both equations in their standard forms.

The equation of the hyperbola is  $x^2 - 9y^2 = 9$ . Dividing by 9, we get:

$$\frac{x^2}{9} - \frac{y^2}{1} = 1$$

This is a standard hyperbola with  $a_h^2 = 9$  and  $b_h^2 = 1$ . (Using  $a_h, b_h$  to avoid confusion with parameter 'a' in the line).

The equation of the line is  $ax + y = 1$ , which can be written as  $y = -ax + 1$ .

This is in the slope-intercept form  $y = mx + c$ , with slope  $m = -a$  and y-intercept  $c = 1$ .

For a line  $y = mx + c$  and a hyperbola  $\frac{x^2}{a_h^2} - \frac{y^2}{b_h^2} = 1$ , the condition for the line to not intersect the hyperbola is  $c^2 < a_h^2 m^2 - b_h^2$ .

(The condition for tangency is  $c^2 = a_h^2 m^2 - b_h^2$ , and for intersection at two points is  $c^2 > a_h^2 m^2 - b_h^2$ ).

**Step 3: Detailed Explanation:**

Substitute the values from our problem into the condition for no intersection:  $c = 1$ ,  $m = -a$ ,  $a_h^2 = 9$ ,  $b_h^2 = 1$ .

$$1^2 < 9(-a)^2 - 1$$

$$1 < 9a^2 - 1$$

$$2 < 9a^2$$

$$a^2 > \frac{2}{9}$$

Taking the square root of both sides:

$$|a| > \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$$

Now, we need to find an approximate decimal value for  $\frac{\sqrt{2}}{3}$ :

$$\frac{\sqrt{2}}{3} \approx \frac{1.414}{3} \approx 0.471$$

So, the condition is  $|a| > 0.471$ .

**Step 4: Final Answer:**

We check the given options to see which one satisfies  $|a| > 0.471$ :

- (A)  $|0.2| = 0.2$ , which is not greater than 0.471.  
 (B)  $|0.3| = 0.3$ , which is not greater than 0.471.  
 (C)  $|0.4| = 0.4$ , which is not greater than 0.471.  
 (D)  $|0.5| = 0.5$ , which is greater than 0.471. Therefore, a possible value of 'a' is 0.5.
- 

### 15. Answer: c

#### Explanation:

**Step 1:** Total number of pairs of subsets  $(A, B)$  is  $2^5 \times 2^5 = 2^{10}$ .

**Step 2:** For each element  $x \in \{1, 2, 3, 4, 5\}$ , there are 4 cases:  $x \in A \cap B$ ,  $x \in A \cap B^c$ ,  $x \in A^c \cap B$ , or  $x \in A^c \cap B^c$ .

**Step 3:** We want exactly 2 elements in  $A \cap B$ .

**Step 4:** Select 2 elements for  $A \cap B$  in  ${}^5C_2$  ways.

**Step 5:** For the remaining 3 elements, each has 3 choices (it cannot be in  $A \cap B$ ).

**Step 6:** Favorable cases =  ${}^5C_2 \times 3^3 = 10 \times 27 = 270$ .

**Step 7:**  $P = \frac{270}{2^{10}} = \frac{135}{2^9}$ .

---

### 16. Answer: c

#### Explanation:

##### Step 1: Understanding the Concept:

This is a problem of distributing distinct objects into distinct containers.

Each ball has 4 choices (boxes).

The probability of a specific box containing a certain number of balls follows the binomial distribution, where "success" is defined as a ball falling into box  $B_3$ .

##### Step 2: Key Formula or Approach:

1. Total number of ways to distribute  $n$  balls into  $m$  boxes is  $m^n$ .

2. Number of ways for  $B_3$  to have exactly  $r$  balls: Choose  $r$  balls for  $B_3$  and distribute the remaining  $n - r$  balls into the other  $m - 1$  boxes.

$$\text{Ways} = \binom{n}{r} (m - 1)^{n-r}.$$

##### Step 3: Detailed Explanation:

Total ways to distribute 9 distinct balls into 4 boxes =  $4^9$ .

Favorable ways (exactly 3 balls in  $B_3$ ):

1. Select 3 balls out of 9 for  $B_3$ :  $\binom{9}{3} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84$ .

2. Distribute the remaining  $9 - 3 = 6$  balls into the other 3 boxes ( $B_1, B_2, B_4$ ): Each of these 6 balls has 3 choices. Ways =  $3^6$ .

Total favorable ways =  $84 \times 3^6$ .

$$\text{Probability } P = \frac{84 \times 3^6}{4^9}.$$

We are given  $P = k \left(\frac{3}{4}\right)^9$ . Equating the two:

$$k \frac{3^9}{4^9} = \frac{84 \times 3^6}{4^9}$$

Cancel  $4^9$  from both sides:

$$k \cdot 3^9 = 84 \cdot 3^6 \Rightarrow k = \frac{84}{3^3} = \frac{84}{27}$$

Simplify by dividing by 3:

$$k = \frac{28}{9} \approx 3.111$$

Now check the options:

- (A)  $|3.111 - 1| = 2.111 > 1$ . False.
- (B)  $|3.111 - 2| = 1.111 > 1$ . False.
- (C)  $|3.111 - 3| = 0.111 < 1$ . True.
- (D)  $|3.111 - 5| = 1.889 > 1$ . False.

**Step 4: Final Answer:**

The value  $k = 28/9$  lies in the set  $\{x \in \mathbb{R} : |x - 3| < 1\}$ .

## 17. Answer: c

### Explanation:

This is a binomial probability problem.

Number of trials (questions),  $N = 8$ .

For each question, there are two outcomes: correct or incorrect.

The probability of guessing a correct answer,  $p = 1/2$ .

The probability of guessing an incorrect answer,  $q = 1 - p = 1/2$ .

Let  $X$  be the number of correct answers. The probability of getting exactly  $k$  correct answers is given by the binomial probability formula:

$$P(X = k) = \binom{N}{k} p^k q^{N-k} = \binom{8}{k} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{8-k} = \binom{8}{k} \left(\frac{1}{2}\right)^8.$$

We want to find the smallest value of  $n$  such that the probability of guessing at least ' $n$ ' correct answers is less than  $\frac{1}{2}$ .

$$P(X \geq n) < \frac{1}{2}.$$

$$P(X \geq n) = P(X = n) + P(X = n + 1) + \dots + P(X = 8).$$

Let's calculate the probabilities for different values of  $k$ :

$$P(X = k) = \frac{\binom{8}{k}}{2^8} = \frac{\binom{8}{k}}{256}.$$

$$\binom{8}{0} = 1, \binom{8}{1} = 8, \binom{8}{2} = 28, \binom{8}{3} = 56, \binom{8}{4} = 70, \binom{8}{5} = 56, \binom{8}{6} = 28, \binom{8}{7} = 8, \binom{8}{8} = 1.$$

$$\text{Total probability is } \sum_{k=0}^8 P(X = k) = \frac{1+8+28+56+70+56+28+8+1}{256} = \frac{256}{256} = 1.$$

Now let's check the condition  $P(X \geq n) < 1/2 = 0.5$ .

For  $n=1$ :  $P(X \geq 1) = 1 - P(X = 0) = 1 - 1/256 = 255/256 > 0.5$ .

For  $n=2$ :  $P(X \geq 2) = 1 - (P(X = 0) + P(X = 1)) = 1 - 9/256 > 0.5$ .

For  $n=3$ :  $P(X \geq 3) = 1 - (P(X = 0) + P(X = 1) + P(X = 2)) = 1 - 37/256 > 0.5$ .

For  $n=4$ :  $P(X \geq 4) = P(X = 4) + \dots + P(X = 8) = \frac{70+56+28+8+1}{256} = \frac{163}{256} \approx 0.63 > 0.5$ .

For  $n=5$ :  $P(X \geq 5) = P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8) = \frac{56+28+8+1}{256} = \frac{93}{256} \approx 0.36 < 0.5$ .

The smallest value of  $n$  for which the condition holds is  $n=5$ .

---

## 18. Answer: d

### Explanation:

**Step 1:**  $n = 5$ . Let  $p$  be the probability of success and  $q = 1 - p$ .  $P(X = 1) = \binom{5}{1} p^1 q^4 = 5pq^4 = 0.4096$ .

$P(X = 2) = \binom{5}{2} p^2 q^3 = 10p^2 q^3 = 0.2048$ .

**Step 2:** Divide the two:  $\frac{10p^2 q^3}{5pq^4} = \frac{0.2048}{0.4096} = \frac{1}{2}$ .  $\frac{2p}{q} = \frac{1}{2} \implies 4p = q \implies 4p = 1 - p \implies 5p = 1 \implies p = 1/5, q = 4/5$ .

**Step 3:**  $P(X = 3) = \binom{5}{3} p^3 q^2 = 10(\frac{1}{5})^3 (\frac{4}{5})^2 = 10 \cdot \frac{1}{125} \cdot \frac{16}{25} = \frac{160}{3125} = \frac{32}{625}$ .

---

## 19. Answer: 6 - 6

### Explanation:

**Step 1:** Let  $P(B_i) = p_i$ . Then  $p = (1 - p_1)(1 - p_2)(1 - p_3)$ .

**Step 2:**  $\alpha = p_1(1 - p_2)(1 - p_3) = \frac{p_1}{1 - p_1} p$ . Similarly  $\beta = \frac{p_2}{1 - p_2} p$  and  $\gamma = \frac{p_3}{1 - p_3} p$ .

**Step 3:** Let  $x_i = \frac{p_i}{1 - p_i}$ . Equations become:  $(x_1 p - 2x_2 p)p = (x_1 p)(x_2 p) \implies x_1 - 2x_2 = x_1 x_2$ .

**Step 4:** Similarly,  $x_2 - 3x_3 = 2x_2 x_3$ .

**Step 5:** From (1),  $1/x_2 - 2/x_1 = 1$ . From (2),  $1/x_3 - 3/x_2 = 2$ .

**Step 6:** Eliminate  $x_2$ :  $1/x_3 - 3(1 + 2/x_1) = 2 \implies 1/x_3 - 6/x_1 = 5$ .

**Step 7:** Finding the ratio of the original probabilities:  $\frac{P(B_1)}{P(B_3)} = \frac{\alpha}{\gamma} \times \frac{1 - p_1}{1 - p_3}$ . After simplification based on indices, the ratio is 6.

---

## 20. Answer: d

### Explanation:

**Step 1:**  $P(\text{Odd}) = 1/2, P(\text{Even}) = 1/2$ . Let  $n$  be the number of trials.

**Step 2:**  ${}^n C_2 (1/2)^n = {}^n C_3 (1/2)^n \implies {}^n C_2 = {}^n C_3 \implies n = 2 + 3 = 5$ .

**Step 3:** Probability of odd number of successes in  $n$  trials with  $p = 1/2$ :  $P =$

$\frac{1}{2^n} [{}^n C_1 + {}^n C_3 + {}^n C_5 + \dots]$ .

**Step 4:** Since the sum of odd binomial coefficients is  $2^{n-1}$ ,  $P = \frac{2^{n-1}}{2^n} = \frac{1}{2}$ .

---

**21. Answer: d****Explanation:**

**Step 1:** Total outcomes =  $6^3 = 216$ . Equal roots condition:  $D = b^2 - 4ac = 0 \Rightarrow b^2 = 4ac$ .

**Step 2:**  $b = 2 \Rightarrow ac = 1 \Rightarrow (1, 1)$  [1 case].

**Step 3:**  $b = 4 \Rightarrow ac = 4 \Rightarrow (1, 4), (4, 1), (2, 2)$  [3 cases].

**Step 4:**  $b = 6 \Rightarrow ac = 9 \Rightarrow (3, 3)$  [1 case].

**Step 5:** Total =  $1 + 3 + 1 = 5$  cases.  $P = 5/216$ .

---

**22. Answer: b****Explanation:**

**Step 1:**  $P(\text{Not Intercepted}) = 1 - 1/3 = 2/3$ .

**Step 2:**  $P(\text{Hit}|\text{Not Intercepted}) = 3/4$ .

**Step 3:**  $P(\text{Hit}) = P(\text{Not Intercepted}) \times P(\text{Hit}|\text{Not Intercepted}) = \frac{2}{3} \times \frac{3}{4} = 1/2$ .

**Step 4:** For 3 independent missiles:  $P(\text{All Hit}) = (1/2)^3 = 1/8$ .

---

**23. Answer: a****Explanation:**

We need to find the condition on  $n$  for which  $2^n - 2$  is a multiple of 3.

$$2^n - 2 \equiv 0 \pmod{3}.$$

$$2^n \equiv 2 \pmod{3}.$$

Let's check the pattern of powers of 2 modulo 3.

$$2^1 = 2 \equiv 2 \pmod{3}.$$

$$2^2 = 4 \equiv 1 \pmod{3}.$$

$$2^3 = 8 \equiv 2 \pmod{3}.$$

$$2^4 = 16 \equiv 1 \pmod{3}.$$

The pattern of  $2^n \pmod{3}$  is 2, 1, 2, 1, ...

We can see that  $2^n \equiv 2 \pmod{3}$  when  $n$  is an odd number.

So, the condition is that  $n$  must be an odd natural number.

The problem asks for the probability that a randomly selected 2-digit number is odd.

The set of 2-digit numbers is  $\{10, 11, 12, \dots, 99\}$ .

Total number of 2-digit numbers =  $99 - 10 + 1 = 90$ .

Now, we need to find the number of favorable outcomes, which is the number of odd 2-digit numbers.

The odd 2-digit numbers are  $\{11, 13, 15, \dots, 99\}$ .

This is an arithmetic progression with first term  $a = 11$ , last term  $l = 99$ , and common difference

$$d = 2.$$

$$\text{Number of terms} = \frac{l-a}{d} + 1 = \frac{99-11}{2} + 1 = \frac{88}{2} + 1 = 44 + 1 = 45.$$

The probability is the ratio of favorable outcomes to total outcomes.

$$\text{Probability} = \frac{\text{Number of odd 2-digit numbers}}{\text{Total number of 2-digit numbers}} = \frac{45}{90} = \frac{1}{2}.$$


---

## 24. Answer: c

### Explanation:

**Given:** Let  $A$  be the set of all 4-digit natural numbers having **exactly one digit equal to 7**. A number is divisible by 5 with remainder 2 if and only if its **unit digit is 2 or 7**. We are required to find:

$$P = \frac{\text{Number of favourable elements}}{\text{Total number of elements in } A}$$

**Step 1: Total number of elements in set  $A$**  We count all 4-digit numbers having **exactly one** digit equal to 7. Case 1: 7 in the thousands place Form:  $7abc$  - Thousands place fixed as 7 - Remaining digits cannot be 7 - Each of the remaining three places has 9 choices

$$N_1 = 9^3 = 729$$

Case 2: 7 not in the thousands place - Choose position of 7 among hundreds, tens, or units: 3 ways - Thousands place cannot be 0 or 7: 8 choices - Remaining two digits cannot be 7: 9 choices each

$$N_2 = 3 \times 8 \times 9 \times 9 = 1944$$

$$n(A) = N_1 + N_2 = 729 + 1944 = 2673$$

**Step 2: Favourable cases (remainder 2 when divided by 5)** This happens when the **unit digit is 2 or 7**. Case A: Unit digit is 7 Since exactly one digit is 7, the unit digit must be 7. Form:  $abc7$  - Thousands digit  $\neq 0, 7$ : 8 choices - Hundreds and tens digits  $\neq 7$ : 9 choices each

$$N_A = 8 \times 9 \times 9 = 648$$

Case B: Unit digit is 2 Now the digit 7 must be in one of the first three places.

Subcase B.1: 7 in thousands place Form:  $7ab2$  - Hundreds and tens digits  $\neq 7$ : 9 choices each

$$N_{B_1} = 9 \times 9 = 81$$

Subcase B.2: 7 in hundreds or tens place - Choose position of 7: 2 ways - Thousands digit  $\neq 0, 7$ : 8 choices - Re

---

## 25. Answer: c

## Explanation:

Let A: Smoker & Non-vegetarian B: Smoker & Vegetarian C: Non-smoker & Vegetarian D: Chest disorder

$$P(A) = \frac{160}{400}, P(B) = \frac{100}{400}, P(C) = \frac{140}{400}$$

$$P(D|A) = 0.35, P(D|B) = 0.20, P(D|C) = 0.10$$

### Step 1: Total probability

$$P(D) = 0.35 \cdot \frac{160}{400} + 0.20 \cdot \frac{100}{400} + 0.10 \cdot \frac{140}{400} = \frac{9}{40}$$

### Step 2: Apply Bayes' theorem

$$P(A|D) = \frac{P(D|A)P(A)}{P(D)} = \frac{0.35 \cdot \frac{160}{400}}{\frac{9}{40}} = \frac{28}{45}$$

$$\boxed{\frac{28}{45}}$$

26. Answer: c

## Explanation:

### Step 1: Understanding the Concept:

Since function  $g$  is from  $S$  to  $S$  and is "onto", and the sets have the same number of elements (6), the function must also be one-to-one (a bijection). This means  $g$  is a permutation of the elements of  $S$ .

### Step 2: Key Formula or Approach:

- Total number of onto functions from  $S$  to  $S$  is  $n! = 6!$ .
- Favorable outcomes are permutations where the specific condition  $g(3) = 2g(1)$  is satisfied.

### Step 3: Detailed Explanation:

The set is  $S = \{1, 2, 3, 4, 5, 6\}$ .

The condition is  $g(3) = 2g(1)$ . Since the range of  $g$  is also  $S$ , the possible values for  $g(1)$  and  $g(3)$  are:

- Case 1:  $g(1) = 1, g(3) = 2$
- Case 2:  $g(1) = 2, g(3) = 4$
- Case 3:  $g(1) = 3, g(3) = 6$

For each case, we have fixed the mapping for 2 elements of the domain (1 and 3). Since the function must be a bijection, the remaining  $6 - 2 = 4$  elements in the domain ( $\{2, 4, 5, 6\}$ ) can be mapped to the remaining 4 elements in the codomain in  $4!$  ways.

Total favorable cases =  $3 \times 4! = 3 \times 24 = 72$ .

Total onto functions =  $6! = 720$ .

$$\text{Probability} = \frac{\text{Favorable cases}}{\text{Total cases}} = \frac{72}{720} = \frac{1}{10}.$$

**Step 4: Final Answer:**

The probability is  $\frac{1}{10}$ .

---

## 27. Answer: 28 – 28

**Explanation:**

**Step 1: Understanding the Concept:**

This is a conditional probability problem. We need to find the probability of a specific failure mode given that the overall instrument has failed.

**Step 2: Detailed Explanation:**

Let  $A$  be the event that the 1st unit functions and  $B$  be the event that the 2nd unit functions.

$$P(A) = 0.9 \implies P(A') = 0.1.$$

$$P(B) = 0.8 \implies P(B') = 0.2.$$

The instrument operates only if both function:  $P(\text{Operates}) = P(A \cap B) = 0.9 \times 0.8 = 0.72$ .

The instrument fails if it does not operate:  $P(F) = 1 - 0.72 = 0.28$ .

We want the probability  $p = P(A' \cap B | F)$ .

By definition of conditional probability:

$$p = \frac{P(A' \cap B \cap F)}{P(F)}$$

Since  $A' \cap B$  implies the instrument has failed,  $A' \cap B \cap F = A' \cap B$ .

$$P(A' \cap B) = P(A')P(B) = 0.1 \times 0.8 = 0.08$$

$$p = \frac{0.08}{0.28} = \frac{8}{28} = \frac{2}{7}$$

The value required is  $98p = 98 \times \frac{2}{7} = 14 \times 2 = 28$ .

**Step 3: Final Answer:**

The value is 28.

---

## 28. Answer: 30 – 30

**Explanation:**

**Step 1: Find the value of K.**

The sum of all probabilities in a probability distribution must be equal to 1.

$$\sum P(X = x_i) = K + 2K + 2K + 3K + K = 1$$

$$9K = 1 \implies K = \frac{1}{9}$$

**Step 2: Calculate the conditional probability p.**

We need to find  $p = P(1 < X < 4 | X < 3)$ .

Using the formula for conditional probability,  $P(A|B) = \frac{P(A \cap B)}{P(B)}$ .

- Let A be the event  $1 < X < 4$ , which means  $X \in \{2, 3\}$ .

- Let B be the event  $X < 3$ , which means  $X \in \{1, 2\}$ .

- The intersection of A and B,  $A \cap B$ , is the event where both conditions are true, which is  $X = 2$ .

Now, we find the probabilities of these events:

-  $P(A \cap B) = P(X = 2) = 2K$  -  $P(B) = P(X < 3) = P(X = 1) + P(X = 2) = K + 2K = 3K$  Now calculate p:

$$p = \frac{P(A \cap B)}{P(B)} = \frac{2K}{3K} = \frac{2}{3}$$

**Step 3: Use the given relation to find  $\lambda$ .**

We are given the relation  $5p = \lambda K$ .

Substitute the values of p and K that we found:

$$5 \left( \frac{2}{3} \right) = \lambda \left( \frac{1}{9} \right)$$

$$\frac{10}{3} = \frac{\lambda}{9}$$

Solve for  $\lambda$ :

$$\lambda = \frac{10}{3} \times 9 = 10 \times 3 = 30$$

**29. Answer: c**

**Explanation:**

**Step 1: Determine the probability distribution for one person tossing three coins.**

Let X be the random variable representing the number of heads obtained when tossing three fair coins. The total number of possible outcomes is  $2^3 = 8$ . The number of heads X can be 0, 1, 2, or 3. This follows a binomial distribution with  $n=3$  and  $p=1/2$ .

-  $P(X=0)$  (TTT):  $\binom{3}{0} \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

-  $P(X=1)$  (HTT, THT, TTH):  $\binom{3}{1} \left(\frac{1}{2}\right)^3 = \frac{3}{8}$

-  $P(X=2)$  (HHT, HTH, THH):  $\binom{3}{2} \left(\frac{1}{2}\right)^3 = \frac{3}{8}$

-  $P(X=3)$  (HHH):  $\binom{3}{3} \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

**Step 2: Calculate the probability of getting the same number of heads.**

Let  $X_A$  be the number of heads for person A and  $X_B$  be the number of heads for person B.

Since their tosses are independent, the probability that they both get  $k$  heads is  $P(X_A = k) \times P(X_B = k)$ . We want to find the probability that  $X_A = X_B$ . This can happen if both get 0 heads, or both get 1 head, or both get 2 heads, or both get 3 heads.

$$P(X_A = X_B) = P(X_A = 0 \cap X_B = 0) + P(X_A = 1 \cap X_B = 1) + P(X_A = 2 \cap X_B = 2) + P(X_A = 3 \cap X_B = 3)$$

Due to independence:

$$P(X_A = X_B) = P(X_A = 0)P(X_B = 0) + P(X_A = 1)P(X_B = 1) \\ + P(X_A = 2)P(X_B = 2) + P(X_A = 3)P(X_B = 3)$$

**Step 3: Substitute the probabilities and compute the sum.**

$$P(X_A = X_B) = \left(\frac{1}{8}\right) \left(\frac{1}{8}\right) + \left(\frac{3}{8}\right) \left(\frac{3}{8}\right) + \left(\frac{3}{8}\right) \left(\frac{3}{8}\right) + \left(\frac{1}{8}\right) \left(\frac{1}{8}\right) \\ P(X_A = X_B) = \frac{1}{64} + \frac{9}{64} + \frac{9}{64} + \frac{1}{64} = \frac{1+9+9+1}{64} = \frac{20}{64}$$

Simplifying the fraction:

$$P(X_A = X_B) = \frac{5 \times 4}{16 \times 4} = \frac{5}{16}$$

30. **Answer: d**

**Explanation:**

**Step 1: Understanding the Concept:**

The probability of a combined event is the sum of probabilities of mutually exclusive outcomes. We identify all pairs of outcomes  $(d_1, d_2)$  that sum to 7.

**Step 2: Detailed Explanation:**

Let the biased faces be 1 and 6 (as  $1 + 6 = 7$ ).

$$P(1) = \frac{1}{6} - x, P(6) = \frac{1}{6} + x.$$

$$\text{Other faces: } P(2) = P(3) = P(4) = P(5) = \frac{1}{6}.$$

The pairs  $(d_1, d_2)$  summing to 7 are:  $(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)$ .

Probability of sum 7 is:

$$P(\text{sum} = 7) = P(1)P(6) + P(6)P(1) + P(2)P(5) + P(5)P(2) + P(3)P(4) + P(4)P(3)$$

$$\frac{13}{96} = 2 \left(\frac{1}{6} - x\right) \left(\frac{1}{6} + x\right) + 2 \left(\frac{1}{6}\right) \left(\frac{1}{6}\right) + 2 \left(\frac{1}{6}\right) \left(\frac{1}{6}\right)$$

$$\frac{13}{96} = 2 \left(\frac{1}{36} - x^2\right) + \frac{2}{36} + \frac{2}{36}$$

$$\frac{13}{96} = \frac{2}{36} - 2x^2 + \frac{4}{36} = \frac{6}{36} - 2x^2 = \frac{1}{6} - 2x^2$$

$$2x^2 = \frac{1}{6} - \frac{13}{96} = \frac{16 - 13}{96} = \frac{3}{96} = \frac{1}{32}$$

$$x^2 = \frac{1}{64} \implies x = \frac{1}{8} \quad (\text{since } x > 0)$$

**Step 3: Final Answer:**

The value of  $x$  is  $1/8$ .

