Rajasthan Board Class 12, 2026 Mathematics Question Paper with Solutions

Time Allowed : 3 Hours	Maximum Marks : 100	Total questions : 24

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The paper is divided into Section A and Section B.
2. Section A includes objective-type, short answer, and long answer questions.
3. All questions in Section A are compulsory.
4. Section B contains elective questions based on the chosen topic.
5. Answers must be written legibly within the word limit.
6. Use of unfair means or electronic devices is prohibited.
7. Follow the correct format and instructions for each section.

1. If $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then $f(f(x))$ is equal to

(A) $x^{\frac{1}{3}}$

(B) x^3

(C) x

(D) $(3 - x^3)$

Correct Answer: (C) x

Solution:

Step 1: Understanding the given function.

We are given:

$$f(x) = (3 - x^3)^{\frac{1}{3}}$$

Step 2: Finding $f(f(x))$.

$f(f(x))$ means we substitute $f(x)$ into the function f.

$$f(f(x)) = [3 - (f(x))^3]^{\frac{1}{3}}$$

Step 3: Calculate $(f(x))^3$.

Since $f(x) = (3 - x^3)^{\frac{1}{3}}$, cubing both sides:

$$(f(x))^3 = 3 - x^3$$

Step 4: Substitute back.

$$f(f(x)) = [3 - (3 - x^3)]^{\frac{1}{3}}$$

Step 5: Simplify.

$$f(f(x)) = [3 - 3 + x^3]^{\frac{1}{3}}$$

$$f(f(x)) = [x^3]^{\frac{1}{3}}$$

$$f(f(x)) = x$$

Step 6: Analysis of options.

- (A) $x^{\frac{1}{3}}$: Incorrect. This would be $f(x)$, not $f(f(x))$.
- (B) x^3: Incorrect. This is the cube of x.
- (C) x: **Correct.** As derived above, $f(f(x)) = x$.
- (D) $(3 - x^3)$: Incorrect. This is $(f(x))^3$, not $f(f(x))$.

Final Answer: (C) x

Quick Tip

When a function satisfies $f(f(x)) = x$, it is called an involution. Such functions are their own inverse. Here, f is its own inverse function.

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2. The principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Correct Answer: (A) $\frac{\pi}{4}$

Solution:

Step 1: Understanding inverse sine function.

The inverse sine function, denoted as $\sin^{-1} x$ or $\arcsin x$, gives the angle whose sine is x.

Step 2: Principal value branch.

For $\sin^{-1} x$, the principal value range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$. The answer must lie in this interval.

Step 3: Recall standard angle values.

We know that:

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

Step 4: Check if it lies in principal range.

$\frac{\pi}{4} = 45^\circ$ lies within $[-\frac{\pi}{2}, \frac{\pi}{2}] = [-90^\circ, 90^\circ]$.

Step 5: Analysis of options.

- (A) $\frac{\pi}{4}$: **Correct.** Since $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $\frac{\pi}{4}$ is in the principal range.
- (B) $\frac{\pi}{6}$: Incorrect. $\sin\frac{\pi}{6} = \frac{1}{2}$, not $\frac{1}{\sqrt{2}}$.
- (C) $\frac{\pi}{3}$: Incorrect. $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, not $\frac{1}{\sqrt{2}}$.
- (D) $\frac{\pi}{2}$: Incorrect. $\sin\frac{\pi}{2} = 1$, not $\frac{1}{\sqrt{2}}$.

Final Answer: (A) $\frac{\pi}{4}$

Quick Tip

Remember standard values:

- $\sin^{-1}(0) = 0$
- $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$
- $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$
- $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$
- $\sin^{-1}(1) = \frac{\pi}{2}$

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3. A = [a]× is a square matrix, if

(A) m < n

(B) m > n

(C) m = n

(D) None of these

Correct Answer: (C) m = n

Solution:

Step 1: Understanding matrix notation.

A matrix $A = [a_{ij}]_{m\times n}$ has m rows and n columns. The order of the matrix is $m \times n$.

Step 2: Definition of a square matrix.

A square matrix is a matrix that has the same number of rows and columns.

Step 3: Condition for square matrix.

For a matrix to be square:

$$\text{Number of rows} = \text{Number of columns}$$

$$m = n$$

Step 4: Analysis of options.

- (A) m < n: Incorrect. This means the matrix has fewer rows than columns, giving a rectangular matrix (horizontal rectangle).

- (B) m > n: Incorrect. This means the matrix has more rows than columns, giving a rectangular matrix (vertical rectangle).
- (C) m = n: **Correct.** When the number of rows equals the number of columns, the matrix is square.
- (D) None of these: Incorrect. Since option (C) is correct, this option is invalid.

Step 5: Examples.

- Square matrix: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (2 × 2)
- Rectangular matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ (2 × 3, where m < n)
- Rectangular matrix: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ (3 × 2, where m > n)

Final Answer: (C) m = n

> **Quick Tip**
>
> Square matrix Number of rows = Number of columns (m = n). Examples: 2×2, 3×3, 4×4 matrices. Only square matrices have determinants and inverses!

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4. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$**, then the** $(2A - B)$ **will be**

(A) $\begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 5 & -1 & 3 \\ 5 & 6 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \end{bmatrix}$

(D) $\begin{bmatrix} -3 & 1 & -3 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$

Solution:

Step 1: Understanding the given matrices.

We have:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

Both matrices are of order 2×3, so matrix operations are possible.

Step 2: Calculate $2A$.

$$2A = 2 \times \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 \times 1 & 2 \times 2 & 2 \times 3 \\ 2 \times 2 & 2 \times 3 & 2 \times 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \end{bmatrix}$$

Step 3: Calculate $2A - B$.

Subtract corresponding elements:

$$2A - B = \begin{bmatrix} 2-3 & 4-(-1) & 6-3 \\ 4-(-1) & 6-0 & 2-2 \end{bmatrix}$$

Step 4: Simplify each element.

First row:

$$2 - 3 = -1$$

$$4 - (-1) = 4 + 1 = 5$$

$$6 - 3 = 3$$

Second row:

$$4 - (-1) = 4 + 1 = 5$$

$$6 - 0 = 6$$

$$2 - 2 = 0$$

Step 5: Resulting matrix.

$$2A - B = \begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$$

Step 6: Analysis of options.

- (A) $\begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$: **Correct.** Matches our calculated result.
- (B) $\begin{bmatrix} 5 & -1 & 3 \\ 5 & 6 & 0 \end{bmatrix}$: Incorrect. First element is 5 instead of -1.
- (C) $\begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \end{bmatrix}$: Incorrect. This is $2A$, not $2A - B$.
- (D) $\begin{bmatrix} -3 & 1 & -3 \end{bmatrix}$: Incorrect. This is a row matrix of wrong order.

Final Answer: (A) $\begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$

> **Quick Tip**
>
> Matrix subtraction: Subtract corresponding elements. Order of matrices must be same for addition/subtraction. Scalar multiplication: Multiply each element by the scalar.

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5. Value of $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$ **will be**

(A) $x^2 - x + 2$

(B) $x^3 + x^2 - 2$

(C) $x^3 - x^2 + 2$

(D) $x^3 + x^2 + 4$

Correct Answer: (C) $x^3 - x^2 + 2$

Solution:

Step 1: Recall determinant formula for 2×2 matrix.

For a 2×2 matrix $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$, the determinant is:

$$\text{det} = ad - bc$$

Step 2: Identify elements.

Here:

$$a = x^2 - x + 1, \quad b = x - 1, \quad c = x + 1, \quad d = x + 1$$

Step 3: Apply the formula.

$$\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} = (x^2 - x + 1)(x + 1) - (x - 1)(x + 1)$$

Step 4: Factor common term.

Notice that $(x - 1)(x + 1)$ is common in the subtraction:

$$= (x + 1)[(x^2 - x + 1) - (x - 1)]$$

Step 5: Simplify inside the brackets.

$$(x^2 - x + 1) - (x - 1) = x^2 - x + 1 - x + 1$$

$$= x^2 - 2x + 2$$

Step 6: Multiply.

$$= (x + 1)(x^2 - 2x + 2)$$

Step 7: Expand.

$$= x(x^2 - 2x + 2) + 1(x^2 - 2x + 2)$$

$$= x^3 - 2x^2 + 2x + x^2 - 2x + 2$$

$$= x^3 - x^2 + 2$$

Step 8: Analysis of options.

- (A) $x^2 - x + 2$: Incorrect. This is not the expanded form.
- (B) $x^3 + x^2 - 2$: Incorrect. Sign of x^2 is positive, but should be negative.
- (C) $x^3 - x^2 + 2$: **Correct.** Matches our calculated result.
- (D) $x^3 + x^2 + 4$: Incorrect. Both signs and constant term are wrong.

Final Answer: (C) $x^3 - x^2 + 2$

Quick Tip

For 2×2 determinants: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$. Factor common terms when possible to simplify calculation.

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6. If $2x + 3y = \sin y$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{3}{\sin y - 2}$

(B) $\frac{2}{\cos y - 3}$

(C) $\frac{\cos y + 3}{2}$

(D) $\frac{2}{\cos y}$

Correct Answer: (B) $\frac{2}{\cos y - 3}$

Solution:

Step 1: Given equation.

We have:

$$2x + 3y = \sin y$$

Step 2: Differentiate both sides with respect to x.

Note that y is a function of x, so we use implicit differentiation.

$$\frac{d}{dx}(2x) + \frac{d}{dx}(3y) = \frac{d}{dx}(\sin y)$$

Step 3: Apply derivatives.

$$2 + 3\frac{dy}{dx} = \cos y \cdot \frac{dy}{dx}$$

Step 4: Collect terms with $\frac{dy}{dx}$.

$$2 = \cos y \cdot \frac{dy}{dx} - 3\frac{dy}{dx}$$

$$2 = \frac{dy}{dx}(\cos y - 3)$$

Step 5: Solve for $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{2}{\cos y - 3}$$

Step 6: Analysis of options.

- (A) $\frac{3}{\sin y - 2}$: Incorrect. This involves $\sin y$ instead of $\cos y$ and wrong numerator.
- (B) $\frac{2}{\cos y - 3}$: **Correct.** Matches our calculated result.
- (C) $\frac{\cos y + 3}{2}$: Incorrect. This is the reciprocal with wrong sign.
- (D) $\frac{2}{\cos y}$: Incorrect. Missing the -3 term in denominator.

Final Answer: (B) $\frac{2}{\cos y - 3}$

Quick Tip

In implicit differentiation, differentiate each term carefully. Remember:

$$\frac{d}{dx}(\sin y) = \cos y \cdot \frac{dy}{dx}$$

Then collect terms containing $\frac{dy}{dx}$ to solve.

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7. $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$, **then** $\frac{dy}{dx}$ **is equal to**

(A) $\frac{1}{1+x^2}$

(B) $\frac{2}{4+x}$

(C) $\frac{2}{1+x^2}$

(D) $\frac{2}{x+x^2}$

Correct Answer: (C) $\frac{2}{1+x^2}$

Solution:

Step 1: Recall the standard formula.

We know the identity:

$$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1} x \quad \text{for } x > 0$$

This is a standard substitution used in integration and differentiation problems.

Step 2: Verify the domain.

Given $0 < x < 1$, the identity holds true.

Step 3: Rewrite the function.

Using the identity:

$$y = 2\tan^{-1} x$$

Step 4: Differentiate with respect to x.

$$\frac{dy}{dx} = 2 \cdot \frac{d}{dx}(\tan^{-1} x)$$

We know that:

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

Step 5: Apply the derivative.

$$\frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2} = \frac{2}{1+x^2}$$

Step 6: Analysis of options.

- (A) $\frac{1}{1+x^2}$: Incorrect. This is the derivative of $\tan^{-1} x$ without the factor 2.
- (B) $\frac{2}{4+x}$: Incorrect. This does not match our derived expression.

- (C) $\frac{2}{1+x^2}$: **Correct.** Matches our calculated result.
- (D) $\frac{2}{x+x^2}$: Incorrect. This would be $\frac{2}{x(1+x)}$, which is not correct.

Final Answer: (C) $\frac{2}{1+x^2}$

Quick Tip

Important identities:

- $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$ for $x > 0$
- $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$ for $|x| \leq 1$
- $\tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$ for $|x| < 1$

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8. The rate of change of the area of a circle with respect to its radius r at $r = 5$ cm is

(A) 12π

(B) 8π

(C) 5π

(D) 10π

Correct Answer: (D) 10π

Solution:

Step 1: Recall the formula for area of a circle.

The area A of a circle with radius r is given by:

$$A = \pi r^2$$

Step 2: Rate of change of area with respect to radius.

The rate of change of area with respect to radius is the derivative $\frac{dA}{dr}$.

$$\frac{dA}{dr} = \frac{d}{dr}(\pi r^2) = \pi \cdot 2r = 2\pi r$$

Step 3: Evaluate at $r = 5\,\text{cm}$.

$$\left.\frac{dA}{dr}\right|_{r=5} = 2\pi \times 5 = 10\pi$$

Step 4: Analysis of options.

- (A) 12π: Incorrect. This would be for $r = 6$.
- (B) 8π: Incorrect. This would be for $r = 4$.
- (C) 5π: Incorrect. This would be for $r = 2.5$.
- (D) 10π: **Correct.** Matches our calculated result.

Final Answer: (D) 10π

> Quick Tip
>
> Rate of change of area of circle w.r.t radius = circumference $(2\pi r)$.
>
> Rate of change of area w.r.t radius at $r = R$ = $2\pi R$.

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9. $\int \frac{dx}{\sin^2 x \cos^2 x}$ **equals**

(A) $\tan x + \sin x + c$

(B) $\tan x - \cot x + c$

(C) $\tan x \cot x + c$

(D) $2\tan x - \cot 2x + c$

Correct Answer: (B) $\tan x - \cot x + c$

Solution:

Step 1: Rewrite the integrand using trigonometric identities.

We know that:

$$\sin^2 x \cos^2 x = (\sin x \cos x)^2$$

Also, $\sin x \cos x = \frac{\sin 2x}{2}$

Therefore:

$$\sin^2 x \cos^2 x = \left(\frac{\sin 2x}{2}\right)^2 = \frac{\sin^2 2x}{4}$$

Step 2: Alternative approach using standard identity.

A more efficient method is to use:

$$\frac{1}{\sin^2 x \cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$$

Step 3: Split the fraction.

$$\frac{1}{\sin^2 x \cos^2 x} = \frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x}$$

$$= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$$

$$= \sec^2 x + \csc^2 x$$

Step 4: Integrate.

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \int \sec^2 x \, dx + \int \csc^2 x \, dx$$

We know:

$$\int \sec^2 x \, dx = \tan x + c_1$$

$$\int \csc^2 x \, dx = -\cot x + c_2$$

Step 5: Combine the results.

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \tan x - \cot x + c$$

Step 6: Analysis of options.

- (A) $\tan x + \sin x + c$: Incorrect. $\sin x$ is not obtained from integration.
- (B) $\tan x - \cot x + c$: **Correct.** Matches our calculated result.
- (C) $\tan x \cot x + c$: Incorrect. $\tan x \cot x = 1$, so this would be constant + c, which is wrong.
- (D) $2 \tan x - \cot 2x + c$: Incorrect. This does not match our result.

Final Answer: (B) $\tan x - \cot x + c$

Quick Tip

Useful identity: $1 = \sin^2 x + \cos^2 x$ to split fractions.

Remember: $\int \sec^2 x\, dx = \tan x + c$ and $\int \csc^2 x\, dx = -\cot x + c$

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10. Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

(A) 2

(B) $\frac{4}{9}$

(C) $\frac{9}{4}$

(D) $\frac{9}{2}$

Correct Answer: (C) $\frac{9}{4}$

Solution:

Step 1: Understand the region.

The curve is $y^2 = 4x$, which is a rightward opening parabola with vertex at (0,0).

The y-axis is the line $x = 0$.

The line $y = 3$ is a horizontal line.

The region is bounded by:

- The parabola $y^2 = 4x$
- The y-axis $(x = 0)$
- The horizontal line $y = 3$

Step 2: Express x in terms of y.

From $y^2 = 4x$, we get:

$$x = \frac{y^2}{4}$$

Step 3: Determine the limits of integration.

The region extends from the y-axis $(x = 0)$ to the curve $x = \frac{y^2}{4}$.

The y-values vary from $y = 0$ (vertex) to $y = 3$ (given line).

Step 4: Set up the integral for area.

Area = $\int_{y=0}^{3} x\,dy = \int_0^3 \frac{y^2}{4}\,dy$

Step 5: Evaluate the integral.

$$\text{Area} = \frac{1}{4}\int_0^3 y^2\,dy = \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$$

$$= \frac{1}{4}\cdot\frac{27}{3} = \frac{1}{4}\cdot 9 = \frac{9}{4}$$

Step 6: Analysis of options.

- (A) 2: Incorrect. This would be $\frac{8}{4}$, not matching.
- (B) $\frac{4}{9}$: Incorrect. This is the reciprocal of the correct answer.
- (C) $\frac{9}{4}$: **Correct.** Matches our calculated result.
- (D) $\frac{9}{2}$: Incorrect. This would be $\frac{18}{4}$, double the correct answer.

Final Answer: (C) $\frac{9}{4}$

Quick Tip

When finding area bounded by a curve and the y-axis, integrate with respect to y:

$$\text{Area} = \int_{y=a}^{y=b} x\,dy = \int_{y=a}^{y=b} f(y)\,dy$$

where $x = f(y)$ is the equation of the curve.

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11. The order of the differential equation $2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ **is**

(A) 2

(B) 1

(C) 0

(D) Not defined

Correct Answer: (A) 2

Solution:

Step 1: Recall the definition of order of a differential equation.

The order of a differential equation is the highest order derivative present in the equation.

Step 2: Identify the derivatives in the given equation.

The given differential equation is:

$$2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$$

The derivatives present are:

- $\frac{d^2y}{dx^2}$ (second derivative)
- $\frac{dy}{dx}$ (first derivative)

Step 3: Determine the highest order derivative.

The highest order derivative in the equation is $\frac{d^2y}{dx^2}$, which is of order 2.

Step 4: Analysis of options.

- (A) 2: **Correct.** The highest derivative is of order 2.
- (B) 1: Incorrect. This would be the order if only first derivative was present.
- (C) 0: Incorrect. Order 0 would mean no derivatives, which is not the case.
- (D) Not defined: Incorrect. The order is clearly defined as 2.

Final Answer: (A) 2

Quick Tip

Order = Highest order derivative present in the equation.

Degree = Power of the highest order derivative (after rationalizing).

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12. Which of the following is a vector quantity?

(A) Time

(B) Volume

(C) Force

(D) Speed

Correct Answer: (C) Force

Solution:

Step 1: Understanding scalar and vector quantities.

- **Scalar quantities:** Have only magnitude (size or amount). Examples: mass, time, temperature, volume, speed.
- **Vector quantities:** Have both magnitude and direction. Examples: force, velocity, displacement, acceleration, momentum.

Step 2: Analysis of each option.

- (A) Time: **Scalar quantity.** Time has only magnitude (e.g., 5 seconds) and no direction.
- (B) Volume: **Scalar quantity.** Volume has only magnitude (e.g., 10 liters) and no direction.
- (C) Force: **Vector quantity.** Force has both magnitude (e.g., 10 Newtons) and direction (e.g., upward, eastward).
- (D) Speed: **Scalar quantity.** Speed has only magnitude (e.g., 50 km/h) and no direction. (Velocity is the vector counterpart.)

Step 3: Conclusion.

Among the given options, force is the only vector quantity as it requires both magnitude and direction for complete description.

Final Answer: (C) Force

Quick Tip

Common vector quantities: Force, Velocity, Displacement, Acceleration, Momentum, Weight.

Common scalar quantities: Mass, Time, Volume, Speed, Distance, Temperature, Energy.

13. The sum of the vectors $a = i - 2j + k$, $b = -2i + 4j + 5k$ and $c = i - 6j - 7k$ is

(A) $-4j - k$

(B) $4i - k$

(C) $4j + 5k$

(D) $i + 4j - k$

Correct Answer: (A) $-4j - k$

Solution:

Step 1: Write the vectors in component form.

$$a = i - 2j + k = (1, -2, 1)$$

$$b = -2i + 4j + 5k = (-2, 4, 5)$$

$$c = i - 6j - 7k = (1, -6, -7)$$

Step 2: Add the vectors component-wise.

$$a + b + c = (1 + (-2) + 1,\ -2 + 4 + (-6),\ 1 + 5 + (-7))$$

Step 3: Calculate each component.

For i component (x-coordinate):

$$1 + (-2) + 1 = 1 - 2 + 1 = 0$$

For j component (y-coordinate):

$$-2 + 4 + (-6) = -2 + 4 - 6 = -4$$

For k component (z-coordinate):

$$1 + 5 + (-7) = 1 + 5 - 7 = -1$$

Step 4: Write the resultant vector.

$$a + b + c = 0i - 4j - k = -4j - k$$

Step 5: Analysis of options.

- (A) $-4j - k$: **Correct.** Matches our calculated result.
- (B) $4i - k$: Incorrect. This has an i component of 4, but we got 0.
- (C) $4j + 5k$: Incorrect. Both j and k components are wrong.
- (D) $i + 4j - k$: Incorrect. This has i component of 1 and j component of +4, both incorrect.

Final Answer: (A) $-4j - k$

Quick Tip

Vector addition: Add corresponding components separately.

$(a_1 i + a_2 j + a_3 k) + (b_1 i + b_2 j + b_3 k) = (a_1 + b_1)i + (a_2 + b_2)j + (a_3 + b_3)k$

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14. The unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ is

(A) $\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{5}}$
(B) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$
(C) $\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$
(D) $\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$

Correct Answer: (D) $\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$

Solution:

Step 1: Recall the formula for unit vector.

The unit vector in the direction of a vector $\vec{a}$ is given by:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

where $|\vec{a}|$ is the magnitude (length) of the vector.

Step 2: Find the magnitude of $\vec{a}$.

Given $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$, the components are:

$$a_x = 1, \quad a_y = 1, \quad a_z = 2$$

The magnitude is:

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (2)^2} = \sqrt{1+1+4} = \sqrt{6}$$

Step 3: Form the unit vector.

$$\hat{a} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}$$

Step 4: Analysis of options.

- (A) $\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{5}}$: Incorrect. Magnitude used is $\sqrt{5}$, but correct magnitude is $\sqrt{6}$.
- (B) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$: Incorrect. This is for vector $\hat{i} + \hat{j} + \hat{k}$, not our given vector.
- (C) $\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$: Incorrect. This has different components.
- (D) $\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$: **Correct.** Matches our calculated result.

Final Answer: (D) $\frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$

Quick Tip

Unit vector = Vector / Magnitude.

Magnitude of $a\hat{i} + b\hat{j} + c\hat{k}$ = $\sqrt{a^2 + b^2 + c^2}$.

Always divide by the correct magnitude!

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15. If the straight lines $\frac{x+1}{1} = \frac{y+2}{\lambda} = \frac{z-1}{-1}$ and $\frac{x-1}{-\lambda} = \frac{y+1}{2} = \frac{z+1}{1}$ are perpendicular to each other, then the value of "λ" is

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Step 1: Identify direction ratios of the lines.

For a line in symmetric form $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$, the direction ratios are (a, b, c).

For the first line:

$$\frac{x+1}{1} = \frac{y+2}{\lambda} = \frac{z-1}{-1}$$

Direction ratios: $\vec{d_1} = (1, \lambda, -1)$

For the second line:

$$\frac{x-1}{-\lambda} = \frac{y+1}{2} = \frac{z+1}{1}$$

Direction ratios: $\vec{d_2} = (-\lambda, 2, 1)$

Step 2: Condition for perpendicular lines.

Two lines are perpendicular if the dot product of their direction vectors is zero:

$$\vec{d_1} \cdot \vec{d_2} = 0$$

Step 3: Calculate the dot product.

$$(1)(-\lambda) + (\lambda)(2) + (-1)(1) = 0$$

$$-\lambda + 2\lambda - 1 = 0$$

Step 4: Solve for λ.

$$\lambda - 1 = 0$$

$$\lambda = 1$$

Step 5: Analysis of options.

- (A) 0: Incorrect. If $\lambda = 0$, dot product = $-1 \neq 0$.
- (B) 1: **Correct.** $\lambda = 1$ satisfies the perpendicular condition.
- (C) 2: Incorrect. If $\lambda = 2$, dot product = $-2 + 4 - 1 = 1 \neq 0$.
- (D) 3: Incorrect. If $\lambda = 3$, dot product = $-3 + 6 - 1 = 2 \neq 0$.

Final Answer: (B) 1

Quick Tip

For perpendicular lines: $\vec{d_1} \cdot \vec{d_2} = 0$

For parallel lines: $\vec{d_1} = k\vec{d_2}$ for some scalar k.

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16. Two cards are drawn at random without replacement from a pack of 52 playing cards, then the probability that both the cards are black in color is:

(A) $\frac{1}{2}$

(B) $\frac{1}{12}$

(C) $\frac{25}{102}$

(D) $\frac{1}{4}$

Correct Answer: (C) $\frac{25}{102}$

Solution:

Step 1: Understand the composition of a deck.

A standard deck of 52 playing cards has:

- 26 red cards (13 hearts + 13 diamonds)
- 26 black cards (13 spades + 13 clubs)

Step 2: Probability using combination method.

Total number of ways to draw 2 cards from 52 cards:

$$\text{Total outcomes} = \binom{52}{2}$$

Number of ways to draw 2 black cards from 26 black cards:

$$\text{Favorable outcomes} = \binom{26}{2}$$

Step 3: Calculate the probability.

$$P(\text{both black}) = \frac{\binom{26}{2}}{\binom{52}{2}}$$

$$\binom{26}{2} = \frac{26 \times 25}{2} = 325$$

$$\binom{52}{2} = \frac{52 \times 51}{2} = 1326$$

$$P(\text{both black}) = \frac{325}{1326}$$

Step 4: Simplify the fraction.

Divide numerator and denominator by 13:

$$\frac{325 \div 13}{1326 \div 13} = \frac{25}{102}$$

Step 5: Alternative method - sequential probability.

Probability first card is black = $\frac{26}{52} = \frac{1}{2}$

Probability second card is black (given first was black) = $\frac{25}{51}$

$$P(\text{both black}) = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

Step 6: Analysis of options.

- (A) $\frac{1}{2}$: Incorrect. This is probability of first card being black only.
- (B) $\frac{1}{12}$: Incorrect. This does not match our calculation.
- (C) $\frac{25}{102}$: **Correct.** Matches our calculated result.
- (D) $\frac{1}{4}$: Incorrect. This would be $\frac{26}{104}$, not correct.

Final Answer: (C) $\frac{25}{102}$

Quick Tip

For "without replacement" problems:

- Use combinations: $\frac{\binom{\text{favorable}}{\text{draws}}}{\binom{\text{total}}{\text{draws}}}$
- Or multiply sequential probabilities: $\frac{26}{52} \times \frac{25}{51}$

17. If a pair of dice is thrown, then the probability of getting an even prime number on each die will be

(A) $\frac{1}{3}$

(B) $\frac{1}{12}$

(C) $\frac{1}{36}$

(D) 0

Correct Answer: (D) 0

Solution:

Step 1: Understand what is an even prime number.

Prime numbers are numbers greater than 1 that have exactly two factors: 1 and itself.

The prime numbers are: 2, 3, 5, 7, 11, 13, ...

Even numbers are numbers divisible by 2.

Step 2: Identify the only even prime number.

The only even prime number is 2, because:

- 2 is divisible by 2 (so it's even)
- 2 has exactly two factors: 1 and 2 (so it's prime)
- Any other even number greater than 2 is divisible by 2 and at least one other number, hence not prime

Step 3: Possible outcomes when throwing a pair of dice.

When a pair of dice is thrown, each die shows a number from 1 to 6.

The possible numbers on each die are: 1, 2, 3, 4, 5, 6.

Step 4: Check if "even prime number" appears on dice.

The only even prime number is 2.

The number 2 appears on a standard die.

So, getting an even prime number on a die means getting the number 2.

Step 5: Probability of getting 2 on each die.

The event "getting an even prime number on each die" means getting (2, 2).

Total number of outcomes when throwing two dice = $6 \times 6 = 36$

Number of favorable outcomes = 1 (only (2, 2))

$$P = \frac{1}{36}$$

Wait, this gives $\frac{1}{36}$, but let's double-check the question carefully.

Step 6: Re-examine the question.

The question asks: "probability of getting an even prime number on each die"

But there is a catch: Is 2 considered an even prime number? Yes, mathematically it is.

However, sometimes in such questions, the trick is that the only even prime is 2, and getting 2 on both dice gives probability $\frac{1}{36}$, which is option (C).

But option (D) is 0. Which one is correct?

Step 7: Check if there's any trick.

Actually, the number 2 is indeed an even prime number. So (2, 2) is a valid outcome.

But let's verify: On a standard die, the numbers are 1, 2, 3, 4, 5, 6. Among these, the prime numbers are 2, 3, 5. Among these primes, the only even number is 2.

So "even prime number" on a die means the number 2.

Therefore, probability = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

This suggests option (C) is correct.

However, I notice in some contexts, people mistakenly think there is no even prime, but mathematically there is (2).

Step 8: Conclusion based on mathematical fact.

Since 2 is an even prime number, the probability is $\frac{1}{36}$.

Final Answer: (C) $\frac{1}{36}$

Quick Tip

Remember: 2 is the only even prime number!

When a pair of dice is thrown, total outcomes = 36.

Favorable outcome for (2,2) = 1, so probability = $\frac{1}{36}$.

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18. $\sin^{-1} x$ **is a function whose domain is ________.**

Solution:

Step 1: Recall the definition of inverse sine function.

The inverse sine function, denoted by $\sin^{-1} x$ or $\arcsin x$, is defined as the inverse of the sine function restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Step 2: Determine the domain of $\sin^{-1} x$.

The domain of a function is the set of all possible input values (x) for which the function is defined.

For $\sin^{-1} x$ to be defined, x must be in the range of the sine function. The sine function $\sin\theta$ takes values between -1 and 1 inclusive for all real θ.

Therefore, the domain of $\sin^{-1} x$ is:

$$x \in [-1, 1]$$

Step 3: Verify with examples.

- $\sin^{-1}(0.5)$ is defined (equals $\frac{\pi}{6}$)
- $\sin^{-1}(1)$ is defined (equals $\frac{\pi}{2}$)
- $\sin^{-1}(-1)$ is defined (equals $-\frac{\pi}{2}$)
- $\sin^{-1}(2)$ is not defined because 2 is outside $[-1, 1]$

Step 4: Final answer.

$$\boxed{[-1, 1]}$$

Quick Tip

Domain of $\sin^{-1} x$ is $[-1, 1]$ (all real numbers from -1 to 1 inclusive). Range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Remember: input to inverse trig functions must be within the range of the original trig function.

19. The value of determinant $\Delta = \begin{vmatrix} 1 & -2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$ **is** -5**.**

Solution:

Step 1: Recall the formula for determinant of a 3×3 matrix.

For a matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, the determinant can be expanded along any row or column. Expanding along the first row:

$$|A| = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Step 2: Identify the elements of the given determinant.

$$\Delta = \begin{vmatrix} 1 & -2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$$

Here:

$$a_{11} = 1, \quad a_{12} = -2, \quad a_{13} = 4$$

$$a_{21} = -1, \quad a_{22} = 3, \quad a_{23} = 0$$

$$a_{31} = 4, \quad a_{32} = 1, \quad a_{33} = 0$$

Step 3: Choose the easiest method for calculation.

Notice that the third column has two zeros ($a_{23} = 0$ and $a_{33} = 0$). It will be easiest to expand along the third column, as it will simplify calculations.

Expanding along column 3:

$$\Delta = a_{13} \cdot C_{13} + a_{23} \cdot C_{23} + a_{33} \cdot C_{33}$$

where C_{ij} is the cofactor of element a_{ij}.

Since $a_{23} = 0$ and $a_{33} = 0$, only the first term remains:

$$\Delta = a_{13} \cdot C_{13}$$

Step 4: Find the cofactor C_{13}.

The cofactor $C_{13} = (-1)^{1+3} \times M_{13} = (+1) \times M_{13}$, where M_{13} is the minor (determinant of the matrix obtained by removing row 1 and column 3).

Remove row 1 and column 3:

$$M_{13} = \begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix}$$

Step 5: Calculate M_{13}.

For a 2×2 matrix $\begin{vmatrix} p & q \\ r & s \end{vmatrix}$, the determinant is $ps - qr$.

$$M_{13} = (-1)(1) - (3)(4) = -1 - 12 = -13$$

Therefore:

$$C_{13} = (-1)^{1+3} \times (-13) = 1 \times (-13) = -13$$

Step 6: Calculate Δ.

$$\Delta = a_{13} \cdot C_{13} = 4 \times (-13) = -52$$

Step 7: Compare with the given value.

We found $\Delta = -52$, but the problem states that the value is -5. This suggests there might be an error in the problem statement or in our understanding.

Let's verify by expanding along the first row as a check.

Step 8: Verify by expanding along the first row.

$$\Delta = 1 \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} - (-2) \begin{vmatrix} -1 & 0 \\ 4 & 0 \end{vmatrix} + 4 \begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix}$$

Calculate each 2×2 determinant:

$$\begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = (3 \times 0) - (0 \times 1) = 0 - 0 = 0$$

$$\begin{vmatrix} -1 & 0 \\ 4 & 0 \end{vmatrix} = (-1 \times 0) - (0 \times 4) = 0 - 0 = 0$$

$$\begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix} = (-1 \times 1) - (3 \times 4) = -1 - 12 = -13$$

Now substitute:

$$\Delta = 1(0) + 2(0) + 4(-13) = 0 + 0 - 52 = -52$$

This confirms our calculation: $\Delta = -52$.

Step 9: Conclusion.

The given statement ”The value of determinant is -5” is **false**. The correct value is -52.

$$\boxed{\text{The correct value is } -52, \text{ not } -5.}$$

Quick Tip

When a column (or row) has many zeros, expand along that column/row to simplify determinant calculation. Always verify by expanding along another row/column to ensure accuracy.

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20. The edge of a variable cube is increasing at the rate of 3 cm/s. The volume of the cube is increasing at the rate of ________________ while the edge is 10 cm long.

Solution:

Step 1: Identify the given information.

Let x be the length of the edge of the cube (in cm). Given: The rate of increase of the edge:

$$\frac{dx}{dt} = 3\,\text{cm/s}$$

We need to find the rate of increase of volume V when $x = 10$ cm.

Step 2: Recall the formula for volume of a cube.

Volume of a cube with edge length x is:

$$V = x^3$$

Step 3: Differentiate with respect to time t.

Using the chain rule:

$$\frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt}$$
$$\frac{dV}{dx} = \frac{d}{dx}(x^3) = 3x^2$$

Therefore:

$$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$$

Step 4: Substitute the given values.

When $x = 10$ cm and $\frac{dx}{dt} = 3$ cm/s:

$$\frac{dV}{dt} = 3(10)^2 \times 3$$

$$\frac{dV}{dt} = 3 \times 100 \times 3$$

$$\frac{dV}{dt} = 900\,\text{cm}^3/\text{s}$$

Step 5: Final answer.

The volume of the cube is increasing at the rate of:

$$\boxed{900\,\text{cm}^3/\text{s}}$$

Quick Tip

For a cube with edge x, volume $V = x^3$. Rate of change of volume: $\frac{dV}{dt} = 3x^2\frac{dx}{dt}$. When edge increases at 3 cm/s and $x = 10$ cm, $\frac{dV}{dt} = 3(100)(3) = 900$ cm³/s.

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21. $\int (2x - 3\cos x + e^x)\,dx =$ ____________________.

Solution:

Step 1: Recall the basic integration formulas.

We need to integrate each term separately using standard formulas:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$$

$$\int \cos x\,dx = \sin x + C$$

$$\int e^x\,dx = e^x + C$$

Step 2: Apply the linearity property of integration.

The integral of a sum/difference is the sum/difference of the integrals:

$$\int (2x - 3\cos x + e^x)\,dx = \int 2x\,dx - \int 3\cos x\,dx + \int e^x\,dx$$

Step 3: Integrate each term.

$$\int 2x\,dx = 2\int x\,dx = 2\cdot\frac{x^2}{2} + C_1 = x^2 + C_1$$

$$\int 3\cos x\,dx = 3\int \cos x\,dx = 3\sin x + C_2$$

$$\int e^x\,dx = e^x + C_3$$

Step 4: Combine the results.

$$\int (2x - 3\cos x + e^x)\,dx = x^2 - 3\sin x + e^x + C$$

where $C = C_1 + C_2 + C_3$ is the constant of integration.

Step 5: Final answer.

$$\boxed{x^2 - 3\sin x + e^x + C}$$

Quick Tip

Integrate term by term: $\int 2x\,dx = x^2$ $\int -3\cos x\,dx = -3\sin x$ $\int e^x\,dx = e^x$ Don't forget the constant of integration $+C$!
