Rajasthan JET Mathematics Sample Paper-10

Duration: 40 Minutes **Maximum Marks: 160**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **−1 marks**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $A^2 - kA - 11I = 0$, where I is the identity matrix of order 2, then the value of k is:

(A) -2

(B) 2

(C) -6

(D) 6

Q2. The value of $\lim_{x\to 0} \frac{e^{\sin x} - 1 - x}{x^2}$ is:

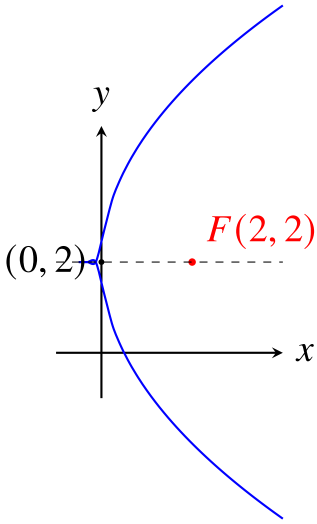
(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

Q3. The coordinates of the focus of the parabola $y^2 - 4y - 8x + 4 = 0$ are:



(A) $(0, 2)$

(B) $(2, 2)$

(C) $(2, 0)$

(D) $(-2, 2)$

Q4. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{x}{1+x^2}$. Then the range of the function $f(x)$ is:

(A) $[-1, 1]$

(B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(C) $(-\infty, \infty)$

(D) $[0, 1]$

Q5. A box contains 6 red and 4 black balls. Two balls are drawn at random one after another without replacement. The probability that both balls are red is:

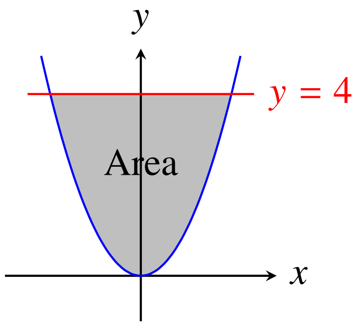
(A) $\frac{1}{3}$

(B) $\frac{3}{5}$

(C) $\frac{1}{15}$

(D) $\frac{5}{12}$

Q6. The area bounded by the curve $y = x^2$ and the line $y = 4$ is:



(A) $\frac{32}{3}$ sq. units

(B) $\frac{16}{3}$ sq. units

(C) $\frac{8}{3}$ sq. units

(D) 16 sq. units

Q7. If the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar, then the value of λ is:

(A) -2

(B) -3

(C) -4

(D) -5

Q8. The principal value of $\sin^{-1}\left(\sin\frac{4\pi}{3}\right)$ is:

(A) $\frac{4\pi}{3}$

(B) $\frac{\pi}{3}$

(C) $-\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Q9. The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observations are 1, 2, and 6, then the other two observations are:

(A) 4, 9

(B) 3, 10

(C) 5, 8

(D) 6, 7

Q10. The value of the integral $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x+\cos^{3/2} x}\, dx$ is:

(A) π

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) 0

Q11. If the n-th term of an Arithmetic Progression (A.P.) is $3n + 5$, then the sum of its first 20 terms is:

(A) 610

(B) 730

(C) 740

(D) 820

Q12. The general solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

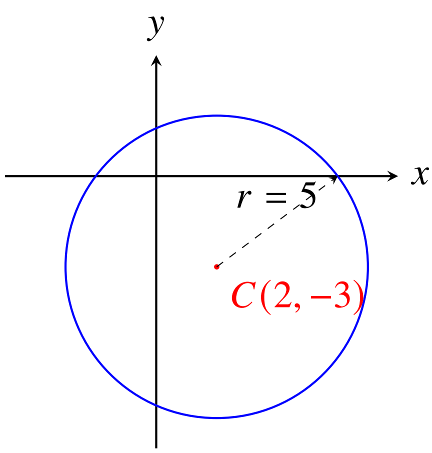
(A) $xy = \frac{x^3}{3} + C$

(B) $xy = \frac{x^4}{4} + C$

(C) $y = \frac{x^3}{4} + C$

(D) $xy^2 = \frac{x^4}{4} + C$

Q13. The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is:



(A) 5

(B) $\sqrt{13}$

(C) 25

(D) $\sqrt{3}$

Q14. If $z = \frac{1+i}{1-i}$, then the value of z^{4n} (where $n \in \mathbb{N}$) is:

(A) 1

(B) -1

(C) i

(D) $-i$

Q15. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to x is:

(A) $\frac{1}{1+x^2}$

(B) $\frac{1}{2(1+x^2)}$

(C) $\frac{2}{1+x^2}$

(D) $\frac{x}{1+x^2}$

Q16. Two dice are thrown simultaneously. The probability of getting a total score of 7 is:

(A) $\frac{1}{6}$

(B) $\frac{1}{12}$

(C) $\frac{5}{36}$

(D) $\frac{7}{36}$

Q17. The equation of the line passing through the point $(1, 2, 3)$ and parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$ is:

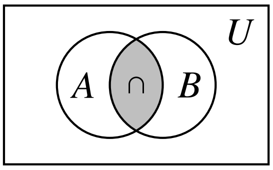
(A) $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2}$

(B) $\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+2}{3}$

(C) $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{-2}$

(D) $\frac{x-1}{-3} = \frac{y-2}{2} = \frac{z-3}{2}$

Q18. If A and B are two sets such that $n(A) = 17$, $n(B) = 23$, and $n(A \cup B) = 38$, then $n(A \cap B)$ is equal to:



(A) 2

(B) 4

(C) 6

(D) 8

Q19. The maximum value of the function $f(x) = x^3 - 3x$ on the interval $[0, 2]$ is achieved at $x =$:

(A) 0

(B) 1

(C) 2

(D) $\sqrt{3}$

Q20. If the standard deviation of a distribution is 5, then its variance is (σ^2):

(A) $\sqrt{5}$

(B) 10

(C) 25

(D) 50

Q21. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is:

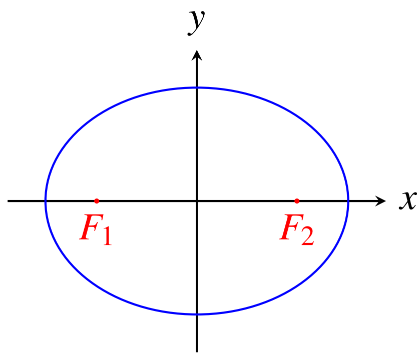
(A) 0

(B) $a + b + c$

(C) abc

(D) $(a - b)(b - c)(c - a)$

Q22. The eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is:



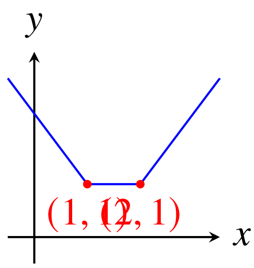
(A) $\frac{\sqrt{7}}{4}$

(B) $\frac{7}{16}$

(C) $\frac{3}{4}$

(D) $\frac{\sqrt{7}}{3}$

Q23. The function $f(x) = |x - 1| + |x - 2|$ is not differentiable at:



(A) $x = 1$ only

(B) $x = 2$ only

(C) Both $x = 1$ and $x = 2$

(D) $x = 1.5$

Q24. If $\tan\theta = \frac{3}{4}$ and θ lies in the third quadrant, then the value of $\cos\theta$ is:

(A) $\frac{4}{5}$

(B) $-\frac{4}{5}$

(C) $\frac{3}{5}$

(D) $-\frac{3}{5}$

Q25. The angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ is:

(A) $\cos^{-1}\left(\frac{1}{3}\right)$

(B) $\cos^{-1}\left(-\frac{1}{3}\right)$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Q26. The value of $\int e^x(\tan x + \ln(\sec x))\,dx$ is:

(A) $e^x \tan x + C$

(B) $e^x \ln(\sec x) + C$

(C) $e^x \sec x + C$

(D) $e^x \ln(\tan x) + C$

Q27. If the sum of an infinite geometric progression is 4 and its first term is 2, then the common ratio is:

(A) $\frac{1}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

Q28. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ are respectively:

(A) 2, 3

(B) 2, 2

(C) 1, 2

(D) 2, 1

Q29. A card is drawn at random from a well-shuffled pack of 52 cards. The probability that it is either a king or a spade is:

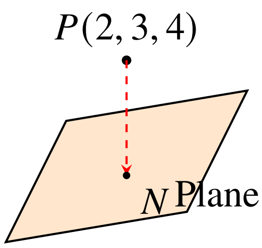
(A) $\frac{17}{52}$

(B) $\frac{4}{13}$

(C) $\frac{1}{4}$

(D) $\frac{16}{52}$

Q30. The distance of the point $(2, 3, 4)$ from the plane $3x - 6y + 2z + 11 = 0$ is:



(A) 0 units

(B) 1 unit

(C) 2 units

(D) 3 units

Q31. If ω is an imaginary cube root of unity, then the value of $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$ is:

(A) 32

(B) -32

(C) 64

(D) -64

Q32. The interval in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly decreasing is:

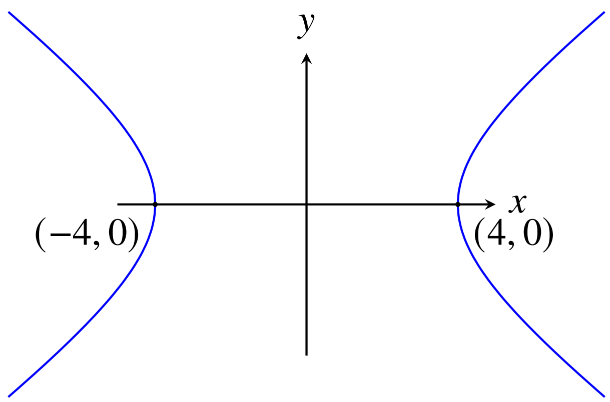
(A) $(2, 3)$

(B) $(-\infty, 2)$

(C) $(3, \infty)$

(D) $(-\infty, 2) \cup (3, \infty)$

Q33. If the length of the major axis of a hyperbola is 8 and its eccentricity is $\frac{5}{4}$, then the equation of the hyperbola in standard form is:



(A) $\frac{x^2}{16} - \frac{y^2}{9} = 1$
(B) $\frac{x^2}{9} - \frac{y^2}{16} = 1$
(C) $\frac{x^2}{16} - \frac{y^2}{25} = 1$
(D) $\frac{x^2}{25} - \frac{y^2}{16} = 1$

Q34. The domain of the function $f(x) = \sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$ is:

(A) $[1, 4]$

(B) $(1, 4)$

(C) $[0, 5]$

(D) $[-1, 4]$

Q35. The coefficient of variation for a data set is 60% and its arithmetic mean is 15. The standard deviation of the data is:

(A) 9

(B) 6

(C) 4

(D) 2.5

Q36. The value of $\int_1^2 \frac{1}{x^2+2x}\,dx$ is:

(A) $\frac{1}{2}\ln\left(\frac{3}{2}\right)$

(B) $\frac{1}{2}\ln\left(\frac{8}{3}\right)$

(C) $\frac{1}{2}\ln\left(\frac{4}{3}\right)$

(D) $\ln\left(\frac{4}{3}\right)$

Q37. If the matrix $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal, then the values of a, b, c can be:

(A) $a = \pm\frac{1}{\sqrt{2}}, b = \pm\frac{1}{\sqrt{6}}, c = \pm\frac{1}{\sqrt{3}}$

(B) $a = \pm\frac{1}{\sqrt{3}}, b = \pm\frac{1}{\sqrt{3}}, c = \pm\frac{1}{\sqrt{3}}$

(C) $a = \pm\frac{1}{2}, b = \pm\frac{1}{2}, c = \pm\frac{1}{2}$

(D) $a = \pm\frac{1}{\sqrt{2}}, b = \pm\frac{1}{\sqrt{2}}, c = \pm\frac{1}{\sqrt{2}}$

Q38. The value of $\cos 20^\circ \cos 40^\circ \cos 80^\circ$ is:

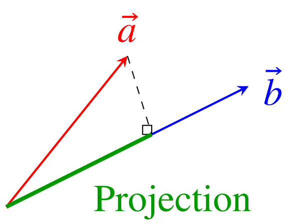
(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{8}$

(D) $\frac{1}{16}$

Q39. The projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is:



(A) $\frac{10}{\sqrt{6}}$

(B) $\frac{10}{\sqrt{17}}$

(C) $\frac{5}{\sqrt{6}}$

(D) $\sqrt{6}$

Q40. The integrating factor of the differential equation $(x\ln x)\frac{dy}{dx} + y = 2\ln x$ is:

(A) x

(B) $\ln x$

(C) e^x

(D) $\ln(\ln x)$

Detailed Solutions

Q1.

Solution

Concept: A square matrix satisfies its own characteristic equation (Cayley-Hamilton theorem), or we can directly find A^2 via matrix multiplication and solve the linear matrix equation for the unknown constant k.

Solution: Step 1: Compute A^2 by multiplying matrix A by itself:

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} = \begin{bmatrix} 2(2)+3(1) & 2(3)+3(-4) \\ 1(2)-4(1) & 1(3)-4(-4) \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ -2 & 19 \end{bmatrix}$$

Step 2: Substitute A^2, A, and the identity matrix I into $A^2 - kA - 11I = 0$:

$$\begin{bmatrix} 7 & -6 \\ -2 & 19 \end{bmatrix} - k \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} - 11 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Step 3: Simplify the matrices element-by-element into a single matrix:

$$\begin{bmatrix} -4-2k & -6-3k \\ -2-k & 8+4k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Step 4: Equate the element from the second row and first column to zero to find k:

$$-2 - k = 0 \implies k = -2$$

Step 5: Verify with the first row, second column element: $-6 - 3(-2) = 0$, confirming the solution is consistent.

Final Answer: -2

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: The limit of a $\frac{0}{0}$ indeterminate form can be evaluated systematically using L'Hopital's Rule by differentiating the numerator and denominator with respect to x until a determinate value is reached.

Solution: Step 1: Substitute $x = 0$ to verify the indeterminate form:

$$\text{Form: } \frac{e^{\sin 0} - 1 - 0}{0^2} = \frac{1-1}{0} = \frac{0}{0}$$

Step 2: Apply L'Hopital's Rule by differentiating the numerator and denominator once:

$$\lim_{x \to 0} \frac{\frac{d}{dx}\left(e^{\sin x} - 1 - x\right)}{\frac{d}{dx}\left(x^2\right)} = \lim_{x \to 0} \frac{e^{\sin x}\cos x - 1}{2x}$$

Step 3: Test $x = 0$ again. The expression yields $\frac{1(1)-1}{0} = \frac{0}{0}$, requiring a second application of L'Hopital's Rule.

Step 4: Differentiate the numerator and denominator a second time using the product rule:

$$\lim_{x \to 0} \frac{\frac{d}{dx}\left(e^{\sin x}\cos x - 1\right)}{\frac{d}{dx}(2x)} = \lim_{x \to 0} \frac{e^{\sin x}\cos^2 x - e^{\sin x}\sin x}{2}$$

Step 5: Substitute $x = 0$ into this determinate form to obtain the final value:

$$\text{Limit} = \frac{e^0(1)^2 - e^0(0)}{2} = \frac{1-0}{2} = \frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: To find the focus of a parabola whose axis is parallel to a coordinate axis, convert the given general quadratic equation into its standard parabolic form $(y-k)^2 = 4a(x-h)$ by completing the square. The focus is then given by $(h+a, k)$.

Solution: Step 1: Write down the given general equation of the parabola.

$$y^2 - 4y - 8x + 4 = 0$$

Step 2: Rearrange the terms to group the quadratic y terms together on one side of the equation.

$$y^2 - 4y = 8x - 4$$

Step 3: Complete the square on the left-hand side. Add the square of half the coefficient of y, which is $\left(\frac{-4}{2}\right)^2 = 4$, to both sides of the equation.

$$y^2 - 4y + 4 = 8x - 4 + 4$$
$$(y-2)^2 = 8x$$

Step 4: Express the right-hand side in the standard form $4a(x-h)$ to identify the parameters.

$$(y-2)^2 = 4(2)(x-0)$$

By comparing with $(y-k)^2 = 4a(x-h)$, we extract the values:

$$\text{Vertex } (h, k) = (0, 2)$$

$$\text{Focal parameter } a = 2$$

Step 5: Determine the coordinates of the focus. For a horizontal parabola opening to the right, the focus formula is defined as $F = (h+a, k)$.

$$F = (0+2, 2) = (2, 2)$$

Final Answer: $\boxed{(2, 2)}$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: The range of a real-valued rational function $y = f(x)$ can be determined by expressing the independent variable x explicitly in terms of the dependent variable y. This sets up a quadratic equation in terms of x where the discriminant must be non-negative for real solutions to exist.

Solution: Step 1: Set the function equal to y.

$$y = \frac{x}{1 + x^2}$$

Step 2: Cross-multiply to clear the denominator and rearrange the equation into a standard quadratic form in terms of x.

$$y(1 + x^2) = x$$

$$y + yx^2 = x$$

$$yx^2 - x + y = 0$$

Step 3: For x to be a real number, the discriminant ($\Delta = b^2 - 4ac$) of this quadratic equation must be greater than or equal to zero. Identify the coefficients:

$$a = y, \quad b = -1, \quad c = y$$

$$\Delta = (-1)^2 - 4(y)(y) \geq 0$$

$$1 - 4y^2 \geq 0$$

Step 4: Solve the quadratic inequality for y.

$$4y^2 \leq 1$$

$$y^2 \leq \frac{1}{4}$$

Taking the square root on both sides yields the interval:

$$-\frac{1}{2} \leq y \leq \frac{1}{2}$$

Step 5: Verify if $y = 0$ is a valid value. Substituting $y = 0$ in the quadratic expression gives $-x = 0 \implies x = 0$, which is a valid real input. Hence, $y = 0$ is included. Thus, the complete range is $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

Final Answer: $\boxed{\left[-\frac{1}{2}, \frac{1}{2}\right]}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: The probability of dependent successive events can be found using the multiplication rule of probability without replacement. The total number of favorable outcomes and the total sample space decrease progressively after each successful draw.

Solution: Step 1: Identify the initial distribution of balls in the box.

$$\text{Number of Red balls} = 6$$
$$\text{Number of Black balls} = 4$$
$$\text{Total number of balls} = 6 + 4 = 10$$

Step 2: Find the probability of drawing a red ball on the first attempt, denoted as $P(R_1)$.

$$P(R_1) = \frac{\text{Number of Red balls}}{\text{Total number of balls}} = \frac{6}{10}$$

Step 3: Since the ball is drawn without replacement, update the count of balls remaining in the box for the second draw.

$$\text{Remaining Red balls} = 6 - 1 = 5$$
$$\text{Remaining Total balls} = 10 - 1 = 9$$

Step 4: Find the probability of drawing a red ball on the second attempt given that the first was red, denoted as $P(R_2|R_1)$.

$$P(R_2|R_1) = \frac{5}{9}$$

Step 5: Compute the combined probability that both balls drawn are red using the compound probability formula.

$$P(R_1 \cap R_2) = P(R_1) \times P(R_2|R_1)$$
$$P(R_1 \cap R_2) = \frac{6}{10} \times \frac{5}{9}$$
$$P(R_1 \cap R_2) = \frac{30}{90} = \frac{1}{3}$$

Final Answer: $\boxed{\frac{1}{3}}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: The area bounded by a curve and a horizontal line can be computed using definite integration. The total area is found by integrating the difference between the line and the function with respect to x between their points of intersection, capitalizing on the geometric symmetry about the y-axis.

Solution: Step 1: Determine the intersection points between the curve $y = x^2$ and the line $y = 4$.

$$x^2 = 4 \implies x = \pm 2$$

The limits of integration are from $x = -2$ to $x = 2$.

Step 2: Set up the definite integral for the area. The upper boundary is the line $y = 4$ and the lower boundary is the parabola $y = x^2$.

$$\text{Area} = \int_{-2}^{2} (4 - x^2)\, dx$$

Step 3: Use the symmetry of the even function to rewrite the integral with a lower limit of zero.

$$\text{Area} = 2\int_{0}^{2} (4 - x^2)\, dx$$

Step 4: Find the antiderivative and evaluate it at the limits of integration.

$$\text{Area} = 2\left[4x - \frac{x^3}{3}\right]_0^2$$

$$\text{Area} = 2\left[\left(4(2) - \frac{2^3}{3}\right) - 0\right]$$

$$\text{Area} = 2\left[8 - \frac{8}{3}\right]$$

Step 5: Simplify the terms inside the bracket to get the final numerical area.

$$\text{Area} = 2\left[\frac{24 - 8}{3}\right] = 2\left[\frac{16}{3}\right] = \frac{32}{3} \text{ sq. units}$$

Final Answer: $\boxed{\frac{32}{3} \text{ sq. units}}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: Three vectors are coplanar if and only if their scalar triple product is equal to zero. This condition can be mathematically evaluated by setting the determinant formed by the components of the three vectors to zero.

Solution: Step 1: Write down the component forms of the three given vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$.

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$$

Step 2: Formulate the condition for coplanarity using the scalar triple product determinant.

$$[\vec{a} \quad \vec{b} \quad \vec{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

Step 3: Expand the determinant along the first row.

$$2\begin{vmatrix} 2 & -3 \\ \lambda & 5 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 3 & \lambda \end{vmatrix} = 0$$

Step 4: Evaluate each of the 2×2 sub-determinants explicitly.

$$2(10 - (-3\lambda)) + 1(5 - (-9)) + 1(\lambda - 6) = 0$$

$$2(10 + 3\lambda) + 1(5 + 9) + (\lambda - 6) = 0$$

$$20 + 6\lambda + 14 + \lambda - 6 = 0$$

Step 5: Combine like linear terms and solve for the unknown parameter λ.

$$7\lambda + 28 = 0$$

$$7\lambda = -28$$

$$\lambda = -4$$

Final Answer: -4

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: The principal value branch of the inverse sine function, $\sin^{-1}(x)$, is strictly restricted to the closed interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. If the given angle lies outside this domain, trigonometric reduction formulas must be used to find its equivalent angle within the standard branch.

Solution: Step 1: Analyze the given expression $\sin^{-1}\left(\sin\frac{4\pi}{3}\right)$. The angle $\frac{4\pi}{3}$ is in the third quadrant and does not lie within the principal value range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Step 2: Use trigonometric identities to rewrite $\sin\left(\frac{4\pi}{3}\right)$ using an angle related to the horizontal axis.

$$\frac{4\pi}{3} = \pi + \frac{\pi}{3}$$

Step 3: Apply the third quadrant reduction identity $\sin(\pi + \theta) = -\sin\theta$.

$$\sin\left(\pi + \frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$$

Step 4: Use the property of the sine function where negative signs can be moved inside the angle, $-\sin\theta = \sin(-\theta)$.

$$-\sin\left(\frac{\pi}{3}\right) = \sin\left(-\frac{\pi}{3}\right)$$

Step 5: Substitute this equivalent form back into the original inverse function expression.

$$\sin^{-1}\left(\sin\frac{4\pi}{3}\right) = \sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$$

Since $-\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the inverse sine cancels the sine function directly.

$$\text{Principal Value} = -\frac{\pi}{3}$$

Final Answer: $\boxed{-\frac{\pi}{3}}$

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept: The mean and variance of a dataset provide two independent equations that can be used to solve for two unknown values. The mean deals with the sum of all terms, while the variance deals with the sum of the squares of all terms.

Solution: Step 1: Let the two unknown observations be x and y. Write the formula for the mean of the 5 numbers.

$$\text{Mean } (\bar{x}) = \frac{1+2+6+x+y}{5} = 4.4$$

$$9 + x + y = 4.4 \times 5 = 22$$

$$x + y = 22 - 9 = 13 \quad \text{— (Equation 1)}$$

Step 2: Set up the formula for the variance of the data.

$$\text{Variance } (\sigma^2) = \frac{\sum x_i^2}{5} - (\bar{x})^2 = 8.24$$

Step 3: Calculate the sum of squares of the known terms and write out the full variance equation.

$$\frac{1^2 + 2^2 + 6^2 + x^2 + y^2}{5} - (4.4)^2 = 8.24$$

$$\frac{1 + 4 + 36 + x^2 + y^2}{5} - 19.36 = 8.24$$

$$\frac{41 + x^2 + y^2}{5} = 8.24 + 19.36 = 27.6$$

Step 4: Solve for the sum of squares $x^2 + y^2$.

$$41 + x^2 + y^2 = 27.6 \times 5 = 138$$

$$x^2 + y^2 = 138 - 41 = 97 \quad \text{— (Equation 2)}$$

Step 5: Use the identity $(x+y)^2 = x^2 + y^2 + 2xy$ to find the product xy.

$$(13)^2 = 97 + 2xy \implies 169 = 97 + 2xy$$

$$2xy = 72 \implies xy = 36$$

We look for two numbers whose sum is 13 and product is 36. Solving the quadratic $t^2 - 13t + 36 = 0$ gives $t = 4$ and $t = 9$. Thus, the remaining values are 4 and 9.

Final Answer: 4, 9

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: Definite integrals of this form can be efficiently solved using King's Property of integration: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$. This transforms the integrand by converting sine functions to cosines and vice-versa, allowing for basic algebraic simplification when added together.

Solution: Step 1: Let the given integral be denoted as I.

$$I = \int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\,dx \quad \text{— (Equation 1)}$$

Step 2: Apply King's Property by substituting x with $\left(0 + \frac{\pi}{2} - x\right) = \frac{\pi}{2} - x$ into the integral.

$$I = \int_0^{\pi/2} \frac{\sin^{3/2}\left(\frac{\pi}{2} - x\right)}{\sin^{3/2}\left(\frac{\pi}{2} - x\right) + \cos^{3/2}\left(\frac{\pi}{2} - x\right)}\,dx$$

Step 3: Use the complementary trigonometric identities $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ and $\cos\left(\frac{\pi}{2} - x\right) = \sin x$ to simplify the expression.

$$I = \int_0^{\pi/2} \frac{\cos^{3/2} x}{\cos^{3/2} x + \sin^{3/2} x}\,dx \quad \text{— (Equation 2)}$$

Step 4: Add Equation 1 and Equation 2 together. Because they share identical limits and denominators, their numerators can be directly combined.

$$2I = \int_0^{\pi/2} \frac{\sin^{3/2} x + \cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\,dx$$

$$2I = \int_0^{\pi/2} 1\,dx$$

Step 5: Integrate the simplified expression and solve for I.

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (C)

Go Back to Question 10

Q11.

Solution

Concept: The sum of the first n terms of an Arithmetic Progression (A.P.) can be found using the formula $S_n = \frac{n}{2}[a + a_n]$, where a represents the first term and a_n is the n-th term. Alternatively, the formula $S_n = \frac{n}{2}[2a + (n-1)d]$ can be utilized.

Solution: Step 1: Identify the given expression for the general n-th term of the series.

$$a_n = 3n + 5$$

Step 2: Compute the first term (a_1) by substituting $n = 1$ into the general formula.

$$a = a_1 = 3(1) + 5 = 8$$

Step 3: Compute the twentieth term (a_{20}) by substituting $n = 20$ into the general formula.

$$a_{20} = 3(20) + 5 = 60 + 5 = 65$$

Step 4: Substitute the values of $n = 20$, $a = 8$, and $a_{20} = 65$ into the dynamic arithmetic series sum formula.

$$S_{20} = \frac{20}{2} \cdot [a_1 + a_{20}]$$
$$S_{20} = 10 \cdot [8 + 65]$$

Step 5: Perform the final scalar multiplication to obtain the numerical sum.

$$S_{20} = 10 \cdot 73 = 730$$

Final Answer: $\boxed{730}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept: The given equation is a first-order linear differential equation matching the standard structure $\frac{dy}{dx} + P(x)y = Q(x)$. The solution is systematically obtained by calculating the Integrating Factor, $I.F. = e^{\int P(x)\,dx}$, and using the formula $y \cdot (I.F.) = \int Q(x) \cdot (I.F.)\,dx$.

Solution: Step 1: Compare the given equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ with the standard linear form to identify the function components.

$$P(x) = \frac{1}{x}, \quad Q(x) = x^2$$

Step 2: Compute the Integrating Factor $(I.F.)$.

$$I.F. = e^{\int \frac{1}{x}\,dx} = e^{\ln x} = x$$

Step 3: Write out the general solution structure utilizing the calculated $I.F.$ value.

$$y \cdot (I.F.) = \int Q(x) \cdot (I.F.)\,dx + C$$
$$y \cdot x = \int x^2 \cdot x\,dx + C$$
$$xy = \int x^3\,dx + C$$

Step 4: Perform the explicit integration of the right-hand term using the standard power rule.

$$\int x^3\,dx = \frac{x^4}{4}$$

Step 5: Assemble the integrated parts back into the complete algebraic expression.

$$xy = \frac{x^4}{4} + C$$

Final Answer: $\boxed{xy = \frac{x^4}{4} + C}$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: The general equation of a circle is expressed as $x^2 + y^2 + 2gx + 2fy + c = 0$. The radius r can be directly computed from this standard representation using the square root relation $r = \sqrt{g^2 + f^2 - c}$.

Solution: Step 1: Write down the given algebraic equation of the circle.

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

Step 2: Match the given coefficients with the general equation parameters to determine g, f, and c.

$$2g = -4 \implies g = -2$$

$$2f = 6 \implies f = 3$$

$$\text{Constant term } c = -12$$

Step 3: Write down the analytic formula used for computing the radius of a circle.

$$r = \sqrt{g^2 + f^2 - c}$$

Step 4: Substitute the values of g, f, and c into the equation. Take note of any negative signs.

$$r = \sqrt{(-2)^2 + (3)^2 - (-12)}$$

$$r = \sqrt{4 + 9 + 12}$$

Step 5: Compute the sum inside the radical and evaluate the final square root value.

$$r = \sqrt{25} = 5$$

Final Answer: $\boxed{5}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: To evaluate higher powers of a complex number fraction, simplify the base expression into its standard algebraic Cartesian form $(a + ib)$ by rationalizing the denominator. Once the base is simplified, standard properties of the imaginary unit i are used.

Solution: Step 1: Take the base complex number z and multiply both its numerator and denominator by the complex conjugate of the denominator.

$$z = \frac{1+i}{1-i}$$

The conjugate of $(1-i)$ is $(1+i)$.

$$z = \frac{(1+i)(1+i)}{(1-i)(1+i)}$$

Step 2: Expand the numerator and denominator expressions using basic algebra.

$$\text{Numerator: } (1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$$

$$\text{Denominator: } (1-i)(1+i) = 1^2 - i^2 = 1 - (-1) = 2$$

Step 3: Simplify the resulting fraction to determine the final compact form of z.

$$z = \frac{2i}{2} = i$$

Step 4: Substitute $z = i$ into the required expression z^{4n}.

$$z^{4n} = (i)^{4n}$$

Step 5: Apply standard exponent rules to separate the terms. Since $i^4 = 1$, any integer multiple power of 4 will yield unity.

$$(i)^{4n} = (i^4)^n = (1)^n = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: Trigonometric substitution simplifies inverse trigonometric expressions before differentiating. Substituting $x = \tan\theta$ allows the use of trigonometric identities to reduce the internal rational term into a linear form.

Solution: Step 1: Let the given function be $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$. Introduce the substitution $x = \tan\theta$, which implies $\theta = \tan^{-1} x$.

Step 2: Substitute $x = \tan\theta$ into the function and simplify using the fundamental identity $1 + \tan^2\theta = \sec^2\theta$.

$$y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

Step 3: Convert the secant and tangent functions into basic sine and cosine components.

$$y = \tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right) = \tan^{-1}\left(\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right)$$

$$y = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

Step 4: Apply half-angle formulas: $1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$ and $\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$.

$$y = \tan^{-1}\left(\frac{2\sin^2(\theta/2)}{2\sin(theta/2)\cos(\theta/2)}\right) = \tan^{-1}\left(\tan\left(\frac{\theta}{2}\right)\right)$$

$$y = \frac{\theta}{2}$$

Step 5: Substitute back $\theta = \tan^{-1} x$ and take the derivative with respect to x.

$$y = \frac{1}{2}\tan^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{1+x^2} = \frac{1}{2(1+x^2)}$$

Final Answer: $\boxed{\frac{1}{2(1+x^2)}}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: The probability of an event is computed as the number of favorable outcomes divided by the total number of equally likely outcomes in the sample space. For rolling two distinct six-sided dice, the sample space size is $6 \times 6 = 36$.

Solution: Step 1: Calculate the total number of outcomes when two standard dice are rolled simultaneously.

$$\text{Total Outcomes } (n(S)) = 6 \times 6 = 36$$

Step 2: List all ordered pairs (d_1, d_2) where the sum of the numbers equals exactly 7.

$$\text{Favorable pairs: } (1, 6),\ (2, 5),\ (3, 4),\ (4, 3),\ (5, 2),\ (6, 1)$$

Step 3: Count the total number of elements present in the favorable event set.

$$\text{Number of favorable outcomes } (n(E)) = 6$$

Step 4: Apply the classical probability definition formula.

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{6}{36}$$

Step 5: Reduce the fraction to its lowest terms.

$$P(E) = \frac{1}{6}$$

Final Answer: $\boxed{\frac{1}{6}}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: The standard symmetrical Cartesian equation of a line passing through a given point (x_1, y_1, z_1) and aligned parallel to a direction vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by the ratio formula $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$.

Solution: Step 1: Identify the coordinates of the fixed point through which the 3D line passes.

$$(x_1, y_1, z_1) = (1, 2, 3)$$

Step 2: Extract the direction ratios from the given parallel vector $\vec{v} = 3\hat{i} + 2\hat{j} - 2\hat{k}$.

$$\text{Direction ratios: } a = 3, \quad b = 2, \quad c = -2$$

Step 3: Write down the standard formula for the symmetrical Cartesian equation of a line.

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Step 4: Substitute the values of (x_1, y_1, z_1) and (a, b, c) into the general formula.

$$\frac{x - 1}{3} = \frac{y - 2}{2} = \frac{z - 3}{-2}$$

Step 5: Confirm that the signs match the standard format conventions precisely. The denominators represent the vector directions while the numerators represent the point offsets.

Final Answer: $\boxed{\frac{x - 1}{3} = \frac{y - 2}{2} = \frac{z - 3}{-2}}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: The Principle of Inclusion-Exclusion for two finite sets states that the size of the union of two sets is equal to the sum of the individual sizes minus the size of their intersection: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Solution: Step 1: Identify the given cardinal values of the sets from the problem statement.

$$n(A) = 17$$

$$n(B) = 23$$

$$n(A \cup B) = 38$$

Step 2: State the standard formula relating the set unions and intersections.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Step 3: Rearrange the algebraic formula to isolate the intersection term $n(A \cap B)$ on one side.

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

Step 4: Substitute the given numerical values into the rewritten equation.

$$n(A \cap B) = 17 + 23 - 38$$

Step 5: Complete the arithmetic operations to find the value.

$$n(A \cap B) = 40 - 38 = 2$$

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: To find the absolute maximum value of a continuous function on a closed interval, locate all critical points within the interval by finding where the first derivative is zero. Then, evaluate and compare the function values at these critical points and at the endpoints.

Solution: Step 1: Find the first derivative of the given cubic function $f(x) = x^3 - 3x$.

$$f'(x) = \frac{d}{dx}(x^3 - 3x) = 3x^2 - 3$$

Step 2: Set the derivative equal to zero to identify the critical points.

$$3x^2 - 3 = 0 \implies 3x^2 = 3 \implies x^2 = 1 \implies x = \pm 1$$

Step 3: Filter the critical points based on the given closed interval domain $[0, 2]$.
The point $x = -1$ lies outside the interval and is discarded. The valid critical point is $x = 1$.

Step 4: Evaluate the function $f(x)$ at the valid critical point and at the two boundary endpoints ($x = 0$ and $x = 2$).

$$\text{At } x = 0: \quad f(0) = (0)^3 - 3(0) = 0$$
$$\text{At } x = 1: \quad f(1) = (1)^3 - 3(1) = 1 - 3 = -2$$
$$\text{At } x = 2: \quad f(2) = (2)^3 - 3(2) = 8 - 6 = 2$$

Step 5: Compare the values to find the maximum value. The values are $0, -2$, and 2. The maximum value is 2, which occurs exactly at the endpoint $x = 2$.

Final Answer: $\boxed{2}$

Answer: (C)

Go Back to Question 19

Q20.

Solution

Concept: In statistics, the variance of a given population or sample distribution is mathematically defined as the square of its standard deviation (σ). Therefore, if the standard deviation is known, the variance is computed by squaring that value.

Solution: Step 1: Identify the given statistical parameter from the problem text.

$$\text{Standard deviation } (\sigma) = 5$$

Step 2: Recall the fundamental formula connecting standard deviation and variance.

$$\text{Variance } = (\text{Standard deviation})^2$$
$$\text{Variance } = \sigma^2$$

Step 3: Substitute the given value of 5 into the formula.

$$\text{Variance } = 5^2$$

Step 4: Perform the squaring operation.

$$\text{Variance } = 5 \times 5 = 25$$

Step 5: Verify that the calculation represents the variance, which is always non-negative.

Final Answer: $\boxed{25}$

Answer: (C)

Go Back to Question 20

Q21.

Solution

Concept: Determinants can be simplified using standard row or column operations. If an operation results in two identical or proportional rows/columns, or allows a scalar combination that creates a column of zeros, the value of the determinant is automatically zero.

Solution: Step 1: Write down the given determinant.

$$\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

Step 2: Apply the column operation $C_3 \to C_3 + C_2$. This replaces the third column with the sum of the elements of the second and third columns.

$$\Delta = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

Step 3: Factor out the common expression $(a + b + c)$ from the third column.

$$\Delta = (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

Step 4: Analyze the columns of the new determinant. Notice that the first column (C_1) and the third column (C_3) are completely identical:

$$C_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Step 5: Use the determinant property which states that if any two rows or columns are identical, the value of the determinant is zero.

$$\Delta = (a+b+c) \cdot 0 = 0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: The eccentricity e of a standard ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (where $a > b$) measures its flatness and is calculated using the formula $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution: Step 1: Write down the given equation of the ellipse.

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Step 2: Identify the major and minor semi-axes parameters by comparing with the standard form.

$$a^2 = 16 \implies a = 4$$

$$b^2 = 9 \implies b = 3$$

Since $a^2 > b^2$, the ellipse has a horizontal major axis.

Step 3: State the mathematical formula used to find the eccentricity of a horizontal ellipse.

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Step 4: Substitute the values of $a^2 = 16$ and $b^2 = 9$ into the eccentricity equation.

$$e = \sqrt{1 - \frac{9}{16}}$$

$$e = \sqrt{\frac{16 - 9}{16}}$$

Step 5: Simplify the fraction inside the square root to get the final answer.

$$e = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$$

Final Answer: $\boxed{\frac{\sqrt{7}}{4}}$

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: An absolute value function $|x - c|$ is continuous everywhere but fails to be differentiable at the point $x = c$ because the graph forms a sharp corner or "kink" at that point, causing the left-hand and right-hand derivatives to differ.

Solution: Step 1: Analyze the structure of the given function.

$$f(x) = |x - 1| + |x - 2|$$

Step 2: Identify individual components that could cause non-differentiability. The absolute value function $|x - 1|$ is non-differentiable at $x = 1$. Similarly, the component $|x - 2|$ is non-differentiable at $x = 2$.

Step 3: Evaluate the behavior of the combined function by breaking it into piecewise regions.

$$\text{For } x < 1: \quad f(x) = -(x-1) - (x-2) = -2x + 3 \implies f'(x) = -2$$

$$\text{For } 1 \le x < 2: \quad f(x) = (x-1) - (x-2) = 1 \implies f'(x) = 0$$

$$\text{For } x \ge 2: \quad f(x) = (x-1) + (x-2) = 2x - 3 \implies f'(x) = 2$$

Step 4: Examine differentiability at the transition point $x = 1$ by comparing one-sided derivatives.

$$\text{Left-hand derivative at } x = 1 = -2$$

$$\text{Right-hand derivative at } x = 1 = 0$$

Since they are unequal, the function is not differentiable at $x = 1$.

Step 5: Examine differentiability at the transition point $x = 2$.

$$\text{Left-hand derivative at } x = 2 = 0$$

$$\text{Right-hand derivative at } x = 2 = 2$$

Since they are unequal, the function is also not differentiable at $x = 2$. Thus, the function is not differentiable at both $x = 1$ and $x = 2$.

Final Answer: Both $x = 1$ and $x = 2$

Answer: (C)

Go Back to Question 23

Q24.

Solution

Concept: Trigonometric ratios in different quadrants have specific signs (ASTC rule). In the third quadrant, tangent and cotangent are positive, while sine, cosine, secant, and cosecant are negative. The fundamental identities are used to compute magnitudes.

Solution: Step 1: Identify the given value and quadrant constraint.

$$\tan\theta = \frac{3}{4}, \quad \theta \in \text{ Third Quadrant (III)}$$

Step 2: Use the standard right-angled triangle definition for the tangent function ($\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$) to find the hypotenuse.

$$\text{Perpendicular } = 3, \quad \text{Base } = 4$$

$$\text{Hypotenuse } = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

Step 3: Determine the magnitude of the cosine function using the definition $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$.

$$|\cos\theta| = \frac{4}{5}$$

Step 4: Apply the quadrant sign rule. Since θ lies in the third quadrant, the cosine function must take a negative sign.

$$\cos\theta = -\frac{4}{5}$$

Step 5: Double check using identities. $\sec^2\theta = 1 + \tan^2\theta = 1 + \frac{9}{16} = \frac{25}{16} \implies \sec\theta = -\frac{5}{4}$ (since secant is negative in quadrant III). Taking the reciprocal gives $\cos\theta = -\frac{4}{5}$.

Final Answer: $-\frac{4}{5}$

Answer: (B)

Go Back to Question 24

Q25.

Solution

Concept: The angle θ between two vectors $\vec{a}$ and $\vec{b}$ is computed using the dot product formula: $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$, where $\vec{a}\cdot\vec{b}$ is the scalar product and $|\vec{a}|, |\vec{b}|$ represent the vector magnitudes.

Solution: Step 1: Compute the dot product of the two given vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

$$\vec{a}\cdot\vec{b} = (1)(1) + (1)(-1) + (-1)(1)$$

$$\vec{a}\cdot\vec{b} = 1 - 1 - 1 = -1$$

Step 2: Compute the magnitude of the first vector $\vec{a}$.

$$|\vec{a}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{1+1+1} = \sqrt{3}$$

Step 3: Compute the magnitude of the second vector $\vec{b}$.

$$|\vec{b}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{1+1+1} = \sqrt{3}$$

Step 4: Substitute the calculated values into the standard cosine formula.

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\cos\theta = \frac{-1}{\sqrt{3}\cdot\sqrt{3}} = -\frac{1}{3}$$

Step 5: Solve explicitly for the angle θ by taking the inverse cosine.

$$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$$

Final Answer: $\boxed{\cos^{-1}\left(-\frac{1}{3}\right)}$

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept: Integrals involving an exponential factor multiplied by a sum of functions can often be resolved using the special integration identity $\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + C$. We must correctly identify which term represents the function and which represents its derivative.

Solution: Step 1: Write down the given expression inside the integral.

$$I = \int e^x(\tan x + \ln(\sec x))\,dx$$

Step 2: Test the terms to check for a function-derivative pair relationship. Let us define a function:

$$f(x) = \ln(\sec x)$$

Step 3: Differentiate $f(x)$ with respect to x using the chain rule.

$$f'(x) = \frac{1}{\sec x} \cdot \frac{d}{dx}(\sec x)$$
$$f'(x) = \frac{1}{\sec x} \cdot (\sec x \tan x)$$
$$f'(x) = \tan x$$

Step 4: Compare this result with the remaining term inside the integrand. We see that the integrand matches the standard formula structure precisely:

$$\text{Integrand} = e^x[f'(x) + f(x)] = e^x[\tan x + \ln(\sec x)]$$

Step 5: Apply the standard theorem integration rule directly to write down the final expression, appending the constant of integration C.

$$\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + C$$
$$I = e^x \ln(\sec x) + C$$

Final Answer: $\boxed{e^x \ln(\sec x) + C}$

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept: The sum of an infinite geometric progression (G.P.) is given by the formula $S_\infty = \frac{a}{1-r}$, where a is the first term and r is the common ratio, with the convergence condition that $|r| < 1$.

Solution: Step 1: Identify the given values from the problem description.

$$\text{Sum of the infinite G.P. } (S_\infty) = 4$$

$$\text{First term } (a) = 2$$

Step 2: Write down the standard formula for the sum of an infinite geometric series.

$$S_\infty = \frac{a}{1-r}$$

Step 3: Substitute the known parameters into the formula to create an equation for r.

$$4 = \frac{2}{1-r}$$

Step 4: Solve the equation by cross-multiplying and isolating the variable term.

$$4(1-r) = 2$$

$$1 - r = \frac{2}{4}$$

$$1 - r = \frac{1}{2}$$

Step 5: Rearrange terms to find the exact numerical value of r.

$$r = 1 - \frac{1}{2} = \frac{1}{2}$$

Since $\left|\frac{1}{2}\right| < 1$, the convergence criterion is satisfied.

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: The order of a differential equation is defined as the highest derivative present in the expression. The degree is the power of this highest-order derivative after the equation is cleared of all fractional exponents and radical signs.

Solution: Step 1: Write down the given differential equation expression.

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$$

Step 2: Identify the highest derivative order. The expression contains a first derivative $\frac{dy}{dx}$ and a second derivative $\frac{d^2y}{dx^2}$. Thus, the highest derivative is the second derivative.

$$\text{Order} = 2$$

Step 3: Eliminate the fractional exponent $\frac{3}{2}$ on the left-hand side to put the equation in standard polynomial form. Square both sides of the equation.

$$\left(\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}\right)^2 = \left(\frac{d^2y}{dx^2}\right)^2$$

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3} = \left(\frac{d^2y}{dx^2}\right)^2$$

Step 4: Identify the power or exponent of the highest-order derivative term $(\frac{d^2y}{dx^2})$ in this cleared format. The term has an explicit exponent of 2.

$$\text{Degree} = 2$$

Step 5: Combine the values into the required ordered pair format. The order is 2 and the degree is 2.

Final Answer: 2, 2

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: The probability of the union of two events can be calculated using the addition rule of probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. This accounts for any overlap between categories to prevent double-counting.

Solution: Step 1: Determine the total number of items in the sample space of a standard deck of cards.

$$n(S) = 52$$

Step 2: Find the number of favorable outcomes for drawing a King (A). There are 4 Kings in a deck.

$$n(A) = 4 \implies P(A) = \frac{4}{52}$$

Step 3: Find the number of favorable outcomes for drawing a Spade (B). There are 13 Spades in a deck.

$$n(B) = 13 \implies P(B) = \frac{13}{52}$$

Step 4: Identify the intersection set, which is the card that is both a King and a Spade (the King of Spades). There is exactly 1 such card.

$$n(A \cap B) = 1 \implies P(A \cap B) = \frac{1}{52}$$

Step 5: Combine these values using the probabilistic addition formula to determine the total probability.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$$

Reducing the fraction by dividing the numerator and denominator by 4 gives:

$$P(A \cup B) = \frac{4}{13}$$

Final Answer: $\boxed{\frac{4}{13}}$

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept: The shortest perpendicular distance from a specific point (x_1, y_1, z_1) to a three-dimensional plane given by the equation $Ax + By + Cz + D = 0$ is computed using the coordinate substitution formula $d = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$.

Solution: Step 1: Identify the coordinates of the point and the parameters of the plane equation.

$$\text{Point: } (x_1, y_1, z_1) = (2, 3, 4)$$

$$\text{Plane: } 3x - 6y + 2z + 11 = 0 \implies A = 3,\ B = -6,\ C = 2,\ D = 11$$

Step 2: Write down the analytic formula for the distance from a point to a plane.

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Step 3: Substitute the coordinate values and equation constants into the numerator expression.

$$\text{Numerator} = |3(2) + (-6)(3) + 2(4) + 11|$$

$$\text{Numerator} = |6 - 18 + 8 + 11| = |7| = 7$$

Step 4: Substitute the plane coefficients into the radical expression in the denominator.

$$\text{Denominator} = \sqrt{3^2 + (-6)^2 + 2^2}$$

$$\text{Denominator} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$$

Step 5: Divide the computed numerator value by the denominator value to find the final distance.

$$d = \frac{7}{7} = 1 \text{ unit}$$

Final Answer: 1 unit

Answer: (B)

Go Back to Question 30

Q31.

Solution

Concept: The complex properties of the imaginary cube roots of unity are governed by two algebraic relations: $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$. These properties allow for the systematic reduction of complex multi-term polynomial expressions.

Solution: Step 1: Write out the individual algebraic parts from the given equation.

$$\text{Term 1: } (1 - \omega + \omega^2)^5$$

$$\text{Term 2: } (1 + \omega - \omega^2)^5$$

Step 2: Use the identity $1 + \omega^2 = -\omega$ to substitute and simplify the first expression.

$$1 - \omega + \omega^2 = (1 + \omega^2) - \omega = (-\omega) - \omega = -2\omega$$

Thus, the first term becomes:

$$(-2\omega)^5 = -32\omega^5$$

Step 3: Use the identity $1 + \omega = -\omega^2$ to substitute and simplify the second expression.

$$1 + \omega - \omega^2 = (1 + \omega) - \omega^2 = (-\omega^2) - \omega^2 = -2\omega^2$$

Thus, the second term becomes:

$$(-2\omega^2)^5 = -32\omega^{10}$$

Step 4: Combine the simplified parts together and factor out the common scalar coefficient.

$$\text{Sum} = -32\omega^5 - 32\omega^{10} = -32(\omega^5 + \omega^{10})$$

Step 5: Reduce higher power terms of ω using the relation $\omega^3 = 1$.

$$\omega^5 = \omega^3 \cdot \omega^2 = 1 \cdot \omega^2 = \omega^2$$

$$\omega^{10} = (\omega^3)^3 \cdot \omega = (1)^3 \cdot \omega = \omega$$

Substitute these back into the expression:

$$\text{Sum} = -32(\omega^2 + \omega)$$

Since $\omega + \omega^2 = -1$, we perform the final substitution:

$$\text{Sum} = -32(-1) = 32$$

Final Answer: $\boxed{32}$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: A differentiable function is strictly decreasing in intervals where its first derivative is strictly less than zero ($f'(x) < 0$). Finding this interval requires differentiating the function and solving the resulting quadratic inequality.

Solution: Step 1: Differentiate the given function $f(x) = 2x^3 - 15x^2 + 36x + 1$ with respect to x.

$$f'(x) = \frac{d}{dx}(2x^3 - 15x^2 + 36x + 1)$$

$$f'(x) = 6x^2 - 30x + 36$$

Step 2: Set up the inequality for a strictly decreasing function.

$$f'(x) < 0 \implies 6x^2 - 30x + 36 < 0$$

Step 3: Divide the entire inequality by the positive scalar 6 to simplify the quadratic coefficients.

$$x^2 - 5x + 6 < 0$$

Step 4: Factor the quadratic expression into linear components. Find two numbers that multiply to 6 and add up to -5, which are -2 and -3.

$$(x - 2)(x - 3) < 0$$

Step 5: Determine the solution interval for the inequality. For the product of two terms to be negative, the variable x must lie strictly between the two critical roots.

$$2 < x < 3$$

In interval notation, this is represented as $(2, 3)$.

Final Answer: $\boxed{(2, 3)}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: The standard Cartesian equation of a horizontal hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The parameter a is obtained from the length of the transverse axis (major axis), and the parameter b is calculated using the eccentricity relation $b^2 = a^2(e^2 - 1)$.

Solution: Step 1: Use the given length of the major (transverse) axis to determine the value of a.

$$\text{Length of major axis } = 2a = 8 \implies a = 4$$

$$a^2 = 16$$

Step 2: Note the given value for the eccentricity parameter.

$$e = \frac{5}{4}$$

Step 3: State the standard geometric identity connecting hyperbola parameters a, b, and e.

$$b^2 = a^2(e^2 - 1)$$

Step 4: Substitute the known values of $a^2 = 16$ and $e = \frac{5}{4}$ into the equation to compute b^2.

$$b^2 = 16\left[\left(\frac{5}{4}\right)^2 - 1\right]$$

$$b^2 = 16\left[\frac{25}{16} - 1\right]$$

$$b^2 = 16\left[\frac{25 - 16}{16}\right] = 16\left[\frac{9}{16}\right] = 9$$

Step 5: Substitute the computed values of $a^2 = 16$ and $b^2 = 9$ into the standard standard form equation.

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Final Answer: $\boxed{\frac{x^2}{16} - \frac{y^2}{9} = 1}$

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: The domain of a real-valued function $f(x) = \sqrt{g(x)}$ requires $g(x) \geq 0$ for real square roots. Furthermore, for a logarithmic expression $\log_{10}(u)$, the input argument must be strictly positive ($u > 0$). When combined, $\log_{10}(u) \geq 0$ implies $u \geq 1$, which inherently satisfies $u > 0$.

Solution: Step 1: Identify the conditions required for the function $f(x) = \sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$ to be valid. The expression inside the radical must be non-negative:

$$\log_{10}\left(\frac{5x-x^2}{4}\right) \geq 0$$

Step 2: Eliminate the logarithm by raising the base 10 to both sides. Since the base is greater than 1, the direction of the inequality remains unchanged.

$$\frac{5x-x^2}{4} \geq 10^0$$

$$\frac{5x-x^2}{4} \geq 1$$

Step 3: Clear the fraction by multiplying by 4 and rearrange terms to form a standard quadratic inequality.

$$5x - x^2 \geq 4$$

$$0 \geq x^2 - 5x + 4 \implies x^2 - 5x + 4 \leq 0$$

Step 4: Factor the quadratic expression into linear components. Find two numbers that multiply to 4 and add up to -5. These are -1 and -4.

$$(x-1)(x-4) \leq 0$$

Step 5: Solve the inequality. For the product to be less than or equal to zero, the variable x must lie inside the closed region bounded by the roots.

$$1 \leq x \leq 4$$

In interval notation, the domain is represented as $[1, 4]$. Both endpoints are included because they result in $\log_{10}(1) = 0$, which is a valid input for the square root.

Final Answer: $\boxed{[1, 4]}$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept: The Coefficient of Variation $(C.V.)$ is a normalized measure of the spread of a probability or statistical data distribution. It is calculated as the ratio of the standard deviation (σ) to the arithmetic mean (μ or $\bar{x}$), expressed as a percentage: $C.V. = \left(\frac{\sigma}{\bar{x}}\right) \times 100$.

Solution: Step 1: Write down the given values from the problem statement.

$$\text{Coefficient of Variation } (C.V.) = 60\%$$

$$\text{Arithmetic Mean } (\bar{x}) = 15$$

Step 2: State the formal statistical formula used to define the Coefficient of Variation.

$$C.V. = \left(\frac{\sigma}{\bar{x}}\right) \times 100$$

Step 3: Substitute the known values into the equation to set up an algebraic expression for the unknown standard deviation σ.

$$60 = \left(\frac{\sigma}{15}\right) \times 100$$

Step 4: Isolate the standard deviation parameter σ by rearranging the terms.

$$60 = \frac{100\sigma}{15}$$

$$60 = \frac{20\sigma}{3}$$

Step 5: Solve the linear equation using scalar arithmetic.

$$\sigma = \frac{60 \times 3}{20}$$

$$\sigma = 3 \times 3 = 9$$

The standard deviation of the data set is 9.

Final Answer: 9

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept: Integrals of rational functions where the denominator can be factored into linear components can be evaluated using the method of partial fractions. This splits the complex fraction into a sum of simpler terms that integrate directly into logarithmic functions.

Solution: Step 1: Write down the given definite integral.

$$I = \int_1^2 \frac{1}{x^2 + 2x}\,dx$$

Step 2: Factor the polynomial expression in the denominator.

$$x^2 + 2x = x(x + 2)$$

Step 3: Decompose the integrand using the method of partial fractions.

$$\frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$

Multiplying through gives $1 = A(x + 2) + Bx$.

$$\text{Set } x = 0: \quad 1 = 2A \implies A = \frac{1}{2}$$

$$\text{Set } x = -2: \quad 1 = -2B \implies B = -\frac{1}{2}$$

Thus, the integral becomes:

$$I = \int_1^2 \left(\frac{1/2}{x} - \frac{1/2}{x+2}\right) dx = \frac{1}{2}\int_1^2 \left(\frac{1}{x} - \frac{1}{x+2}\right) dx$$

Step 4: Integrate each term using the standard log integration rule $\int \frac{1}{u}\,du = \ln|u|$.

$$I = \frac{1}{2}\left[\ln|x| - \ln|x+2|\right]_1^2 = \frac{1}{2}\left[\ln\left|\frac{x}{x+2}\right|\right]_1^2$$

Step 5: Substitute the upper and lower limits of integration and simplify using log rules.

$$I = \frac{1}{2}\left[\ln\left(\frac{2}{4}\right) - \ln\left(\frac{1}{3}\right)\right] = \frac{1}{2}\left[\ln\left(\frac{1}{2}\right) - \ln\left(\frac{1}{3}\right)\right]$$

$$I = \frac{1}{2}\ln\left(\frac{1/2}{1/3}\right) = \frac{1}{2}\ln\left(\frac{3}{2}\right)$$

Final Answer: $\boxed{\frac{1}{2}\ln\left(\frac{3}{2}\right)}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept: A square matrix A is classified as an orthogonal matrix if it satisfies the structural relationship $A \cdot A^T = I$, where A^T represents the transpose matrix and I is the identity matrix. This implies that the row vectors must be mutually orthogonal unit vectors.

Solution: Step 1: Write out the given matrix A and construct its transpose A^T.

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}, \quad A^T = \begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix}$$

Step 2: Multiply matrix A and its transpose A^T, and set the result equal to the identity matrix I.

$$AA^T = \begin{bmatrix} 0+4b^2+c^2 & 0+2b^2-c^2 & 0-2b^2+c^2 \\ 0+2b^2-c^2 & a^2+b^2+c^2 & a^2-b^2-c^2 \\ 0-2b^2+c^2 & a^2-b^2-c^2 & a^2+b^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3: Set up independent algebraic equations by equating the corresponding diagonal and off-diagonal elements.

$$4b^2 + c^2 = 1 \quad \text{— (Equation 1)}$$
$$2b^2 - c^2 = 0 \quad \text{— (Equation 2)}$$
$$a^2 + b^2 + c^2 = 1 \quad \text{— (Equation 3)}$$

Step 4: Solve for b^2 and c^2 using Equations 1 and 2. From Equation 2, we have $c^2 = 2b^2$. Substitute this into Equation 1:

$$4b^2 + 2b^2 = 1 \implies 6b^2 = 1 \implies b^2 = \frac{1}{6} \implies b = \pm\frac{1}{\sqrt{6}}$$

Now substitute b^2 back to find c^2:

$$c^2 = 2\left(\frac{1}{6}\right) = \frac{1}{3} \implies c = \pm\frac{1}{\sqrt{3}}$$

Step 5: Substitute the values of b^2 and c^2 into Equation 3 to determine the value of a^2.

$$a^2 + \frac{1}{6} + \frac{1}{3} = 1 \implies a^2 + \frac{1+2}{6} = 1$$
$$a^2 + \frac{3}{6} = 1 \implies a^2 + \frac{1}{2} = 1 \implies a^2 = \frac{1}{2} \implies a = \pm\frac{1}{\sqrt{2}}$$

Final Answer: $\boxed{a = \pm\frac{1}{\sqrt{2}}, b = \pm\frac{1}{\sqrt{6}}, c = \pm\frac{1}{\sqrt{3}}}$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept: Products of cosine terms with doubling angles can be simplified systematically using the product identity $\cos\theta \cdot \cos 2\theta \cdot \cos 4\theta \ldots \cos 2^{n-1}\theta = \frac{\sin(2^n\theta)}{2^n \sin\theta}$. Alternatively, it can be evaluated by repeatedly applying the sine double-angle identity.

Solution: Step 1: Write down the expression to be evaluated. Let it be denoted as P.

$$P = \cos 20^\circ \cos 40^\circ \cos 80^\circ$$

Step 2: Multiply and divide the entire expression by $2\sin 20^\circ$ to set up the double-angle identity.

$$P = \frac{2\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 80^\circ}{2\sin 20^\circ}$$

Step 3: Apply the product identity $2\sin\theta\cos\theta = \sin 2\theta$ to combine the first two terms in the numerator. Here, $\theta = 20^\circ$, so $2\theta = 40^\circ$.

$$P = \frac{\sin 40^\circ \cos 40^\circ \cos 80^\circ}{2\sin 20^\circ}$$

Step 4: Multiply the numerator and denominator by 2 again to combine the next set of terms.

$$P = \frac{2\sin 40^\circ \cos 40^\circ \cos 80^\circ}{4\sin 20^\circ} = \frac{\sin 80^\circ \cos 80^\circ}{4\sin 20^\circ}$$

Multiply by 2 once more:

$$P = \frac{2\sin 80^\circ \cos 80^\circ}{8\sin 20^\circ} = \frac{\sin 160^\circ}{8\sin 20^\circ}$$

Step 5: Use supplementary angle properties to simplify the numerator. Since $\sin(180^\circ - \theta) = \sin\theta$:

$$\sin 160^\circ = \sin(180^\circ - 20^\circ) = \sin 20^\circ$$

Substitute this back into the expression for P:

$$P = \frac{\sin 20^\circ}{8\sin 20^\circ} = \frac{1}{8}$$

Final Answer: $\boxed{\frac{1}{8}}$

Answer: (C)

Go Back to Question 38

Q39.

Solution

Concept: The scalar projection of a vector $\vec{a}$ onto another non-zero vector $\vec{b}$ measures the component of $\vec{a}$ aligned in the direction of $\vec{b}$. It is mathematically calculated using the geometric formula Projection = $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Solution: Step 1: Write down the component values of the given vectors.

$$\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$$

Step 2: Calculate the dot product $\vec{a} \cdot \vec{b}$ of the two vectors.

$$\vec{a} \cdot \vec{b} = (2)(1) + (3)(2) + (2)(1)$$

$$\vec{a} \cdot \vec{b} = 2 + 6 + 2 = 10$$

Step 3: Calculate the magnitude of the target base vector $\vec{b}$.

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2}$$

$$|\vec{b}| = \sqrt{1 + 4 + 1} = \sqrt{6}$$

Step 4: Formulate the projection expression by substituting the computed dot product and magnitude into the standard formula.

$$\text{Projection} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$\text{Projection} = \frac{10}{\sqrt{6}}$$

Step 5: Confirm that the final term matches the required format options exactly. No further radical rationalization is required.

Final Answer: $\boxed{\frac{10}{\sqrt{6}}}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: To find the Integrating Factor ($I.F.$) of a first-order linear differential equation, it must first be rearranged into its standard mathematical form: $\frac{dy}{dx} + P(x)y = Q(x)$. The Integrating Factor is then calculated using the exponential integral formula $I.F. = e^{\int P(x)\,dx}$.

Solution: Step 1: Write down the given differential equation.

$$(x \ln x)\frac{dy}{dx} + y = 2 \ln x$$

Step 2: Divide every term in the equation by the lead coefficient $(x \ln x)$ to isolate the $\frac{dy}{dx}$ term and convert it to standard form.

$$\frac{dy}{dx} + \frac{1}{x \ln x}y = \frac{2 \ln x}{x \ln x}$$
$$\frac{dy}{dx} + \frac{1}{x \ln x}y = \frac{2}{x}$$

Step 3: Extract the coefficient function $P(x)$ that multiplies the dependent variable y.

$$P(x) = \frac{1}{x \ln x}$$

Step 4: Set up the integral of $P(x)$ to calculate the Integrating Factor.

$$\int P(x)\,dx = \int \frac{1}{x \ln x}\,dx$$

Evaluate this using substitution: set $u = \ln x \implies du = \frac{1}{x}\,dx$.

$$\int \frac{1}{x \ln x}\,dx = \int \frac{1}{u}\,du = \ln|u| = \ln(\ln x)$$

Step 5: Compute the final Integrating Factor ($I.F.$) by raising e to the power of this integrated result.

$$I.F. = e^{\int P(x)\,dx} = e^{\ln(\ln x)}$$

Since exponential and natural logarithm functions are inverses, they cancel each other out:

$$I.F. = \ln x$$

Final Answer: $\boxed{\ln x}$

Answer: (B)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	B	4	B	5	A
6	A	7	C	8	C	9	A	10	C
11	B	12	B	13	A	14	A	15	B
16	A	17	A	18	A	19	C	20	C
21	A	22	A	23	C	24	B	25	B
26	B	27	A	28	B	29	B	30	B
31	A	32	A	33	A	34	A	35	A
36	A	37	A	38	C	39	A	40	B