Rajasthan JET Mathematics Sample Paper-1

Duration: 40 Minutes **Maximum Marks: 160**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **−1 marks**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. If the matrix $A = \begin{bmatrix} \lambda & 1 & 2 \\ 0 & \lambda - 1 & 3 \\ 0 & 0 & \lambda - 2 \end{bmatrix}$ is singular, then the sum of all possible real values of λ is

(A) 3

(B) 2

(C) 1

(D) 0

Q2. If $y = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, then $\frac{dy}{dx}$ is equal to

(A) 1

(B) −1

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Q3. The equation of the line passing through the point $(1, 2, 3)$ and parallel to the vector $2\hat{i} - \hat{j} + 3\hat{k}$ is

(A) $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$

(B) $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{3}$

(C) $\frac{x+1}{2} = \frac{y+2}{-1} = \frac{z+3}{3}$

(D) $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{3}$

Q4. Let A and B be two sets such that $n(A) = 4$ and $n(B) = 3$. The total number of non-empty relations that can be defined from A to B is

(A) 2^{12}

(B) $2^{12} - 1$

(C) 12

(D) $2^7 - 1$

Q5. If the variance of a data set containing 5 observations is 4, and each observation is multiplied by 3, then the new variance of the data will be

(A) 12

(B) 36

(C) 6

(D) 2

Q6. The value of $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx$ is

(A) π

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) 0

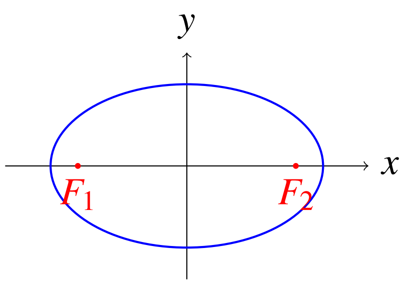
Q7. The modulus of the complex number $z = \frac{1+2i}{1-3i}$ is

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{1}{2}$

(C) $\sqrt{2}$ D 1

Q8. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is



(A) $\frac{3}{5}$

(B) $\frac{4}{5}$

(C) $\frac{16}{25}$

(D) $\frac{2}{5}$

Q9. Two dice are thrown simultaneously. The probability of getting a total score of 7 is

(A) $\frac{1}{6}$

(B) $\frac{5}{36}$

(C) $\frac{7}{36}$

(D) $\frac{1}{12}$

Q10. The value of $\lim_{x\to 0} \frac{e^{\sin x}-1}{x}$ is

(A) 0

(B) 1

(C) e

(D) Does not exist

Q11. If the n-th term of an Arithmetic Progression is given by $a_n = 3n + 5$, then the common difference of the AP is

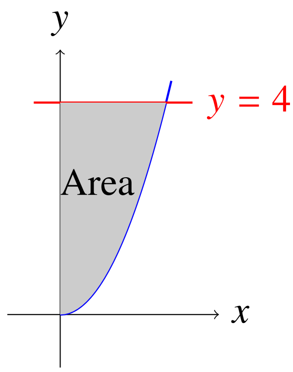
(A) 3

(B) 5

(C) 8

(D) 2

Q12. The area bounded by the curve $y = x^2$ and the line $y = 4$ in the first quadrant is



(A) $\frac{16}{3}$

(B) $\frac{8}{3}$

(C) $\frac{4}{3}$

(D) 4

Q13. The domain of the function $f(x) = \frac{1}{\sqrt{x^2-9}}$ is

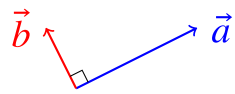
(A) $[-3, 3]$

(B) $(-\infty, -3] \cup [3, \infty)$

(C) $(-\infty, -3) \cup (3, \infty)$

(D) $\mathbb{R}$

Q14. If vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other, then the value of λ is



(A) 2

(B) $\frac{5}{2}$

(C) $-\frac{5}{2}$

(D) 5

Q15. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(A) $y = cx$

(B) $xy = c$

(C) $y = ce^x$

(D) $x^2 + y^2 = c$

Q16. The value of $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$ is

(A) $\frac{2\pi}{3}$

(B) $\frac{\pi}{3}$

(C) $-\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

Q17. The mean of 10 observations is 20. If an observation 30 is replaced by 40, the new mean will be

(A) 21

(B) 20.1

(C) 25

(D) 22

Q18. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A + A^T$ is

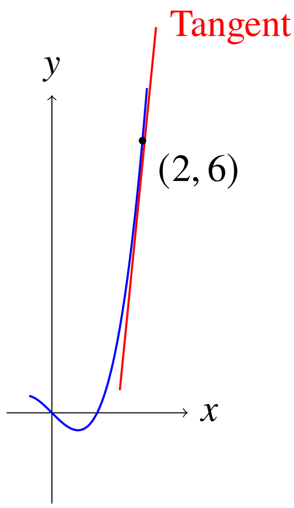
(A) A symmetric matrix

(B) A skew-symmetric matrix

(C) An identity matrix

(D) A null matrix

Q19. The slope of the tangent to the curve $y = x^3 - x$ at the point $x = 2$ is



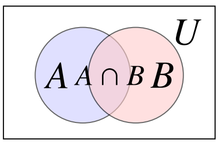
(A) A 11

(B) B 12

(C) C 6

(D) D 10

Q20. If $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$, then $P(A|B)$ is



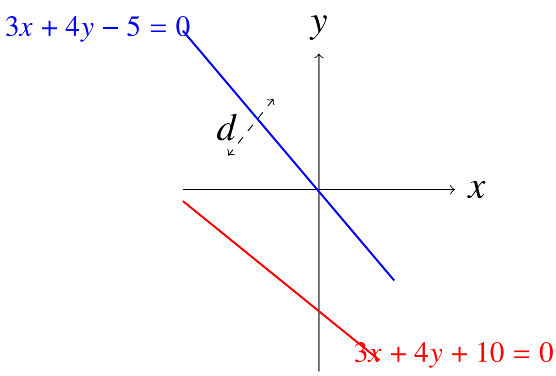
(A) 0.4

(B) 0.5

(C) 0.2

(D) 0.8

Q21. The distance between the parallel lines $3x + 4y - 5 = 0$ and $3x + 4y + 10 = 0$ is



(A) 3

(B) 5

(C) 15

(D) 1

Q22. The solution of the inequality $|x - 1| \leq 3$ is

(A) $x \in [-2, 4]$

(B) $x \in [-4, 2]$

(C) $x \in (2, 4)$

(D) $x \in [-3, 3]$

Q23. The value of $\int e^x(\tan x + \sec^2 x)\, dx$ is

(A) $e^x \sec x + c$

(B) $e^x \tan x + c$

(C) $e^x \sec^2 x + c$

(D) $e^x + \tan x + c$

Q24. The distance of the point $(2, 3, 4)$ from the xy-plane is

(A) 2

(B) 3

(C) 4

(D) $\sqrt{29}$

Q25. If A is a square matrix of order 3 and $|A| = 4$, then the value of $|2A|$ is

(A) 8

(B) 16

(C) 32

(D) 12

Q26. The maximum value of $f(x) = \sin x + \cos x$ is

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Q27. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y = 0$ are respectively

(A) 2, 3

(B) 3, 2

(C) 2, 4

(D) 4, 3

Q28. The number of terms in the sequence $5, 9, 13, \ldots, 81$ is

(A) 19

(B) 20

(C) 21

(D) 22

Q29. A card is drawn at random from a well-shuffled pack of 52 cards. The probability that it is a king or a club is

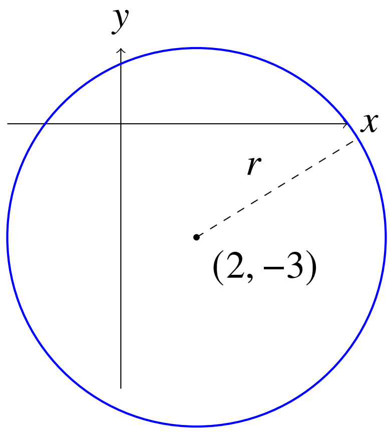
(A) $\frac{17}{52}$

(B) $\frac{4}{13}$

(C) $\frac{16}{52}$

(D) $\frac{1}{26}$

Q30. The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is



(A) 5

(B) $\sqrt{13}$

(C) 25

(D) 7

Q31. If $f(x) = 2x + 3$ and $g(x) = x^2$, then the value of $(g \circ f)(1)$ is

(A) 5

(B) 25

(C) 7

(D) 10

Q32. The value of $\int \frac{1}{x \ln x}\, dx$ is

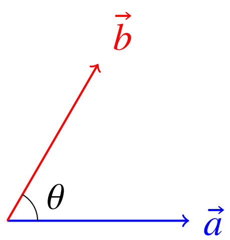
(A) $\ln|x| + c$

(B) $\ln|\ln x| + c$

(C) $\frac{1}{2}(\ln x)^2 + c$

(D) $x \ln x + c$

Q33. If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 6$, then the angle between $\vec{a}$ and $\vec{b}$ is



(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Q34. The value of $\cos(15°)$ is

(A) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(B) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(C) $\frac{\sqrt{3}+1}{2}$

(D) $\frac{\sqrt{3}-1}{2}$

Q35. The standard deviation of the first 5 natural numbers is

(A) 2

(B) $\sqrt{2}$

(C) $\sqrt{5}$

(D) 10

Q36. The integrating factor of the linear differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is

(A) e^x

(B) x

(C) $\ln x$

(D) $\frac{1}{x}$

Q37. If ω is an imaginary cube root of unity, then the value of $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$ is

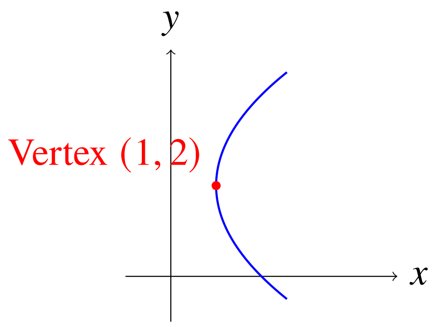
(A) 32

(B) -32

(C) 64

(D) 0

Q38. The vertex of the parabola $y^2 - 4y - 4x + 8 = 0$ is at the point



(A) $(1, 2)$

(B) $(2, 1)$

(C) $(0, 2)$

(D) $(1, 0)$

Q39. If A and B are mutually exclusive events such that $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cup B)$ is

(A) 0.7

(B) 0.12

(C) 0.1

(D) 0.5

Q40. The value of $\lim_{x\to 3} \frac{x^3-27}{x-3}$ is

(A) 9

(B) 27

(C) 0

(D) 3

Detailed Solutions

Q1.

Solution

Concept: A square matrix is singular if and only if its determinant is equal to zero. For an upper triangular matrix, the determinant is simply the product of its diagonal elements.

Solution: Step 1: Identify the structure of the given matrix A. The matrix $A = \begin{bmatrix} \lambda & 1 & 2 \\ 0 & \lambda - 1 & 3 \\ 0 & 0 & \lambda - 2 \end{bmatrix}$ has all elements below the principal diagonal equal to zero, which means it is an upper triangular matrix.

Step 2: Express the determinant of an upper triangular matrix as the product of its diagonal entries. Therefore, we have:

$$|A| = \lambda(\lambda - 1)(\lambda - 2)$$

Step 3: Set the determinant to zero because the problem states that the matrix A is singular:

$$\lambda(\lambda - 1)(\lambda - 2) = 0$$

Step 4: Solve the equation for all possible real values of λ. The roots of this polynomial equation are:

$$\lambda = 0, \quad \lambda = 1, \quad \text{and} \quad \lambda = 2$$

Step 5: Calculate the sum of all these possible real values of λ as requested by the question:

$$\text{Sum} = 0 + 1 + 2 = 3$$

Final Answer: $\boxed{3}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: To differentiate an inverse trigonometric expression containing trigonometric functions, we use algebraic and trigonometric identities to simplify the inner function before finding the derivative.

Solution: Step 1: Write down the given function:

$$y = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$$

Step 2: Divide both the numerator and the denominator inside the parentheses by $\cos x$ to express the inner term in terms of $\tan x$:

$$y = \tan^{-1}\left(\frac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}\right) = \tan^{-1}\left(\frac{1 - \tan x}{1 + \tan x}\right)$$

Step 3: Substitute $1 = \tan\left(\frac{\pi}{4}\right)$ into the expression to rewrite it using the compound angle formula for tangent, $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$:

$$y = \tan^{-1}\left(\frac{\tan\left(\frac{\pi}{4}\right) - \tan x}{1 + \tan\left(\frac{\pi}{4}\right)\tan x}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - x\right)\right)$$

Step 4: Use the property of inverse trigonometric functions $\tan^{-1}(\tan\theta) = \theta$ to simplify the expression completely:

$$y = \frac{\pi}{4} - x$$

Step 5: Differentiate both sides with respect to x to find the required derivative:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{4}\right) - \frac{d}{dx}(x) = 0 - 1 = -1$$

Final Answer: $\boxed{-1}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: The equation of a straight line in three-dimensional space passing through a given point (x_1, y_1, z_1) and parallel to a vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by the symmetrical form $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$.

Solution: Step 1: Identify the coordinates of the given point through which the line passes. The coordinates are:

$$(x_1, y_1, z_1) = (1, 2, 3)$$

Step 2: Identify the direction ratios of the line from the parallel vector. The line is parallel to the vector $\vec{v} = 2\hat{i} - \hat{j} + 3\hat{k}$, so its direction ratios are:

$$a = 2, \quad b = -1, \quad c = 3$$

Step 3: Substitute the coordinates of the point and the direction ratios into the standard symmetrical Cartesian equation of a straight line:

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$$

Step 4: Verify the options to match this exact representation. Option (A) matches our derived equation perfectly.

Final Answer: $\boxed{\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{3}}$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept: A relation from set A to set B is defined as any subset of the Cartesian product $A \times B$. If set A has m elements and set B has n elements, the total number of relations is 2^{mn}, and the number of non-empty relations is $2^{mn} - 1$.

Solution: Step 1: Determine the number of elements in the Cartesian product $A \times B$ using the formula $n(A \times B) = n(A) \times n(B)$. Given $n(A) = 4$ and $n(B) = 3$:

$$n(A \times B) = 4 \times 3 = 12$$

Step 2: Use the formula for the total number of subsets of a set to find the total number of relations. Since $A \times B$ has 12 elements, the total number of subsets (relations) is:

$$\text{Total Relations} = 2^{12}$$

Step 3: Exclude the empty set from the total number of relations to find the number of non-empty relations. The empty set is exactly one unique relation:

$$\text{Non-empty Relations} = 2^{12} - 1$$

Final Answer: $\boxed{2^{12} - 1}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: Variance is a measure of dispersion that describes how far a set of numbers are spread out from their average. If every observation in a data set is multiplied by a constant k, the new variance becomes k^2 times the original variance.

Solution: Step 1: Identify the given information from the problem statement. The original variance of the data set is given as:

$$\text{Variance}_{\text{old}} = 4$$

Step 2: Note the constant factor by which each observation is multiplied. Here, every individual value is scaled by:

$$k = 3$$

Step 3: Apply the property of variance regarding scaling. The new variance after scaling every data point by k is given by the formula:

$$\text{Variance}_{\text{new}} = k^2 \times \text{Variance}_{\text{old}}$$

Step 4: Substitute the values of k and the original variance into the formula to compute the final result:

$$\text{Variance}_{\text{new}} = 3^2 \times 4 = 9 \times 4 = 36$$

Final Answer: 36

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept: Definite integrals can be evaluated efficiently by using the integral property $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$, which often simplifies fractional expressions involving sine and cosine.

Solution: Step 1: Let the given integral be denoted as I:

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx \quad \text{— (Equation 1)}$$

Step 2: Apply the integration property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ to rewrite the variable x as $\frac{\pi}{2} - x$:

$$I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}\,dx$$

Step 3: Use the standard trigonometric identities $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ and $\cos\left(\frac{\pi}{2} - x\right) = \sin x$ to simplify the integrand:

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\,dx \quad \text{— (Equation 2)}$$

Step 4: Add Equation 1 and Equation 2 together to combine their numerators over the common denominator:

$$2I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx + \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\,dx$$

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx = \int_0^{\pi/2} 1\,dx$$

Step 5: Integrate and evaluate between the upper and lower limits to find the value of I:

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2} \implies I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (C)

Go Back to Question 6

Q7.

Solution

Concept: The modulus of a complex number division $\left|\frac{z_1}{z_2}\right|$ is equal to the division of their individual moduli $\frac{|z_1|}{|z_2|}$. The modulus of any complex number $x + iy$ is given by $\sqrt{x^2 + y^2}$.

Solution: Step 1: Write down the given complex number expression:

$$z = \frac{1 + 2i}{1 - 3i}$$

Step 2: Apply the modulus property for quotients to separate the numerator and denominator:

$$|z| = \left|\frac{1 + 2i}{1 - 3i}\right| = \frac{|1 + 2i|}{|1 - 3i|}$$

Step 3: Calculate the modulus of the complex number in the numerator $(1 + 2i)$:

$$|1 + 2i| = \sqrt{1^2 + 2^2} = \sqrt{1 + 4} = \sqrt{5}$$

Step 4: Calculate the modulus of the complex number in the denominator $(1 - 3i)$:

$$|1 - 3i| = \sqrt{1^2 + (-3)^2} = \sqrt{1 + 9} = \sqrt{10}$$

Step 5: Divide the numerator modulus by the denominator modulus and simplify the radical fraction:

$$|z| = \frac{\sqrt{5}}{\sqrt{10}} = \sqrt{\frac{5}{10}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Final Answer: $\boxed{\frac{1}{\sqrt{2}}}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: The standard equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. When $a > b$, the eccentricity e is calculated using the formula $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution: Step 1: Express the given equation of the ellipse $9x^2 + 25y^2 = 225$ in standard form by dividing both sides by 225:

$$\frac{9x^2}{225} + \frac{25y^2}{225} = 1 \implies \frac{x^2}{25} + \frac{y^2}{9} = 1$$

Step 2: Compare this with the standard ellipse equation to find the values of a^2 and b^2:

$$a^2 = 25 \quad \text{and} \quad b^2 = 9$$

Step 3: Observe that $a^2 > b^2$ $(25 > 9)$, indicating that the major axis lies along the x-axis. Use the standard eccentricity formula:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Step 4: Substitute the values of a^2 and b^2 into the formula and solve:

$$e = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Final Answer: $\boxed{\frac{4}{5}}$

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept: The probability of an event is the ratio of the number of favorable outcomes to the total number of outcomes in the sample space. For two fair dice, the total number of sample outcomes is $6 \times 6 = 36$.

Solution: Step 1: Calculate the total number of outcomes when two distinct dice are rolled simultaneously. Each die has 6 faces, so:

$$\text{Total Outcomes } (n(S)) = 6 \times 6 = 36$$

Step 2: List all the possible ordered pairs (d_1, d_2) where the sum of the two faces equals 7:

$$\text{Favorable Outcomes } (E) = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$$

Step 3: Count the total number of pairs that satisfy this condition:

$$\text{Number of Favorable Outcomes } (n(E)) = 6$$

Step 4: Compute the final probability using the fundamental definition of probability:

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Final Answer: $\boxed{\frac{1}{6}}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: Limits involving exponential functions can be solved by multiplying and dividing by appropriate terms to use the standard limit theorem $\lim_{u\to 0}\frac{e^u-1}{u}=1$.

Solution: Step 1: Write down the given limit problem:

$$\lim_{x\to 0}\frac{e^{\sin x}-1}{x}$$

Step 2: Modify the expression by multiplying and dividing the denominator by $\sin x$ to create the structure matching the standard exponential limit form:

$$\lim_{x\to 0}\left(\frac{e^{\sin x}-1}{\sin x}\times\frac{\sin x}{x}\right)$$

Step 3: Apply the product rule for limits to split the expression into two separate limits:

$$\left(\lim_{x\to 0}\frac{e^{\sin x}-1}{\sin x}\right)\times\left(\lim_{x\to 0}\frac{\sin x}{x}\right)$$

Step 4: Evaluate the first limit. As $x\to 0$, we know that $\sin x\to 0$. Let $u=\sin x$, then $\lim_{u\to 0}\frac{e^u-1}{u}=1$.

Step 5: Evaluate the second limit using the standard trigonometric limit theorem $\lim_{x\to 0}\frac{\sin x}{x}=1$. Multiply the results:

$$\text{Value}=1\times 1=1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: In an Arithmetic Progression (AP), the common difference d can be found by taking the difference between any two consecutive terms, i.e., $d = a_n - a_{n-1}$, or by identifying the coefficient of n in the linear expression of the general term.

Solution: Step 1: Write down the given expression for the n-th term of the arithmetic progression:

$$a_n = 3n + 5$$

Step 2: Find the first term (a_1) of the sequence by substituting $n = 1$ into the general formula:

$$a_1 = 3(1) + 5 = 3 + 5 = 8$$

Step 3: Find the second term (a_2) of the sequence by substituting $n = 2$ into the general formula:

$$a_2 = 3(2) + 5 = 6 + 5 = 11$$

Step 4: Calculate the common difference d by subtracting the first term from the second term:

$$d = a_2 - a_1 = 11 - 8 = 3$$

Step 5: Alternatively, observe that for any linear n-th term expression $a_n = pn + q$, the common difference is always the coefficient of n, which confirms $d = 3$.

Final Answer: $\boxed{3}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: The area under a curve in the first quadrant bounded by a function and a horizontal line can be computed using definite integration with respect to either the x-axis or the y-axis.

Solution: Step 1: Identify the bounding equations. The curve is given by $y = x^2$ (a parabola opening upwards) and the horizontal line is $y = 4$. We are restricted to the first quadrant where $x \geq 0$ and $y \geq 0$.

Step 2: Find the intersection point of the curve and the line in the first quadrant by equating them:

$$x^2 = 4 \implies x = 2 \quad (\text{since } x \geq 0)$$

Step 3: Set up the integral with respect to y from $y = 0$ to $y = 4$. Rewriting the curve equation gives $x = \sqrt{y}$:

$$\text{Area} = \int_0^4 \sqrt{y}\, dy$$

Step 4: Integrate the function using the power rule for integration $\int y^{1/2}\, dy = \frac{2}{3}y^{3/2}$:

$$\text{Area} = \left[\frac{2}{3}y^{3/2}\right]_0^4$$

Step 5: Evaluate the definite integral at the upper and lower boundaries:

$$\text{Area} = \frac{2}{3}(4)^{3/2} - 0 = \frac{2}{3}(2^2)^{3/2} = \frac{2}{3} \times 8 = \frac{16}{3}$$

Final Answer: $\boxed{\frac{16}{3}}$

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept: The domain of a real-valued function is the set of all input values for which the expression is real and well-defined. For a function involving $\frac{1}{\sqrt{g(x)}}$, the quantity inside the square root must be strictly positive, i.e., $g(x) > 0$.

Solution: Step 1: Examine the given function expression:

$$f(x) = \frac{1}{\sqrt{x^2 - 9}}$$

Step 2: Set up the necessary mathematical condition for the square root in the denominator to be real and non-zero:

$$x^2 - 9 > 0$$

Step 3: Factor the algebraic expression using the difference of squares identity:

$$(x - 3)(x + 3) > 0$$

Step 4: Determine the intervals that satisfy this quadratic inequality by analyzing the signs on a real number line (wavy curve method). The expression is positive outside the roots -3 and 3:

$$x < -3 \quad \text{or} \quad x > 3$$

Step 5: Represent this solution set in proper interval notation to describe the domain:

$$\text{Domain} = (-\infty, -3) \cup (3, \infty)$$

Final Answer: $\boxed{(-\infty, -3) \cup (3, \infty)}$

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept: Two non-zero vectors are perpendicular (orthogonal) to each other if and only if their scalar dot product is equal to zero. For components, $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$.

Solution: Step 1: Write down the given component vectors from the problem:

$$\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$$

Step 2: Apply the mathematical condition for perpendicular vectors, which specifies that their dot product must vanish:

$$\vec{a} \cdot \vec{b} = 0$$

Step 3: Compute the dot product by multiplying the corresponding coefficients of $\hat{i}$, $\hat{j}$, and $\hat{k}$ together:

$$(2)(1) + (\lambda)(-2) + (1)(3) = 0$$

Step 4: Simplify the resulting linear algebraic equation:

$$2 - 2\lambda + 3 = 0$$

$$5 - 2\lambda = 0$$

Step 5: Solve for the unknown parameter λ:

$$2\lambda = 5 \implies \lambda = \frac{5}{2}$$

Final Answer: $\boxed{\frac{5}{2}}$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept: A first-order differential equation can be solved using the variable separable method if it can be rearranged such that all terms involving y are on one side and all terms involving x are on the other side.

Solution: Step 1: Write down the given differential equation:

$$\frac{dy}{dx} = \frac{y}{x}$$

Step 2: Rearrange the terms to separate the variables y and x onto opposite sides of the equation:

$$\frac{1}{y}\,dy = \frac{1}{x}\,dx$$

Step 3: Integrate both sides of the equation simultaneously:

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

Step 4: Compute the standard logarithmic integrals and add an arbitrary constant of integration, which can be conveniently written as $\ln c$:

$$\ln|y| = \ln|x| + \ln c$$

Step 5: Use the logarithmic property $\ln A + \ln B = \ln(AB)$ to combine the right-hand side, then remove logs from both sides:

$$\ln|y| = \ln|cx| \implies y = cx$$

Final Answer: $\boxed{y = cx}$

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept: The identity $\sin^{-1}(\sin\theta) = \theta$ holds true if and only if θ belongs to the principal value branch of the inverse sine function, which is the closed interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Solution: Step 1: Identify the given expression:

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$$

Step 2: Check if the angle $\frac{2\pi}{3}$ lies within the principal value interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Since $\frac{2\pi}{3}$ is in the second quadrant, it does not belong to this interval.

Step 3: Apply trigonometric reduction formulas to rewrite the angle inside the sine function in terms of an equivalent first-quadrant angle:

$$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right)$$

Step 4: Use the identity $\sin(\pi - \theta) = \sin\theta$ to simplify the inner expression:

$$\sin\left(\pi - \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$$

Step 5: Substitute this back into the original inverse function expression. Now, since $\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, we can evaluate it directly:

$$\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: The mean of a data set is given by the sum of all observations divided by the total number of observations. When an entry is replaced, the new sum is obtained by subtracting the old value and adding the new value.

Solution: Step 1: Calculate the initial total sum of the 10 observations using the formula Sum = Mean × Number of observations:

$$\text{Old Sum} = 20 \times 10 = 200$$

Step 2: Adjust the total sum to account for the replaced observation. Subtract the incorrect or old value (30) and add the new replacement value (40):

$$\text{New Sum} = \text{Old Sum} - 30 + 40$$

$$\text{New Sum} = 200 - 30 + 40 = 210$$

Step 3: Calculate the new arithmetic mean by dividing the updated sum by the total number of observations, which remains unchanged at 10:

$$\text{New Mean} = \frac{\text{New Sum}}{10} = \frac{210}{10} = 21$$

Final Answer: $\boxed{21}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: A square matrix M is defined as symmetric if it is equal to its transpose ($M^T = M$). For any square matrix A, the sum matrix $A + A^T$ always possesses this property.

Solution: Step 1: Write down the given matrix A:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Step 2: Find the transpose of matrix A by interchanging its rows and columns:

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Step 3: Add matrix A and its transpose A^T together by combining corresponding elements:

$$A + A^T = \begin{bmatrix} 1+1 & 2+3 \\ 3+2 & 4+4 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}$$

Step 4: Check the transpose of the resulting sum matrix. Let $M = A + A^T$:

$$M^T = \begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix} = M$$

Step 5: Since $M^T = M$, the matrix $A + A^T$ is classified as a symmetric matrix.

Final Answer: A symmetric matrix

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: The slope of the tangent line to a curve $y = f(x)$ at any given point is equal to the value of the first derivative $\frac{dy}{dx}$ evaluated at that specific point.

Solution: Step 1: Write down the given equation of the curve:

$$y = x^3 - x$$

Step 2: Differentiate the function with respect to x using the basic power rules of differentiation:

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(x) = 3x^2 - 1$$

Step 3: Identify the value of the x-coordinate where the slope needs to be calculated, which is given as:

$$x = 2$$

Step 4: Substitute $x = 2$ into the derivative expression to find the numerical value of the slope:

$$\text{Slope } (m) = \left.\frac{dy}{dx}\right|_{x=2} = 3(2)^2 - 1$$

Step 5: Complete the arithmetic evaluation:

$$m = 3(4) - 1 = 12 - 1 = 11$$

Final Answer: $\boxed{11}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: Conditional probability measures the probability of an event occurring given that another event has already occurred. The formula for the conditional probability of A given B is $P(A|B) = \frac{P(A\cap B)}{P(B)}$.

Solution: Step 1: Extract the given probabilities from the problem description:

$$P(A) = 0.4, \quad P(B) = 0.5, \quad P(A \cap B) = 0.2$$

Step 2: Recall the standard formula for conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Step 3: Substitute the corresponding numerical values directly into the expression:

$$P(A|B) = \frac{0.2}{0.5}$$

Step 4: Simplify the fraction by eliminating the decimals:

$$P(A|B) = \frac{2}{5} = 0.4$$

Final Answer: 0.4

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: The perpendicular distance d between two parallel lines given by equations $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is evaluated using the formula $d = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$.

Solution: Step 1: Identify the coefficients from the given equations of the parallel lines:

$$\text{Line 1: } 3x + 4y - 5 = 0 \implies A = 3,\ B = 4,\ C_1 = -5$$

$$\text{Line 2: } 3x + 4y + 10 = 0 \implies A = 3,\ B = 4,\ C_2 = 10$$

Step 2: Use the standard formula for the distance between parallel lines:

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Step 3: Substitute the extracted constants and coefficients into the numerator and denominator:

$$d = \frac{|-5 - 10|}{\sqrt{3^2 + 4^2}}$$

Step 4: Simplify the values inside the absolute value brackets and the radical root:

$$d = \frac{|-15|}{\sqrt{9 + 16}} = \frac{15}{\sqrt{25}}$$

Step 5: Compute the final numerical division:

$$d = \frac{15}{5} = 3$$

Final Answer: $\boxed{3}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: A modular inequality of the form $|u| \leq k$ (where $k > 0$) can be unraveled into a continuous compound linear inequality represented as $-k \leq u \leq k$.

Solution: Step 1: Write down the given absolute value inequality:

$$|x - 1| \leq 3$$

Step 2: Convert the modular inequality into its equivalent compound inequality format:

$$-3 \leq x - 1 \leq 3$$

Step 3: Isolate the variable x by adding 1 to all three parts of the inequality series:

$$-3 + 1 \leq x - 1 + 1 \leq 3 + 1$$

Step 4: Simplify the boundary numbers to find the range of values for x:

$$-2 \leq x \leq 4$$

Step 5: Convert the solution set into interval notation. Since the inequality signs include equality, we use closed brackets:

$$x \in [-2, 4]$$

Final Answer: $\boxed{x \in [-2, 4]}$

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: Integrals of the specific form $\int e^x(f(x) + f'(x))\,dx$ can be solved instantly using the standard calculus integration rule which yields $e^x f(x) + c$.

Solution: Step 1: Write down the given indefinite integral expression:

$$\int e^x(\tan x + \sec^2 x)\,dx$$

Step 2: Identify the components inside the integrand to see if they fit the special template function plus its derivative. Let:

$$f(x) = \tan x$$

Step 3: Differentiate $f(x)$ with respect to x to find its corresponding derivative:

$$f'(x) = \frac{d}{dx}(\tan x) = \sec^2 x$$

Step 4: Notice that the integrand matches the standard form perfectly:

$$\int e^x(f(x) + f'(x))\,dx = e^x f(x) + c$$

Step 5: Substitute $f(x) = \tan x$ back into the formula to obtain the final integral result:

$$\text{Result} = e^x \tan x + c$$

Final Answer: $\boxed{e^x \tan x + c}$

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept: The perpendicular distance of any given point $P(x, y, z)$ from a coordinate plane is equal to the absolute value of the coordinate missing from that plane's name. For the xy-plane, the distance is simply $|z|$.

Solution: Step 1: Identify the coordinates of the given point in three-dimensional space:

$$P(x, y, z) = (2, 3, 4)$$

Step 2: Note the individual component values:

$$x = 2, \quad y = 3, \quad z = 4$$

Step 3: Recall the geometric rule for calculating distances from coordinate planes. The perpendicular distance of a point (x, y, z) from the xy-plane is determined by its height along the vertical axis:

$$\text{Distance} = |z|$$

Step 4: Substitute the value of z into this simple relation:

$$\text{Distance} = |4| = 4$$

Final Answer: $\boxed{4}$

Answer: (C)

Go Back to Question 24

Q25.

Solution

Concept: For any square matrix A of order n, multiplying the entire matrix by a scalar k scales its determinant by a factor of k^n. The property is expressed mathematically as $|kA| = k^n|A|$.

Solution: Step 1: Identify the given information from the problem. The order of the square matrix A is:

$$n = 3$$

Step 2: Note the given determinant value of matrix A:

$$|A| = 4$$

Step 3: Identify the scalar factor applied to the matrix inside the determinant expression, which is:

$$k = 2$$

Step 4: Apply the determinant property for scalar multiplication of matrices:

$$|2A| = 2^n|A|$$

Step 5: Substitute $n = 3$ and $|A| = 4$ into the formula and evaluate the numerical expression:

$$|2A| = 2^3 \times 4 = 8 \times 4 = 32$$

Final Answer: $\boxed{32}$

Answer: (C)

Go Back to Question 25

Q26.

Solution

Concept: An expression of the form $a \sin x + b \cos x$ varies between a maximum value of $\sqrt{a^2 + b^2}$ and a minimum value of $-\sqrt{a^2 + b^2}$.

Solution: Step 1: Write down the given trigonometric function:

$$f(x) = \sin x + \cos x$$

Step 2: Compare the given function with the standard linear combination form $a \sin x + b \cos x$ to find the coefficients:

$$a = 1, \quad b = 1$$

Step 3: Recall the mathematical theorem specifying the peak boundaries for this function type:

$$\text{Maximum Value} = \sqrt{a^2 + b^2}$$

Step 4: Substitute the coefficients $a = 1$ and $b = 1$ into the formula:

$$\text{Maximum Value} = \sqrt{1^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2}$$

Final Answer: $\boxed{\sqrt{2}}$

Answer: (C)

Go Back to Question 26

Q27.

Solution

Concept: The order of a differential equation is the highest derivative present in it. The degree is the power of that highest derivative after the equation is cleared of fractional or radical exponents.

Solution: Step 1: Analyze the given differential equation:

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y = 0$$

Step 2: Identify all the derivatives present in the expression. The terms are $\frac{d^2y}{dx^2}$ (second derivative) and $\frac{dy}{dx}$ (first derivative).

Step 3: Determine the highest order derivative among them. The highest derivative is $\frac{d^2y}{dx^2}$, which means the order is:

$$\text{Order} = 2$$

Step 4: Look at the exponent raised on this specific highest derivative term. The term $\left(\frac{d^2y}{dx^2}\right)$ is raised to the power of 3, so:

$$\text{Degree} = 3$$

Final Answer: $\boxed{2, 3}$

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: The number of terms n in a finite Arithmetic Progression (AP) can be calculated using the general formula for the n-th term: $a_n = a + (n-1)d$, where a is the first term and d is the common difference.

Solution: Step 1: Identify the components of the given arithmetic progression sequence $5, 9, 13, \ldots, 81$. The first term is:

$$a = 5$$

Step 2: Calculate the common difference d by subtracting the first term from the second term:

$$d = 9 - 5 = 4$$

Step 3: Identify the last term (a_n) of the finite progression:

$$a_n = 81$$

Step 4: Substitute these values into the standard linear equation for the general term of an AP:

$$81 = 5 + (n-1)4$$

Step 5: Solve the equation step-by-step to find the value of n:

$$81 - 5 = 4(n-1) \implies 76 = 4(n-1)$$

$$n - 1 = \frac{76}{4} \implies n - 1 = 19 \implies n = 20$$

Final Answer: $\boxed{20}$

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: The probability of the union of two events is calculated using the addition rule of probability: $P(K \cup C) = P(K) + P(C) - P(K \cap C)$, which avoids double counting overlapping outcomes.

Solution: Step 1: Establish the size of the total sample space for a standard deck of cards:

$$n(S) = 52$$

Step 2: Find the probability of drawing a King (K). There are 4 Kings in a deck:

$$P(K) = \frac{4}{52}$$

Step 3: Find the probability of drawing a Club card (C). There are 13 Clubs in a deck:

$$P(C) = \frac{13}{52}$$

Step 4: Find the probability of drawing a card that is both a King and a Club ($K \cap C$). There is exactly 1 King of Clubs:

$$P(K \cap C) = \frac{1}{52}$$

Step 5: Apply the addition theorem of probability to find the total probability of choosing a king or a club:

$$P(K \cup C) = P(K) + P(C) - P(K \cap C) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Final Answer: $\boxed{\frac{4}{13}}$

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept: The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$. The radius r of this circle is computed using the expression $r = \sqrt{g^2 + f^2 - c}$.

Solution: Step 1: Write down the given equation of the circle:

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

Step 2: Compare it with the standard general form to determine the parameters g, f, and c:

$$2g = -4 \implies g = -2$$

$$2f = 6 \implies f = 3$$

$$c = -12$$

Step 3: Recall the algebraic formula used to compute the radius of a circle from these parameters:

$$r = \sqrt{g^2 + f^2 - c}$$

Step 4: Substitute the values of g, f, and c into the equation:

$$r = \sqrt{(-2)^2 + (3)^2 - (-12)}$$

Step 5: Simplify the numbers inside the radical sign:

$$r = \sqrt{4 + 9 + 12} = \sqrt{25} = 5$$

Final Answer: $\boxed{5}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: The composite function $(g \circ f)(x)$ represents the application of function g to the output of function f, which is written mathematically as $g(f(x))$.

Solution: Step 1: Identify the two separate functions provided in the question:

$$f(x) = 2x + 3 \quad \text{and} \quad g(x) = x^2$$

Step 2: Write down the formal definition for the composite expression that needs to be evaluated at $x = 1$:

$$(g \circ f)(1) = g(f(1))$$

Step 3: Compute the inner function value $f(1)$ by substituting $x = 1$ into the definition of $f(x)$:

$$f(1) = 2(1) + 3 = 2 + 3 = 5$$

Step 4: Substitute this result into the outer function g to calculate $g(5)$:

$$g(f(1)) = g(5)$$

Step 5: Use the definition of $g(x)$ to evaluate the final squared integer value:

$$g(5) = 5^2 = 25$$

Final Answer: $\boxed{25}$

Answer: (B)

Go Back to Question 31

0.1

Q32.

Solution

Concept: Indefinite integrals involving logarithmic expressions can often be simplified using the substitution method, where we choose $u = \ln x$ so that $du = \frac{1}{x}\,dx$.

Solution: Step 1: Write down the given integral expression:

$$\int \frac{1}{x \ln x}\,dx$$

Step 2: Choose an appropriate variable substitution to simplify the integral. Let:

$$u = \ln x$$

Step 3: Differentiate both sides with respect to x to find the differential relationship:

$$du = \frac{1}{x}\,dx$$

Step 4: Substitute u and du back into the original integration formula:

$$\int \frac{1}{\ln x}\left(\frac{1}{x}\,dx\right) = \int \frac{1}{u}\,du$$

Step 5: Integrate using the standard logarithmic integration rule and substitute back $u = \ln x$:

$$\int \frac{1}{u}\,du = \ln|u| + c = \ln|\ln x| + c$$

Final Answer: $\boxed{\ln|\ln x| + c}$

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept: The definition of the scalar dot product between two vectors is given by $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where θ represents the angle between them.

Solution: Step 1: Extract the given geometric values from the problem statement:

$$|\vec{a}| = 3, \quad |\vec{b}| = 4, \quad \vec{a}\cdot\vec{b} = 6$$

Step 2: Set up the algebraic definition of the dot product to solve for the cosine of the angle:

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

Step 3: Substitute the known quantities into this equation:

$$6 = (3)(4)\cos\theta$$

$$6 = 12\cos\theta$$

Step 4: Isolate the trigonometric component $\cos\theta$:

$$\cos\theta = \frac{6}{12} = \frac{1}{2}$$

Step 5: Find the principal angle value θ whose cosine equals $\frac{1}{2}$:

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (C)

Go Back to Question 33

Q34.

Solution

Concept: The exact value of a non-standard trigonometric angle like 15° can be found by expressing it as the difference of two standard angles and using the identity $\cos(A - B) = \cos A \cos B + \sin A \sin B$.

Solution: Step 1: Express the angle 15° as a difference of two well-known standard angles:

$$15^\circ = 45^\circ - 30^\circ$$

Step 2: Apply the standard cosine subtraction compound angle formula:

$$\cos(15^\circ) = \cos(45^\circ - 30^\circ) = \cos(45^\circ)\cos(30^\circ) + \sin(45^\circ)\sin(30^\circ)$$

Step 3: Substitute the exact values for the standard sine and cosine functions:

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}, \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}, \quad \sin(30^\circ) = \frac{1}{2}$$

Step 4: Substitute these individual values back into the expanded expression:

$$\cos(15^\circ) = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

Step 5: Multiply the terms together and combine them over the common denominator:

$$\cos(15^\circ) = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Final Answer: $\boxed{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

Answer: (B)

Go Back to Question 34

Q35.

Solution

Concept: The standard deviation of the first n natural numbers can be computed directly using the specialized statistical formula $\sigma = \sqrt{\frac{n^2-1}{12}}$.

Solution: Step 1: Identify the number of terms in the sequence of consecutive starting integers. For the first 5 natural numbers $(1, 2, 3, 4, 5)$, we have:

$$n = 5$$

Step 2: Recall the derived standard formula for the standard deviation (σ) of the first n natural numbers:

$$\sigma = \sqrt{\frac{n^2 - 1}{12}}$$

Step 3: Substitute the count value $n = 5$ into the algebraic formula:

$$\sigma = \sqrt{\frac{5^2 - 1}{12}}$$

Step 4: Compute the squared values and perform the internal subtraction:

$$\sigma = \sqrt{\frac{25 - 1}{12}} = \sqrt{\frac{24}{12}}$$

Step 5: Complete the arithmetic fraction division to find the simplified radical answer:

$$\sigma = \sqrt{2}$$

Final Answer: $\boxed{\sqrt{2}}$

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept: A first-order linear differential equation written in the standard form $\frac{dy}{dx} + P(x)y = Q(x)$ has an Integrating Factor (IF) defined by the exponential formula $\text{IF} = e^{\int P(x)\,dx}$.

Solution: Step 1: Write down the given differential equation:

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

Step 2: Compare this expression with the standard linear differential format $\frac{dy}{dx} + P(x)y = Q(x)$ to isolate the function coefficient of y:

$$P(x) = \frac{1}{x}$$

Step 3: Set up the definition equation for finding the integrating factor:

$$\text{IF} = e^{\int P(x)\,dx} = e^{\int \frac{1}{x}\,dx}$$

Step 4: Integrate the function inside the exponent. The standard integral of $\frac{1}{x}$ is $\ln x$:

$$\text{IF} = e^{\ln x}$$

Step 5: Use the exponential-logarithmic identity property $e^{\ln(\theta)} = \theta$ to simplify the expression completely:

$$\text{IF} = x$$

Final Answer: $\boxed{x}$

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept: The complex cube roots of unity satisfy the two fundamental algebraic properties: $1 + \omega + \omega^2 = 0$ (hence $1 + \omega^2 = -\omega$ and $1 + \omega = -\omega^2$) and $\omega^3 = 1$.

Solution: Step 1: Write down the expression that needs simplification:

$$(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5$$

Step 2: Group the terms to apply the identity $1 + \omega + \omega^2 = 0$. For the first term, replace $1 + \omega^2$ with $-\omega$:

$$(1 + \omega^2 - \omega)^5 = (-\omega - \omega)^5 = (-2\omega)^5$$

Step 3: For the second term, substitute $1 + \omega$ with $-\omega^2$ using the same relation:

$$(1 + \omega - \omega^2)^5 = (-\omega^2 - \omega^2)^5 = (-2\omega^2)^5$$

Step 4: Combine the simplified terms back into the expression and factor out the constant coefficients:

$$\text{Value} = (-2\omega)^5 + (-2\omega^2)^5 = -32\omega^5 - 32\omega^{10} = -32(\omega^5 + \omega^{10})$$

Step 5: Reduce the higher powers of ω using the cyclical property $\omega^3 = 1$. Since $\omega^5 = \omega^2$ and $\omega^{10} = \omega$, substitute them back:

$$\text{Value} = -32(\omega^2 + \omega) = -32(-1) = 32$$

Final Answer: $\boxed{32}$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept: The vertex of a parabola can be found by transforming its quadratic equation into the standard vertex form $(y-k)^2 = 4a(x-h)$, where the coordinates of the vertex are given by (h, k).

Solution: Step 1: Write down the given equation of the parabola:

$$y^2 - 4y - 4x + 8 = 0$$

Step 2: Rearrange the equation to keep all the terms involving y on the left side and move the other terms to the right side:

$$y^2 - 4y = 4x - 8$$

Step 3: Complete the square on the left-hand side by adding $\left(\frac{-4}{2}\right)^2 = 4$ to both sides of the equation:

$$y^2 - 4y + 4 = 4x - 8 + 4$$

Step 4: Express the left side as a perfect square and simplify the linear expression on the right side:

$$(y-2)^2 = 4x - 4 \implies (y-2)^2 = 4(x-1)$$

Step 5: Compare this rearranged equation with the standard form $(y-k)^2 = 4a(x-h)$ to identify the vertex coordinates (h, k):

$$h = 1, \quad k = 2 \implies \text{Vertex} = (1, 2)$$

Final Answer: $\boxed{(1, 2)}$

Answer: (A)

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Q39.

Solution

Concept: Two events A and B are said to be mutually exclusive if they cannot occur at the same time, which means their intersection is an empty set and $P(A \cap B) = 0$. The addition rule simplifies to $P(A \cup B) = P(A) + P(B)$.

Solution: Step 1: Identify the given probabilities from the problem text:

$$P(A) = 0.3 \quad \text{and} \quad P(B) = 0.4$$

Step 2: Note the crucial condition given that the two events are mutually exclusive. By definition, this property implies:

$$P(A \cap B) = 0$$

Step 3: Recall the general probability addition formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Step 4: Substitute the zero value for the intersection term into the equation:

$$P(A \cup B) = P(A) + P(B) - 0 = P(A) + P(B)$$

Step 5: Add the individual numerical values together to find the final probability:

$$P(A \cup B) = 0.3 + 0.4 = 0.7$$

Final Answer: 0.7

Answer: (A)

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Q40.

Solution

Concept: Limits of algebraic fractions exhibiting the indeterminate form $\frac{0}{0}$ can be evaluated by factoring the polynomials to cancel out the common zero-producing factor using the identity $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

Solution: Step 1: Write down the given limit expression:

$$\lim_{x \to 3} \frac{x^3 - 27}{x - 3}$$

Step 2: Substitute $x = 3$ into the function to check its initial form. Since $3^3 - 27 = 0$ and $3 - 3 = 0$, it produces the indeterminate form $\frac{0}{0}$.

Step 3: Factor the numerator using the difference of cubes algebraic identity, noting that $27 = 3^3$:

$$x^3 - 3^3 = (x - 3)(x^2 + 3x + 3^2) = (x - 3)(x^2 + 3x + 9)$$

Step 4: Substitute the factored expression back into the limit and cancel out the common factor $(x - 3)$ from the numerator and denominator:

$$\lim_{x \to 3} \frac{(x - 3)(x^2 + 3x + 9)}{x - 3} = \lim_{x \to 3} (x^2 + 3x + 9)$$

Step 5: Evaluate the limit by directly substituting $x = 3$ into the simplified polynomial:

$$\text{Value} = 3^2 + 3(3) + 9 = 9 + 9 + 9 = 27$$

Final Answer: $\boxed{27}$

Answer: (B)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	A	4	B	5	B
6	C	7	A	8	B	9	A	10	B
11	A	12	A	13	C	14	B	15	A
16	B	17	A	18	A	19	A	20	A
21	A	22	A	23	B	24	C	25	C
26	C	27	A	28	B	29	B	30	A
31	B	32	B	33	C	34	B	35	B
36	B	37	A	38	A	39	A	40	B