Rajasthan JET Mathematics Sample Paper-2

Duration: 40 Minutes **Maximum Marks: 160**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **−1 marks**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. Let A and B be two non-singular matrices of order 3 such that $A + B = I$ and $A^{-1} + B^{-1} = 2I$. Then the value of $|4AB|$ is

(A) 1

(B) 2

(C) 4

(D) 8

Q2. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \frac{x}{1+x^2}$, then the range of the function is

(A) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(B) $[-1, 1]$

(C) $\mathbb{R}$

(D) $[0, \infty)$

Q3. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x}\, dx$ is

(A) π

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) 0

Q4. The focus of the parabola $y^2 - 4y - 8x + 4 = 0$ is located at the point

(A) $(0, 2)$

(B) $(2, 2)$

(C) $(2, 0)$

(D) $(0, 0)$

Q5. A bag contains 4 white and 6 black balls. Two balls are drawn at random one after the other without replacement. What is the probability that both drawn balls are white?

(A) $\frac{4}{25}$

(B) $\frac{2}{15}$

(C) $\frac{1}{6}$

(D) $\frac{8}{45}$

Q6. The value of $\lim_{x\to 0} \frac{e^{\sin x}-1}{x}$ is

(A) 0

(B) 1

(C) e

(D) Does not exist

Q7. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then the value of $\cos^{-1} x + \cos^{-1} y$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) π

Q8. The value of $(1 + i)^{10} + (1 - i)^{10}$, where $i = \sqrt{-1}$, is

(A) 0

(B) 64

(C) -64

(D) $32i$

Q9. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$ are respectively

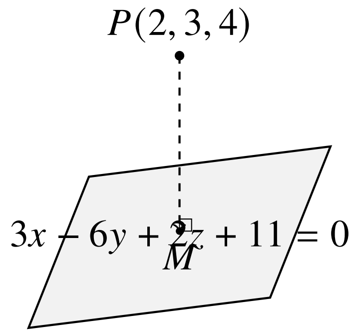
(A) $2, 3$

(B) $2, 2$

(C) $3, 2$

(D) $1, 2$

Q10. The distance of the point $(2, 3, 4)$ from the plane $3x-6y+2z+11=0$ is



(A) 1 unit

(B) 2 units

(C) 3 units

(D) 0 units

Q11. The mean of 5 observations is 4 and their variance is 5.2. If three of the observations are $1, 2$, and 6, then the other two observations are

(A) $2, 9$

(B) $3, 8$

(C) $4, 7$

(D) $5, 6$

Q12. If the n-th term of an Arithmetic Progression is $3n+5$, then the sum of its first 20 terms is

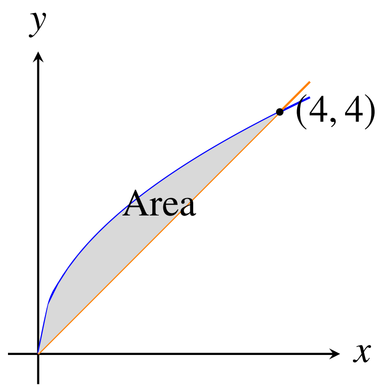
(A) 610

(B) 690

(C) 730

(D) 740

Q13. The area of the region bounded by the curve $y^2 = 4x$ and the line $y = x$ is



(A) $\frac{2}{3}$ sq. units

(B) $\frac{4}{3}$ sq. units

(C) $\frac{8}{3}$ sq. units

(D) $\frac{16}{3}$ sq. units

Q14. If the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar, then the value of λ is

(A) -4

(B) -2

(C) 2

(D) 4

Q15. The derivative of $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ with respect to x is

(A) 1

(B) -1

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Q16. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.8$, and $P(B|A) = 0.6$, then $P(A \cup B)$ is equal to

(A) 0.96

(B) 0.24

(C) 0.84

(D) 0.56

Q17. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is

(A) $xy = \frac{x^3}{3} + C$

(B) $xy = \frac{x^4}{4} + C$

(C) $y = x^3 + Cx$

(D) $y = \frac{x^4}{4} + C$

Q18. The number of non-empty subsets of a set containing 6 elements is

(A) 64

(B) 63

(C) 32

(D) 31

Q19. The equation of the line passing through the point $(1, 2, 3)$ and parallel to the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-5}{2}$ is

(A) $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{2}$

(B) $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-2}{3}$

(C) $\frac{x+1}{3} = \frac{y+2}{4} = \frac{z+3}{2}$

(D) $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{5}$

Q20. If $\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$, then the positive value of x is

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{3}{2}$

(D) 3

Q21. The maximum value of the function $f(x) = x^3 - 3x$ on the interval $[0, 2]$ is attained at $x =$

(A) 0

(B) 1

(C) $\sqrt{3}$

(D) 2

Q22. If the third term of a Geometric Progression is 4, then the product of its first 5 terms is

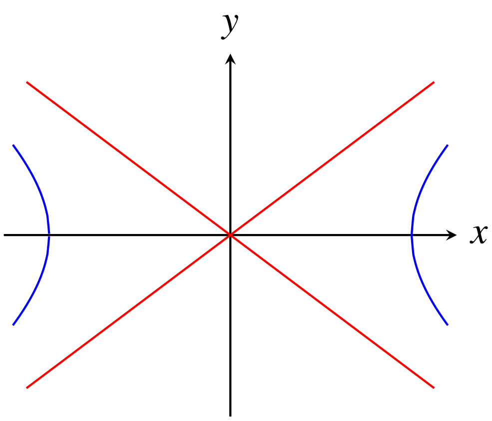
(A) 4^3

(B) 4^4

(C) 4^5

(D) 4^6

Q23. The angle between the asymptotes of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is



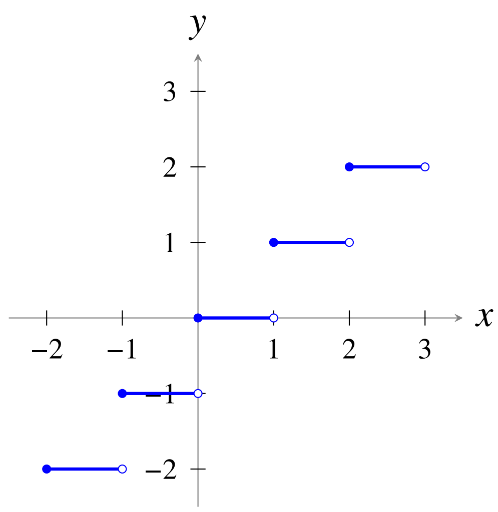
(A) $2\tan^{-1}\left(\frac{3}{4}\right)$

(B) $2\tan^{-1}\left(\frac{4}{3}\right)$

(C) $\tan^{-1}\left(\frac{3}{4}\right)$

(D) $\tan^{-1}\left(\frac{4}{3}\right)$

Q24. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer function, is discontinuous at



(A) $x = 0.5$

(B) $x = 1.2$

(C) $x = 3$

(D) $x = -1.5$

Q25. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $|\vec{a} + \vec{b}| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) $\frac{2\pi}{3}$

Q26. The value of $\cos 20° \cos 40° \cos 80°$ is equal to

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{8}$

(D) $\frac{1}{16}$

Q27. The line $y = mx + 1$ is a tangent to the parabola $y^2 = 4x$ if the value of m is

(A) 1

(B) 2

(C) 3

(D) $\frac{1}{2}$

Q28. If the coefficient of variation of a distribution is 60% and its standard deviation is 12, then the arithmetic mean of the distribution is

(A) 20

(B) 5

(C) 7.2

(D) 18

Q29. The value of $\int e^x(\tan x + \ln(\sec x))\,dx$ is

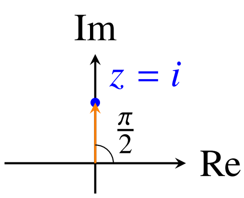
(A) $e^x \tan x + C$

(B) $e^x \ln(\sec x) + C$

(C) $e^x \sec x + C$

(D) $-e^x \ln(\cos x) + C$

Q30. If $z = \frac{1+i}{1-i}$, then the principal argument of z is



(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

Q31. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is

(A) 3

(B) $\frac{1}{3}$

(C) -3

(D) $-\frac{1}{3}$

Q32. If A is a square matrix of order 3 and $|A| = 5$, then the value of $|\text{adj}(A)|$ is

(A) 5

(B) 25

(C) 125

(D) $\frac{1}{5}$

Q33. A die is tossed twice. A 'success' is getting an odd number on a toss. The variance of the number of successes is

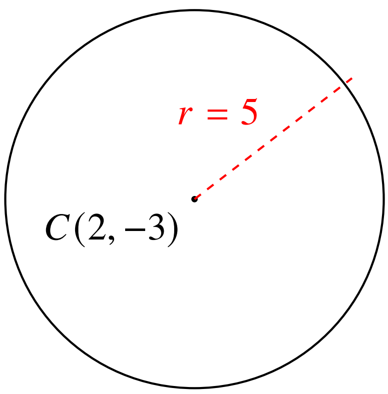
(A) $\frac{1}{2}$

(B) 1

(C) $\frac{1}{4}$

(D) 2

Q34. The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is



(A) 3 units

(B) 4 units

(C) 5 units

(D) 6 units

Q35. The fundamental period of the function $f(x) = \sin\left(\frac{2\pi x}{3}\right) + \cos\left(\frac{\pi x}{2}\right)$ is

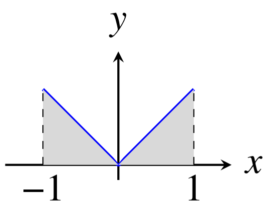
(A) 3

(B) 4

(C) 6

(D) 12

Q36. The value of $\int_{-1}^{1} |x|\, dx$ is



(A) 0

(B) 1

(C) 2

(D) $\frac{1}{2}$

Q37. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$ for two vectors $\vec{a}$ and $\vec{b}$, then it implies that

(A) $\vec{a}$ is parallel to $\vec{b}$

(B) $\vec{a}$ is perpendicular to $\vec{b}$

(C) At least one of $\vec{a}$ or $\vec{b}$ is a null vector

(D) $\vec{a}$ and $\vec{b}$ are unit vectors

Q38. The value of $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$ is equal to

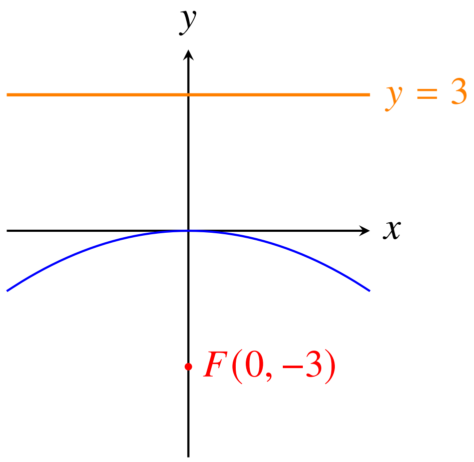
(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{3\pi}{4}$

(D) 2π

Q39. If the vertex of a parabola is at $(0, 0)$ and its focus is at $(0, -3)$, then the equation of its directrix is



(A) $y = 3$

(B) $y = -3$

(C) $x = 3$

(D) $x = -3$

Q40. The integrating factor of the differential equation $x\frac{dy}{dx} - y = x^2$ is

(A) x

(B) e^{-x}

(C) $\frac{1}{x}$

(D) $\ln x$

Detailed Solutions

Q1.

Solution

Concept: For two non-singular matrices of order 3, we use matrix algebraic identities along with the properties of determinants, specifically that $|kM| = k^3|M|$ for a matrix M of order 3, and $|MN| = |M||N|$.

Solution: Step 1: We are given the matrix equation $A^{-1} + B^{-1} = 2I$. Pre-multiplying both sides by A and post-multiplying both sides by B, we can simplify the expression:

$$A(A^{-1} + B^{-1})B = A(2I)B$$

$$(I + AB^{-1})B = 2AB$$

$$B + A(B^{-1}B) = 2AB$$

$$B + A = 2AB$$

Step 2: We are given that $A + B = I$. Substituting this into our simplified equation gives:

$$I = 2AB$$

$$AB = \frac{1}{2}I$$

Step 3: Now, we apply the determinant operator to both sides of the equation $AB = \frac{1}{2}I$:

$$|AB| = \left|\frac{1}{2}I\right|$$

Since I is an identity matrix of order 3, factoring out the scalar constant $\frac{1}{2}$ requires raising it to the power of 3:

$$|AB| = \left(\frac{1}{2}\right)^3 |I| = \frac{1}{8} \times 1 = \frac{1}{8}$$

Step 4: We need to evaluate the determinant value of the matrix $4AB$. Since the matrix AB has an order of 3, the scalar multiple 4 must be factored out by cubing it:

$$|4AB| = 4^3|AB|$$

$$|4AB| = 64 \times \frac{1}{8} = 8$$

Final Answer: The value of $|4AB|$ is 8.

Answer: (D)

Go Back to Question 1

Q2.

Solution

Concept: To find the range of a real-valued function $y = f(x)$, we set up the equation $y = \frac{x}{1+x^2}$ and convert it into a quadratic equation in terms of x. For x to be real, the discriminant of this quadratic equation must be greater than or equal to zero.

Solution: Step 1: Set the function equal to y and clear the denominator:

$$y = \frac{x}{1+x^2}$$

$$y(1+x^2) = x$$

$$yx^2 - x + y = 0$$

Step 2: We analyze the equation based on the value of y. If $y = 0$, the equation reduces to $-x = 0$, which gives $x = 0$. Since $x = 0$ is a valid real number, $y = 0$ belongs to the range of the function.

Step 3: If $y \neq 0$, the equation $yx^2 - x + y = 0$ is a quadratic equation in x. Since x is a real number, the discriminant (D) of this quadratic equation must satisfy $D \geq 0$:

$$(-1)^2 - 4(y)(y) \geq 0$$

$$1 - 4y^2 \geq 0$$

$$4y^2 \leq 1$$

$$y^2 \leq \frac{1}{4}$$

Step 4: Taking the square root on both sides gives the interval for y:

$$-\frac{1}{2} \leq y \leq \frac{1}{2}$$

Combining this with $y = 0$, the complete range of the function is $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

Final Answer: The range of the function is $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept: This definite integral can be solved efficiently by applying the modular property of definite integrals, $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$, which is commonly referred to as the King's property.

Solution: Step 1: Let the given integral be represented as I:

$$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}\,dx \quad \text{— (Equation 1)}$$

Step 2: Apply the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ by replacing x with $\left(\frac{\pi}{2} - x\right)$:

$$I = \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2} - x\right)}{\sin^3\left(\frac{\pi}{2} - x\right) + \cos^3\left(\frac{\pi}{2} - x\right)}\,dx$$

Step 3: Use the standard trigonometric reduction identities $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ and $\cos\left(\frac{\pi}{2} - x\right) = \sin x$:

$$I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x}\,dx \quad \text{— (Equation 2)}$$

Step 4: Add Equation 1 and Equation 2 together to simplify the integrand:

$$2I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}\,dx + \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x}\,dx$$

$$2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x}\,dx$$

$$2I = \int_0^{\pi/2} 1\,dx$$

Step 5: Integrate and apply the lower and upper limits of integration:

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Final Answer: The value of the integral is $\frac{\pi}{4}$.

Answer: (C)

Go Back to Question 3

Q4.

Solution

Concept: To find the focus of a parabola given in its general quadratic form, we convert the equation into the standard form $(y-k)^2 = 4a(x-h)$ by completing the square for the quadratic terms. The focus is then given by $(h+a, k)$.

Solution: Step 1: Group the y terms on one side and move the other terms to the right-hand side:

$$y^2 - 4y = 8x - 4$$

Step 2: Complete the square on the left-hand side by adding $\left(\frac{-4}{2}\right)^2 = 4$ to both sides:

$$y^2 - 4y + 4 = 8x - 4 + 4$$

$$(y-2)^2 = 8x$$

Step 3: Compare this with the standard shifting vertex form of a parabola $(y-k)^2 = 4a(x-h)$. From comparison, we get:

$$\text{Vertex } (h, k) = (0, 2)$$

$$4a = 8 \implies a = 2$$

Step 4: The focus of the standard horizontal parabola opening to the right is shifted relative to the vertex and is located at $(h+a, k)$.

Step 5: Substitute the values of h, k, and a into the focus coordinate formula:

$$\text{Focus} = (0+2, 2) = (2, 2)$$

Final Answer: The focus of the parabola is $(2, 2)$.

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: The probability of independent sequential events without replacement is solved by multiplying the conditional probabilities of each step, or by using combinations to choose a subset of desired items from the total sample space.

Solution: Step 1: Compute the total number of balls in the bag initially:

$$\text{Total balls} = 4 \text{ white} + 6 \text{ black} = 10 \text{ balls}$$

Step 2: Find the probability of drawing a white ball on the first attempt:

$$P(W_1) = \frac{\text{Number of white balls}}{\text{Total number of balls}} = \frac{4}{10}$$

Step 3: Since the ball is drawn without replacement, the composition of the bag changes for the second draw. There are now $4 - 1 = 3$ white balls left, and the total number of balls is reduced to $10 - 1 = 9$ balls.

Step 4: Find the conditional probability of drawing a white ball on the second attempt given that the first was white:

$$P(W_2|W_1) = \frac{3}{9} = \frac{1}{3}$$

Step 5: Multiply the two sequential probabilities to obtain the compound probability that both balls are white:

$$P(W_1 \cap W_2) = P(W_1) \times P(W_2|W_1) = \frac{4}{10} \times \frac{1}{3} = \frac{4}{30} = \frac{2}{15}$$

Final Answer: The probability is $\frac{2}{15}$.

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept: This limit problem exhibits an indeterminate form of $\frac{0}{0}$. It can be resolved by using standard fundamental limits, specifically $\lim_{u\to 0} \frac{e^u-1}{u} = 1$, or by applying L'Hopital's Rule.

Solution: Step 1: Check the direct substitution value at $x = 0$:

$$\text{Numerator: } e^{\sin 0} - 1 = e^0 - 1 = 0$$

$$\text{Denominator: } 0$$

Since it forms $\frac{0}{0}$, we proceed with algebraic manipulation to use a standard limit form.

Step 2: Multiply and divide the expression by $\sin x$ to structurally align it with the exponential limit identity:

$$\lim_{x\to 0} \frac{e^{\sin x} - 1}{x} = \lim_{x\to 0} \left(\frac{e^{\sin x} - 1}{\sin x} \times \frac{\sin x}{x} \right)$$

Step 3: Split the limits using the product limit rule:

$$\left(\lim_{x\to 0} \frac{e^{\sin x} - 1}{\sin x} \right) \times \left(\lim_{x\to 0} \frac{\sin x}{x} \right)$$

Step 4: Evaluate each part. Let $u = \sin x$. As $x \to 0$, $u \to 0$, so the first limit becomes $\lim_{u\to 0} \frac{e^u-1}{u} = 1$. The second part is the standard trigonometric limit $\lim_{x\to 0} \frac{\sin x}{x} = 1$.

Step 5: Multiply the evaluated limits:

$$1 \times 1 = 1$$

Final Answer: The value of the limit is 1.

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept: We use the basic complementary inverse trigonometric identity, which states that for any $k \in [-1, 1]$, $\sin^{-1} k + \cos^{-1} k = \frac{\pi}{2}$.

Solution: Step 1: Write down the identity for both variables x and y:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \quad \text{— (Equation 1)}$$

$$\sin^{-1} y + \cos^{-1} y = \frac{\pi}{2} \quad \text{— (Equation 2)}$$

Step 2: Add Equation 1 and Equation 2 together:

$$(\sin^{-1} x + \sin^{-1} y) + (\cos^{-1} x + \cos^{-1} y) = \frac{\pi}{2} + \frac{\pi}{2}$$

$$(\sin^{-1} x + \sin^{-1} y) + (\cos^{-1} x + \cos^{-1} y) = \pi$$

Step 3: Substitute the given value $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$ into the consolidated equation:

$$\frac{2\pi}{3} + (\cos^{-1} x + \cos^{-1} y) = \pi$$

Step 4: Solve for the target expression $(\cos^{-1} x + \cos^{-1} y)$ by subtraction:

$$\cos^{-1} x + \cos^{-1} y = \pi - \frac{2\pi}{3}$$

$$\cos^{-1} x + \cos^{-1} y = \frac{\pi}{3}$$

Final Answer: The value of $\cos^{-1} x + \cos^{-1} y$ is $\frac{\pi}{3}$.

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept: Complex numbers raised to high powers are simplified by converting them to polar/Euler form or by calculating small integer powers first to find an easier base value.

Solution: Step 1: Let us evaluate the squares of the complex expressions $(1+i)$ and $(1-i)$:

$$(1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$$

$$(1-i)^2 = 1 - 2i + i^2 = 1 - 2i - 1 = -2i$$

Step 2: Rewrite the 10th power expressions as powers of their squares using exponent index laws:

$$(1+i)^{10} = \left((1+i)^2\right)^5 = (2i)^5$$

$$(1-i)^{10} = \left((1-i)^2\right)^5 = (-2i)^5$$

Step 3: Expand the terms using power distribution rules:

$$(2i)^5 = 2^5 \times i^5 = 32 \times i^4 \times i = 32 \times 1 \times i = 32i$$

$$(-2i)^5 = (-2)^5 \times i^5 = -32 \times i = -32i$$

Step 4: Add the two simplified values together:

$$(1+i)^{10} + (1-i)^{10} = 32i + (-32i) = 0$$

Final Answer: The value of the expression is 0.

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: The order of a differential equation is the highest derivative present. The degree is the power of this highest derivative after the equation is cleared of fractional indices and radicals regarding the derivatives.

Solution: Step 1: Write down the given differential equation:

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$$

Step 2: Identify the highest derivative in the expression. The terms are $\frac{dy}{dx}$ (first derivative) and $\frac{d^2y}{dx^2}$ (second derivative). Therefore, the highest derivative is $\frac{d^2y}{dx^2}$, which means:

$$\text{Order} = 2$$

Step 3: To find the degree, we must eliminate the fractional exponent $\frac{3}{2}$. Square both sides of the equation:

$$\left(\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}\right)^2 = \left(\frac{d^2y}{dx^2}\right)^2$$

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

Step 4: Now, the equation is a polynomial in terms of its derivatives. The exponent raised on the highest order derivative $\left(\frac{d^2y}{dx^2}\right)$ is 2. Thus:

$$\text{Degree} = 2$$

Final Answer: The order and degree are 2, 2 respectively.

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: The perpendicular distance d from a 3D point $P(x_1, y_1, z_1)$ to a plane described by the algebraic equation $Ax + By + Cz + D = 0$ is given by the formula $d = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$.

Solution: Step 1: Identify the components from the coordinates of the point and the coefficients of the plane:

$$\text{Point: } x_1 = 2,\ y_1 = 3,\ z_1 = 4$$

$$\text{Plane: } A = 3,\ B = -6,\ C = 2,\ D = 11$$

Step 2: Substitute these values into the numerator of the distance formula:

$$\text{Numerator} = |3(2) + (-6)(3) + 2(4) + 11|$$

$$\text{Numerator} = |6 - 18 + 8 + 11|$$

$$\text{Numerator} = |7| = 7$$

Step 3: Substitute the coefficients into the denominator formula:

$$\text{Denominator} = \sqrt{3^2 + (-6)^2 + 2^2}$$

$$\text{Denominator} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$$

Step 4: Divide the numerator value by the denominator value to find the distance:

$$d = \frac{7}{7} = 1 \text{ unit}$$

Final Answer: The distance from the point is 1 unit.

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: We use the fundamental statistical definitions of mean ($\bar{x} = \frac{\sum x_i}{n}$) and variance ($\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$) to set up two simultaneous equations for the two missing values.

Solution: Step 1: Let the two unknown observations be a and b. The complete set of 5 observations is $\{1, 2, 6, a, b\}$.

Step 2: Use the mean formula to establish the first equation:

$$\bar{x} = \frac{1 + 2 + 6 + a + b}{5} = 4$$

$$9 + a + b = 20 \implies a + b = 11 \quad \text{— (Equation 1)}$$

Step 3: Use the variance formula to establish the second equation:

$$\sigma^2 = \frac{1^2 + 2^2 + 6^2 + a^2 + b^2}{5} - (4)^2 = 5.2$$

$$\frac{1 + 4 + 36 + a^2 + b^2}{5} - 16 = 5.2$$

$$\frac{41 + a^2 + b^2}{5} = 21.2$$

$$41 + a^2 + b^2 = 106 \implies a^2 + b^2 = 65 \quad \text{— (Equation 2)}$$

Step 4: Use the identity $(a + b)^2 = a^2 + b^2 + 2ab$ to find ab:

$$11^2 = 65 + 2ab \implies 121 - 65 = 2ab \implies 56 = 2ab \implies ab = 28$$

Step 5: Solve the system. We look for two numbers whose sum is 11 and product is 28. These numbers are the roots of $t^2 - 11t + 28 = 0$, which factors into $(t - 4)(t - 7) = 0$. Thus, the numbers are 4 and 7.

Final Answer: The other two observations are 4, 7.

Answer: (C)

Go Back to Question 11

Q12.

Solution

Concept: The sum of the first n terms of an Arithmetic Progression can be determined using the formula $S_n = \frac{n}{2}(a + a_n)$, where a is the first term and a_n is the n-th term.

Solution: Step 1: The general n-th term expression is given as $a_n = 3n + 5$. Find the first term (a) by setting $n = 1$:

$$a = a_1 = 3(1) + 5 = 8$$

Step 2: Find the 20th term (a_{20}) by substituting $n = 20$ into the expression:

$$a_{20} = 3(20) + 5 = 60 + 5 = 65$$

Step 3: Use the sum formula for $n = 20$ terms:

$$S_{20} = \frac{20}{2}(a_1 + a_{20})$$

$$S_{20} = 10 \times (8 + 65)$$

Step 4: Compute the final multiplication value:

$$S_{20} = 10 \times 73 = 730$$

Final Answer: The sum of the first 20 terms is 730.

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: The area bounded between two intersecting curves $y_1(x)$ and $y_2(x)$ from $x = a$ to $x = b$ is given by the definite integral $\int_a^b (y_{\text{upper}} - y_{\text{lower}})\, dx$.

Solution: Step 1: Find the points of intersection by solving the two equations $y^2 = 4x$ and $y = x$ simultaneously. Substitute $y = x$ into the parabola equation:

$$x^2 = 4x \implies x^2 - 4x = 0 \implies x(x-4) = 0$$

The intersection points occur at $x = 0$ and $x = 4$. The corresponding points are $(0, 0)$ and $(4, 4)$.

Step 2: Determine which curve lies upper in the interval $[0, 4]$. For $x = 1$, $y_{\text{parabola}} = 2$ and $y_{\text{line}} = 1$. Thus, the parabola is the upper curve ($y = 2\sqrt{x}$) and the line is the lower curve ($y = x$).

Step 3: Set up the area definite integral:

$$\text{Area} = \int_0^4 (2\sqrt{x} - x)\, dx$$

Step 4: Integrate the individual terms:

$$\text{Area} = \left[2 \cdot \frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^4 = \left[\frac{4}{3}x^{3/2} - \frac{x^2}{2}\right]_0^4$$

Step 5: Substitute the upper limit 4 and lower limit 0:

$$\text{Area} = \left(\frac{4}{3}(4)^{3/2} - \frac{4^2}{2}\right) - 0 = \left(\frac{4}{3}(8) - \frac{16}{2}\right) = \frac{32}{3} - 8 = \frac{32 - 24}{3} = \frac{8}{3}$$

Final Answer: The bounded area is $\frac{8}{3}$ sq. units.

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept: Three vectors are coplanar if and only if their scalar triple product is equal to zero. This can be evaluated by setting the determinant of the matrix formed by their components to zero.

Solution: Step 1: Write out the component matrix for the given vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

Step 2: Expand the determinant along the first row:

$$2\begin{vmatrix} 2 & -3 \\ \lambda & 5 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 3 & \lambda \end{vmatrix} = 0$$

Step 3: Compute the 2×2 minor values:

$$2(10 - (-3\lambda)) + 1(5 - (-9)) + 1(\lambda - 6) = 0$$

$$2(10 + 3\lambda) + 1(14) + (\lambda - 6) = 0$$

Step 4: Expand terms and combine like linear expressions:

$$20 + 6\lambda + 14 + \lambda - 6 = 0$$

$$7\lambda + 28 = 0$$

Step 5: Solve for λ:

$$7\lambda = -28 \implies \lambda = -4$$

Final Answer: The value of λ is -4.

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: To differentiate an inverse trigonometric expression, we first simplify the internal trigonometric terms using compound angle identities before executing differentiation.

Solution: Step 1: Let the internal expression be y. Divide the numerator and denominator inside the parenthesis by $\cos x$:

$$y = \tan^{-1}\left(\frac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}\right)$$

$$y = \tan^{-1}\left(\frac{1 - \tan x}{1 + \tan x}\right)$$

Step 2: Recognize that $1 = \tan\left(\frac{\pi}{4}\right)$, allowing us to reframe it using the compound subtraction identity $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$:

$$y = \tan^{-1}\left(\frac{\tan(\pi/4) - \tan x}{1 + \tan(\pi/4)\tan x}\right)$$

$$y = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - x\right)\right)$$

Step 3: Simplify the inverse function cancellation:

$$y = \frac{\pi}{4} - x$$

Step 4: Differentiate y with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{4}\right) - \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = 0 - 1 = -1$$

Final Answer: The derivative value is -1.

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: We use the definition of conditional probability $P(B|A) = \frac{P(A\cap B)}{P(A)}$ to discover the intersection value, and then apply the general addition rule $P(A\cup B) = P(A)+P(B)-P(A\cap B)$.

Solution: Step 1: Use the given conditional probability value to find $P(A \cap B)$:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$0.6 = \frac{P(A \cap B)}{0.4}$$

$$P(A \cap B) = 0.6 \times 0.4 = 0.24$$

Step 2: Use the probability set addition formula to compute the union value:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Step 3: Substitute the corresponding given values into the formulation:

$$P(A \cup B) = 0.4 + 0.8 - 0.24$$

Step 4: Calculate the final arithmetic combination:

$$P(A \cup B) = 1.2 - 0.24 = 0.96$$

Final Answer: The value of $P(A \cup B)$ is 0.96.

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: This is a standard first-order linear differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$. It is solved by finding the integrating factor $IF = e^{\int P(x)\,dx}$ and using the solution form $y \cdot IF = \int Q(x) \cdot IF\,dx$.

Solution: Step 1: Compare the given equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ with the standard linear form:

$$P(x) = \frac{1}{x}, \quad Q(x) = x^2$$

Step 2: Calculate the Integrating Factor (IF):

$$IF = e^{\int \frac{1}{x}\,dx} = e^{\ln x} = x$$

Step 3: Write out the structural general solution equation:

$$y \cdot (IF) = \int Q(x) \cdot (IF)\,dx$$

$$y \cdot x = \int x^2 \cdot x\,dx$$

$$xy = \int x^3\,dx$$

Step 4: Integrate the right-hand side using the power rule:

$$xy = \frac{x^4}{4} + C$$

Final Answer: The solution is $xy = \frac{x^4}{4} + C$.

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept: A set containing n elements has a total of 2^n subsets. This total includes the empty set. Therefore, the number of non-empty subsets is found by subtracting 1 from the total number of subsets.

Solution: Step 1: Identify the number of elements given in the problem statement:

$$n = 6$$

Step 2: Calculate the total number of possible subsets using the formula 2^n:

$$\text{Total subsets} = 2^6 = 64$$

Step 3: To find the number of non-empty subsets, exclude the unique empty set ($\emptyset$) from the count:

$$\text{Non-empty subsets} = 2^n - 1$$

$$\text{Non-empty subsets} = 64 - 1 = 63$$

Final Answer: The number of non-empty subsets is 63.

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept: The equation of a straight line in 3D space passing through a point (x_1, y_1, z_1) with direction ratios (a, b, c) is given in symmetrical format by $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$. Parallel lines share identical or proportional direction ratios.

Solution: Step 1: Extract the direction ratios of the given line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-5}{2}$ from its denominators:

$$\text{Direction ratios } (a, b, c) = (3, 4, 2)$$

Step 2: Since the target line is parallel to the given line, it will use the exact same direction ratios:

$$(a, b, c) = (3, 4, 2)$$

Step 3: Identify the given point coordinates through which the line passes:

$$(x_1, y_1, z_1) = (1, 2, 3)$$

Step 4: Substitute the point coordinates and direction ratios into the symmetrical formula:

$$\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{2}$$

Final Answer: The equation is $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{2}$.

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: The value of a 2×2 determinant matrix $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is expanded using the standard cross-multiplication equation $ad - bc$.

Solution: Step 1: Expand the given determinant equation:

$$\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$$

$$(x-2)(2x) - (-3)(3x) = 3$$

Step 2: Distribute and expand the expressions:

$$2x^2 - 4x + 9x = 3$$

$$2x^2 + 5x = 3$$

Step 3: Bring all terms to one side to construct a standard quadratic equation:

$$2x^2 + 5x - 3 = 0$$

Step 4: Factor the quadratic equation by splitting the middle term:

$$2x^2 + 6x - x - 3 = 0$$

$$2x(x+3) - 1(x+3) = 0$$

$$(2x-1)(x+3) = 0$$

Step 5: Solve for potential roots:

$$x = \frac{1}{2} \quad \text{or} \quad x = -3$$

The problem explicitly requests the positive value, which is $\frac{1}{2}$.

Final Answer: The positive value of x is $\frac{1}{2}$.

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: To find the global maximum of a continuous function on a closed interval $[a, b]$, we determine the critical points where $f'(x) = 0$ inside the interval, and then evaluate the function at these critical points and the endpoints.

Solution: Step 1: Compute the first derivative of the given function $f(x) = x^3 - 3x$:

$$f'(x) = 3x^2 - 3$$

Step 2: Find critical points by setting $f'(x) = 0$:

$$3x^2 - 3 = 0 \implies x^2 = 1 \implies x = 1 \text{ or } x = -1$$

Step 3: Check which critical points fall inside the specified domain interval $[0, 2]$. Only $x = 1$ is within the range $[0, 2]$.

Step 4: Evaluate the function value at the relevant critical point $x = 1$ and the boundaries $x = 0, x = 2$:

$$\text{At } x = 0 : f(0) = 0^3 - 3(0) = 0$$

$$\text{At } x = 1 : f(1) = 1^3 - 3(1) = -2$$

$$\text{At } x = 2 : f(2) = 2^3 - 3(2) = 8 - 6 = 2$$

Step 5: Compare the values: $0, -2$, and 2. The maximum value is 2, which is attained at the endpoint $x = 2$.

Final Answer: The maximum value is attained at $x = 2$.

Answer: (D)

Go Back to Question 21

Q22.

Solution

Concept: The general terms of a Geometric Progression can be denoted as a, ar, ar^2, ar^3, ar^4. We write the product of the terms in terms of a and r and substitute the known middle term.

Solution: Step 1: Let the first 5 terms of the Geometric Progression be represented as:

$$t_1 = a,\ t_2 = ar,\ t_3 = ar^2,\ t_4 = ar^3,\ t_5 = ar^4$$

Step 2: We are given that the third term is 4:

$$t_3 = ar^2 = 4$$

Step 3: Write out the equation for the product of the first 5 terms:

$$\text{Product} = a \cdot (ar) \cdot (ar^2) \cdot (ar^3) \cdot (ar^4)$$

$$\text{Product} = a^5 \cdot r^{1+2+3+4} = a^5 r^{10}$$

Step 4: Restructure the product expression using laws of exponents to match our known value:

$$\text{Product} = (ar^2)^5$$

Step 5: Substitute $ar^2 = 4$ into this expression:

$$\text{Product} = 4^5$$

Final Answer: The product of the first 5 terms is 4^5.

Answer: (C)

Go Back to Question 22

Q23.

Solution

Concept: For a standard hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the equations of the asymptotes are $y = \pm\frac{b}{a}x$. The angle θ between the two asymptotes is given by the formula $\theta = 2\tan^{-1}\left(\frac{b}{a}\right)$.

Solution: Step 1: Identify the values of a^2 and b^2 from the given hyperbola equation $\frac{x^2}{16} - \frac{y^2}{9} = 1$:

$$a^2 = 16 \implies a = 4$$

$$b^2 = 9 \implies b = 3$$

Step 2: The slopes of the two asymptotes are given by $m_1 = \frac{b}{a} = \frac{3}{4}$ and $m_2 = -\frac{b}{a} = -\frac{3}{4}$.

Step 3: The semi-angle that an asymptote makes with the principal focal axis (x-axis) is given by $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$.

Step 4: Since the hyperbola is symmetric about the axes, the total angle θ between the two intersecting asymptotes is equal to 2α:

$$\theta = 2\tan^{-1}\left(\frac{b}{a}\right) = 2\tan^{-1}\left(\frac{3}{4}\right)$$

Final Answer: The angle is $2\tan^{-1}\left(\frac{3}{4}\right)$.

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: The greatest integer function $f(x) = [x]$ outputs the largest integer less than or equal to x. This step function is continuous at all non-integer real values but contains jump discontinuities at every integer value.

Solution: Step 1: Analyze the properties of the greatest integer function $[x]$. For any non-integer value, the function remains constant in a small local neighborhood, meaning its left-hand limit, right-hand limit, and functional value are all equal.

Step 2: At any integer point $x = n$, the value of the function is $f(n) = n$.

Step 3: Evaluate the left-hand limit (LHL) as x approaches an integer n from the left:

$$\text{LHL} = \lim_{x \to n^-} [x] = n - 1$$

Step 4: Evaluate the right-hand limit (RHL) as x approaches an integer n from the right:

$$\text{RHL} = \lim_{x \to n^+} [x] = n$$

Since LHL $\neq$ RHL at every integer value, the function is discontinuous at all integers.

Step 5: Examine the given options: $0.5, 1.2, 3$, and -1.5. Among these options, only 3 is an integer. Thus, the function is discontinuous at $x = 3$.

Final Answer: The function is discontinuous at $x = 3$.

Answer: (C)

Go Back to Question 24

Q25.

Solution

Concept: We use the vector magnitude property $|\vec{v}|^2 = \vec{v}\cdot\vec{v}$ to expand the magnitude of the sum of two vectors in terms of their individual dot products.

Solution: Step 1: We are given that $\vec{a}$ and $\vec{b}$ are unit vectors, which means:

$$|\vec{a}| = 1, \quad |\vec{b}| = 1$$

Step 2: Square both sides of the given magnitude equation $|\vec{a}+\vec{b}| = 1$:

$$|\vec{a}+\vec{b}|^2 = 1^2$$

Step 3: Expand the left-hand side using the vector dot product expansion identity:

$$|\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a}\cdot\vec{b}) = 1$$

Step 4: Substitute $|\vec{a}| = 1$ and $|\vec{b}| = 1$ into the expanded equation:

$$1^2 + 1^2 + 2|\vec{a}||\vec{b}|\cos\theta = 1$$

$$1 + 1 + 2(1)(1)\cos\theta = 1$$

$$2 + 2\cos\theta = 1$$

Step 5: Isolate and solve for $\cos\theta$:

$$2\cos\theta = 1 - 2 = -1$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

Final Answer: The angle between the vectors is $\frac{2\pi}{3}$.

Answer: (D)

Go Back to Question 25

Q26.

Solution

Concept: To solve a continuous product of cosines where the angles double successively, we can use the product identity $\cos\theta\cos 2\theta\cos 4\theta\ldots\cos(2^{n-1}\theta) = \frac{\sin(2^n\theta)}{2^n\sin\theta}$.

Solution: Step 1: Let the given expression be denoted as P:

$$P = \cos 20^\circ \cos 40^\circ \cos 80^\circ$$

Step 2: Multiply and divide the expression by $2\sin 20^\circ$:

$$P = \frac{2\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 80^\circ}{2\sin 20^\circ}$$

Step 3: Apply the double angle identity $2\sin\theta\cos\theta = \sin 2\theta$ to the first part of the numerator:

$$P = \frac{\sin 40^\circ \cos 40^\circ \cos 80^\circ}{2\sin 20^\circ}$$

Step 4: Multiply the numerator and denominator by 2 again to combine the next set of terms:

$$P = \frac{2\sin 40^\circ \cos 40^\circ \cos 80^\circ}{4\sin 20^\circ} = \frac{\sin 80^\circ \cos 80^\circ}{4\sin 20^\circ}$$

Step 5: Repeat the process one more time by multiplying the numerator and denominator by 2:

$$P = \frac{2\sin 80^\circ \cos 80^\circ}{8\sin 20^\circ} = \frac{\sin 160^\circ}{8\sin 20^\circ}$$

Step 6: Simplify the numerator using the complementary angle relation $\sin 160^\circ = \sin(180^\circ - 20^\circ) = \sin 20^\circ$:

$$P = \frac{\sin 20^\circ}{8\sin 20^\circ} = \frac{1}{8}$$

Final Answer: The value of the expression is $\frac{1}{8}$.

Answer: (C)

Go Back to Question 26

Q27.

Solution

Concept: The condition for a line $y = mx + c$ to be a tangent to a standard right-opening parabola $y^2 = 4ax$ is given by the algebraic relation $c = \frac{a}{m}$.

Solution: Step 1: Compare the given parabola equation $y^2 = 4x$ with the standard form $y^2 = 4ax$:

$$4a = 4 \implies a = 1$$

Step 2: Identify the components of the given line equation $y = mx + 1$ by comparing it with the slope-intercept form $y = mx + c$:

$$\text{Intercept } c = 1$$

Step 3: State the mathematical condition for tangency:

$$c = \frac{a}{m}$$

Step 4: Substitute the identified values of a and c into this condition:

$$1 = \frac{1}{m}$$

Step 5: Solve the equation for the unknown parameter m:

$$m = 1$$

Final Answer: The value of m is 1.

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: The Coefficient of Variation (CV) is a statistical measure of the relative dispersion of a data distribution. It is defined as the ratio of the standard deviation (σ) to the arithmetic mean (μ), expressed as a percentage: $CV = \left(\frac{\sigma}{\mu}\right) \times 100$.

Solution: Step 1: State the given parameter values from the problem description:

$$\text{Coefficient of Variation } (CV) = 60\%$$

$$\text{Standard Deviation } (\sigma) = 12$$

Step 2: Write out the structural formula relating these statistical terms:

$$CV = \frac{\sigma}{\text{Mean}} \times 100$$

Step 3: Substitute the known values into the equation:

$$60 = \frac{12}{\text{Mean}} \times 100$$

Step 4: Rearrange the equation to isolate the unknown Mean parameter:

$$\text{Mean} = \frac{12 \times 100}{60}$$

Step 5: Calculate the final value:

$$\text{Mean} = \frac{1200}{60} = 20$$

Final Answer: The arithmetic mean is 20.

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept: This integral is solved using the standard exponential integration identity $\int e^x(f(x)+f'(x))\,dx = e^x f(x) + C$. We must identify which function serves as $f(x)$ and verify its derivative.

Solution: Step 1: Write down the given indefinite integral:

$$\int e^x(\tan x + \ln(\sec x))\,dx$$

Step 2: Let us define $f(x) = \ln(\sec x)$ and compute its derivative with respect to x using the chain rule:

$$f'(x) = \frac{d}{dx}[\ln(\sec x)] = \frac{1}{\sec x}\cdot\frac{d}{dx}(\sec x)$$

$$f'(x) = \frac{1}{\sec x}\cdot(\sec x\tan x) = \tan x$$

Step 3: Compare this with the integrand structure. The integral matches the format $\int e^x(f'(x)+f(x))\,dx$, where:

$$f(x) = \ln(\sec x) \quad \text{and} \quad f'(x) = \tan x$$

Step 4: Apply the integration theorem directly:

$$\int e^x(\tan x + \ln(\sec x))\,dx = e^x\ln(\sec x) + C$$

Final Answer: The value of the integral is $e^x\ln(\sec x) + C$.

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept: To find the principal argument of a complex number given as a fraction, we first simplify it into its standard rectangular form $x + iy$ by multiplying the numerator and denominator by the complex conjugate of the denominator.

Solution: Step 1: Simplify the given expression for z:

$$z = \frac{1+i}{1-i}$$

Step 2: Rationalize the denominator by multiplying both top and bottom by $(1 + i)$:

$$z = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+2i+i^2}{1^2-i^2}$$

Step 3: Substitute $i^2 = -1$ into the expression:

$$z = \frac{1+2i-1}{1-(-1)} = \frac{2i}{2} = i$$

Step 4: Write the simplified complex number in standard form:

$$z = 0 + 1i$$

This number lies entirely on the positive imaginary axis of the Argand plane.

Step 5: The principal argument θ for a point on the positive imaginary axis is $\frac{\pi}{2}$. Formally:

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{0}\right) = \frac{\pi}{2}$$

Final Answer: The principal argument of z is $\frac{\pi}{2}$.

Answer: (C)

Go Back to Question 30

Q31.

Solution

Concept: The slope of the tangent to a curve at a given point is the value of the derivative $\frac{dy}{dx}$ at that point. The slope of the normal line is the negative reciprocal of the slope of the tangent line ($m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}}$).

Solution: Step 1: Differentiate the given curve equation $y = 2x^2 + 3\sin x$ with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}(2x^2) + \frac{d}{dx}(3\sin x)$$

$$\frac{dy}{dx} = 4x + 3\cos x$$

Step 2: Find the slope of the tangent line at the specified point $x = 0$ by substitution:

$$m_{\text{tangent}} = \left.\frac{dy}{dx}\right|_{x=0} = 4(0) + 3\cos(0)$$

$$m_{\text{tangent}} = 0 + 3(1) = 3$$

Step 3: Use the relationship between the slopes of perpendicular lines to calculate the slope of the normal line:

$$m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}}$$

Step 4: Substitute the tangent slope value into the formula:

$$m_{\text{normal}} = -\frac{1}{3}$$

Final Answer: The slope of the normal is $-\frac{1}{3}$.

Answer: (D)

Go Back to Question 31

Q32.

Solution

Concept: For any square matrix A of order n, the determinant of its adjugate matrix satisfies the structural determinant identity $|\text{adj}(A)| = |A|^{n-1}$.

Solution: Step 1: Extract the parameters provided in the problem statement:

$$\text{Order of the matrix } (n) = 3$$

$$\text{Determinant value } |A| = 5$$

Step 2: State the matrix determinant power property for adjugates:

$$|\text{adj}(A)| = |A|^{n-1}$$

Step 3: Substitute the known values of n and $|A|$ into the algebraic property:

$$|\text{adj}(A)| = 5^{3-1}$$

Step 4: Compute the final power evaluation:

$$|\text{adj}(A)| = 5^2 = 25$$

Final Answer: The value of $|\text{adj}(A)|$ is 25.

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept: Repeated independent trials with two possible outcomes can be modeled using the Binomial Distribution, where the variance is given by the formula $\text{Var}(X) = npq$.

Solution: Step 1: Identify the total number of independent trials (n) from the problem:

$$n = 2 \quad \text{(since the die is tossed twice)}$$

Step 2: Determine the probability of 'success' (p) on a single toss. A standard die contains numbers $\{1, 2, 3, 4, 5, 6\}$, of which three are odd $\{1, 3, 5\}$. Thus:

$$p = \frac{3}{6} = \frac{1}{2}$$

Step 3: Calculate the probability of 'failure' (q) using the complementary probability rule:

$$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

Step 4: Substitute these parameters into the binomial variance formula:

$$\text{Variance} = npq = 2 \times \frac{1}{2} \times \frac{1}{2}$$

Step 5: Perform the multiplication:

$$\text{Variance} = \frac{1}{2}$$

Final Answer: The variance is $\frac{1}{2}$.

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: The general equation of a circle is expressed as $x^2 + y^2 + 2gx + 2fy + c = 0$. The radius r of this circle is calculated using the algebraic relation $r = \sqrt{g^2 + f^2 - c}$.

Solution: Step 1: Match the coefficients of the given circle equation $x^2 + y^2 - 4x + 6y - 12 = 0$ with the general form:

$$2g = -4 \implies g = -2$$

$$2f = 6 \implies f = 3$$

$$c = -12$$

Step 2: State the radius formula:

$$r = \sqrt{g^2 + f^2 - c}$$

Step 3: Substitute the identified values of g, f, and c into the equation:

$$r = \sqrt{(-2)^2 + 3^2 - (-12)}$$

Step 4: Simplify the values inside the radical sign:

$$r = \sqrt{4 + 9 + 12}$$

$$r = \sqrt{25} = 5 \text{ units}$$

Final Answer: The radius of the circle is 5 units.

Answer: (C)

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Q35.

Solution

Concept: The fundamental period of $\sin(kx)$ or $\cos(kx)$ is $\frac{2\pi}{|k|}$. The collective period of the sum of two periodic functions is found by taking the Least Common Multiple (LCM) of their individual periods.

Solution: Step 1: Find the period (T_1) of the first component function $f_1(x) = \sin\left(\frac{2\pi x}{3}\right)$:

$$T_1 = \frac{2\pi}{\frac{2\pi}{3}} = 2\pi \times \frac{3}{2\pi} = 3$$

Step 2: Find the period (T_2) of the second component function $f_2(x) = \cos\left(\frac{\pi x}{2}\right)$:

$$T_2 = \frac{2\pi}{\frac{pi}{2}} = 2\pi \times \frac{2}{\pi} = 4$$

Step 3: The total fundamental period T of the combined function is the LCM of the individual periods T_1 and T_2:

$$T = \text{LCM}(3, 4)$$

Step 4: Since 3 and 4 are co-prime integers, their least common multiple is simply their product:

$$T = 3 \times 4 = 12$$

Final Answer: The fundamental period is 12.

Answer: (D)

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Q36.

Solution

Concept: The absolute value function $|x|$ changes its definition based on the sign of x. To integrate it over an interval containing both negative and positive numbers, we split the integral at the turning point $x = 0$. Alternatively, we can use geometric symmetry since $|x|$ is an even function.

Solution: Step 1: Identify that the integrand $f(x) = |x|$ is an even function because $|-x| = |x|$. Thus, we can apply the definite integral symmetry property $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$:

$$\int_{-1}^{1} |x|\,dx = 2\int_{0}^{1} |x|\,dx$$

Step 2: In the non-negative interval $[0, 1]$, the absolute value function simplifies directly to $|x| = x$:

$$2\int_{0}^{1} |x|\,dx = 2\int_{0}^{1} x\,dx$$

Step 3: Integrate the linear term using the power rule:

$$2\left[\frac{x^2}{2}\right]_0^1 = 2\left(\frac{1^2}{2} - \frac{0^2}{2}\right)$$

Step 4: Simplify the calculated numerical expression:

$$2 \times \frac{1}{2} = 1$$

Final Answer: The value of the integral is 1.

Answer: (B)

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Q37.

Solution

Concept: The dot product $\vec{a} \cdot \vec{b} = 0$ implies that either the vectors are perpendicular ($\theta = 90^\circ$) or at least one is a null vector. The cross product $\vec{a} \times \vec{b} = \vec{0}$ implies that either the vectors are parallel ($\theta = 0^\circ$ or 180°) or at least one is a null vector.

Solution: Step 1: Analyze the implications of the dot product condition:

$$\vec{a} \cdot \vec{b} = 0 \implies |\vec{a}||\vec{b}| \cos\theta = 0$$

This means either $|\vec{a}| = 0$, or $|\vec{b}| = 0$, or $\cos\theta = 0 \implies \theta = 90^\circ$ (perpendicular vectors).

Step 2: Analyze the implications of the cross product condition:

$$\vec{a} \times \vec{b} = \vec{0} \implies |\vec{a}||\vec{b}| \sin\theta\hat{n} = \vec{0} \implies |\vec{a}||\vec{b}| \sin\theta = 0$$

This means either $|\vec{a}| = 0$, or $|\vec{b}| = 0$, or $\sin\theta = 0 \implies \theta = 0^\circ$ or 180° (parallel vectors).

Step 3: For both statements to be true simultaneously, the angle conditions contradict each other because a pair of non-zero straight lines cannot be both parallel and perpendicular at the same time (θ cannot be both 90° and $0^\circ/180^\circ$).

Step 4: Therefore, the only logical way both vector products vanish simultaneously is if at least one of the vectors has a magnitude of zero, meaning it is a null vector.

Final Answer: It implies that at least one of $\vec{a}$ or $\vec{b}$ is a null vector.

Answer: (C)

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Q38.

Solution

Concept: We use the inverse trigonometric addition identity $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ when the product of the arguments satisfies $xy > 1$.

Solution: Step 1: Evaluate the first term directly using standard exact angles:

$$\tan^{-1}(1) = \frac{\pi}{4}$$

Step 2: Group the remaining two terms together to apply the addition formula:

$$S = \tan^{-1}(2) + \tan^{-1}(3)$$

Notice that the product of the arguments is $2 \times 3 = 6$, which is strictly greater than 1.

Step 3: Apply the specific formula version that accounts for an argument product greater than 1:

$$S = \pi + \tan^{-1}\left(\frac{2+3}{1-2\times 3}\right) = \pi + \tan^{-1}\left(\frac{5}{1-6}\right)$$

$$S = \pi + \tan^{-1}\left(\frac{5}{-5}\right) = \pi + \tan^{-1}(-1)$$

Step 4: Use the odd function property $\tan^{-1}(-k) = -\tan^{-1}(k)$:

$$S = \pi - \tan^{-1}(1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

Step 5: Add all component values together to compute the total sum:

$$\text{Total Value} = \tan^{-1}(1) + S = \frac{\pi}{4} + \frac{3\pi}{4} = \frac{4\pi}{4} = \pi$$

Final Answer: The total value is equal to π.

Answer: (B)

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Q39.

Solution

Concept: A parabola with its vertex at $(0, 0)$ and focus along the negative vertical axis at $(0, -a)$ opens downwards and has the standard form $x^2 = -4ay$. Its directrix line is a horizontal line located at $y = a$.

Solution: Step 1: Identify the orientation of the parabola from the coordinates of its focus. Since the vertex is at $(0, 0)$ and the focus is at $(0, -3)$, the focus lies on the negative y-axis.

Step 2: Determine the parameter distance value a by measuring the distance from the vertex to the focus:

$$\text{Focus} = (0, -a) = (0, -3) \implies a = 3$$

Step 3: The directrix of a parabola is located on the opposite side of the vertex at an equal distance a along the axis of symmetry.

Step 4: For a downward-opening vertical parabola, the directrix is a horizontal straight line given by the linear equation:

$$y = a$$

Step 5: Substitute the value $a = 3$ into the directrix equation:

$$y = 3$$

Final Answer: The equation of its directrix is $y = 3$.

Answer: (A)

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Q40.

Solution

Concept: To find the integrating factor of a first-order linear differential equation, we first rearrange it into the standard mathematical format $\frac{dy}{dx} + P(x)y = Q(x)$. The Integrating Factor is then computed as $IF = e^{\int P(x)\,dx}$.

Solution: Step 1: Write down the given differential equation:

$$x\frac{dy}{dx} - y = x^2$$

Step 2: Convert the equation into standard form by dividing all terms by x:

$$\frac{dy}{dx} - \frac{1}{x}y = x$$

Step 3: Identify the coefficient function $P(x)$ that multiplies y:

$$P(x) = -\frac{1}{x}$$

Step 4: Set up the Integrating Factor (IF) calculation:

$$IF = e^{\int P(x)\,dx} = e^{\int -\frac{1}{x}\,dx} = e^{-\ln x}$$

Step 5: Simplify the exponential logarithmic expression using logarithm power rules:

$$IF = e^{\ln(x^{-1})} = x^{-1} = \frac{1}{x}$$

Final Answer: The integrating factor is $\frac{1}{x}$.

Answer: (C)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	A	3	C	4	B	5	B
6	B	7	B	8	A	9	B	10	A
11	C	12	C	13	C	14	A	15	B
16	A	17	B	18	B	19	A	20	A
21	D	22	C	23	A	24	C	25	D
26	C	27	A	28	A	29	B	30	C
31	D	32	B	33	A	34	C	35	D
36	B	37	C	38	B	39	A	40	C