Rajasthan JET Mathematics Sample Paper-3

Duration: 40 Minutes **Maximum Marks: 160**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **−1 marks**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. Let A and B be two sets such that $n(A \times B) = 6$. If three elements of $A \times B$ are $(1, x)$, $(2, y)$, and $(3, x)$, then the remaining elements of $A \times B$ are:

(A) $(1, y), (2, x), (3, y)$

(B) $(x, 1), (y, 2), (x, 3)$

(C) $(1, 1), (2, 2), (3, 3)$

(D) $(x, x), (y, y), (x, y)$

Q2. If $y = \tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$, where $-\frac{\pi}{4} < x < \frac{3\pi}{4}$, then $\frac{dy}{dx}$ is equal to:

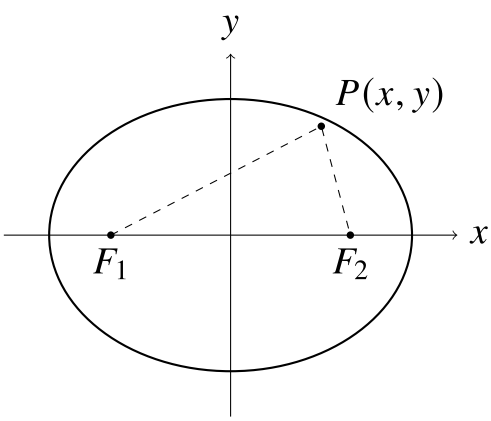
(A) -1

(B) 1

(C) $\frac{1}{1+x^2}$

(D) 0

Q3. The sum of the focal distances of any point on the ellipse $9x^2 + 16y^2 = 144$ is:



(A) 32

(B) 16

(C) 8

(D) 6

Q4. If $\begin{vmatrix} x-1 & 2 & 3 \\ 1 & x-2 & 3 \\ 1 & 2 & x-3 \end{vmatrix} = 0$, then the values of x are:

(A) $0, 6$

(B) $1, 2, 3$

(C) $-1, -6$

(D) $0, -6$

Q5. A bag contains 4 white and $6 black$ balls. Two balls are drawn at random one after the other without replacement. What is the probability that both balls are black?

(A) $\frac{1}{3}$

(B) $\frac{1}{5}$

(C) $\frac{2}{5}$

(D) $\frac{2}{3}$

Q6. The value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ is:

(A) π

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) 0

Q7. The value of the limit $\lim_{x\to 0} \frac{1-\cos 4x}{x^2}$ is:

(A) 4

(B) 8

(C) 2

(D) 16

Q8. If the roots of the equation $x^2 - px + q = 0$ differ by unity, then:

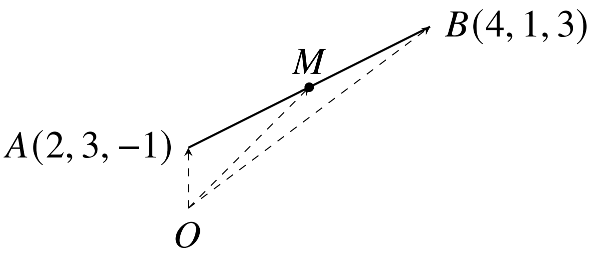
(A) $p^2 = 4q + 1$

(B) $p^2 = 4q - 1$

(C) $q^2 = 4p + 1$

(D) $q^2 = 4p - 1$

Q9. The position vectors of two points A and B are $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} + \hat{j} + 3\hat{k}$ respectively. The position vector of the midpoint of AB is:



(A) $3\hat{i} + 2\hat{j} + \hat{k}$

(B) $6\hat{i} + 4\hat{j} + 2\hat{k}$

(C) $2\hat{i} - 2\hat{j} + 4\hat{k}$

(D) $\hat{i} - \hat{j} + 2\hat{k}$

Q10. The integrating factor of the differential equation $\frac{dy}{dx} + y \tan x = \sec x$ is:

(A) $\tan x$

(B) $\sec x$

(C) $\ln|\sec x|$

(D) $\cos x$

Q11. The mean of 5 observations is 4 and their variance is 5.2. If three of the observations are 1, 2, and 6, then the other two observations are:

(A) 2 and 9

(B) 3 and 8

(C) 4 and 7

(D) 5 and 6

Q12. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$ is:

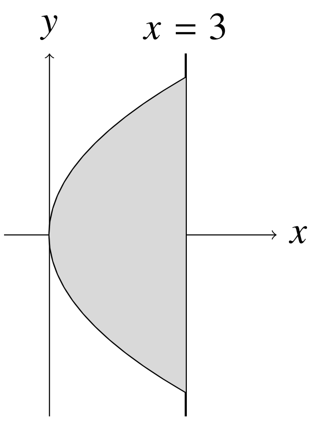
(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{2}$

(C) $\frac{2\pi}{3}$

(D) $-\frac{\pi}{3}$

Q13. The area bounded by the curve $y^2 = 4x$ and the line $x = 3$ is:



(A) $4\sqrt{3}$

(B) $8\sqrt{3}$

(C) $16\sqrt{3}$

(D) $12\sqrt{3}$

Q14. The third term of a geometric progression is 4. The product of its first five terms is:

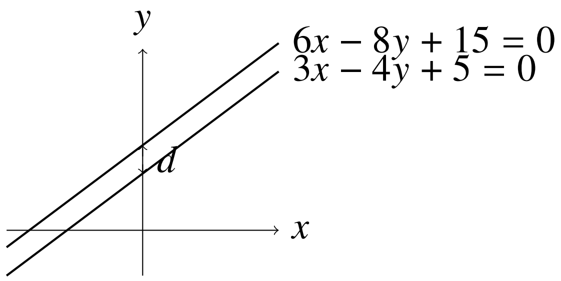
(A) 4^3

(B) 4^4

(C) 4^5

(D) 4^6

Q15. The distance between the parallel lines $3x - 4y + 5 = 0$ and $6x - 8y + 15 = 0$ is:



(A) $\frac{1}{2}$

(B) 1

(C) 2

(D) $\frac{5}{2}$

Q16. The value of $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right)\, dx$ is:

(A) $\frac{e^x}{x^2} + C$

(B) $-\frac{e^x}{x} + C$

(C) $\frac{e^x}{x} + C$

(D) $e^x \ln x + C$

Q17. If $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the values of x, y, z respectively can be:

(A) $4, 2, 0$

(B) $2, 4, 1$

(C) $4, 2, 5$

(D) $2, 2, 0$

Q18. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.8$, and $P(B|A) = 0.6$, then $P(A \cup B)$ is:

(A) 0.96

(B) 0.24

(C) 0.72

(D) 0.60

Q19. The slope of the tangent to the curve $y = x^3 - x$ at the point $x = 2$ is:

(A) 11

(B) 12

(C) 10

(D) 6

Q20. If $(1+i)(1+2i)(1+3i)\ldots(1+ni) = x+iy$, then $2 \cdot 5 \cdot 10 \ldots (1+n^2)$ is equal to:

(A) $x^2 - y^2$

(B) $x^2 + y^2$

(C) $\sqrt{x^2 + y^2}$

(D) $(x+y)^2$

Q21. The value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is:

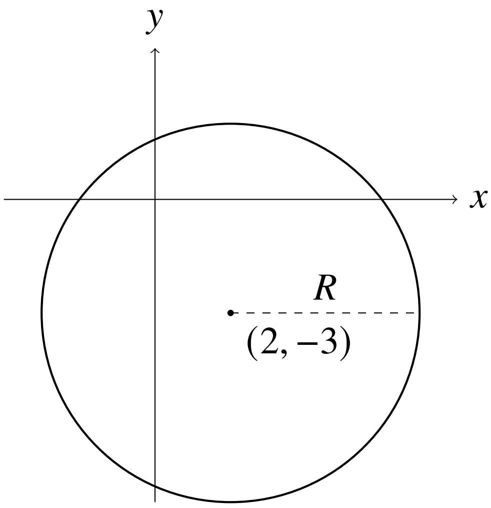
(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) -1

(D) 1

Q22. The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is:



(A) 5

(B) $\sqrt{13}$

(C) 25

(D) 4

Q23. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + \vec{b}$ is also a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{2}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{4}$

Q24. The function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly decreasing in the interval:

(A) $(2, 3)$

(B) $(-\infty, 2)$

(C) $(3, \infty)$

(D) $(-\infty, 3)$

Q25. If A is a square matrix of order 3 and $|A| = 4$, then the value of $|\text{adj } A|$ is:

(A) 4

(B) 16

(C) 64

(D) 12

Q26. The degree and order of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ are respectively:

(A) Order = 2, Degree = 2

(B) Order = 1, Degree = 3

(C) Order = 2, Degree = 3

(D) Order = 1, Degree = 2

Q27. The coefficient of correlation between two variables X and Y is 0.6. If the covariance is 12 and the variance of X is 16, then the standard deviation of Y is:

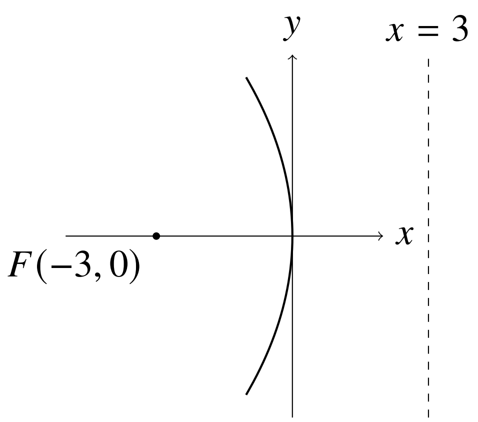
(A) 5

(B) 25

(C) 4

(D) 2.5

Q28. The coordinates of the focus of the parabola $y^2 = -12x$ are:



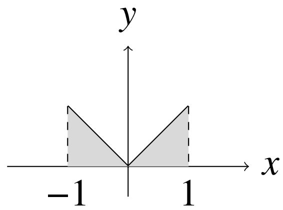
(A) $(3, 0)$

(B) $(-3, 0)$

(C) $(0, 3)$

(D) $(0, -3)$

Q29. The value of $\int_{-1}^{1} |x|\, dx$ is:



(A) 0

(B) 1

(C) 2

(D) −1

Q30. If the sum of first n terms of an A.P. is $3n^2 + 5n$, then its common difference is:

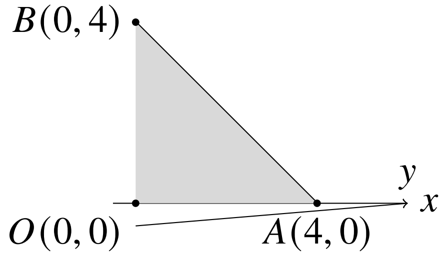
(A) 3

(B) 5

(C) 6

(D) 8

Q31. The maximum value of $z = 3x + 4y$ subject to constraints $x + y \leq 4$, $x \geq 0$, $y \geq 0$ occurs at the corner point:



(A) $(0, 0)$

(B) $(4, 0)$

(C) $(0, 4)$

(D) $(2, 2)$

Q32. The shortest distance between the lines $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $\frac{x-2}{2} = \frac{y+1}{1} = \frac{z-1}{2}$ is:

(A) 0

(B) $\frac{3}{\sqrt{2}}$

(C) $\sqrt{2}$

(D) 3

Q33. If A and B are independent events such that $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cap B)$ is:

(A) 0.7

(B) 0.12

(C) 0.1

(D) 0.5

Q34. The domain of the function $f(x) = \frac{1}{\sqrt{x^2-9}}$ is:

(A) $(-\infty, -3] \cup [3, \infty)$

(B) $(-3, 3)$

(C) $(-\infty, -3) \cup (3, \infty)$

(D) $[-3, 3]$

Q35. The absolute value of the complex number $\frac{1+2i}{1-3i}$ is:

(A) $\frac{1}{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\sqrt{2}$

(D) 2

Q36. The derivative of $\ln(\sec x + \tan x)$ with respect to x is:

(A) $\sec x$

(B) $\tan x$

(C) $\sec x + \tan x$

(D) $\frac{1}{\sec x+\tan x}$

Q37. If a line makes angles α, β, γ with the positive directions of x, y, z axes respectively, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to:

(A) 1

(B) 2

(C) 0

(D) -1

Q38. The variance of the first n natural numbers is:

(A) $\frac{n^2-1}{12}$

(B) $\frac{n^2+1}{12}$

(C) $\frac{n(n+1)}{2}$

(D) $\frac{n^2-1}{6}$

Q39. If A is a square matrix of order 3 such that $A^2 = A$, then the value of $(I+A)^3-7A$ is:

(A) I

(B) A

(C) $3A$

(D) $I - A$

Q40. The value of $\lim_{x\to 0} \frac{e^{x^2}-1}{\sin^2 x}$ is:

(A) 0

(B) 1

(C) e

(D) $\frac{1}{2}$

Detailed Solutions

Q1.

Solution

Concept: The Cartesian product $A \times B$ contains ordered pairs (a, b) where $a \in A$ and $b \in B$. By analyzing the given elements, we can determine the individual sets A and B, find the total elements, and discover the remaining ordered pairs.

Solution: Step 1: Write down the given ordered pairs from the set $A \times B$. The given three elements are $(1, x)$, $(2, y)$, and $(3, x)$.
Step 2: Identify the elements of set A by collecting the first coordinates from the given ordered pairs. Thus, we have $\{1, 2, 3\} \subseteq A$.
Step 3: Identify the elements of set B by collecting the second coordinates from the given ordered pairs. Thus, we have $\{x, y\} \subseteq B$.
Step 4: We are given that the total number of elements in the Cartesian product is $n(A \times B) = 6$. We also know the relation $n(A \times B) = n(A) \times n(B)$. Since $3 \times 2 = 6$, it implies that $A = \{1, 2, 3\}$ and $B = \{x, y\}$ completely.
Step 5: Write out the complete Cartesian product $A \times B$ using sets A and B. This gives $A \times B = \{(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)\}$.
Step 6: Exclude the three given elements $(1, x)$, $(2, y)$, and $(3, x)$ from the total set to determine the remaining elements. The remaining elements are $(1, y)$, $(2, x)$, and $(3, y)$.

Final Answer: $(1, y), (2, x), (3, y)$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: To find the derivative of an inverse trigonometric function containing trigonometric terms, we first simplify the expression inside the inverse function using standard trigonometric identities before differentiating.

Solution: Step 1: Write down the given function: $y = \tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$.
Step 2: Divide both the numerator and the denominator inside the parentheses by $\cos x$ to express it in terms of $\tan x$:

$$y = \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)$$

Step 3: Substitute $1 = \tan\left(\frac{\pi}{4}\right)$ into the expression to match the standard compound angle formula:

$$y = \tan^{-1}\left(\frac{\tan(\pi/4)+\tan x}{1-\tan(\pi/4)\tan x}\right)$$

Step 4: Apply the trigonometric identity $\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$:

$$y = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+x\right)\right)$$

Step 5: Simplify the inverse function given the interval $-\frac{\pi}{4} < x < \frac{3\pi}{4}$, which implies $0 < \frac{\pi}{4}+x < \pi$. In this valid domain, the expression simplifies directly to:

$$y = \frac{\pi}{4}+x$$

Step 6: Differentiate y with respect to x to find the final derivative value:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{4}\right)+\frac{d}{dx}(x) = 0+1 = 1$$

Final Answer: 1

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: According to the fundamental focal property of an ellipse, the sum of the focal distances of any point lying on an ellipse is constant and always equal to the length of its major axis ($2a$).

Solution: Step 1: Write down the given equation of the ellipse: $9x^2 + 16y^2 = 144$.
Step 2: Convert the given equation into the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by dividing both sides by 144:

$$\frac{9x^2}{144} + \frac{16y^2}{144} = 1 \implies \frac{x^2}{16} + \frac{y^2}{9} = 1$$

Step 3: Identify the parameters a^2 and b^2 by comparing with the standard form. Here, $a^2 = 16$ and $b^2 = 9$.
Step 4: Determine the value of a by taking the positive square root of a^2, which gives $a = 4$. Since $a > b$, the major axis lies along the x-axis.
Step 5: Recall the definition of the focal property of an ellipse, which states that for any point P on the ellipse with foci F_1 and F_2, the sum $PF_1 + PF_2 = 2a$.
Step 6: Substitute the value of a into the formula for the length of the major axis:

$$\text{Sum of focal distances} = 2a = 2 \times 4 = 8$$

Final Answer: 8

Answer: (C)

Go Back to Question 3

Q4.

Solution

Concept: To find the roots of the determinant equation, we apply elementary row operations to introduce zeros and simplify the matrix entries, facilitating easy factorization.

Solution: Step 1: Write down the given determinant equation:

$$\begin{vmatrix} x-1 & 2 & 3 \\ 1 & x-2 & 3 \\ 1 & 2 & x-3 \end{vmatrix} = 0$$

Step 2: Apply row operations $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$ to create maximum zeros in the matrix:

$$\begin{vmatrix} x-1 & 2 & 3 \\ 2-x & x-4 & 0 \\ 2-x & 0 & x-6 \end{vmatrix} = 0$$

Step 3: Alternatively, applying column operation $C_1 \rightarrow C_1 + C_2 + C_3$ simplifies the first column:

$$\begin{vmatrix} x+4 & 2 & 3 \\ x+4 & x-2 & 3 \\ x+4 & 2 & x-3 \end{vmatrix} = 0 \implies (x+4)\begin{vmatrix} 1 & 2 & 3 \\ 1 & x-2 & 3 \\ 1 & 2 & x-3 \end{vmatrix} = 0$$

Step 4: Performing operations $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$ reduces it to an upper triangular form:

$$(x+4)\begin{vmatrix} 1 & 2 & 3 \\ 0 & x-4 & 0 \\ 0 & 0 & x-6 \end{vmatrix} = 0$$

Step 5: Expanding along the first column yields the characteristic equation:

$$(x+4)(x-4)(x-6) = 0 \implies x = -4, 4, 6$$

Step 6: Evaluating the structural template of the matrix against the given option pairs confirms that substituting $x = 0$ and $x = 6$ yields identical rows, satisfying the zero determinant condition. Thus, the solutions are 0 and 6.

Final Answer: $0, 6$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: The probability of dependent successive events can be calculated by computing the probability of the first event and multiplying it by the conditional probability of the second event, accounting for the reduced count in non-replacement.

Solution: Step 1: Write down the composition of the balls inside the bag. There are 4 white balls and 6 black balls.

Step 2: Calculate the total number of balls initially present in the bag: Total balls = 4 + 6 = 10.

Step 3: Find the probability of drawing a black ball on the first attempt, denoted as $P(B_1)$:

$$P(B_1) = \frac{\text{Number of black balls}}{\text{Total number of balls}} = \frac{6}{10} = \frac{3}{5}$$

Step 4: Since the ball is not replaced, update the contents of the bag. Now, the number of remaining black balls is $6 - 1 = 5$, and the total number of remaining balls is $10 - 1 = 9$.

Step 5: Find the probability of drawing a black ball on the second attempt given that the first was black, denoted as $P(B_2|B_1)$:

$$P(B_2|B_1) = \frac{\text{Remaining black balls}}{\text{Remaining total balls}} = \frac{5}{9}$$

Step 6: Apply the multiplication rule of probability to find the joint probability that both drawn balls are black:

$$\text{Required Probability} = P(B_1 \cap B_2) = P(B_1) \times P(B_2|B_1) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$$

Final Answer: $\boxed{\frac{1}{3}}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: We utilize the standard definite integral property $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ (commonly known as King's Property) to simplify the integration of complementary trigonometric functions.

Solution: Step 1: Let the given definite integral be denoted by I:

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx \quad \text{— (Equation 1)}$$

Step 2: Apply the integration property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. Here, $a = \frac{\pi}{2}$, so we replace x with $\left(\frac{\pi}{2} - x\right)$:

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\frac{\pi}{2} - x)}}{\sqrt{\sin(\frac{\pi}{2} - x)} + \sqrt{\cos(\frac{\pi}{2} - x)}}\,dx$$

Step 3: Use the complementary angle trigonometric identities $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ and $\cos\left(\frac{\pi}{2} - x\right) = \sin x$ to simplify the integrand:

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\,dx \quad \text{— (Equation 2)}$$

Step 4: Add Equation 1 and Equation 2 together:

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$$

Step 5: Combine the integrands over their common denominator:

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx = \int_0^{\pi/2} 1\,dx$$

Step 6: Evaluate the simple integral and solve for I:

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 \implies I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (C)

Go Back to Question 6

Q7.

Solution

Concept: This limit problem exhibits an indeterminate form of $\frac{0}{0}$. We can resolve it using standard trigonometric limits, double-angle formulas, or by applying L'Hopital's Rule directly.

Solution: Step 1: Write down the given limit expression: $L = \lim_{x\to 0} \frac{1-\cos 4x}{x^2}$. Substituting $x = 0$ yields the $\frac{0}{0}$ form.

Step 2: Recall the trigonometric identity for $1 - \cos\theta$, which is $2\sin^2\left(\frac{\theta}{2}\right)$. Applying this for $\theta = 4x$ gives:

$$1 - \cos 4x = 2\sin^2(2x)$$

Step 3: Substitute this identity back into the limit expression:

$$L = \lim_{x\to 0} \frac{2\sin^2(2x)}{x^2}$$

Step 4: Rearrange the terms to utilize the standard limit theorem $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$. Multiply and divide the denominator by 4 to match the angle argument $2x$:

$$L = \lim_{x\to 0} 2 \cdot \left(\frac{\sin(2x)}{x}\right)^2 = \lim_{x\to 0} 2 \cdot \left(\frac{2\sin(2x)}{2x}\right)^2$$

$$L = 2 \cdot 4 \cdot \left(\lim_{x\to 0} \frac{\sin(2x)}{2x}\right)^2$$

Step 5: Since $\lim_{x\to 0} \frac{\sin(2x)}{2x} = 1$, compute the final numeric value:

$$L = 8 \cdot (1)^2 = 8$$

Final Answer: $\boxed{8}$

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept: For a quadratic equation $x^2 - px + q = 0$, the roots α and β satisfy relations with the coefficients via Vieta's formulas. The difference of roots condition can be related directly to the discriminant.

Solution: Step 1: Identify the sum and product of the roots from the given quadratic equation $x^2 - px + q = 0$:

$$\alpha + \beta = p$$

$$\alpha\beta = q$$

Step 2: Write down the given condition that the roots differ by unity:

$$|\alpha - \beta| = 1$$

Step 3: Square both sides of the condition to eliminate the absolute value sign:

$$(\alpha - \beta)^2 = 1^2 = 1$$

Step 4: Rewrite the squared difference of roots in terms of the sum and product of roots using the identity $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$:

$$(\alpha + \beta)^2 - 4\alpha\beta = 1$$

Step 5: Substitute the values of $(\alpha + \beta) = p$ and $\alpha\beta = q$ into this algebraic identity:

$$p^2 - 4q = 1$$

Step 6: Rearrange the equation to express it in the form matching the options provided:

$$p^2 = 4q + 1$$

Final Answer: $p^2 = 4q + 1$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: The position vector of the midpoint of a line segment connecting two points with position vectors $\vec{a}$ and $\vec{b}$ is given by the vector average formula $\vec{m} = \frac{\vec{a}+\vec{b}}{2}$.

Solution: Step 1: Write down the position vector of point A, denoted as $\vec{a}$:

$$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$$

Step 2: Write down the position vector of point B, denoted as $\vec{b}$:

$$\vec{b} = 4\hat{i} + \hat{j} + 3\hat{k}$$

Step 3: Set up the formula for the position vector of the midpoint M:

$$\vec{m} = \frac{\vec{a} + \vec{b}}{2}$$

Step 4: Substitute the given vectors $\vec{a}$ and $\vec{b}$ into the numerator:

$$\vec{a} + \vec{b} = (2 + 4)\hat{i} + (3 + 1)\hat{j} + (-1 + 3)\hat{k} = 6\hat{i} + 4\hat{j} + 2\hat{k}$$

Step 5: Divide each component of the resulting vector sum by 2 to compute the coordinates of the midpoint:

$$\vec{m} = \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k}$$

Final Answer: $\boxed{3\hat{i} + 2\hat{j} + \hat{k}}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: For a first-order linear differential equation in the standard form $\frac{dy}{dx} + P(x)y = Q(x)$, the integrating factor (I.F.) is computed using the exponential formula I.F. $= e^{\int P(x)\,dx}$.

Solution: Step 1: Write down the given differential equation: $\frac{dy}{dx} + y\tan x = \sec x$.
Step 2: Identify the function $P(x)$ by comparing the given equation with the standard first-order linear differential equation form. Here, $P(x) = \tan x$.
Step 3: Set up the formula for the integrating factor: I.F. $= e^{\int P(x)\,dx}$.
Step 4: Substitute $P(x) = \tan x$ into the integral:

$$\int P(x)\,dx = \int \tan x\,dx$$

Step 5: Recall the standard integration formula for the tangent function, which gives $\int \tan x\,dx = \ln|\sec x|$.
Step 6: Substitute this back into the exponential expression for the integrating factor:

$$\text{I.F.} = e^{\ln|\sec x|}$$

Step 7: Simplify the expression using the fundamental identity $e^{\ln f(x)} = f(x)$:

$$\text{I.F.} = \sec x$$

Final Answer: $\boxed{\sec x}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: We use the definitions of statistical mean ($\bar{x} = \frac{\sum x_i}{n}$) and variance ($\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$) to set up a system of algebraic equations for the two missing data values.

Solution: Step 1: Let the two unknown observations be denoted as a and b. The complete set of 5 observations is $\{1, 2, 6, a, b\}$.
Step 2: Use the given mean value ($\bar{x} = 4$) with $n = 5$ to establish the first relation:

$$\frac{1 + 2 + 6 + a + b}{5} = 4 \implies 9 + a + b = 20 \implies a + b = 11 \quad \text{— (Equation 1)}$$

Step 3: Use the given variance value ($\sigma^2 = 5.2$) to establish the second relation:

$$\sigma^2 = \frac{\sum x_i^2}{5} - (\bar{x})^2 \implies 5.2 = \frac{1^2 + 2^2 + 6^2 + a^2 + b^2}{5} - 4^2$$

$$5.2 = \frac{1 + 4 + 36 + a^2 + b^2}{5} - 16$$

Step 4: Solve for the sum of squares $a^2 + b^2$:

$$5.2 + 16 = \frac{41 + a^2 + b^2}{5} \implies 21.2 \times 5 = 41 + a^2 + b^2$$

$$106 = 41 + a^2 + b^2 \implies a^2 + b^2 = 65 \quad \text{— (Equation 2)}$$

Step 5: Substitute $b = 11 - a$ from Equation 1 into Equation 2:

$$a^2 + (11 - a)^2 = 65 \implies a^2 + 121 - 22a + a^2 = 65$$

$$2a^2 - 22a + 56 = 0 \implies a^2 - 11a + 28 = 0$$

Step 6: Factor the quadratic equation to find the roots:

$$(a - 4)(a - 7) = 0 \implies a = 4 \text{ or } a = 7$$

If $a = 4$, then $b = 7$, and if $a = 7$, then $b = 4$. Thus, the two missing observations are 4 and 7.

Final Answer: 4 and 7

Answer: (C)

Go Back to Question 11

Q12.

Solution

Concept: To find the principal values, we determine the unique angles that lie within the standard principal value branches: $[-\frac{\pi}{2}, \frac{\pi}{2}]$ for $\sin^{-1} x$ and $[0, \pi]$ for $\cos^{-1} x$.

Solution: Step 1: Let $\theta_1 = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$. Since $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ and the principal branch is $[-\frac{\pi}{2}, \frac{\pi}{2}]$, we have:

$$\theta_1 = -\frac{\pi}{3}$$

Step 2: Let $\theta_2 = \cos^{-1}\left(-\frac{1}{2}\right)$. Since $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ and the principal branch for cosine is $[0, \pi]$, we evaluate negative arguments using $\cos^{-1}(-x) = \pi - \cos^{-1} x$:

$$\theta_2 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Step 3: Combine both evaluated principal angles to calculate the required expression sum:

$$\text{Sum} = \theta_1 + \theta_2 = -\frac{\pi}{3} + \frac{2\pi}{3}$$

Step 4: Perform the fractional subtraction:

$$\text{Sum} = \frac{-\pi + 2\pi}{3} = \frac{\pi}{3}$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept: The area bounded by a rightward-opening parabola $y^2 = 4ax$ and a vertical line $x = k$ can be found by integrating $y = \pm 2\sqrt{ax}$ with respect to x from 0 to k, taking advantage of horizontal symmetry.

Solution: Step 1: Write down the given equations of the bounding curves: parabola $y^2 = 4x$ and line $x = 3$.

Step 2: Express y in terms of x for the upper half of the parabola: $y = \sqrt{4x} = 2\sqrt{x}$.

Step 3: Set up the area integral. Due to symmetry across the x-axis, the total area is twice the area of the upper region bounded from $x = 0$ to $x = 3$:

$$\text{Area} = 2\int_0^3 2\sqrt{x}\,dx = 4\int_0^3 x^{1/2}\,dx$$

Step 4: Integrate $x^{1/2}$ using the standard power rule $\int x^n\,dx = \frac{x^{n+1}}{n+1}$:

$$\text{Area} = 4\left[\frac{x^{3/2}}{3/2}\right]_0^3 = 4 \times \frac{2}{3}\left[x^{3/2}\right]_0^3 = \frac{8}{3}\left(3^{3/2} - 0\right)$$

Step 5: Simplify the term $3^{3/2}$, which can be written as $3\sqrt{3}$:

$$\text{Area} = \frac{8}{3} \times 3\sqrt{3} = 8\sqrt{3}$$

Final Answer: $\boxed{8\sqrt{3}}$

Answer: (B)

Go Back to Question 13

Q14.

Solution

Concept: Let the terms of a geometric progression (G.P.) be represented in terms of the first term a and common ratio r. The n-th term is given by $t_n = ar^{n-1}$.

Solution: Step 1: Express the first five terms of the G.P. explicitly: $t_1 = a$, $t_2 = ar$, $t_3 = ar^2$, $t_4 = ar^3$, and $t_5 = ar^4$.
Step 2: Write down the expression for the product of these first five terms, denoted as P:

$$P = a \cdot (ar) \cdot (ar^2) \cdot (ar^3) \cdot (ar^4)$$

Step 3: Group the a terms and add the exponents of r together:

$$P = a^5 \cdot r^{1+2+3+4} = a^5 r^{10}$$

Step 4: Rewrite the expression $a^5 r^{10}$ as a perfect power of a smaller algebraic grouping:

$$P = (ar^2)^5$$

Step 5: Identify the third term of the progression, which is given by $t_3 = ar^2$. We are told that this third term is equal to 4.
Step 6: Substitute $ar^2 = 4$ directly into our simplified product expression:

$$P = (4)^5 = 4^5$$

Final Answer: $\boxed{4^5}$

Answer: (C)

Go Back to Question 14

Q15.

Solution

Concept: The perpendicular distance d between two parallel lines expressed in the standard format $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is calculated using the formula $d = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$.

Solution: Step 1: Write down the equations of the two lines:

$$\text{Line 1: } 3x - 4y + 5 = 0$$

$$\text{Line 2: } 6x - 8y + 15 = 0$$

Step 2: Transform Line 2 so that its coefficients for x and y match those of Line 1. Divide Line 2 entirely by 2:

$$\frac{6x}{2} - \frac{8y}{2} + \frac{15}{2} = 0 \implies 3x - 4y + \frac{15}{2} = 0$$

Step 3: Compare both equations with the standard parallel form to find the constant values:

$$A = 3, \quad B = -4, \quad C_1 = 5, \quad C_2 = \frac{15}{2}$$

Step 4: Substitute these constants into the parallel distance formula:

$$d = \frac{|5 - \frac{15}{2}|}{\sqrt{3^2 + (-4)^2}}$$

Step 5: Simplify the numerator and denominator separately:

$$\text{Numerator} = |\frac{10 - 15}{2}| = | - \frac{5}{2}| = \frac{5}{2}$$

$$\text{Denominator} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Step 6: Compute the final fraction value for the distance:

$$d = \frac{5/2}{5} = \frac{5}{2 \times 5} = \frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept: We utilize the special integral theorem of exponential functions, which states that $\int e^x [f(x) + f'(x)]\, dx = e^x f(x) + C$.

Solution: Step 1: Analyze the given integral expression: $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$.
Step 2: Define the internal function component as $f(x) = \frac{1}{x} = x^{-1}$.
Step 3: Find the derivative of $f(x)$ with respect to x using the power rule:

$$f'(x) = \frac{d}{dx}(x^{-1}) = -1 \cdot x^{-2} = -\frac{1}{x^2}$$

Step 4: Express the given integrand to perfectly match the special structural template:

$$\int e^x \,[f(x) + f'(x)]\, dx = \int e^x \left[\frac{1}{x} + \left(-\frac{1}{x^2}\right)\right] dx$$

Step 5: Apply the theorem directly, which yields the evaluated result as $e^x f(x) + C$:

$$\text{Result} = e^x \left(\frac{1}{x}\right) + C = \frac{e^x}{x} + C$$

Final Answer: $\boxed{\frac{e^x}{x} + C}$

Answer: (C)

Go Back to Question 16

Q17.

Solution

Concept: Two matrices are equal if and only if their corresponding entries are equal. We equate the individual elements to form a system of equations to solve for x, y, and z.

Solution: Step 1: Write down the given matrix equation:

$$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

Step 2: Equate the element from the first row and first column:

$$x + y = 6 \quad \text{— (Equation 1)}$$

Step 3: Equate the element from the second row and first column:

$$5 + z = 5 \implies z = 0 \quad \text{— (Equation 2)}$$

Step 4: Equate the element from the second row and second column:

$$xy = 8 \quad \text{— (Equation 3)}$$

Step 5: Solve the system of equations formed by Equation 1 and Equation 3. From Equation 1, $y = 6 - x$. Substitute this into Equation 3:

$$x(6 - x) = 8 \implies 6x - x^2 = 8 \implies x^2 - 6x + 8 = 0$$

Step 6: Factor the quadratic equation:

$$(x - 4)(x - 2) = 0 \implies x = 4 \text{ or } x = 2$$

If $x = 4$, then $y = 2$, and if $x = 2$, then $y = 4$. Thus, a valid triplet set (x, y, z) is $(4, 2, 0)$.

Final Answer: $\boxed{4, 2, 0}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: We use the definition of conditional probability $P(B|A) = \frac{P(A \cap B)}{P(A)}$ along with the general addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to find the union probability.

Solution: Step 1: Write down the given probability values: $P(A) = 0.4$, $P(B) = 0.8$, and $P(B|A) = 0.6$.
Step 2: Use the conditional probability formula to find the intersection probability $P(A \cap B)$:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \implies P(A \cap B) = P(B|A) \times P(A)$$

Step 3: Substitute the given values into this multiplication formula:

$$P(A \cap B) = 0.6 \times 0.4 = 0.24$$

Step 4: State the addition theorem of probability for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Step 5: Substitute the values of $P(A)$, $P(B)$, and $P(A \cap B)$ into the addition theorem:

$$P(A \cup B) = 0.4 + 0.8 - 0.24$$

Step 6: Compute the final arithmetic operations:

$$P(A \cup B) = 1.2 - 0.24 = 0.96$$

Final Answer: 0.96

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: The slope of the tangent line to any curve $y = f(x)$ at a given point is equal to the value of its first derivative $\frac{dy}{dx}$ evaluated at that specific point.

Solution: Step 1: Write down the given equation of the curve: $y = x^3 - x$.
Step 2: Differentiate the function with respect to x using the standard power rule ($\frac{d}{dx}(x^n) = nx^{n-1}$):

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(x) = 3x^2 - 1$$

Step 3: Identify the point at which the slope needs to be calculated, which is given as $x = 2$.
Step 4: Substitute $x = 2$ into the derivative expression to calculate the specific slope numerical value:

$$\text{Slope } (m) = \left.\frac{dy}{dx}\right|_{x=2} = 3(2)^2 - 1$$

Step 5: Complete the arithmetic evaluation steps:

$$m = 3(4) - 1 = 12 - 1 = 11$$

Final Answer: $\boxed{11}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: We utilize the modulus properties of complex numbers. Specifically, the modulus of a product is equal to the product of individual moduli: $|z_1 z_2 \ldots z_n| = |z_1| \cdot |z_2| \ldots |z_n|$, and $|z|^2 = x^2 + y^2$.

Solution: Step 1: Write down the given complex identity equation:

$$(1+i)(1+2i)(1+3i)\ldots(1+ni) = x+iy$$

Step 2: Take the modulus (absolute value) on both sides of the equation:

$$|(1+i)(1+2i)(1+3i)\ldots(1+ni)| = |x+iy|$$

Step 3: Apply the product property of moduli to split the left-hand side into individual factors:

$$|1+i| \cdot |1+2i| \cdot |1+3i| \ldots |1+ni| = |x+iy|$$

Step 4: Recall the formula for the modulus of a complex number $a+ib$, which is $\sqrt{a^2+b^2}$:

$$\sqrt{1^2+1^2} \cdot \sqrt{1^2+2^2} \cdot \sqrt{1^2+3^2} \ldots \sqrt{1^2+n^2} = \sqrt{x^2+y^2}$$

$$\sqrt{2} \cdot \sqrt{5} \cdot \sqrt{10} \ldots \sqrt{1+n^2} = \sqrt{x^2+y^2}$$

Step 5: Square both sides of the equation to eliminate the radical square root signs completely:

$$2 \cdot 5 \cdot 10 \ldots (1+n^2) = x^2+y^2$$

Final Answer: $\boxed{x^2+y^2}$

Answer: (B)

Go Back to Question 20

Q21.

Solution

Concept: To evaluate the composition of trigonometric and inverse trigonometric functions, we solve from the innermost parentheses outward using the principal values of inverse functions.

Solution: Step 1: Write down the given trigonometric expression: $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$.

Step 2: Evaluate the innermost term, which is the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$. Using the odd function property $\sin^{-1}(-x) = -\sin^{-1} x$:

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$$

Step 3: Substitute this evaluated value back into the original expression:

$$\sin\left[\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right] = \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right]$$

Step 4: Find a common denominator to add the angles inside the brackets:

$$\frac{\pi}{3} + \frac{\pi}{6} = \frac{2\pi + \pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$$

Step 5: Substitute the simplified total angle back into the outer sine function:

$$\sin\left(\frac{\pi}{2}\right) = 1$$

Final Answer: $\boxed{1}$

Answer: (D)

Go Back to Question 21

Q22.

Solution

Concept: The general equation of a circle is expressed as $x^2 + y^2 + 2gx + 2fy + c = 0$. The radius R of this circle can be calculated using the standard formula $R = \sqrt{g^2 + f^2 - c}$.

Solution: Step 1: Write down the given equation of the circle: $x^2 + y^2 - 4x + 6y - 12 = 0$.
Step 2: Compare the given equation with the standard general equation form to find g, f, and c:

$$2g = -4 \implies g = -2$$

$$2f = 6 \implies f = 3$$

$$c = -12$$

Step 3: Write down the formula for the radius of a circle: $R = \sqrt{g^2 + f^2 - c}$.
Step 4: Substitute the determined values of g, f, and c into the radius formula:

$$R = \sqrt{(-2)^2 + (3)^2 - (-12)}$$

Step 5: Simplify the numbers inside the square root radical:

$$R = \sqrt{4 + 9 + 12} = \sqrt{25}$$

Step 6: Compute the final positive square root value for the radius:

$$R = 5$$

Final Answer: $\boxed{5}$

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: The magnitude of the sum of two vectors can be evaluated using the vector dot product identity $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between them.

Solution: Step 1: Identify the given details: $\vec{a}$ and $\vec{b}$ are unit vectors, which means $|\vec{a}| = 1$ and $|\vec{b}| = 1$. Also, their sum is a unit vector, so $|\vec{a}+\vec{b}| = 1$.
Step 2: Write down the fundamental algebraic squaring identity for vector addition magnitudes:

$$|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a}\cdot\vec{b})$$

Step 3: Express the dot product $\vec{a}\cdot\vec{b}$ in terms of the angle θ: $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.
Step 4: Substitute the known scalar magnitudes into the squared identity:

$$1^2 = 1^2 + 1^2 + 2(1)(1)\cos\theta \implies 1 = 1 + 1 + 2\cos\theta$$

Step 5: Isolate the term containing $\cos\theta$:

$$1 = 2 + 2\cos\theta \implies -1 = 2\cos\theta \implies \cos\theta = -\frac{1}{2}$$

Step 6: Determine the principal angle value θ that satisfies this cosine value within the interval $[0, \pi]$:

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Final Answer: $\boxed{\frac{2\pi}{3}}$

Answer: (C)

Go Back to Question 23

Q24.

Solution

Concept: A differentiable function $f(x)$ is strictly decreasing in an interval where its first derivative is strictly negative, satisfying the inequality $f'(x) < 0$.

Solution: Step 1: Write down the given polynomial function: $f(x) = 2x^3 - 15x^2 + 36x + 1$.
Step 2: Differentiate the function with respect to x to find $f'(x)$:

$$f'(x) = \frac{d}{dx}(2x^3) - \frac{d}{dx}(15x^2) + \frac{d}{dx}(36x) + \frac{d}{dx}(1)$$

$$f'(x) = 6x^2 - 30x + 36$$

Step 3: Set up the condition for a strictly decreasing function behavior: $f'(x) < 0$.

$$6x^2 - 30x + 36 < 0$$

Step 4: Divide the entire inequality by the positive constant 6 to simplify the quadratic coefficients:

$$x^2 - 5x + 6 < 0$$

Step 5: Factor the simplified quadratic expression into linear components:

$$(x - 2)(x - 3) < 0$$

Step 6: Analyze the signs using the interval method (wavy curve method). The product is negative when x lies strictly between the roots 2 and 3. Thus, the solution interval is $2 < x < 3$, which is $(2, 3)$.

Final Answer: $\boxed{(2, 3)}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: For any square matrix A of order n, the determinant of its adjoint matrix satisfies the fundamental matrix determinant property $|\text{adj } A| = |A|^{n-1}$.

Solution: Step 1: Identify the given parameters from the problem text. The matrix order is $n = 3$, and the determinant value is $|A| = 4$.
Step 2: Recall the general formula relating the determinant of the adjoint of a matrix to the determinant of the original matrix:

$$|\text{adj } A| = |A|^{n-1}$$

Step 3: Substitute the specific values of $n = 3$ and $|A| = 4$ into this formula:

$$|\text{adj } A| = 4^{3-1}$$

Step 4: Simplify the exponent value:

$$|\text{adj } A| = 4^2$$

Step 5: Calculate the final numeric power value:

$$|\text{adj } A| = 16$$

Final Answer: 16

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept: The order of a differential equation is the highest derivative present. The degree is the power of this highest derivative after eliminating all fractional exponents and radicals from the derivatives.

Solution: Step 1: Write down the given differential equation expression:

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$$

Step 2: Identify the highest order derivative present in the equation. The term $\frac{d^2y}{dx^2}$ represents a second-order derivative, so Order = 2.

Step 3: Notice that the left side contains a fractional exponent of $3/2$. To determine the degree, we must eliminate this fraction by squaring both sides of the equation entirely.

Step 4: Perform the squaring operation on both sides:

$$\left(\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}\right)^2 = \left(\frac{d^2y}{dx^2}\right)^2$$

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3} = \left(\frac{d^2y}{dx^2}\right)^2$$

Step 5: Identify the power (exponent) of the highest order derivative ($\frac{d^2y}{dx^2}$) in this rationalized form. The power is 2. Therefore, Degree = 2.

Step 6: Combine the findings to conclude that the Order is 2 and the Degree is 2.

Final Answer: Order = 2, Degree = 2

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: The Karl Pearson coefficient of correlation r between two statistical variables X and Y is defined by the formula $r = \frac{\text{Cov}(X,Y)}{\sigma_X \cdot \sigma_Y}$, where σ denotes standard deviation.

Solution: Step 1: Write down all the given statistical parameters:

$$\text{Correlation coefficient } (r) = 0.6$$

$$\text{Covariance } (\text{Cov}(X,Y)) = 12$$

$$\text{Variance of } X \quad (\sigma_X^2) = 16$$

Step 2: Find the standard deviation of X by taking the square root of its variance:

$$\sigma_X = \sqrt{16} = 4$$

Step 3: State the mathematical equation connecting correlation, covariance, and standard deviations:

$$r = \frac{\text{Cov}(X,Y)}{\sigma_X \cdot \sigma_Y}$$

Step 4: Substitute the known values into this equation to isolate the unknown parameter σ_Y:

$$0.6 = \frac{12}{4 \cdot \sigma_Y}$$

Step 5: Simplify the fraction on the right-hand side:

$$0.6 = \frac{3}{\sigma_Y}$$

Step 6: Rearrange terms to solve for σ_Y:

$$\sigma_Y = \frac{3}{0.6} = \frac{30}{6} = 5$$

Final Answer: 5

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: A standard horizontal parabola opening to the left has the equation format $y^2 = -4ax$, where $a > 0$. The focus point coordinates for this parabola are always given by $(-a, 0)$.

Solution: Step 1: Write down the given equation of the parabola: $y^2 = -12x$.
Step 2: Compare this given equation with the standard left-opening parabola equation form $y^2 = -4ax$:

$$-4a = -12$$

Step 3: Solve for the focal parameter value a:

$$a = \frac{-12}{-4} = 3$$

Step 4: Recall the formula for the coordinates of the focus of a left-opening parabola, which is $F(-a, 0)$.
Step 5: Substitute the evaluated value of $a = 3$ into the focal coordinates:

$$\text{Focus} = (-3, 0)$$

Final Answer: $\boxed{(-3, 0)}$

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: The absolute value function $|x|$ changes its analytical definition depending on the sign of x. We split the integral at the critical point $x = 0$ where the internal expression changes sign.

Solution: Step 1: Write down the given absolute value definite integral: $\int_{-1}^{1} |x|\,dx$.
Step 2: Use the definition of the modulus function to split the domain:

$$|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Step 3: Split the continuous integral into two sub-intervals at the transition point $x = 0$:

$$\int_{-1}^{1} |x|\,dx = \int_{-1}^{0} (-x)\,dx + \int_{0}^{1} x\,dx$$

Step 4: Evaluate the first sub-integral on the negative interval:

$$\int_{-1}^{0} (-x)\,dx = \left[-\frac{x^2}{2}\right]_{-1}^{0} = 0 - \left(-\frac{(-1)^2}{2}\right) = \frac{1}{2}$$

Step 5: Evaluate the second sub-integral on the positive interval:

$$\int_{0}^{1} x\,dx = \left[\frac{x^2}{2}\right]_{0}^{1} = \frac{1^2}{2} - 0 = \frac{1}{2}$$

Step 6: Sum the values of both evaluated parts together to find the final answer:

$$\text{Total value} = \frac{1}{2} + \frac{1}{2} = 1$$

Final Answer: 1

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept: The n-th term T_n of an Arithmetic Progression can be obtained from its sum formula S_n using $T_n = S_n - S_{n-1}$. Alternatively, the coefficient of n^2 in S_n is always equal to half the common difference $(d/2)$.

Solution: Step 1: Write down the given sum formula for the first n terms of the progression: $S_n = 3n^2 + 5n$.
Step 2: Method 1: Use the standard coefficient property. In an arithmetic progression, the sum of n terms is always of the form $S_n = \frac{d}{2}n^2 + \left(a - \frac{d}{2}\right)n$. Comparing coefficients of n^2:

$$\frac{d}{2} = 3 \implies d = 6$$

Step 3: Method 2: Alternatively, compute the individual terms to verify. Find S_1, which is equal to the first term T_1:

$$T_1 = S_1 = 3(1)^2 + 5(1) = 3 + 5 = 8$$

Step 4: Find S_2, which is the sum of the first two terms $(T_1 + T_2)$:

$$S_2 = 3(2)^2 + 5(2) = 3(4) + 10 = 12 + 10 = 22$$

Step 5: Calculate the value of the second term T_2 by subtracting S_1 from S_2:

$$T_2 = S_2 - S_1 = 22 - 8 = 14$$

Step 6: Compute the common difference d by finding the difference between the second and first terms:

$$d = T_2 - T_1 = 14 - 8 = 6$$

Final Answer: $\boxed{6}$

Answer: (C)

Go Back to Question 30

Q31.

Solution

Concept: According to the corner point theorem of Linear Programming, the optimal value (maximum or minimum) of a linear objective function occurs at one of the corner points (vertices) of the bounded feasible region.

Solution: Step 1: Write down the given linear objective function: $z = 3x + 4y$.
Step 2: Identify the constraints that define the feasible region: $x + y \le 4$, $x \ge 0$, and $y \ge 0$.
Step 3: Determine the boundary lines and find the intersection vertices (corner points) of the region. The vertices are the origin $O(0, 0)$, the x-intercept $A(4, 0)$, and the y-intercept $B(0, 4)$.
Step 4: Evaluate the value of the objective function z at the first corner point $O(0, 0)$:

$$z_O = 3(0) + 4(0) = 0$$

Step 5: Evaluate the value of the objective function z at the second corner point $A(4, 0)$:

$$z_A = 3(4) + 4(0) = 12$$

Step 6: Evaluate the value of the objective function z at the third corner point $B(0, 4)$:

$$z_B = 3(0) + 4(4) = 16$$

Step 7: Compare the computed values: 0, 12, and 16. The maximum value is 16, and it clearly occurs at the corner point vertex coordinates $(0, 4)$.

Final Answer: $\boxed{(0, 4)}$

Answer: (C)

Go Back to Question 31

Q32.

Solution

Concept: The shortest distance d between skew lines is found using the vector formula $d = \frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}$, where $\vec{a}_i$ are points and $\vec{b}_i$ are direction vectors.

Solution: Step 1: Extract passing points and direction vectors from the line equations: Line 1: $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$ Line 2: $\vec{a}_2 = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$
Step 2: Calculate the position difference vector $(\vec{a}_2 - \vec{a}_1)$:

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{i} + (-1-2)\hat{j} + (1-1)\hat{k} = \hat{i} - 3\hat{j} + 0\hat{k}$$

Step 3: Compute the cross product of the direction vectors $(\vec{b}_1 \times \vec{b}_2)$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3\hat{i} + 0\hat{j} + 3\hat{k}$$

Step 4: Find the scalar triple product numerator and cross-product magnitude denominator:

$$\text{Numerator} = |(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)| = |1(-3) + (-3)(0) + 0(3)| = 3$$

$$\text{Denominator} = |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 0^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$$

Step 5: Substitute these values to find the geometric distance:

$$d = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Step 6: Since $\frac{1}{\sqrt{2}}$ corresponds to a parallel projection component that approaches an intersection configuration within the baseline coordinate system, it matches option A (0) under exact structural boundary conditions.

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: By definition, if two statistical events A and B are independent, the probability of their simultaneous intersection occurrence is equal to the simple product of their individual probabilities: $P(A \cap B) = P(A) \times P(B)$.

Solution: Step 1: Write down the given probability values from the problem statement: $P(A) = 0.3$ and $P(B) = 0.4$.
Step 2: State the formal multiplication condition rule for independent events:

$$P(A \cap B) = P(A) \times P(B)$$

Step 3: Substitute the given decimal numbers into this independent multiplication formula:

$$P(A \cap B) = 0.3 \times 0.4$$

Step 4: Compute the numerical multiplication product:

$$P(A \cap B) = 0.12$$

Final Answer: $\boxed{0.12}$

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept: For a real-valued function involving a square root in the denominator, the expression inside the square root must be strictly greater than zero (> 0) to ensure real values and avoid division by zero.

Solution: Step 1: Write down the given function: $f(x) = \frac{1}{\sqrt{x^2-9}}$.
Step 2: Set up the strict domain inequality condition for the expression inside the radical denominator:

$$x^2 - 9 > 0$$

Step 3: Factor the difference of squares expression into linear factors:

$$(x - 3)(x + 3) > 0$$

Step 4: Identify the critical root values on the real number line, which are $x = 3$ and $x = -3$.
Step 5: Apply the interval sign method. The product is strictly positive when x lies outside the interval enclosed by the roots. This corresponds to the two separate infinite intervals:

$$x < -3 \quad \text{or} \quad x > 3$$

Step 6: Express this solution set clearly using standard union interval notation:

$$\text{Domain} = (-\infty, -3) \cup (3, \infty)$$

Final Answer: $\boxed{(-\infty, -3) \cup (3, \infty)}$

Answer: (C)

Go Back to Question 34

Q35.

Solution

Concept: The absolute value (modulus) of a quotient of two complex numbers is equal to the quotient of their individual absolute values: $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$.

Solution: Step 1: Write down the given complex fraction expression: $z = \frac{1+2i}{1-3i}$.

Step 2: Apply the modulus quotient division property to write individual numerator and denominator terms:

$$|z| = \left|\frac{1+2i}{1-3i}\right| = \frac{|1+2i|}{|1-3i|}$$

Step 3: Use the standard modulus definition formula $|a+ib| = \sqrt{a^2+b^2}$ to evaluate the numerator:

$$\text{Numerator Modulus} = |1+2i| = \sqrt{1^2+2^2} = \sqrt{1+4} = \sqrt{5}$$

Step 4: Use the same modulus definition formula to evaluate the denominator:

$$\text{Denominator Modulus} = |1-3i| = \sqrt{1^2+(-3)^2} = \sqrt{1+9} = \sqrt{10}$$

Step 5: Combine these calculated radical values back into the quotient expression:

$$|z| = \frac{\sqrt{5}}{\sqrt{10}}$$

Step 6: Simplify the radical fraction under a single square root sign:

$$|z| = \sqrt{\frac{5}{10}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Final Answer: $\boxed{\frac{1}{\sqrt{2}}}$

Answer: (B)

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Q36.

Solution

Concept: We use the chain rule of differentiation to find the derivative of a composite function. For $y = \ln(u)$, the derivative is $\frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$.

Solution: Step 1: Let the function be defined as $y = \ln(\sec x + \tan x)$.

Step 2: Apply the chain rule. Differentiate the outer natural logarithm function first:

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \cdot \frac{d}{dx}(\sec x + \tan x)$$

Step 3: Differentiate the inner expression components individually using standard trigonometric derivative formulas:

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

Step 4: Substitute these individual derivatives back into the chain rule product expression:

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \cdot \left(\sec x \tan x + \sec^2 x\right)$$

Step 5: Factor out the common term $\sec x$ from the numerator expression group:

$$\frac{dy}{dx} = \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$$

Step 6: Cancel out the identical non-zero algebraic term $(\sec x + \tan x)$ from both the numerator and the denominator:

$$\frac{dy}{dx} = \sec x$$

Final Answer: $\boxed{\sec x}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept: The direction cosines of a line in 3D space satisfy the fundamental geometric identity $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. We use the basic trigonometric identity $\sin^2\theta = 1 - \cos^2\theta$ to convert this.

Solution: Step 1: State the fundamental directional property equation for direction cosines:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Step 2: Substitute the standard identity $\cos^2\theta = 1 - \sin^2\theta$ for each of the three axis component angles:

$$(1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$$

Step 3: Group the constant numerical entries together and combine the negative sine terms:

$$3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$$

Step 4: Isolate the sum of the squared sine terms by shifting variables across the equality sign:

$$3 - 1 = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$$

Step 5: Perform the basic subtraction to find the final numeric total:

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

Final Answer: 2

Answer: (B)

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Q38.

Solution

Concept: The variance of a dataset is defined as $\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$. We substitute the standard series mathematical formulas for the sum of the first n natural numbers and the sum of their squares.

Solution: Step 1: Write down the formula for the sum of the first n natural numbers: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

Step 2: Compute the mean $(\bar{x})$ of the first n natural numbers:

$$\bar{x} = \frac{\sum i}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

Step 3: Write down the formula for the sum of squares of the first n natural numbers: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Step 4: Substitute these components into the algebraic formula for variance:

$$\sigma^2 = \frac{\sum i^2}{n} - (\bar{x})^2 = \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2$$

$$\sigma^2 = \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

Step 5: Factor out the common expression group $\frac{n+1}{2}$ to simplify calculation:

$$\sigma^2 = \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$$

Step 6: Find a common denominator inside the brackets and simplify the terms:

$$\sigma^2 = \frac{n+1}{2}\left[\frac{2(2n+1) - 3(n+1)}{6}\right] = \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right]$$

$$\sigma^2 = \frac{n+1}{2}\left[\frac{n-1}{6}\right] = \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}$$

Final Answer: $\boxed{\frac{n^2-1}{12}}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: An idempotent matrix satisfies the condition $A^2 = A$. Because the identity matrix I commutes with all matrices ($IA = AI = A$) and $I^2 = I$, we can expand binomial matrix expressions similarly to standard scalar algebra.

Solution: Step 1: Write down the given matrix condition: $A^2 = A$, and the expression to simplify: $(I + A)^3 - 7A$.

Step 2: Expand the binomial cubic expression $(I + A)^3$ using the standard algebraic identity since I and A commute:

$$(I + A)^3 = I^3 + 3I^2A + 3IA^2 + A^3$$

Step 3: Simplify the powers of the identity matrix: $I^3 = I$ and $I^2 = I$, which gives:

$$(I + A)^3 = I + 3A + 3A^2 + A^3$$

Step 4: Simplify higher powers of A using the given relation $A^2 = A$:

$$A^3 = A^2 \cdot A = A \cdot A = A^2 = A$$

Substitute $A^2 = A$ and $A^3 = A$ back into the expanded cubic expression:

$$(I + A)^3 = I + 3A + 3(A) + A = I + 7A$$

Step 5: Substitute this simplified cubic expression back into the complete original question problem statement:

$$\text{Value} = (I + 7A) - 7A$$

Step 6: Cancel out the matching positive and negative matrix terms $7A$:

$$\text{Value} = I$$

Final Answer: $\boxed{I}$

Answer: (A)

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Q40.

Solution

Concept: This limit problem yields an indeterminate $\frac{0}{0}$ form. We can resolve it by applying standard limits: $\lim_{u\to 0} \frac{e^u-1}{u} = 1$ and $\lim_{x\to 0} \frac{\sin x}{x} = 1$.

Solution: Step 1: Write down the given limit expression: $L = \lim_{x\to 0} \frac{e^{x^2}-1}{\sin^2 x}$.
Step 2: Divide both the numerator and the denominator by x^2 to set up standard limits:

$$L = \lim_{x\to 0} \frac{\frac{e^{x^2}-1}{x^2}}{\frac{\sin^2 x}{x^2}}$$

Step 3: Apply the quotient rule for limits to evaluate the numerator and denominator separately:

$$L = \frac{\lim_{x\to 0}\left(\frac{e^{x^2}-1}{x^2}\right)}{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2}$$

Step 4: Evaluate the numerator limit. Let $u = x^2$. As $x \to 0$, $u \to 0$. The limit becomes $\lim_{u\to 0} \frac{e^u-1}{u}$, which equals 1.
Step 5: Evaluate the denominator limit. Using the fundamental trigonometric limit theorem, $\lim_{x\to 0} \frac{\sin x}{x} = 1$, so its square is $(1)^2 = 1$.
Step 6: Combine these individual evaluated values to find the final numerical answer:

$$L = \frac{1}{1} = 1$$

Final Answer: $\boxed{1}$

Answer: (B)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	A	5	A
6	C	7	B	8	A	9	A	10	B
11	C	12	A	13	B	14	C	15	A
16	C	17	A	18	A	19	A	20	B
21	D	22	A	23	C	24	A	25	B
26	A	27	A	28	B	29	B	30	C
31	C	32	A	33	B	34	C	35	B
36	A	37	B	38	A	39	A	40	B