

Rajasthan JET Mathematics Sample Paper-5

Duration: 40 Minutes

Maximum Marks: 160

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **-1 marks**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. Let A and B be two square matrices of order 3 such that $|A| = 3$ and $|B| = -2$. Then the value of $|2A^T B^{-1}|$ is

- (A) -12
- (B) 12
- (C) -6
- (D) 6

Q2. The domain of the function $f(x) = \sqrt{\log_{0.5}(x^2 - 5x + 7)}$ is

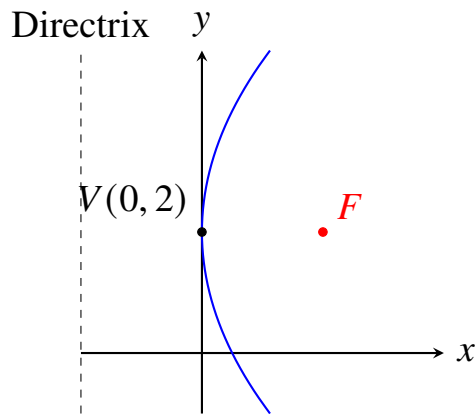
- (A) (2, 3)
- (B) [2, 3]
- (C) $(-\infty, 2] \cup [3, \infty)$
- (D) $(-\infty, 2) \cup (3, \infty)$

Q3. The value of $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$ is

- (A) $\frac{\pi^2}{2}$
- (B) $\frac{\pi^2}{4}$
- (C) $\frac{\pi}{4}$
- (D) π^2



Q4. The focus of the parabola $y^2 - 4y - 8x + 4 = 0$ is located at the point



- (A) (2, 2)
- (B) (0, 2)
- (C) (2, 0)
- (D) (-2, 2)

Q5. A bag contains 5 red and 4 black balls. A ball is drawn at random from the bag and its color is noted. Then it is returned to the bag along with 2 additional balls of the same color. Now a ball is drawn at random from the bag. The probability that the second ball drawn is red is

- (A) $\frac{5}{9}$
- (B) $\frac{19}{33}$
- (C) $\frac{7}{11}$
- (D) $\frac{4}{9}$

Q6. If the n -th term of an arithmetic progression is given by $T_n = 3n + 5$, then the sum of the first 20 terms of this progression is

- (A) 710
- (B) 730
- (C) 690
- (D) 750



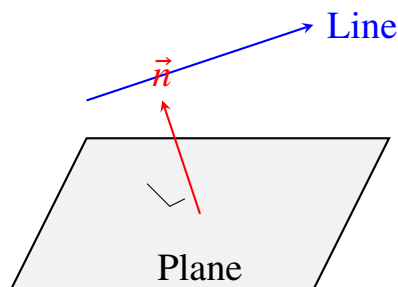
Q7. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ under the condition $y(1) = \frac{1}{4}$ is

- (A) $4xy = x^4$
- (B) $xy = x^4$
- (C) $4xy = x^4 + 3$
- (D) $y = x^3$

Q8. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then the value of $\cos^{-1} x + \cos^{-1} y$ is

- (A) $\frac{\pi}{6}$
- (B) $\frac{\pi}{3}$
- (C) π
- (D) $\frac{\pi}{3}$

Q9. The line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is parallel to which of the following planes?



- (A) $2x + 3y + 4z = 5$
- (B) $3x + 2y - 3z = 1$
- (C) $x + y + z = 6$
- (D) $2x - y + z = 4$

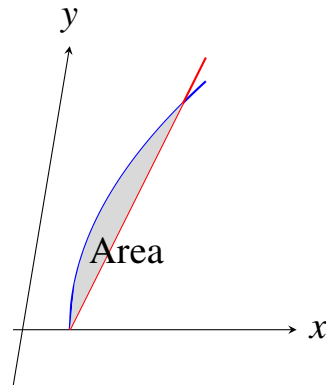
Q10. The mean of 5 observations is 4 and their variance is 5.2. If three of these observations are 1, 2, and 6, then the other two observations are

- (A) 3 and 8



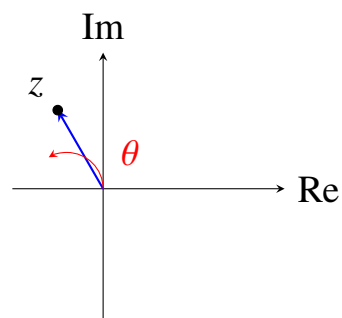
- (B) 4 and 7
 (C) 5 and 6
 (D) 2 and 9

Q11. The area bounded by the curve $y^2 = 4x$ and the line $y = 2x$ is



- (A) $\frac{2}{3}$
 (B) $\frac{1}{3}$
 (C) $\frac{4}{3}$
 (D) $\frac{1}{6}$

Q12. If $z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$, then the principal argument of z is



- (A) $\frac{\pi}{3}$
 (B) $\frac{2\pi}{3}$
 (C) $-\frac{\pi}{3}$



(D) $-\frac{2\pi}{3}$

Q13. Let $f(x) = |x - 1| + |x - 2|$. At $x = 1$, the function is

- (A) Continuous but not differentiable
- (B) Differentiable but not continuous
- (C) Both continuous and differentiable
- (D) Neither continuous nor differentiable

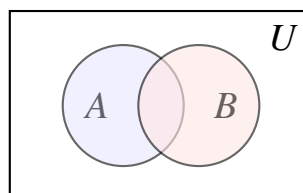
Q14. The angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ is

- (A) $\cos^{-1}\left(\frac{1}{3}\right)$
- (B) $\cos^{-1}\left(-\frac{1}{3}\right)$
- (C) $\frac{\pi}{3}$
- (D) $\frac{2\pi}{3}$

Q15. The maximum value of the function $f(x) = x^3 - 3x$ on the interval $[-2, 2]$ is

- (A) 2
- (B) -2
- (C) 1
- (D) 0

Q16. If A and B are independent events such that $P(A) = 0.3$ and $P(B) = 0.4$, then the value of $P(A \cup B)$ is



- (A) 0.70
- (B) 0.12



(C) 0.58

(D) 0.46

Q17. The value of $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x^2}$ is

(A) 2

(B) 4

(C) 8

(D) 16

Q18. The number of terms in the expansion of $(x + y + z)^{10}$ after simplification is

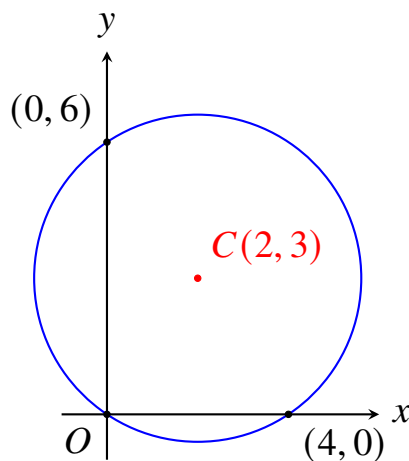
(A) 11

(B) 22

(C) 55

(D) 66

Q19. The equation of the circle passing through the origin and making intercepts of 4 and 6 on the positive x and y axes respectively is



(A) $x^2 + y^2 - 4x - 6y = 0$

(B) $x^2 + y^2 + 4x + 6y = 0$

(C) $x^2 + y^2 - 8x - 12y = 0$



(D) $x^2 + y^2 - 2x - 3y = 0$

Q20. If $y = \tan^{-1} \left(\frac{\cos x + \sin x}{\cos x - \sin x} \right)$, then $\frac{dy}{dx}$ is equal to

(A) 1

(B) -1

(C) $\frac{1}{1+x^2}$

(D) 0

Q21. The value of $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$ is

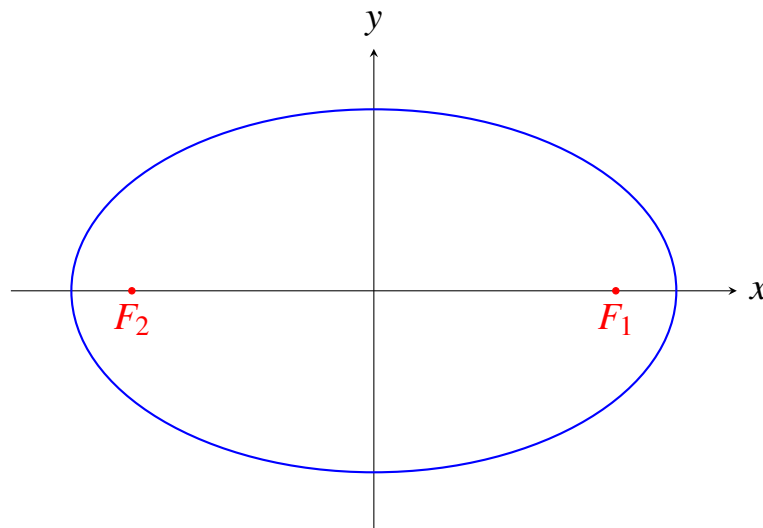
(A) $\frac{e^x}{x^2} + C$

(B) $-\frac{e^x}{x} + C$

(C) $\frac{e^x}{x} + C$

(D) $e^x \ln x + C$

Q22. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is



(A) $\frac{4}{5}$

(B) $\frac{3}{5}$

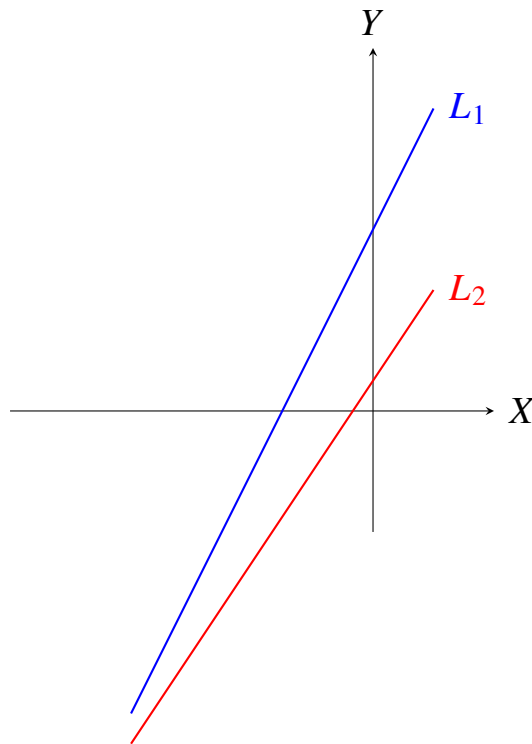
(C) $\frac{16}{25}$

(D) $\frac{5}{4}$



- Q23.** If α and β are the roots of the equation $x^2 - 5x + 6 = 0$, then the value of $\alpha^3 + \beta^3$ is
- (A) 35
(B) 45
(C) 19
(D) 9
- Q24.** The general solution of the differential equation $\frac{dy}{dx} = e^{x-y}$ is
- (A) $e^y = e^x + C$
(B) $e^{-y} = e^x + C$
(C) $e^y = e^{-x} + C$
(D) $e^{-y} = e^{-x} + C$
- Q25.** If the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar, then the value of λ is
- (A) -4
(B) 4
(C) -2
(D) 2
- Q26.** The coefficient of correlation between two variables X and Y when the two lines of regression are $2X - Y + 3 = 0$ and $3X - 2Y + 1 = 0$ is





- (A) $\sqrt{\frac{3}{4}}$
- (B) $\sqrt{\frac{2}{3}}$
- (C) $\frac{3}{4}$
- (D) $\frac{1}{2}$

Q27. The range of the function $f(x) = \frac{x}{1+x^2}$ for all real x is

- (A) $[-1, 1]$
- (B) $(-\infty, \infty)$
- (C) $[-\frac{1}{2}, \frac{1}{2}]$
- (D) $[0, \frac{1}{2}]$

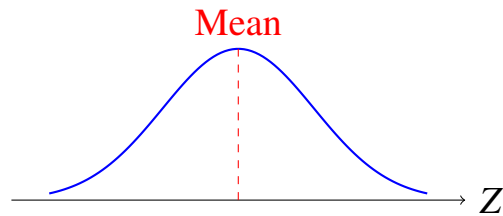
Q28. The derivative of $\sec x$ with respect to x at $x = \frac{\pi}{4}$ is

- (A) 1
- (B) $\sqrt{2}$



- (C) 2
(D) $\frac{1}{\sqrt{2}}$

Q29. If standard deviation of a distribution is 4.5, then its mean deviation about mean for a normal distribution is approximately



- (A) 3.6
(B) 4.0
(C) 5.4
(D) 3.0

Q30. The distance between the parallel lines $3x + 4y - 9 = 0$ and $6x + 8y + 15 = 0$ is

- (A) $\frac{24}{5}$
(B) $\frac{33}{10}$
(C) $\frac{33}{5}$
(D) $\frac{6}{5}$

Q31. The value of $\cos 20^\circ \cos 40^\circ \cos 80^\circ$ is

- (A) $\frac{1}{2}$
(B) $\frac{1}{4}$
(C) $\frac{1}{8}$
(D) $\frac{1}{16}$

Q32. If a pair of fair dice is rolled, the probability of obtaining a sum of 8 given that the numbers shown on both dice are different is



- (A) $\frac{5}{36}$
- (B) $\frac{1}{9}$
- (C) $\frac{2}{15}$
- (D) $\frac{1}{6}$

Q33. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^2 - 5A$ is equal to

- (A) $2I$
- (B) $-2I$
- (C) I
- (D) O

Q34. The value of $\int_1^2 \frac{1}{x(1+x^4)} dx$ is

- (A) $\frac{1}{4} \ln \left(\frac{32}{17} \right)$
- (B) $\frac{1}{4} \ln \left(\frac{17}{2} \right)$
- (C) $\frac{1}{4} \ln \left(\frac{34}{17} \right)$
- (D) $\frac{1}{4} \ln \left(\frac{16}{17} \right)$

Q35. The shortest distance between the lines $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$ is

- (A) 0
- (B) $\frac{3}{\sqrt{2}}$
- (C) $3\sqrt{2}$
- (D) $\frac{9}{\sqrt{2}}$

Q36. The value of $(1+i)^{10} + (1-i)^{10}$ is

- (A) 64
- (B) -64



(C) $64i$

(D) 0

Q37. The vector projection of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is

(A) $\frac{10}{\sqrt{6}}$

(B) $\frac{5}{3}(\hat{i} + 2\hat{j} + \hat{k})$

(C) $\frac{5}{6}(2\hat{i} + 3\hat{j} + 2\hat{k})$

(D) $10(\hat{i} + 2\hat{j} + \hat{k})$

Q38. The value of $\tan\left(2 \tan^{-1} \frac{1}{5}\right)$ is

(A) $\frac{5}{12}$

(B) $\frac{5}{24}$

(C) $\frac{12}{5}$

(D) $\frac{7}{12}$

Q39. If $y = x^x$, then $\frac{dy}{dx}$ at $x = 1$ is

(A) 0

(B) 1

(C) e

(D) -1

Q40. The sum of the infinite geometric series $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \dots$ is

(A) 2

(B) 3

(C) $\frac{3}{2}$

(D) 4



Detailed Solutions

Q1.

Solution

Concept: For a square matrix A of order n and scalar k , the determinant property states $|kA| = k^n|A|$. Additionally, for matrix products, transposes, and inverses, the properties are $|AB| = |A||B|$, $|A^T| = |A|$, and $|B^{-1}| = \frac{1}{|B|}$ respectively.

Solution: Step 1: Identify given parameters of order $n = 3$: $|A| = 3$ and $|B| = -2$.

Step 2: Factor out the scalar $k = 2$ from the expression using the property $|2A^T B^{-1}| = 2^3|A^T B^{-1}|$.

Step 3: Simplify the scalar multiplier coefficient to obtain:

$$8|A^T B^{-1}|$$

Step 4: Expand the product of the determinants inside using the multiplicative property:

$$8|A^T||B^{-1}|$$

Step 5: Substitute the identities $|A^T| = |A|$ and $|B^{-1}| = \frac{1}{|B|}$ into the formula:

$$8 \times |A| \times \frac{1}{|B|}$$

Step 6: Substitute the given numerical values and calculate the final quantity:

$$8 \times 3 \times \left(\frac{1}{-2}\right) = -12$$

Final Answer:

Answer: (A)

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Q2.

Solution

Concept: For a real-valued radical function, the radicand must be non-negative. For logarithms $\log_b(g(x))$ with base $0 < b < 1$, the argument must be strictly positive, and the inequality sign reverses when converting out of logarithmic form.

Solution: Step 1: Establish the non-negative constraint for the expression inside the square root:

$$\log_{0.5}(x^2 - 5x + 7) \geq 0$$

Step 2: Check the logarithmic domain condition $x^2 - 5x + 7 > 0$. The discriminant is $\Delta = 25 - 28 = -3 < 0$, meaning the quadratic expression is strictly positive for all real x .

Step 3: Solve the logarithmic inequality. Since the base $0.5 < 1$, reverse the inequality direction:

$$x^2 - 5x + 7 \leq (0.5)^0 \implies x^2 - 5x + 7 \leq 1$$

Step 4: Rearrange the terms into standard quadratic form by subtracting 1 from both sides:

$$x^2 - 5x + 6 \leq 0$$

Step 5: Factor the expression to determine the critical boundary roots:

$$(x - 2)(x - 3) \leq 0$$

Step 6: Apply the sign interval method. The expression is non-positive inside the roots, yielding the closed interval $[2, 3]$.

Final Answer:

Answer: (B)

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Q3.

Solution

Concept: Definite integrals of the form $\int_0^a f(x) dx$ can be solved using the reduction property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$, which helps eliminate an explicit algebraic variable x in the numerator.

Solution: Step 1: Write down the primary integral expression and label it as equation (1):

$$I = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx \quad \text{--- (1)}$$

Step 2: Substitute $(\pi - x)$ for x using the definite integral property:

$$I = \int_0^{\pi} \frac{(\pi - x) \sin(\pi - x)}{1 + \cos^2(\pi - x)} dx$$

Step 3: Apply standard trigonometric reduction identities to form equation (2):

$$I = \int_0^{\pi} \frac{(\pi - x) \sin x}{1 + \cos^2 x} dx \quad \text{--- (2)}$$

Step 4: Add equations (1) and (2) together to eliminate x :

$$2I = \int_0^{\pi} \frac{\pi \sin x}{1 + \cos^2 x} dx \implies I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx$$

Step 5: Substitute $u = \cos x$, giving $du = -\sin x dx$. The evaluation limits shift from $[0, \pi]$ to $[1, -1]$:

$$I = \frac{\pi}{2} \int_1^{-1} \frac{-du}{1 + u^2} = \frac{\pi}{2} \int_{-1}^1 \frac{du}{1 + u^2}$$

Step 6: Solve the standard integral using the inverse tangent form:

$$I = \pi \int_0^1 \frac{du}{1 + u^2} = \pi [\tan^{-1} u]_0^1 = \pi \left(\frac{\pi}{4} \right) = \frac{\pi^2}{4}$$

Final Answer:

Answer: (B)

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Q4.

Solution

Concept: The standard equation of a horizontal parabola opening to the right is $(y-k)^2 = 4a(x-h)$, where (h, k) is the vertex coordinate. The coordinates of its focus are given by $(h + a, k)$.

Solution: Step 1: Write down the given quadratic equation of the parabola:

$$y^2 - 4y - 8x + 4 = 0$$

Step 2: Group the y -terms on the left side and transfer the remaining components to the right:

$$y^2 - 4y = 8x - 4$$

Step 3: Complete the perfect square on the left side by adding 4 to both sides:

$$y^2 - 4y + 4 = 8x \implies (y - 2)^2 = 8x$$

Step 4: Rewrite the right side into the standard format $4a(x - h)$:

$$(y - 2)^2 = 4(2)(x - 0)$$

Step 5: Compare parameters with the standard form to find the values of h , k , and a :

$$h = 0, \quad k = 2, \quad a = 2$$

Step 6: Calculate the focus coordinates using the translation formula:

$$\text{Focus} = (h + a, k) = (0 + 2, 2) = (2, 2)$$

Final Answer:

Answer: (A)

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Q5.

Solution

Concept: By using the Law of Total Probability, the probability of selecting a red ball on the second draw (R_2) depends on whether a red (R_1) or black (B_1) ball was chosen first:
 $P(R_2) = P(R_1)P(R_2|R_1) + P(B_1)P(R_2|B_1)$.

Solution: Step 1: Count the initial contents of the bag containing 5 red and 4 black balls, totaling 9 balls.

Step 2: Determine the initial probabilities for the first draw:

$$P(R_1) = \frac{5}{9}, \quad P(B_1) = \frac{4}{9}$$

Step 3: Find the conditional probability $P(R_2|R_1)$ after adding 2 extra red balls (7 red, 4 black):

$$P(R_2|R_1) = \frac{7}{11}$$

Step 4: Find the conditional probability $P(R_2|B_1)$ after adding 2 extra black balls (5 red, 6 black):

$$P(R_2|B_1) = \frac{5}{11}$$

Step 5: Combine these parameters using the total probability theorem:

$$P(R_2) = \left(\frac{5}{9} \times \frac{7}{11}\right) + \left(\frac{4}{9} \times \frac{5}{11}\right) = \frac{35}{99} + \frac{20}{99} = \frac{55}{99}$$

Step 6: Reduce the fraction to its lowest terms by dividing by 11:

$$P(R_2) = \frac{5}{9}$$

Final Answer:

$$\frac{5}{9}$$

Answer: (A)

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Q6.

Solution

Concept: The sum of the first n terms of an arithmetic progression can be evaluated using the simplified formula $S_n = \frac{n}{2} [T_1 + T_n]$, where T_1 represents the initial term and T_n represents the final term.

Solution: Step 1: Write down the explicit formula for the general term of the sequence:

$$T_n = 3n + 5$$

Step 2: Calculate the first term (T_1) by setting $n = 1$:

$$T_1 = 3(1) + 5 = 8$$

Step 3: Calculate the twentieth term (T_{20}) by setting $n = 20$:

$$T_{20} = 3(20) + 5 = 65$$

Step 4: Set up the sum formula for the first $n = 20$ terms:

$$S_{20} = \frac{20}{2} [T_1 + T_{20}]$$

Step 5: Substitute the computed values of the boundaries into the expression:

$$S_{20} = 10 \times [8 + 65]$$

Step 6: Complete the basic arithmetic multiplication to obtain the final sum:

$$S_{20} = 10 \times 73 = 710$$

Final Answer:

Answer: (A)

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Q7.

Solution

Concept: A first-order linear differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is solved by determining an integrating factor I.F. = $e^{\int P(x) dx}$ and applying the general solution form $y \times \text{I.F.} = \int (Q(x) \times \text{I.F.}) dx + C$.

Solution: Step 1: Match the given expression $\frac{dy}{dx} + \frac{1}{x}y = x^2$ with standard components: $P(x) = \frac{1}{x}$ and $Q(x) = x^2$.

Step 2: Determine the integrating factor (I.F.) value:

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

Step 3: Structure the general integration equation using the computed factor:

$$y \times x = \int (x^2 \times x) dx \implies xy = \int x^3 dx$$

Step 4: Integrate the right-hand side expression using standard power rules:

$$xy = \frac{x^4}{4} + C$$

Step 5: Apply the given boundary initial condition $y(1) = \frac{1}{4}$ to solve for C :

$$(1) \left(\frac{1}{4} \right) = \frac{(1)^4}{4} + C \implies C = 0$$

Step 6: Substitute $C = 0$ back into the solution structure and simplify:

$$xy = \frac{x^4}{4} \implies 4xy = x^4$$

Final Answer:

Answer: (A)

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Q8.

Solution

Concept: The standard complementary relationship between inverse trigonometric functions states that for any variable $k \in [-1, 1]$, the identity $\sin^{-1} k + \cos^{-1} k = \frac{\pi}{2}$ always holds true.

Solution: Step 1: Write down the primary inverse trigonometric identity for both variables x and y independently:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\sin^{-1} y + \cos^{-1} y = \frac{\pi}{2}$$

Step 2: Combine these two independent linear equations by adding them together directly:

$$\left(\sin^{-1} x + \sin^{-1} y\right) + \left(\cos^{-1} x + \cos^{-1} y\right) = \frac{\pi}{2} + \frac{\pi}{2}$$

$$\left(\sin^{-1} x + \sin^{-1} y\right) + \left(\cos^{-1} x + \cos^{-1} y\right) = \pi \quad \text{--- (1)}$$

Step 3: Identify the value provided in the problem statement, which gives $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$.

Step 4: Substitute this given value into equation (1):

$$\frac{2\pi}{3} + \left(\cos^{-1} x + \cos^{-1} y\right) = \pi$$

Step 5: Isolate the target expression $\cos^{-1} x + \cos^{-1} y$ by subtracting $\frac{2\pi}{3}$ from both sides of the equation:

$$\cos^{-1} x + \cos^{-1} y = \pi - \frac{2\pi}{3}$$

Step 6: Find a common denominator and perform the fractional subtraction:

$$\cos^{-1} x + \cos^{-1} y = \frac{3\pi - 2\pi}{3} = \frac{\pi}{3}$$

Final Answer:

$$\frac{\pi}{3}$$

Answer: (B)

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Q9.

Solution

Concept: A straight line with a parallel direction vector $\vec{m} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ is completely parallel to a plane with a normal vector $\vec{n} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ if and only if the line is perpendicular to the normal vector of that plane. This geometric condition is satisfied when their dot product is zero: $\vec{m} \cdot \vec{n} = 0$, which translates to $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

Solution: Step 1: Extract the direction ratios of the given line from the denominators of its symmetric Cartesian equation. The given equation is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Thus, the direction vector of the line is:

$$\vec{m} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

Step 2: Test Option (A) where the plane is $2x + 3y + 4z = 5$. The normal vector is $\vec{n}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$. Compute the dot product: $\vec{m} \cdot \vec{n}_1 = 2(2) + 3(3) + 4(4) = 4 + 9 + 16 = 29 \neq 0$. This option is incorrect.

Step 3: Test Option (B) where the plane is $3x + 2y - 3z = 1$. The normal vector is $\vec{n}_2 = 3\hat{i} + 2\hat{j} - 3\hat{k}$. Compute the dot product: $\vec{m} \cdot \vec{n}_2 = 2(3) + 3(2) + 4(-3) = 6 + 6 - 12 = 0$. Since the dot product is exactly zero, the line is perpendicular to the normal, which means the line is parallel to this plane.

Step 4: For completeness, double check Option (C) where the plane is $x + y + z = 6$. The normal vector is $\vec{n}_3 = \hat{i} + \hat{j} + \hat{k}$. Compute the dot product: $\vec{m} \cdot \vec{n}_3 = 2(1) + 3(1) + 4(1) = 9 \neq 0$. This option is incorrect.

Step 5: Double check Option (D) where the plane is $2x - y + z = 4$. The normal vector is $\vec{n}_4 = 2\hat{i} - \hat{j} + \hat{k}$. Compute the dot product: $\vec{m} \cdot \vec{n}_4 = 2(2) + 3(-1) + 4(1) = 4 - 3 + 4 = 5 \neq 0$. This option is incorrect.

Step 6: Conclude from Step 3 that only Option (B) satisfies the required dot product condition.

Final Answer: $3x + 2y - 3z = 1$

Answer: (B)

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Q10.

Solution

Concept: The mean (μ) of n observations is $\mu = \frac{\sum x_i}{n}$, and the variance (σ^2) is defined by $\sigma^2 = \frac{\sum x_i^2}{n} - \mu^2$. We can solve for the two unknown observations by establishing a system of equations.

Solution: Step 1: Let the two missing observations be a and b . The sample size is $n = 5$, and the known values are 1, 2, and 6.

Step 2: Use the given mean ($\mu = 4$) to set up the sum equation:

$$\frac{1 + 2 + 6 + a + b}{5} = 4 \implies 9 + a + b = 20 \implies a + b = 11 \quad \text{--- (1)}$$

Step 3: Use the given variance ($\sigma^2 = 5.2$) to set up the sum of squares equation:

$$\frac{1^2 + 2^2 + 6^2 + a^2 + b^2}{5} - 4^2 = 5.2 \implies \frac{41 + a^2 + b^2}{5} - 16 = 5.2$$

Step 4: Isolate the sum of squares variable terms:

$$\frac{41 + a^2 + b^2}{5} = 21.2 \implies 41 + a^2 + b^2 = 106 \implies a^2 + b^2 = 65 \quad \text{--- (2)}$$

Step 5: Substitute $b = 11 - a$ from equation (1) into equation (2) to create a quadratic equation:

$$a^2 + (11 - a)^2 = 65 \implies 2a^2 - 22a + 121 = 65 \implies 2a^2 - 22a + 56 = 0$$

Divide by 2 to simplify:

$$a^2 - 11a + 28 = 0$$

Step 6: Factor the equation to determine the values of the missing elements:

$$(a - 4)(a - 7) = 0 \implies a = 4 \text{ or } a = 7$$

If $a = 4$, then $b = 7$ (and vice versa). Thus, the missing observations are 4 and 7.

Final Answer:

Answer: (B)

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Q11.

Solution

Concept: The area bounded between two intersecting curves $y_1 = f(x)$ and $y_2 = g(x)$ from $x = x_1$ to $x = x_2$ is given by the definite integral $\int_{x_1}^{x_2} (y_1 - y_2) dx$, where y_1 represents the upper curve on the given interval.

Solution: Step 1: Determine the intersection points of the two given functions $y^2 = 4x$ and $y = 2x$. Substitute the linear equation directly into the parabolic equation:

$$(2x)^2 = 4x \implies 4x^2 = 4x$$

$$4x^2 - 4x = 0 \implies 4x(x - 1) = 0$$

This yields two horizontal boundary values: $x = 0$ and $x = 1$.

Step 2: Find the corresponding y -coordinates for these limits. For $x = 0$, $y = 2(0) = 0$. For $x = 1$, $y = 2(1) = 2$. The curves intersect at the points $(0, 0)$ and $(1, 2)$.

Step 3: Identify the upper curve and the lower curve within the bounding interval $x \in [0, 1]$. For a test value like $x = 0.25$, the parabola gives $y = 2\sqrt{0.25} = 1$, while the line gives $y = 2(0.25) = 0.5$. Since $1 > 0.5$, the upper curve is the parabola $y = 2\sqrt{x}$ and the lower curve is the line $y = 2x$.

Step 4: Set up the definite area integral using the discovered bounds and curve order:

$$\text{Area} = \int_0^1 (2\sqrt{x} - 2x) dx$$

Step 5: Integrate each term independently using standard power rules:

$$\text{Area} = \left[2 \cdot \frac{x^{3/2}}{3/2} - \frac{2x^2}{2} \right]_0^1 = \left[\frac{4}{3}x^{3/2} - x^2 \right]_0^1$$

Step 6: Evaluate the definite integral by substituting the upper limit 1 and lower limit 0:

$$\text{Area} = \left(\frac{4}{3}(1)^{3/2} - (1)^2 \right) - (0) = \frac{4}{3} - 1 = \frac{1}{3}$$

Final Answer:

$$\frac{1}{3}$$

Answer: (B)

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Q12.

Solution

Concept: The principal argument of a complex number $z = x + iy$, denoted as $\arg(z)$, lies in the interval $(-\pi, \pi]$. For a complex fraction, we can use the property of arguments: $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$, adjusted to fit within the principal value range if necessary.

Solution: Step 1: Break down the complex expression into its numerator and denominator parts. Let $z_1 = 1 + i\sqrt{3}$ and $z_2 = 1 - i\sqrt{3}$.

Step 2: Find the principal argument of the numerator $z_1 = 1 + i\sqrt{3}$. Since both the real part (1) and imaginary part ($\sqrt{3}$) are positive, z_1 lies in the first quadrant:

$$\arg(z_1) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

Step 3: Find the principal argument of the denominator $z_2 = 1 - i\sqrt{3}$. The real part is positive and the imaginary part is negative, placing z_2 in the fourth quadrant:

$$\arg(z_2) = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\frac{\pi}{3}$$

Step 4: Use the subtraction property of arguments to find the total argument of z :

$$\arg(z) = \arg(z_1) - \arg(z_2) = \frac{\pi}{3} - \left(-\frac{\pi}{3}\right)$$

Step 5: Perform the fractional addition to simplify the result:

$$\arg(z) = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

Step 6: Verify that the result $\frac{2\pi}{3}$ falls within the standard principal argument range $(-\pi, \pi]$. Since it satisfies the condition, $\frac{2\pi}{3}$ is the final correct answer.

Final Answer:

Answer: (B)

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Q13.

Solution

Concept: A function involving absolute values like $|x - c|$ is continuous everywhere because it is a composition of continuous functions. However, it fails to be differentiable at the critical point $x = c$ where the expression inside the absolute value becomes zero, creating a sharp corner or vertex on its graph.

Solution: Step 1: Write down the given expression for the function:

$$f(x) = |x - 1| + |x - 2|$$

Step 2: Analyze the continuity of the function at the target value $x = 1$. The individual absolute value components $|x - 1|$ and $|x - 2|$ are well-defined and continuous for all real numbers. Since the sum of two continuous functions is always continuous, $f(x)$ is continuous at $x = 1$. Step 3: Define the function explicitly in the neighborhood of $x = 1$ to check for differentiability. Consider a small interval around 1, specifically $1 \leq x < 2$. In this region, $(x - 1) \geq 0$ and $(x - 2) < 0$. Step 4: Remove the absolute value bars based on the signs in this neighborhood. For $1 \leq x < 2$:

$$f(x) = (x - 1) - (x - 2) = x - 1 - x + 2 = 1$$

Thus, the right-hand derivative (RHD) at $x = 1$ is the derivative of a constant function 1, which gives:

$$\text{RHD} = \frac{d}{dx}(1) = 0$$

Step 5: Define the function in the region just to the left of 1, specifically $0 < x < 1$. In this region, both $(x - 1) < 0$ and $(x - 2) < 0$. Remove the absolute value bars:

$$f(x) = -(x - 1) - (x - 2) = -x + 1 - x + 2 = -2x + 3$$

Thus, the left-hand derivative (LHD) at $x = 1$ is the derivative of this linear expression:

$$\text{LHD} = \frac{d}{dx}(-2x + 3) = -2$$

Step 6: Compare the two directional derivatives. Since $\text{LHD} = -2$ and $\text{RHD} = 0$, they are not equal ($\text{LHD} \neq \text{RHD}$). Therefore, the function is continuous but not differentiable at $x = 1$.

Final Answer:

Answer: (A)

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Q14.

Solution

Concept: The angle θ between two non-zero vectors \vec{a} and \vec{b} is determined using the dot product formula: $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$.

Solution: Step 1: Identify the given vectors from the problem description:

$$\vec{a} = \hat{i} + \hat{j} - \hat{k}, \quad \vec{b} = \hat{i} - \hat{j} + \hat{k}$$

Step 2: Compute the scalar dot product ($\vec{a} \cdot \vec{b}$) by summing the component products:

$$\vec{a} \cdot \vec{b} = (1)(1) + (1)(-1) + (-1)(1) = -1$$

Step 3: Calculate the magnitude of vector \vec{a} :

$$|\vec{a}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$$

Step 4: Calculate the magnitude of vector \vec{b} :

$$|\vec{b}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$

Step 5: Substitute the dot product and magnitudes into the cosine relation:

$$\cos \theta = \frac{-1}{\sqrt{3} \times \sqrt{3}} = -\frac{1}{3}$$

Step 6: Solve for the angle variable θ using the inverse cosine function:

$$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$$

Final Answer: $\cos^{-1}\left(-\frac{1}{3}\right)$ $\cos^{-1}\left(-\frac{1}{3}\right)$ $\cos^{-1}\left(-\frac{1}{3}\right)$ $\cos^{-1}\left(-\frac{1}{3}\right)$

Answer: (B)

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Q15.

Solution

Concept: To locate the absolute maximum of a function on a closed interval $[a, b]$, identify the critical points where $f'(x) = 0$, then evaluate $f(x)$ at these points and at the boundary endpoints. The largest result is the absolute maximum.

Solution: Step 1: Write down the cubic function and compute its first derivative:

$$f(x) = x^3 - 3x \implies f'(x) = 3x^2 - 3$$

Step 2: Equate the derivative to zero to find the critical points:

$$3(x^2 - 1) = 0 \implies x = \pm 1 \quad (\text{both points lie in } [-2, 2])$$

Step 3: Evaluate the function at the critical point $x = -1$:

$$f(-1) = (-1)^3 - 3(-1) = 2$$

Step 4: Evaluate the function at the critical point $x = 1$:

$$f(1) = (1)^3 - 3(1) = -2$$

Step 5: Evaluate the function at the boundary endpoints $x = -2$ and $x = 2$:

$$f(-2) = (-2)^3 - 3(-2) = -2, \quad f(2) = (2)^3 - 3(2) = 2$$

Step 6: Compare all output values $\{2, -2, -2, 2\}$. The largest value obtained is 2.

Final Answer:

Answer: (A)

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Q16.

Solution

Concept: For any two events A and B , the probability of their union is given by the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. If the events are independent, the probability of their intersection simplifies to the product of their individual probabilities: $P(A \cap B) = P(A) \times P(B)$.

Solution: Step 1: Identify the given probabilities from the problem text: $P(A) = 0.3$ and $P(B) = 0.4$.

Step 2: Use the property of independent events to calculate the probability of their intersection:

$$P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.4 = 0.12$$

Step 3: Recall the general addition rule of probability for the union of two sets:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Step 4: Substitute the values calculated in the previous steps into the formula:

$$P(A \cup B) = 0.3 + 0.4 - 0.12$$

Step 5: Perform the basic addition first:

$$0.3 + 0.4 = 0.7$$

Step 6: Complete the subtraction to find the final probability:

$$P(A \cup B) = 0.7 - 0.12 = 0.58$$

Final Answer:

Answer: (C)

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Q17.

Solution

Concept: The standard trigonometric limit states that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$. We can rewrite the identity for $1 - \cos 2A$ as $2 \sin^2 A$ to transform the limit into a form where this standard identity can be applied directly.

Solution: Step 1: Write down the given limit expression that needs to be evaluated:

$$\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x^2}$$

Step 2: Apply the double angle trigonometric identity $1 - \cos(2\theta) = 2 \sin^2(\theta)$. Substituting $\theta = 2x$ gives:

$$1 - \cos 4x = 2 \sin^2(2x)$$

Step 3: Substitute this identity back into the original limit expression:

$$\lim_{x \rightarrow 0} \frac{2 \sin^2(2x)}{x^2}$$

Step 4: Adjust the denominator so it matches the argument of the sine function ($2x$). To create a $(2x)^2 = 4x^2$ term in the denominator, multiply and divide the entire expression by 4:

$$\lim_{x \rightarrow 0} 2 \times 4 \times \frac{\sin^2(2x)}{4x^2} = 8 \times \lim_{x \rightarrow 0} \left(\frac{\sin 2x}{2x} \right)^2$$

Step 5: Apply the standard limit rule. As $x \rightarrow 0$, the term $2x \rightarrow 0$ as well, which means $\lim_{2x \rightarrow 0} \frac{\sin 2x}{2x} = 1$.

Step 6: Compute the final numerical value:

$$8 \times (1)^2 = 8$$

Final Answer:

Answer: (C)

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Q18.

Solution

Concept: The total number of distinct terms in the expansion of a multinomial of the form $(x_1 + x_2 + \dots + x_r)^n$ is given by the combinatorial formula $\binom{n+r-1}{r-1}$.

Solution: Step 1: Identify the parameters from the given expression $(x + y + z)^{10}$. Here, the total power is $n = 10$, and the number of variables inside the parentheses is $r = 3$.

Step 2: Substitute these values into the standard multinomial terms formula:

$$\text{Number of terms} = \binom{10 + 3 - 1}{3 - 1}$$

Step 3: Simplify the numbers inside the binomial coefficient:

$$\text{Number of terms} = \binom{12}{2}$$

Step 4: Expand the binomial coefficient using the standard formula $\binom{m}{k} = \frac{m!}{k!(m-k)!}$:

$$\binom{12}{2} = \frac{12 \times 11}{2 \times 1}$$

Step 5: Perform the basic division and multiplication:

$$\frac{12}{2} = 6 \implies 6 \times 11 = 66$$

Step 6: Conclude that the total number of distinct terms after expansion is 66.

Final Answer:

Answer: (D)

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Q19.

Solution

Concept: The equation of a circle passing through the origin $(0, 0)$ with an x -intercept of g and a y -intercept of h on the positive axes has its center at $\left(\frac{g}{2}, \frac{h}{2}\right)$. The standard equation can be written as $x^2 + y^2 - gx - hy = 0$.

Solution: Step 1: Identify the given intercepts from the problem description. The circle passes through the origin $(0, 0)$, makes a positive x -intercept of 4, and a positive y -intercept of 6.

Step 2: Determine the points where the circle intersects the coordinate axes. It cuts the x -axis at $(4, 0)$ and cuts the y -axis at $(0, 6)$.

Step 3: Since the angle in a semicircle is a right angle ($\angle XOY = 90^\circ$), the line segment joining the points $(4, 0)$ and $(0, 6)$ must form a diameter of this circle.

Step 4: Use the diameter form of a circle equation, which is given by $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$, where the endpoints are $(4, 0)$ and $(0, 6)$:

$$(x - 4)(x - 0) + (y - 0)(y - 6) = 0$$

Step 5: Expand the factored algebraic terms:

$$x(x - 4) + y(y - 6) = 0$$

$$x^2 - 4x + y^2 - 6y = 0$$

Step 6: Rearrange the terms into standard form:

$$x^2 + y^2 - 4x - 6y = 0$$

Final Answer: $x^2 + y^2 - 4x - 6y = 0$

Answer: (A)

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Q20.

Solution

Concept: To differentiate an inverse trigonometric function with a complex argument, it is often helpful to simplify the internal expression first using trigonometric identities. Dividing the numerator and denominator by $\cos x$ transforms the expression into the form $\tan\left(\frac{\pi}{4} + x\right)$, which cancels directly with the outer \tan^{-1} function.

Solution: Step 1: Write down the given function expression:

$$y = \tan^{-1} \left(\frac{\cos x + \sin x}{\cos x - \sin x} \right)$$

Step 2: Simplify the expression inside the parentheses by dividing both the numerator and the denominator by $\cos x$:

$$\frac{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}} = \frac{1 + \tan x}{1 - \tan x}$$

Step 3: Recognize the standard trigonometric identity for the tangent of a sum: $\tan\left(\frac{\pi}{4} + x\right) = \frac{1 + \tan x}{1 - \tan x}$. Substitute this back into the expression:

$$y = \tan^{-1} \left(\tan \left(\frac{\pi}{4} + x \right) \right)$$

Step 4: Use the property of inverse functions where $\tan^{-1}(\tan \theta) = \theta$ within the principal domain:

$$y = \frac{\pi}{4} + x$$

Step 5: Differentiate this simplified linear expression with respect to x :

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{\pi}{4} \right) + \frac{d}{dx} (x)$$

Step 6: Since $\frac{\pi}{4}$ is a constant, its derivative is 0, and the derivative of x is 1:

$$\frac{dy}{dx} = 0 + 1 = 1$$

Final Answer:

Answer: (A)

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Q21.

Solution

Concept: A standard integral identity involving exponential functions states that $\int e^x (f(x) + f'(x)) dx = e^x f(x) + C$. We can solve this problem by identifying the function $f(x)$ and its derivative from the given expression.

Solution: Step 1: Write down the given integral expression:

$$\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$$

Step 2: Compare the integrand with the standard form $e^x (f(x) + f'(x))$. Let us define the function as:

$$f(x) = \frac{1}{x} = x^{-1}$$

Step 3: Differentiate $f(x)$ with respect to x using the power rule:

$$f'(x) = -1 \cdot x^{-2} = -\frac{1}{x^2}$$

Step 4: Rewrite the original integral by substituting $f(x)$ and $f'(x)$ to verify it matches the standard identity:

$$\int e^x (f(x) + f'(x)) dx$$

Step 5: Since it matches the standard identity perfectly, apply the theorem to find the antiderivative:

$$e^x f(x) + C$$

Step 6: Substitute back the actual expression for $f(x) = \frac{1}{x}$ to get the final answer:

$$\frac{e^x}{x} + C$$

Final Answer: $\frac{e^x}{x} + C$

Answer: (C)

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Q22.

Solution

Concept: The standard equation of a horizontal ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$). Its eccentricity e is calculated using the formula $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution: Step 1: Write down the given equation of the ellipse:

$$9x^2 + 25y^2 = 225$$

Step 2: Divide both sides by 225 to convert it into standard form:

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Step 3: Compare with the standard form to identify the parameters:

$$a^2 = 25 \implies a = 5, \quad b^2 = 9 \implies b = 3 \quad (a > b)$$

Step 4: Recall the formula for the eccentricity of a horizontal ellipse:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Step 5: Substitute the values of a^2 and b^2 into the formula:

$$e = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}}$$

Step 6: Simplify the square root to find the final value:

$$e = \frac{4}{5}$$

Final Answer:

$$\frac{4}{5}$$

Answer: (A)

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Q23.

Solution

Concept: For a quadratic equation $ax^2 + bx + c = 0$ with roots α and β , Vieta's formulas state $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$. The sum of cubes is evaluated using the identity $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$.

Solution: Step 1: Identify coefficients from the equation $x^2 - 5x + 6 = 0$: $a = 1$, $b = -5$, $c = 6$.
Step 2: Determine the sum of the roots using Vieta's relations:

$$\alpha + \beta = -\frac{-5}{1} = 5$$

Step 3: Determine the product of the roots using Vieta's relations:

$$\alpha\beta = \frac{6}{1} = 6$$

Step 4: State the algebraic identity for the sum of cubes:

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

Step 5: Substitute the values of the sum and product into the identity:

$$\alpha^3 + \beta^3 = (5)^3 - 3(6)(5)$$

Step 6: Perform the final numerical arithmetic calculations:

$$\alpha^3 + \beta^3 = 125 - 90 = 35$$

Final Answer:

Answer: (A)

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Q24.

Solution

Concept: A first-order differential equation that can be written in the form $\frac{dy}{dx} = f(x)g(y)$ is a separable differential equation. It can be solved by separating the variables such that all y -terms are on one side and all x -terms are on the other, followed by integrating both sides independently.

Solution: Step 1: Write down the given differential equation:

$$\frac{dy}{dx} = e^{x-y}$$

Step 2: Use exponent rules to split the right-hand side into separate functions of x and y :

$$\frac{dy}{dx} = \frac{e^x}{e^y}$$

Step 3: Separate the variables by multiplying both sides by e^y and dx :

$$e^y dy = e^x dx$$

Step 4: Set up the integral on both sides of the equation:

$$\int e^y dy = \int e^x dx$$

Step 5: Integrate both sides using the standard exponential integration rule $\int e^u du = e^u$:

$$e^y = e^x + C$$

where C represents the arbitrary constant of integration.

Step 6: Verify that the expression is in its simplest general form, which matches Option (A).

Final Answer:

Answer: (A)

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Q25.

Solution

Concept: Three vectors \vec{u} , \vec{v} , and \vec{w} are coplanar if and only if their scalar triple product is zero ($[\vec{u} \ \vec{v} \ \vec{w}] = 0$). This condition is evaluated by setting the determinant of the component matrix to zero.

Solution: Step 1: Write down the components of the three given vectors to form a 3×3 matrix:

$$\vec{u} \implies (2, -1, 1), \quad \vec{v} \implies (1, 2, -3), \quad \vec{w} \implies (3, \lambda, 5)$$

Step 2: Set up the determinant condition for coplanarity:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

Step 3: Expand the determinant along the first row:

$$2(10 - (-3\lambda)) - (-1)(5 - (-9)) + 1(\lambda - 6) = 0$$

Step 4: Simplify the individual arithmetic expressions:

$$2(10 + 3\lambda) + 1(14) + 1(\lambda - 6) = 0$$

Step 5: Expand and group the algebraic terms:

$$20 + 6\lambda + 14 + \lambda - 6 = 0 \implies 7\lambda + 28 = 0$$

Step 6: Isolate the unknown variable λ :

$$7\lambda = -28 \implies \lambda = -4$$

Final Answer:

Answer: (A)

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Q26.

Solution

Concept: The correlation coefficient r satisfies $r^2 = b_{yx} \times b_{xy}$, where b_{yx} and b_{xy} are the slopes of the regression lines of Y on X and X on Y . Valid lines must satisfy $|b_{yx} \times b_{xy}| \leq 1$, and r takes the same sign as the coefficients.

Solution: Step 1: Consider the given lines: $2X - Y + 3 = 0$ and $3X - 2Y + 1 = 0$. Assume the first is Y on X and the second is X on Y . Step 2: Rearrange to find the slopes under this assumption:

$$Y = 2X + 3 \implies b_{yx} = 2$$

Step 3: Rearrange the second equation to find b_{xy} :

$$3X = 2Y - 1 \implies X = \frac{2}{3}Y - \frac{1}{3} \implies b_{xy} = \frac{2}{3}$$

Step 4: Check the validity condition: $b_{yx} \times b_{xy} = 2 \times \frac{2}{3} = \frac{4}{3} > 1$. Since this is invalid, reverse the initial assumption. Step 5: Let the first line be X on Y and the second be Y on X :

$$2X = Y - 3 \implies X = \frac{1}{2}Y - \frac{3}{2} \implies b_{xy} = \frac{1}{2}$$

$$2Y = 3X + 1 \implies Y = \frac{3}{2}X + \frac{1}{2} \implies b_{yx} = \frac{3}{2}$$

Step 6: Calculate r^2 and verify: $b_{yx} \times b_{xy} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4} \leq 1$. Since both slopes are positive, r is positive:

$$r = \sqrt{\frac{3}{4}}$$

Final Answer:

$$\sqrt{\frac{3}{4}}$$

Answer: (A)

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Q27.

Solution

Concept: To find the range of a rational function $y = \frac{x}{1+x^2}$, we rewrite it as a quadratic equation in terms of x , and use the condition that for x to be real, the discriminant of this quadratic equation must be greater than or equal to zero ($\Delta \geq 0$).

Solution: Step 1: Set the function expression equal to a variable y :

$$y = \frac{x}{1+x^2}$$

Step 2: Cross-multiply to clear the fraction and group all terms on one side:

$$y(1+x^2) = x \implies y + yx^2 = x$$

$$yx^2 - x + y = 0$$

Step 3: Analyze the equation as a quadratic in terms of x , where the coefficients are $a = y$, $b = -1$, and $c = y$. For x to have real solutions, the discriminant must be non-negative:

$$\Delta = b^2 - 4ac \geq 0$$

Step 4: Substitute the coefficients into the discriminant inequality:

$$(-1)^2 - 4(y)(y) \geq 0$$

$$1 - 4y^2 \geq 0$$

Step 5: Solve the quadratic inequality by factoring:

$$4y^2 - 1 \leq 0 \implies (2y - 1)(2y + 1) \leq 0$$

This means that y must lie between the roots $-\frac{1}{2}$ and $\frac{1}{2}$. Step 6: Check the special case when $y = 0$. If $y = 0$, the original equation gives $0 = \frac{x}{1+x^2} \implies x = 0$, which is a valid real number. Thus, $y = 0$ is included, and the full range is the closed interval $[-\frac{1}{2}, \frac{1}{2}]$.

Final Answer: $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Answer: (C)

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Q28.

Solution

Concept: The standard derivative of the trigonometric function $\sec x$ with respect to x is given by $\frac{d}{dx}(\sec x) = \sec x \tan x$. To find the value at a specific point, we substitute that value into the derivative expression.

Solution: Step 1: Let the given function be $f(x) = \sec x$.

Step 2: Differentiate the function with respect to x using the standard trigonometric derivative identity:

$$f'(x) = \sec x \tan x$$

Step 3: We need to evaluate this derivative at the specific point $x = \frac{\pi}{4}$. Substitute this angle into the derivative expression:

$$f'\left(\frac{\pi}{4}\right) = \sec\left(\frac{\pi}{4}\right) \tan\left(\frac{\pi}{4}\right)$$

Step 4: Recall the exact trigonometric values for the given angle:

$$\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$

$$\tan\left(\frac{\pi}{4}\right) = 1$$

Step 5: Multiply these values together to find the final result:

$$f'\left(\frac{\pi}{4}\right) = \sqrt{2} \times 1 = \sqrt{2}$$

Step 6: Verify that the calculated value matches Option (B).

Final Answer:

Answer: (B)

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Q29.

Solution

Concept: For a perfect continuous normal distribution, there is a fixed mathematical relationship between the standard deviation (σ) and the mean deviation about the mean (M.D.). This relationship is given by the formula $M.D. = \sqrt{\frac{2}{\pi}}\sigma \approx 0.8\sigma$, or more precisely, $M.D. = \frac{4}{5}\sigma$.

Solution: Step 1: Identify the given information from the problem statement. The standard deviation of the normal distribution is provided as $\sigma = 4.5$.

Step 2: Recall the theoretical relationship formula that connects the mean deviation to the standard deviation for a normal curve:

$$M.D. = \frac{4}{5}\sigma$$

Step 3: Substitute the given value of $\sigma = 4.5$ into the formula:

$$M.D. = 0.8 \times 4.5$$

Step 4: Perform the decimal multiplication:

$$4.5 \times 0.8 = 3.6$$

Step 5: Double-check the calculation using the alternative fractional form:

$$M.D. = \frac{4}{5} \times \frac{9}{2} = \frac{36}{10} = 3.6$$

Step 6: Conclude that the mean deviation about the mean is exactly 3.6, which matches Option (A).

Final Answer:

Answer: (A)

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Q30.

Solution

Concept: The shortest perpendicular distance d between two parallel lines written in the forms $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is calculated using the formula $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$. It is necessary to match the coefficients of x and y before using the formula.

Solution: Step 1: Write down the equations of the two parallel lines given in the question:

$$\text{Line 1: } 3x + 4y - 9 = 0$$

$$\text{Line 2: } 6x + 8y + 15 = 0$$

Step 2: Notice that the coefficients of x and y are different between the two equations. Multiply the first equation by 2 so that its leading coefficients match the second equation:

$$2 \times (3x + 4y - 9) = 0 \implies 6x + 8y - 18 = 0$$

Step 3: Identify the parameters to use in the distance formula from our modified lines:

$$A = 6, \quad B = 8$$

$$C_1 = -18, \quad C_2 = 15$$

Step 4: Substitute these values into the parallel lines distance formula:

$$d = \frac{|-18 - 15|}{\sqrt{6^2 + 8^2}}$$

Step 5: Simplify the expression in the numerator and the denominator:

$$\text{Numerator: } |-33| = 33$$

$$\text{Denominator: } \sqrt{36 + 64} = \sqrt{100} = 10$$

Step 6: Combine these results to find the final distance fraction:

$$d = \frac{33}{10}$$

Final Answer:

$$\frac{33}{10}$$

Answer: (B)

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Q31.

Solution

Concept: A standard trigonometric identity states that $\cos \theta \cos(2\theta) \cos(4\theta) \dots \cos(2^{n-1}\theta) = \frac{\sin(2^n \theta)}{2^n \sin \theta}$. Alternatively, this problem can be solved step-by-step by repeatedly applying the sine double-angle formula $\sin 2A = 2 \sin A \cos A$.

Solution: Step 1: Let the given trigonometric expression be denoted as P :

$$P = \cos 20^\circ \cos 40^\circ \cos 80^\circ$$

Step 2: Multiply and divide the expression by $2 \sin 20^\circ$ to create a structure where the double-angle formula can be applied:

$$P = \frac{2 \sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 80^\circ}{2 \sin 20^\circ}$$

Step 3: Replace $2 \sin 20^\circ \cos 20^\circ$ with $\sin 40^\circ$:

$$P = \frac{\sin 40^\circ \cos 40^\circ \cos 80^\circ}{2 \sin 20^\circ}$$

Step 4: Multiply and divide the numerator and denominator by 2 again to group the next pair of terms:

$$P = \frac{2 \sin 40^\circ \cos 40^\circ \cos 80^\circ}{4 \sin 20^\circ} = \frac{\sin 80^\circ \cos 80^\circ}{4 \sin 20^\circ}$$

Step 5: Repeat the process one more time by multiplying and dividing by 2 to combine the final cosine term:

$$P = \frac{2 \sin 80^\circ \cos 80^\circ}{8 \sin 20^\circ} = \frac{\sin 160^\circ}{8 \sin 20^\circ}$$

Step 6: Simplify the numerator using the reduction identity $\sin(180^\circ - \theta) = \sin \theta$. This gives $\sin 160^\circ = \sin(180^\circ - 20^\circ) = \sin 20^\circ$. Substitute this back into the fraction:

$$P = \frac{\sin 20^\circ}{8 \sin 20^\circ} = \frac{1}{8}$$

Final Answer:

$$\frac{1}{8}$$

Answer: (C)

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Q32.

Solution

Concept: Conditional probability is defined by the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$, where $P(B)$ is the probability of the conditioning event and $P(A \cap B)$ is the probability that both events occur simultaneously. Alternatively, we can find the answer by counting the number of favorable outcomes within the reduced sample space of event B .

Solution: Step 1: Define the sample space when a pair of fair dice is rolled. The total number of possible outcomes is $6 \times 6 = 36$.

Step 2: Let B be the conditioning event that the numbers shown on both dice are different. The outcomes where the numbers are the same (doublets) are $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)$, which totals 6 outcomes. Therefore, the number of outcomes in event B is:

$$n(B) = 36 - 6 = 30$$

Step 3: Let A be the event that the sum of the numbers on the two dice is equal to 8. List all possible pairs that sum to 8:

$$\{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$$

There are 5 such outcomes. Step 4: Find the outcomes that belong to the intersection $A \cap B$, meaning they sum to 8 AND have different numbers. We exclude the doublet $(4, 4)$ from the list in Step 3:

$$A \cap B = \{(2, 6), (3, 5), (5, 3), (6, 2)\}$$

The number of favorable outcomes is:

$$n(A \cap B) = 4$$

Step 5: Calculate the conditional probability using the reduced sample space method:

$$P(A|B) = \frac{n(A \cap B)}{n(B)} = \frac{4}{30}$$

Step 6: Simplify the fraction by dividing the numerator and denominator by 2:

$$P(A|B) = \frac{2}{15}$$

Final Answer:

Answer: (C)

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Q33.

Solution

Concept: The Cayley-Hamilton theorem states that every square matrix satisfies its own characteristic equation. For a 2×2 matrix A , the characteristic equation is given by $\det(A - \lambda I) = 0$, which simplifies to the form $\lambda^2 - \text{tr}(A)\lambda + \det(A) = 0$. Therefore, the matrix satisfies $A^2 - \text{tr}(A)A + \det(A)I = O$.

Solution: Step 1: Write down the given matrix A :

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Step 2: Calculate the trace of matrix A , which is defined as the sum of the elements on the main diagonal:

$$\text{tr}(A) = 1 + 4 = 5$$

Step 3: Calculate the determinant of matrix A :

$$\det(A) = (1)(4) - (2)(3) = 4 - 6 = -2$$

Step 4: Recall the matrix identity derived from the Cayley-Hamilton theorem for a 2×2 matrix:

$$A^2 - \text{tr}(A)A + \det(A)I = O$$

Step 5: Substitute the evaluated trace and determinant values into this matrix identity:

$$A^2 - 5A + (-2)I = O$$

$$A^2 - 5A - 2I = O$$

Step 6: Isolate the target expression $A^2 - 5A$ by moving the identity matrix term to the right side of the equation:

$$A^2 - 5A = 2I$$

Final Answer:

Answer: (A)

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Q34.

Solution

Concept: To evaluate an algebraic definite integral of the form $\int \frac{1}{x(1+x^n)} dx$, a common technique is to multiply the numerator and denominator by x^{n-1} . This transforms the integrand into a form where the substitution $u = x^n$ can be applied cleanly.

Solution: Step 1: Write down the given definite integral expression:

$$I = \int_1^2 \frac{1}{x(1+x^4)} dx$$

Step 2: Multiply the numerator and the denominator inside the integrand by x^3 :

$$I = \int_1^2 \frac{x^3}{x^4(1+x^4)} dx$$

Step 3: Apply substitution. Let $u = x^4$, which means that its derivative is $du = 4x^3 dx$, or $\frac{1}{4} du = x^3 dx$. Step 4: Adjust the upper and lower integration limits for the new variable u : When $x = 1$, $u = 1^4 = 1$. When $x = 2$, $u = 2^4 = 16$. Step 5: Substitute the new variable, derivative, and limits into the integral:

$$I = \int_1^{16} \frac{\frac{1}{4} du}{u(1+u)} = \frac{1}{4} \int_1^{16} \frac{1}{u(u+1)} du$$

Use partial fractions to split the integrand: $\frac{1}{u(u+1)} = \frac{1}{u} - \frac{1}{u+1}$.

$$I = \frac{1}{4} \int_1^{16} \left(\frac{1}{u} - \frac{1}{u+1} \right) du = \frac{1}{4} [\ln |u| - \ln |u+1|]_1^{16}$$

$$I = \frac{1}{4} \left[\ln \left| \frac{u}{u+1} \right| \right]_1^{16}$$

Step 6: Evaluate the expressions at the upper and lower limits:

$$I = \frac{1}{4} \left(\ln \left(\frac{16}{17} \right) - \ln \left(\frac{1}{2} \right) \right) = \frac{1}{4} \ln \left(\frac{16}{17} \div \frac{1}{2} \right) = \frac{1}{4} \ln \left(\frac{16}{17} \times 2 \right) = \frac{1}{4} \ln \left(\frac{32}{17} \right)$$

Final Answer: $\frac{1}{4} \ln \left(\frac{32}{17} \right)$ $\frac{1}{4} \ln \left(\frac{32}{17} \right)$ $\frac{1}{4} \ln \left(\frac{32}{17} \right)$ $\frac{1}{4} \ln \left(\frac{32}{17} \right)$

Answer: (A)

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Q35.

Solution

Concept: The shortest distance d between two skew straight lines given by the vector equations $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is calculated using the standard formula $d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$.

Solution: Step 1: Extract the position vectors and direction vectors from the given line equations:

$$\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}, \quad \vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}, \quad \vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$$

Step 2: Calculate the difference vector between the two position points ($\vec{a}_2 - \vec{a}_1$):

$$\vec{a}_2 - \vec{a}_1 = (2 - 1)\hat{i} + (-1 - 2)\hat{j} + (-1 - 1)\hat{k} = \hat{i} - 3\hat{j} - 2\hat{k}$$

Step 3: Compute the cross product of the two direction vectors ($\vec{b}_1 \times \vec{b}_2$) using a determinant:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$\vec{b}_1 \times \vec{b}_2 = \hat{i}(-2 - 1) - \hat{j}(2 - 2) + \hat{k}(1 - (-2)) = -3\hat{i} + 3\hat{k}$$

Step 4: Calculate the magnitude of the cross product vector $|\vec{b}_1 \times \vec{b}_2|$:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 0^2 + 3^2} = \sqrt{9 + 0 + 9} = \sqrt{18} = 3\sqrt{2}$$

Step 5: Calculate the dot product of the difference vector and the cross product vector:

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (1)(-3) + (-3)(0) + (-2)(3) = -3 + 0 - 6 = -9$$

Step 6: Substitute these values into the shortest distance formula:

$$d = \frac{|-9|}{3\sqrt{2}} = \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}}$$

Final Answer:

Answer: (B)

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Q36.

Solution

Concept: Complex expressions of the form $(1 \pm i)$ can be simplified by squaring them first to find a simpler imaginary form. Specifically, $(1 + i)^2 = 2i$ and $(1 - i)^2 = -2i$. This allows large powers to be evaluated easily using index laws.

Solution: Step 1: Write down the expression given in the problem:

$$(1 + i)^{10} + (1 - i)^{10}$$

Step 2: Rewrite the powers of 10 as a square raised to the power of 5 using the index rule $(a^2)^5 = a^{10}$:

$$[(1 + i)^2]^5 + [(1 - i)^2]^5$$

Step 3: Expand the internal squared expressions:

$$(1 + i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$$

$$(1 - i)^2 = 1 - 2i + i^2 = 1 - 2i - 1 = -2i$$

Step 4: Substitute these simplified forms back into the main expression:

$$(2i)^5 + (-2i)^5$$

Step 5: Use the algebraic property that $(-x)^5 = -x^5$ for odd powers:

$$(-2i)^5 = -(2i)^5$$

Substitute this back to see that the two terms cancel each other out:

$$(2i)^5 - (2i)^5 = 0$$

Step 6: Conclude that the total evaluated sum is exactly 0, which matches Option (D).

Final Answer:

Answer: (D)

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Q37.

Solution

Concept: The vector projection of a vector \vec{a} onto another non-zero vector \vec{b} is defined as the scalar projection multiplied by the unit vector in the direction of \vec{b} . The standard formula is given by $\text{Proj}_{\vec{b}}\vec{a} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$.

Solution: Step 1: Identify the components of the two given vectors:

$$\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$$

Step 2: Calculate the dot product ($\vec{a} \cdot \vec{b}$) by multiplying corresponding components:

$$\vec{a} \cdot \vec{b} = (2)(1) + (3)(2) + (2)(1) = 2 + 6 + 2 = 10$$

Step 3: Calculate the square of the magnitude of vector \vec{b} , denoted as $|\vec{b}|^2$:

$$|\vec{b}|^2 = 1^2 + 2^2 + 1^2 = 1 + 4 + 1 = 6$$

Step 4: Set up the vector projection formula using these calculated values:

$$\text{Proj}_{\vec{b}}\vec{a} = \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\right)\vec{b} = \left(\frac{10}{6}\right)(\hat{i} + 2\hat{j} + \hat{k})$$

Step 5: Simplify the fractional scalar factor by dividing the numerator and denominator by 2:

$$\frac{10}{6} = \frac{5}{3}$$

Step 6: Combine the simplified scalar factor with the vector \vec{b} to get the final answer:

$$\frac{5}{3}(\hat{i} + 2\hat{j} + \hat{k})$$

Final Answer: $\frac{5}{3}(\hat{i} + 2\hat{j} + \hat{k})$

Answer: (B)

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Q38.

Solution

Concept: The standard double-angle identity for the inverse tangent function states that $2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$ when $|x| < 1$. This identity can be used to simplify the inner expression before applying the outer tangent function.

Solution: Step 1: Identify the given expression that needs evaluation:

$$\tan \left(2 \tan^{-1} \frac{1}{5} \right)$$

Step 2: Focus on simplifying the inner expression $2 \tan^{-1} \left(\frac{1}{5} \right)$ using the double-angle formula with $x = \frac{1}{5}$:

$$2 \tan^{-1} \left(\frac{1}{5} \right) = \tan^{-1} \left(\frac{2 \times \frac{1}{5}}{1 - \left(\frac{1}{5} \right)^2} \right)$$

Step 3: Simplify the fractions inside the inverse tangent argument:

$$\text{Numerator: } \frac{2}{5}$$

$$\text{Denominator: } 1 - \frac{1}{25} = \frac{24}{25}$$

Step 4: Divide the numerator fraction by the denominator fraction:

$$\frac{\frac{2}{5}}{\frac{24}{25}} = \frac{2}{5} \times \frac{25}{24} = \frac{2}{24} \times \frac{25}{5} = \frac{1}{12} \times 5 = \frac{5}{12}$$

So the expression becomes $\tan^{-1} \left(\frac{5}{12} \right)$. Step 5: Substitute this simplified angle back into the main expression:

$$\tan \left(\tan^{-1} \left(\frac{5}{12} \right) \right)$$

Step 6: Use the standard inverse property $\tan(\tan^{-1} \theta) = \theta$ to get the final numerical answer:

$$\frac{5}{12}$$

Final Answer:

Answer: (A)

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Q39.

Solution

Concept: To differentiate a variable base raised to a variable exponent of the form $y = f(x)^{g(x)}$, we use logarithmic differentiation. Taking the natural logarithm of both sides allows us to use the product rule to find the derivative.

Solution: Step 1: Write down the given function expression:

$$y = x^x$$

Step 2: Take the natural logarithm (ln) on both sides of the equation to simplify the exponent:

$$\ln y = \ln(x^x) \implies \ln y = x \ln x$$

Step 3: Differentiate both sides of the equation with respect to x . Use implicit differentiation on the left side and the product rule on the right side:

$$\frac{1}{y} \frac{dy}{dx} = \frac{d}{dx}(x) \cdot \ln x + x \cdot \frac{d}{dx}(\ln x)$$

$$\frac{1}{y} \frac{dy}{dx} = 1 \cdot \ln x + x \cdot \left(\frac{1}{x}\right) = \ln x + 1$$

Step 4: Isolate the derivative term $\frac{dy}{dx}$ by multiplying both sides by y :

$$\frac{dy}{dx} = y(1 + \ln x)$$

Step 5: Substitute the original expression $y = x^x$ back into the derivative formula:

$$\frac{dy}{dx} = x^x(1 + \ln x)$$

Step 6: Evaluate the derivative at the specific point $x = 1$:

$$\left. \frac{dy}{dx} \right|_{x=1} = 1^1 \times (1 + \ln 1) = 1 \times (1 + 0) = 1$$

Final Answer:

Answer: (B)

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Q40.

Solution

Concept: The sum of an infinite geometric series $S_\infty = a + ar + ar^2 + \dots$ can be calculated using the formula $S_\infty = \frac{a}{1-r}$, provided that the absolute value of the common ratio is strictly less than one ($|r| < 1$).

Solution: Step 1: Write down the given infinite geometric progression series from the problem statement:

$$1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \dots$$

Step 2: Identify the first term (a) of this geometric series:

$$a = 1$$

Step 3: Determine the common ratio (r) by dividing the second term by the first term:

$$r = \frac{\frac{2}{3}}{1} = \frac{2}{3}$$

Step 4: Check if the series converges by verifying that the magnitude of the common ratio is less than 1:

$$|r| = \left| \frac{2}{3} \right| = \frac{2}{3} < 1$$

Since the condition is satisfied, the infinite series converges to a finite sum. Step 5: Substitute the values $a = 1$ and $r = \frac{2}{3}$ into the standard infinite geometric sum formula:

$$S_\infty = \frac{1}{1 - \frac{2}{3}}$$

Step 6: Simplify the fraction in the denominator and perform the final division:

$$\text{Denominator: } 1 - \frac{2}{3} = \frac{1}{3}$$

$$S_\infty = \frac{1}{\frac{1}{3}} = 3$$

Final Answer:

Answer: (B)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	B	4	A	5	A
6	A	7	A	8	B	9	B	10	B
11	B	12	B	13	A	14	B	15	A
16	C	17	C	18	D	19	A	20	A
21	C	22	A	23	A	24	A	25	A
26	A	27	C	28	B	29	A	30	B
31	C	32	C	33	A	34	A	35	B
36	D	37	B	38	A	39	B	40	B

