Rajasthan JET Mathematics Sample Paper-6

Duration: 40 Minutes **Maximum Marks: 160**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **−1 marks**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$. If $A^2 - 5A + kI = 0$, where I is the identity matrix of order 2, then the value of k is

(A) 5

(B) 6

(C) -6

(D) 0

Q2. If $f(x) = \frac{\sin 3x}{x}$ for $x \neq 0$ is continuous at $x = 0$, then the value of $f(0)$ must be

(A) 1

(B) 0

(C) 3

(D) $\frac{1}{3}$

Q3. The coordinates of the focus of the parabola $y^2 - 4y - 8x + 4 = 0$ are

(A) $(2, 2)$

(B) $(0, 2)$

(C) $(2, 0)$

(D) $(-2, 2)$

Q4. Two dice are thrown simultaneously. What is the probability of getting a total sum of 7?

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{5}{36}$

(D) $\frac{1}{9}$

Q5. The value of $\lim_{x\to 0} \frac{1-\cos 4x}{x^2}$ is

(A) 4

(B) 8

(C) 2

(D) 16

Q6. If the n-th term of an arithmetic progression is $3n+5$, then the common difference of the AP is

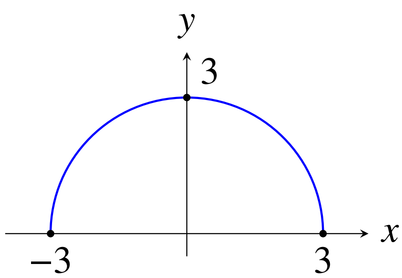
(A) 3

(B) 5

(C) 8

(D) 2

Q7. The domain of the function $f(x) = \sqrt{9-x^2}$ is



(A) $(-3, 3)$

(B) $[0, 3]$

(C) $[-3, 3]$

(D) $(-\infty, -3] \cup [3, \infty)$

Q8. The mean of 5 observations is 4 and their variance is 5.2. If three of the observations are 1, 2, and 6, then the other two observations are

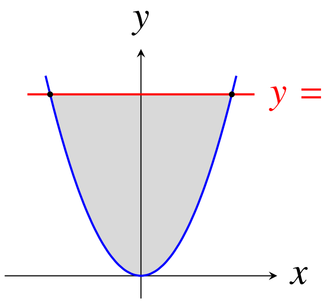
(A) 2 and 9

(B) 4 and 7

(C) 3 and 8

(D) 5 and 6

Q9. The area bounded by the curve $y = x^2$ and the line $y = 4$ is



(A) $\frac{32}{3}$

(B) $\frac{16}{3}$

(C) $\frac{8}{3}$

(D) $\frac{64}{3}$

Q10. The value of $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$ is

(A) $\frac{2\pi}{3}$

(B) $\frac{\pi}{3}$

(C) $-\frac{\pi}{3}$

(D) $\frac{4\pi}{3}$

Q11. The projection of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is

(A) $\frac{10}{\sqrt{6}}$

(B) $\frac{10}{\sqrt{17}}$

(C) $\frac{5}{\sqrt{6}}$

(D) $\sqrt{6}$

Q12. The general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is

(A) $\tan^{-1} y + \tan^{-1} x = C$

(B) $\tan^{-1} y - \tan^{-1} x = C$

(C) $y - x = C(1 + xy)$

(D) $\tan^{-1}\left(\frac{y-x}{1+xy}\right) = C$

Q13. If the value of a determinant of order 3×3 is 4, then the value of the determinant formed by its cofactors is

(A) 4

(B) 16

(C) 64

(D) 2

Q14. The value of $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx$ is

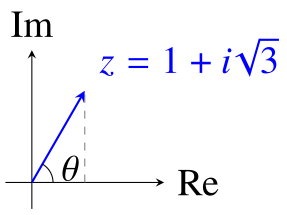
(A) π

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) 0

Q15. If $z = 1 + i\sqrt{3}$, then the amplitude (argument) of z is



(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{4}$

Q16. The length of the perpendicular from the point $(1, 2, 3)$ to the plane $2x - 3y + 6z + 7 = 0$ is

(A) 3

(B) $21_{\overline{7}}$

(B) 4

(C) $18_{\overline{7}}$

Q17. The function $f(x) = 2x^3 - 9x^2 + 12x + 4$ is strictly increasing in the interval

(A) $(1, 2)$

(B) $(-\infty, 1) \cup (2, \infty)$

(C) $(-\infty, 2)$

(D) $(1, \infty)$

Q18. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is

(A) 0.96

(B) 0.24

(C) 0.56

(D) 0.72

Q19. The number of non-empty subsets of a set containing 5 elements is

(A) 32

(B) 31

(C) 25

(D) 16

Q20. The solution of the differential equation $x\frac{dy}{dx} - y = x^2$ is

(A) $y = x^2 + Cx$

(B) $y = x \ln x + Cx$

(C) $y = \frac{x^2}{2} + Cx$

(D) $y = x^2 \ln x + Cx$

Q21. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

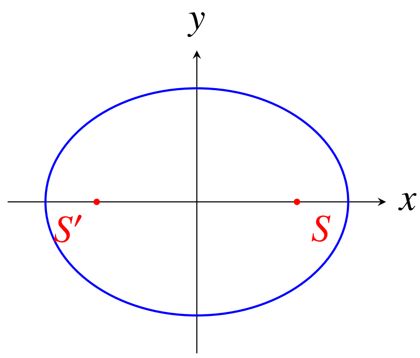
(A) 6

(B) ±6

(C) -6

(D) 0

Q22. The eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is



(A) $\frac{\sqrt{7}}{4}$

(B) $\frac{7}{16}$

(C) $\frac{3}{4}$

(D) $\frac{\sqrt{7}}{3}$

Q23. The derivative of $\ln(\sec x + \tan x)$ with respect to x is

(A) $\sec x$

(B) $\sec x + \tan x$

(C) $\tan x$

(D) $\frac{1}{\sec x + \tan x}$

Q24. If the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar, then the value of λ is

(A) -4

(B) -2

(C) 4

(D) 2

Q25. The maximum value of $f(x) = \sin x + \cos x$ is

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Q26. If the third term of a geometric progression is 4, then the product of its first 5 terms is

(A) 4^3

(B) 4^5

(C) 4^4

(D) Cannot be determined

Q27. The arithmetic mean of the first n natural numbers is

(A) $\frac{n}{2}$

(B) $\frac{n+1}{2}$

(C) $\frac{n(n+1)}{2}$

(D) $n + 1$

Q28. The value of $\int e^x(\tan x + \ln(\sec x))\, dx$ is

(A) $e^x \tan x + C$

(B) $e^x \ln(\sec x) + C$

(C) $e^x \sec x + C$

(D) $e^x(\tan x - \ln(\sec x)) + C$

Q29. The equation of the line passing through $(1, 2, -4)$ and parallel to the line $\frac{x-3}{2} = \frac{y+1}{-3} = \frac{z-2}{6}$ is

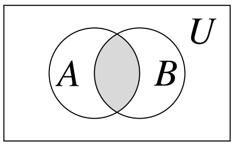
(A) $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+4}{6}$

(B) $\frac{x+1}{2} = \frac{y+2}{-3} = \frac{z-4}{6}$

(C) $\frac{x-2}{1} = \frac{y+3}{2} = \frac{z-6}{-4}$

(D) $\frac{x-1}{3} = \frac{y-2}{-1} = \frac{z+4}{2}$

Q30. If A and B are two sets such that $n(A) = 17$, $n(B) = 23$ and $n(A \cup B) = 38$, then $n(A \cap B)$ is



(A) 2

(B) 4

(C) 6

(D) 0

Q31. The value of $(1 + i)^4 + (1 - i)^4$ is

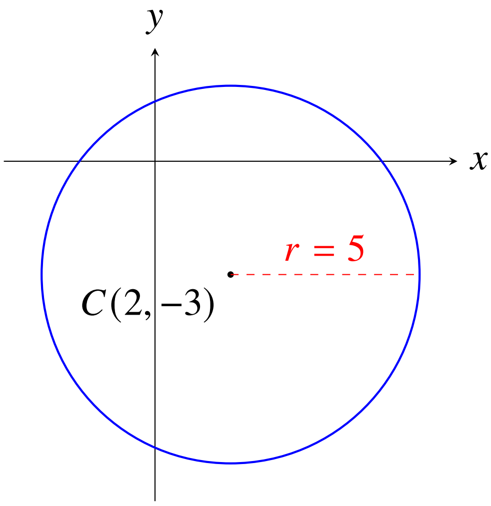
(A) 8

(B) -8

(C) -4

(D) 0

Q32. The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is



(A) 5

(B) $\sqrt{13}$

(C) 25

(D) 1

Q33. A bag contains 4 white and 6 black balls. Two balls are drawn at random one by one without replacement. The probability that both are black is

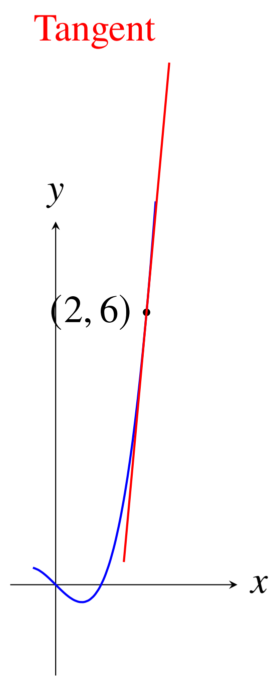
(A) $\frac{1}{3}$

(B) $\frac{5}{15}$

(C) $\frac{1}{5}$

(D) $\frac{4}{15}$

Q34. The slope of the tangent to the curve $y = x^3 - x$ at $x = 2$ is



(A) 11

(B) 12

(C) 13

(D) 6

Q35. If matrix A is both symmetric and skew-symmetric, then

(A) A is a diagonal matrix

(B) A is a zero matrix

(C) A is a square matrix

(D) No such matrix exists

Q36. The value of $\tan\left(2\tan^{-1}\frac{1}{3}\right)$ is

(A) $\frac{3}{4}$

(B) $\frac{4}{3}$

(C) $\frac{3}{5}$

(D) $\frac{1}{3}$

Q37. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$ are respectively

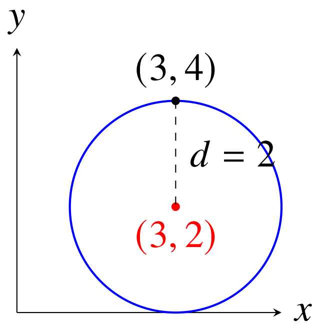
(A) 2, 2

(B) 2, 3

(C) 1, 2

(D) 2, 1

Q38. If the distance between the points $(x, 2)$ and $(3, 4)$ is 2, then the value of x is



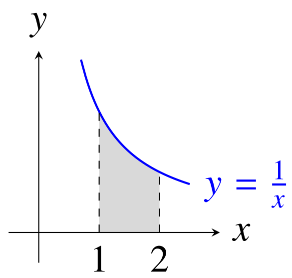
(A) 2

(B) 3

(C) 4

(D) 5

Q39. The value of $\int_1^2 \frac{1}{x}\,dx$ is



(A) 1

(B) $\ln 2$

(C) e^2

(D) 0

Q40. If a card is drawn from a well-shuffled pack of 52 cards, the probability that it is a king or a club is

(A) $\frac{17}{52}$

(B) $\frac{4}{13}$

(C) $\frac{16}{52}$

(D) $\frac{3}{13}$

Detailed Solutions

Q1.

Solution

Concept: For any square matrix A, it satisfies its own characteristic equation given by $\det(A - \lambda I) = 0$. Alternatively, we can evaluate A^2 directly by matrix multiplication and substitute it into the given matrix equation $A^2 - 5A + kI = 0$ to determine the scalar constant k.

Solution: Step 1: Compute the square of matrix A via row-by-column multiplication:

$$A^2 = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 2(2) + 1(0) & 2(1) + 1(3) \\ 0(2) + 3(0) & 0(1) + 3(3) \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 0 & 9 \end{bmatrix}$$

Step 2: Calculate the matrix expression $5A$ by multiplying each element by 5:

$$5A = 5 \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 0 & 15 \end{bmatrix}$$

Step 3: Substitute A^2 and $5A$ back into the given matrix equation $A^2 - 5A + kI = 0$:

$$\begin{bmatrix} 4 & 5 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 10 & 5 \\ 0 & 15 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Step 4: Combine the corresponding components to form a single matrix equation:

$$\begin{bmatrix} 4 - 10 + k & 5 - 5 + 0 \\ 0 - 0 + 0 & 9 - 15 + k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 + k & 0 \\ 0 & -6 + k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Step 5: Equate the corresponding entry to zero to find the value of k:

$$-6 + k = 0 \implies k = 6$$

Final Answer: $\boxed{6}$

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept: A function $f(x)$ is continuous at a point $x = a$ if the limiting value of the function as x approaches a equals the actual defined value of the function at that point, which means $\lim_{x\to a} f(x) = f(a)$. Here, we use the standard trigonometric limit $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$.

Solution: Step 1: Write down the condition for continuity of the function $f(x)$ at the origin $x = 0$:

$$f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin 3x}{x}$$

Step 2: Multiply and divide the expression by 3 to match the standard form of the trigonometric limit:

$$\lim_{x\to 0} \frac{\sin 3x}{x} = \lim_{x\to 0} \left(3 \cdot \frac{\sin 3x}{3x}\right)$$

Step 3: Pull out the constant factor 3 from the limit operator:

$$= 3 \cdot \lim_{x\to 0} \frac{\sin 3x}{3x}$$

Step 4: Since $x \to 0$, it implies that $3x \to 0$. Apply the fundamental limit formula $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$:

$$= 3 \cdot 1 = 3$$

Step 5: Equate the result to $f(0)$ to preserve continuity:

$$f(0) = 3$$

Final Answer: $\boxed{3}$

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept: To find the focus of a parabola whose axis is parallel to the coordinate axes, we rearrange the quadratic equation by completing the square into its standard form $(y-k)^2 = 4a(x-h)$. The coordinates of the focus for this standard form are given by $(h+a, k)$.

Solution: Step 1: Write down the given parabolic equation and group the y terms on one side:

$$y^2 - 4y = 8x - 4$$

Step 2: Complete the square on the left-hand side by adding 4 to both sides of the equation:

$$y^2 - 4y + 4 = 8x - 4 + 4$$

$$(y-2)^2 = 8x$$

Step 3: Express the right-hand side in the standard form $4a(x-h)$:

$$(y-2)^2 = 4 \cdot 2 \cdot (x-0)$$

Step 4: Identify the values of h, k, and a by comparing it with $(y-k)^2 = 4a(x-h)$:

$$h = 0, \quad k = 2, \quad a = 2$$

Step 5: Compute the focus coordinates using the formula $F(h+a, k)$:

$$F = (0+2, 2) = (2, 2)$$

Final Answer: $\boxed{(2, 2)}$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept: The probability of an event is computed as the ratio of the number of favorable outcomes to the total number of possible outcomes in the sample space. When two independent fair dice are rolled, each die has 6 faces, leading to a total sample space size of $n(S) = 6 \times 6 = 36$.

Solution: Step 1: Write down the total number of possible outcomes when two dice are thrown together:

$$n(S) = 6 \times 6 = 36$$

Step 2: List all ordered pairs (x, y) where the sum of the numbers on the top faces satisfies $x + y = 7$:

$$E = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$$

Step 3: Count the total number of elements in the favorable event set E:

$$n(E) = 6$$

Step 4: Apply the classical definition of probability to compute $P(E)$:

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36}$$

Step 5: Simplify the fraction to its lowest terms:

$$P(E) = \frac{1}{6}$$

Final Answer: $\boxed{\frac{1}{6}}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: The limit involves a $0/0$ indeterminate form which can be evaluated either by using standard trigonometric identities or by applying L'Hopital's rule. Using the identity $1 - \cos\theta = 2\sin^2(\theta/2)$ is generally cleaner and less prone to differentiation errors.

Solution: Step 1: Identify the indeterminate form by direct substitution of $x = 0$:

$$\frac{1 - \cos 0}{0^2} = \frac{1 - 1}{0} = \frac{0}{0}$$

Step 2: Substitute the half-angle trigonometric identity $1 - \cos 4x = 2\sin^2(2x)$ into the limit:

$$\lim_{x\to 0} \frac{1 - \cos 4x}{x^2} = \lim_{x\to 0} \frac{2\sin^2(2x)}{x^2}$$

Step 3: Rearrange the expression to group the terms under a single square bracket:

$$= 2 \cdot \lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2$$

Step 4: Multiply and divide inside the bracket by 2 to align with the standard limit form:

$$= 2 \cdot \lim_{x\to 0} \left(2 \cdot \frac{\sin 2x}{2x}\right)^2$$

Step 5: Apply the standard limit formula $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ and compute the final value:

$$= 2 \cdot (2 \cdot 1)^2 = 2 \cdot 4 = 8$$

Final Answer: $\boxed{8}$

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept: In an arithmetic progression (AP), the common difference d represents the constant change between any two consecutive terms, defined as $d = t_n - t_{n-1}$. Alternatively, for any linear general term expression $t_n = an + b$, the coefficient of n always represents the common difference.

Solution: Step 1: Write down the given expression for the n-th term of the arithmetic progression:

$$t_n = 3n + 5$$

Step 2: Calculate the first term t_1 by substituting $n = 1$ into the formula:

$$t_1 = 3(1) + 5 = 8$$

Step 3: Calculate the second term t_2 by substituting $n = 2$ into the formula:

$$t_2 = 3(2) + 5 = 11$$

Step 4: Find the common difference d by subtracting the first term from the second term:

$$d = t_2 - t_1 = 11 - 8 = 3$$

Step 5: Verify using the alternative method where d is the coefficient of n in $3n + 5$, which directly yields 3.

Final Answer: $\boxed{3}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: The domain of a real-valued function $f(x) = \sqrt{g(x)}$ is the set of all real values of x for which the expression inside the square root is non-negative, meaning $g(x) \geq 0$. We then solve the resulting quadratic inequality.

Solution: Step 1: Set up the inequality ensuring that the value inside the square root is non-negative:

$$9 - x^2 \geq 0$$

Step 2: Multiply the entire inequality by -1 and flip the direction of the inequality sign:

$$x^2 - 9 \leq 0$$

Step 3: Factorize the quadratic expression using the difference of squares identity:

$$(x-3)(x+3) \leq 0$$

Step 4: Determine the critical points where the expression equals zero, which are $x = 3$ and $x = -3$. Apply the wavy curve method to test the signs across intervals:

The inequality holds true when x lies between the two closed roots.

Step 5: Write the solution set in interval notation:

$$x \in [-3, 3]$$

Final Answer: $\boxed{[-3, 3]}$

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: We use the basic definitions of mean $\bar{x} = \frac{\sum x_i}{n}$ and variance $\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$. Let the two missing observations be a and b. We establish a system of two algebraic equations to solve for them.

Solution: Step 1: Use the given mean to set up the first equation for the sum of all 5 observations:

$$\bar{x} = \frac{1+2+6+a+b}{5} = 4$$

$$9 + a + b = 20 \implies a + b = 11$$

Step 2: Use the given variance formula to set up the second equation involving the sum of squares:

$$\sigma^2 = \frac{1^2 + 2^2 + 6^2 + a^2 + b^2}{5} - 4^2 = 5.2$$

$$\frac{1 + 4 + 36 + a^2 + b^2}{5} - 16 = 5.2$$

Step 3: Simplify the equation to isolate $a^2 + b^2$:

$$\frac{41 + a^2 + b^2}{5} = 21.2$$

$$41 + a^2 + b^2 = 106 \implies a^2 + b^2 = 65$$

Step 4: Substitute $b = 11 - a$ into the sum of squares equation:

$$a^2 + (11 - a)^2 = 65$$

$$a^2 + 121 - 22a + a^2 = 65$$

$$2a^2 - 22a + 56 = 0 \implies a^2 - 11a + 28 = 0$$

Step 5: Solve the quadratic equation by factoring:

$$(a-4)(a-7) = 0 \implies a = 4 \text{ or } a = 7$$

Thus, the other two observations are 4 and 7.

Final Answer: $\boxed{4 \text{ and } 7}$

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept: The area bounded by a curve and a horizontal line can be computed by integrating with respect to y along the vertical axis, or by integrating with respect to x by calculating the difference between the upper function and the lower function. Due to symmetry across the y-axis, we can double the area evaluated from $x = 0$.

Solution: Step 1: Find the intersection points between the parabola $y = x^2$ and the straight line $y = 4$:

$$x^2 = 4 \implies x = \pm 2$$

Step 2: Set up the integral for the bounded region, using symmetry about the vertical axis:

$$\text{Area} = \int_{-2}^{2} (4 - x^2)\, dx = 2\int_{0}^{2} (4 - x^2)\, dx$$

Step 3: Find the antiderivative of the integrand with respect to x:

$$\text{Area} = 2\left[4x - \frac{x^3}{3}\right]_0^2$$

Step 4: Substitute the upper and lower limits of integration into the expression:

$$\text{Area} = 2\left[\left(4(2) - \frac{2^3}{3}\right) - 0\right]$$

$$\text{Area} = 2\left[8 - \frac{8}{3}\right] = 2\left[\frac{24 - 8}{3}\right]$$

Step 5: Compute the final numerical value:

$$\text{Area} = 2 \cdot \frac{16}{3} = \frac{32}{3}$$

Final Answer: $\boxed{\frac{32}{3}}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: The principal value branch for the inverse sine function $\sin^{-1}(x)$ is restricted to the closed interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Since the angle $\frac{2\pi}{3}$ does not fall within this range, we must use trigonometric identities to map it into the principal domain.

Solution: Step 1: Check if the angle lies in the principal value interval:

$$\frac{2\pi}{3} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Step 2: Rewrite the angle using the second-quadrant identity $\sin(\pi - \theta) = \sin\theta$:

$$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right)$$

Step 3: Simplify the internal expression:

$$\sin\left(\pi - \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$$

Step 4: Substitute this back into the original inverse trigonometric expression:

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right)$$

Step 5: Since $\frac{\pi}{3} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, we can cancel the function with its inverse:

$$\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: The scalar projection of a vector $\vec{a}$ onto another vector $\vec{b}$ is given by the formula $\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$. This represents the length of the shadow cast by vector $\vec{a}$ along the direction of vector $\vec{b}$.

Solution: Step 1: Calculate the dot product (scalar product) of vectors $\vec{a}$ and $\vec{b}$:

$$\vec{a}\cdot\vec{b} = (2\hat{i} + 3\hat{j} + 2\hat{k})\cdot(\hat{i} + 2\hat{j} + \hat{k})$$

$$\vec{a}\cdot\vec{b} = (2\cdot 1) + (3\cdot 2) + (2\cdot 1) = 2 + 6 + 2 = 10$$

Step 2: Compute the magnitude of the target vector $\vec{b}$:

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$$

Step 3: Substitute the dot product and magnitude values into the projection formula:

$$\text{Projection} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{10}{\sqrt{6}}$$

Final Answer: $\boxed{\frac{10}{\sqrt{6}}}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: This is a first-order ordinary differential equation that can be solved by separating the variables. We group all terms containing y with dy and all terms containing x with dx, then integrate both sides using standard integration formulas.

Solution: Step 1: Separate the variables by moving the y-terms to the left side and x-terms to the right side:

$$\frac{1}{1+y^2}\,dy = \frac{1}{1+x^2}\,dx$$

Step 2: Apply the integration operator to both sides of the separated equation:

$$\int \frac{1}{1+y^2}\,dy = \int \frac{1}{1+x^2}\,dx$$

Step 3: Integrate using the standard inverse tangent integral rule $\int \frac{1}{1+t^2}\,dt = \tan^{-1} t$:

$$\tan^{-1} y = \tan^{-1} x + C$$

Step 4: Rearrange the constants and variables to match the given options:

$$\tan^{-1} y - \tan^{-1} x = C$$

Final Answer: $\boxed{\tan^{-1} y - \tan^{-1} x = C}$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: Let A be a square matrix of order n. The determinant of the cofactor matrix, often denoted as $\det(\text{adj}(A))$, is related to the determinant of the original matrix by the standard property $\det(\text{cofactor}(A)) = |A|^{n-1}$.

Solution: Step 1: Identify the parameters given in the problem statement:

$$\text{Order of the determinant, } n = 3$$

$$\text{Value of the original determinant, } |A| = 4$$

Step 2: State the matrix property for the determinant of a cofactor matrix of order n:

$$\det(\text{cofactor}(A)) = |A|^{n-1}$$

Step 3: Substitute the given values $n = 3$ and $|A| = 4$ into the exponent property:

$$\det(\text{cofactor}(A)) = 4^{3-1}$$

Step 4: Calculate the final numerical value:

$$\det(\text{cofactor}(A)) = 4^2 = 16$$

Final Answer: $\boxed{16}$

Answer: (B)

Go Back to Question 13

Q14.

Solution

Concept: We apply the king's property of definite integrals, which states that $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$. This property creates a secondary integral that can be added to the original one to simplify the expression.

Solution: Step 1: Represent the given definite integral as equation (1):

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx \quad \text{— (1)}$$

Step 2: Apply the integration property by replacing x with $\left(0 + \frac{\pi}{2} - x\right) = \frac{\pi}{2} - x$:

$$I = \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2} - x)}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)}\,dx$$

Step 3: Use the trigonometric co-function identities $\sin(\frac{\pi}{2} - x) = \cos x$ and $\cos(\frac{\pi}{2} - x) = \sin x$:

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\,dx \quad \text{— (2)}$$

Step 4: Add equation (1) and equation (2) together since they share identical limits:

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx$$

$$2I = \int_0^{\pi/2} 1\,dx$$

Step 5: Integrate and solve for I:

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} \implies I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (C)

Go Back to Question 14

Q15.

Solution

Concept: The amplitude or argument θ of a complex number $z = x + iy$ located in the first quadrant ($x > 0, y > 0$) is calculated using the formula $\theta = \tan^{-1}\left(\frac{|y|}{|x|}\right)$.

Solution: Step 1: Identify the real component x and the imaginary component y from the complex number:

$$z = 1 + i\sqrt{3} \implies x = 1, \quad y = \sqrt{3}$$

Step 2: Determine the quadrant location of the complex number. Since both $x > 0$ and $y > 0$, z lies in the first quadrant.

Step 3: Set up the formula for the principal argument θ:

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

Step 4: Find the angle whose tangent value equals $\sqrt{3}$:

$$\theta = \frac{\pi}{3}$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: The shortest distance d from a given point $P(x_1, y_1, z_1)$ to a plane described by the general linear equation $Ax + By + Cz + D = 0$ is found using the perpendicular distance formula $d = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$.

Solution: Step 1: Match the given values with the variables in the standard distance equation:

$$\text{Point } (x_1, y_1, z_1) = (1, 2, 3)$$

$$\text{Plane parameters: } A = 2,\ B = -3,\ C = 6,\ D = 7$$

Step 2: Substitute these values into the numerator of the perpendicular distance formula:

$$\text{Numerator} = |2(1) - 3(2) + 6(3) + 7|$$

$$\text{Numerator} = |2 - 6 + 18 + 7| = |21| = 21$$

Step 3: Compute the denominator value using the coefficients of the plane equation:

$$\text{Denominator} = \sqrt{2^2 + (-3)^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

Step 4: Divide the numerator by the denominator to obtain the final distance:

$$d = \frac{21}{7} = 3$$

Final Answer: $\boxed{3}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: A differentiable real function $f(x)$ is strictly increasing in any interval where its first derivative is strictly positive, meaning $f'(x) > 0$. We compute the derivative and determine the solution intervals for the resulting inequality.

Solution: Step 1: Differentiate the polynomial function $f(x)$ with respect to x:

$$f'(x) = \frac{d}{dx}(2x^3 - 9x^2 + 12x + 4)$$

$$f'(x) = 6x^2 - 18x + 12$$

Step 2: Set up the inequality condition for a strictly increasing function:

$$6x^2 - 18x + 12 > 0$$

Step 3: Divide the entire inequality by the positive constant 6 to simplify it:

$$x^2 - 3x + 2 > 0$$

Step 4: Factor the quadratic expression into linear components:

$$(x - 1)(x - 2) > 0$$

Step 5: Use the sign scheme method. The product is positive when x lies outside the root intervals:

$$x \in (-\infty, 1) \cup (2, \infty)$$

Final Answer: $\boxed{(-\infty, 1) \cup (2, \infty)}$

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept: We use the conditional probability formula $P(B|A) = \frac{P(A\cap B)}{P(A)}$ to determine the intersection probability, and then apply the general addition rule of probability $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Solution: Step 1: Write down the definition of conditional probability to solve for the intersection term:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \implies 0.6 = \frac{P(A \cap B)}{0.4}$$

Step 2: Solve for $P(A \cap B)$ by cross-multiplication:

$$P(A \cap B) = 0.6 \times 0.4 = 0.24$$

Step 3: Set up the probability addition formula to calculate the union of the events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Step 4: Substitute the values into the formula:

$$P(A \cup B) = 0.4 + 0.8 - 0.24$$

Step 5: Compute the final numerical answer:

$$P(A \cup B) = 1.2 - 0.24 = 0.96$$

Final Answer: $\boxed{0.96}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: For any finite set containing n distinct elements, the total number of possible subsets is given by 2^n. The term "non-empty subsets" requires us to exclude the empty set ($\emptyset$) from this total count.

Solution: Step 1: Identify the total number of elements in the set from the problem description:

$$n = 5$$

Step 2: Apply the formula to find the total number of possible subsets:

$$\text{Total Subsets} = 2^n = 2^5 = 32$$

Step 3: Recall that the collection of all subsets includes exactly one empty set.
Step 4: Subtract 1 from the total subset count to find the number of non-empty subsets:

$$\text{Non-empty Subsets} = 2^n - 1 = 32 - 1 = 31$$

Final Answer: $\boxed{31}$

Answer: (B)

Go Back to Question 19

Q20.

Solution

Concept: This is a first-order linear differential equation. We can rearrange it into the standard form $\frac{dy}{dx} + P(x)y = Q(x)$, compute the integrating factor I.F. $= e^{\int P(x)\,dx}$, and find the general solution using the formula $y \cdot (\text{I.F.}) = \int Q(x) \cdot (\text{I.F.})\,dx$.

Solution: Step 1: Divide the given equation by x to convert it into the standard linear format:

$$x\frac{dy}{dx} - y = x^2 \implies \frac{dy}{dx} - \frac{1}{x}y = x$$

Step 2: Identify the coefficient functions $P(x)$ and $Q(x)$:

$$P(x) = -\frac{1}{x}, \quad Q(x) = x$$

Step 3: Calculate the integrating factor (I.F.):

$$\text{I.F.} = e^{\int -\frac{1}{x}\,dx} = e^{-\ln x} = e^{\ln(x^{-1})} = \frac{1}{x}$$

Step 4: Set up the general solution equation:

$$y \cdot \left(\frac{1}{x}\right) = \int x \cdot \left(\frac{1}{x}\right)\,dx$$

$$\frac{y}{x} = \int 1\,dx \implies \frac{y}{x} = x + C$$

Step 5: Multiply through by x to solve for y:

$$y = x^2 + Cx$$

Final Answer: $\boxed{y = x^2 + Cx}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: To solve a determinant equation, we evaluate both sides independently using the definition of a 2×2 determinant: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$. We then solve the resulting equation for x.

Solution: Step 1: Expand the determinant on the left-hand side:

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = x \cdot x - 2 \cdot 18 = x^2 - 36$$

Step 2: Expand the determinant on the right-hand side:

$$\begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix} = 6 \cdot 6 - 2 \cdot 18 = 36 - 36 = 0$$

Step 3: Equate the two expanded expressions:

$$x^2 - 36 = 0$$

Step 4: Isolate x^2 by moving the constant to the other side:

$$x^2 = 36$$

Step 5: Take the square root of both sides to find all possible values for x:

$$x = \pm 6$$

Final Answer: $\boxed{\pm 6}$

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept: For a standard horizontal ellipse given by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b$, the eccentricity e measures its deviation from a perfect circle and is calculated using the formula $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution: Step 1: Compare the given equation with the standard form of an ellipse to find a^2 and b^2:

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \implies a^2 = 16, \quad b^2 = 9$$

Step 2: Verify that $a^2 > b^2$ $(16 > 9)$, which confirms it is a horizontal ellipse.
Step 3: Set up the standard eccentricity formula:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Step 4: Substitute the values of a^2 and b^2 into the formula:

$$e = \sqrt{1 - \frac{9}{16}} = \sqrt{\frac{16 - 9}{16}}$$

Step 5: Simplify the radical expression:

$$e = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$$

Final Answer: $\boxed{\frac{\sqrt{7}}{4}}$

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: We use the chain rule for differentiation, which states that $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$. Here, the outer function is the natural logarithm, $\frac{d}{du}(\ln u) = \frac{1}{u}$, and the inner function contains standard trigonometric terms.

Solution: Step 1: Apply the derivative rule for logarithms to the outer function:

$$\frac{d}{dx}[\ln(\sec x + \tan x)] = \frac{1}{\sec x + \tan x} \cdot \frac{d}{dx}(\sec x + \tan x)$$

Step 2: Differentiate the inner trigonometric expressions:

$$\frac{d}{dx}(\sec x) = \sec x \tan x, \quad \frac{d}{dx}(\tan x) = \sec^2 x$$

Step 3: Substitute these back into the chain rule expression:

$$= \frac{1}{\sec x + \tan x} \cdot (\sec x \tan x + \sec^2 x)$$

Step 4: Factor out $\sec x$ from the numerator expression:

$$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$$

Step 5: Cancel the identical common factor from both the numerator and denominator:

$$= \sec x$$

Final Answer: $\boxed{\sec x}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: Three vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ are coplanar if and only if their scalar triple product is zero, which means $[\vec{u}\ \vec{v}\ \vec{w}] = 0$. This condition can be evaluated by setting the determinant of the matrix formed by their components to zero.

Solution: Step 1: Set up the component determinant equation using the coefficients of the three vectors:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

Step 2: Expand the determinant along the first row:

$$2\begin{vmatrix} 2 & -3 \\ \lambda & 5 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 3 & \lambda \end{vmatrix} = 0$$

Step 3: Evaluate each of the 2×2 minor determinants:

$$2(10 - (-3\lambda)) + 1(5 - (-9)) + 1(\lambda - 6) = 0$$

$$2(10 + 3\lambda) + 1(14) + (\lambda - 6) = 0$$

Step 4: Expand the terms and simplify the linear algebraic equation:

$$20 + 6\lambda + 14 + \lambda - 6 = 0$$

$$7\lambda + 28 = 0$$

Step 5: Solve for the unknown parameter λ:

$$7\lambda = -28 \implies \lambda = -4$$

Final Answer: -4

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: An expression of the form $a \sin x + b \cos x$ always varies within a fixed bounded range given by $\left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$. The maximum value corresponds to the positive square root boundary.

Solution: Step 1: Match the given function $f(x) = \sin x + \cos x$ with the standard form $a \sin x + b \cos x$:

$$a = 1, \quad b = 1$$

Step 2: Recall the range formula for this standard linear combination of sine and cosine:

$$-\sqrt{a^2+b^2} \leq a \sin x + b \cos x \leq \sqrt{a^2+b^2}$$

Step 3: Substitute the coefficients $a = 1$ and $b = 1$ into the upper bound expression:

$$\text{Maximum Value} = \sqrt{1^2 + 1^2}$$

Step 4: Calculate the final value:

$$\text{Maximum Value} = \sqrt{1+1} = \sqrt{2}$$

Final Answer: $\boxed{\sqrt{2}}$

Answer: (C)

Go Back to Question 25

Q26.

Solution

Concept: Let the terms of a geometric progression (GP) be represented as $a, ar, ar^2, ar^3, \ldots$. The third term is $t_3 = ar^2$. We write down the product of the first 5 terms and substitute the known value of the third term into it.

Solution: Step 1: Write down the algebraic expressions for the first 5 terms of the GP:

$$t_1 = a, \quad t_2 = ar, \quad t_3 = ar^2, \quad t_4 = ar^3, \quad t_5 = ar^4$$

Step 2: Express the product P of these first five terms:

$$P = a \cdot (ar) \cdot (ar^2) \cdot (ar^3) \cdot (ar^4)$$

Step 3: Combine the base variables by adding their respective exponents:

$$P = a^{1+1+1+1+1} \cdot r^{0+1+2+3+4} = a^5 r^{10}$$

Step 4: Rewrite this expression as a power of the third term component:

$$P = (ar^2)^5$$

Step 5: Substitute the given value of the third term ($t_3 = ar^2 = 4$) into the equation:

$$P = 4^5$$

Final Answer: $\boxed{4^5}$

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept: The arithmetic mean of a set of numbers is the sum of all elements divided by the total count of elements, $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$. We use the standard summation formula for the first n natural numbers: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

Solution: Step 1: Write down the definition of the arithmetic mean for the first n natural numbers:

$$\text{Mean} = \frac{1+2+3+\cdots+n}{n}$$

Step 2: Substitute the standard sum formula for the numerator:

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

Step 3: Set up the fraction for the mean:

$$\text{Mean} = \frac{\frac{n(n+1)}{2}}{n}$$

Step 4: Cancel out the common factor n from both the numerator and the denominator:

$$\text{Mean} = \frac{n+1}{2}$$

Final Answer: $\boxed{\frac{n+1}{2}}$

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept: This integration problem uses the special exponential integral property $\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + C$. We must identify which term acts as the function $f(x)$ and verify its derivative.

Solution: Step 1: Write down the given integral expression:

$$\int e^x(\tan x + \ln(\sec x))\,dx$$

Step 2: Test the functions to determine which one is the derivative of the other. Let $f(x) = \ln(\sec x)$.
Step 3: Differentiate $f(x)$ using the chain rule:

$$f'(x) = \frac{1}{\sec x} \cdot \frac{d}{dx}(\sec x) = \frac{\sec x \tan x}{\sec x} = \tan x$$

Step 4: Match this structure with the special integral identity $\int e^x[f'(x) + f(x)]\,dx$:

$$\text{Here, } f(x) = \ln(\sec x) \quad \text{and} \quad f'(x) = \tan x$$

Step 5: Apply the integration rule directly to find the final result:

$$\int e^x(\tan x + \ln(\sec x))\,dx = e^x \ln(\sec x) + C$$

Final Answer: $\boxed{e^x \ln(\sec x) + C}$

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: A straight line passing through a point (x_1, y_1, z_1) with direction ratios (a, b, c) has a symmetrical equation given by $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$. Parallel lines share identical direction ratios.

Solution: Step 1: Extract the direction ratios from the given parallel line equation $\frac{x-3}{2} = \frac{y+1}{-3} = \frac{z-2}{6}$:

$$(a, b, c) = (2, -3, 6)$$

Step 2: Identify the given point coordinates through which the new line passes:

$$(x_1, y_1, z_1) = (1, 2, -4)$$

Step 3: Substitute the point coordinates and direction ratios into the standard symmetrical line equation form:

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-(-4)}{6}$$

Step 4: Simplify the sign inside the numerator of the third term:

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+4}{6}$$

Final Answer: $\boxed{\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+4}{6}}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: The Principle of Inclusion-Exclusion for two finite sets A and B relates their individual cardinalities to the cardinalities of their union and intersection through the equation $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Solution: Step 1: Write down the set cardinality values given in the problem statement:

$$n(A) = 17, \quad n(B) = 23, \quad n(A \cup B) = 38$$

Step 2: Rearrange the Principle of Inclusion-Exclusion formula to isolate the intersection term $n(A \cap B)$:

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

Step 3: Substitute the given numerical values into the rearranged formula:

$$n(A \cap B) = 17 + 23 - 38$$

Step 4: Simplify the arithmetic expression:

$$n(A \cap B) = 40 - 38 = 2$$

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: To compute higher powers of complex numbers like $(1 \pm i)$, it is often easier to first compute their squares, $(1 \pm i)^2$, and then square that intermediate result to find the fourth power.

Solution: Step 1: Expand the expression $(1+i)^2$ using the algebraic identity $(a+b)^2 = a^2+2ab+b^2$:

$$(1+i)^2 = 1^2 + 2i + i^2 = 1 + 2i - 1 = 2i$$

Step 2: Expand the expression $(1-i)^2$ using the algebraic identity $(a-b)^2 = a^2 - 2ab + b^2$:

$$(1-i)^2 = 1^2 - 2i + i^2 = 1 - 2i - 1 = -2i$$

Step 3: Compute the fourth power by squaring the results from the first two steps:

$$(1+i)^4 = [(1+i)^2]^2 = (2i)^2 = 4i^2 = -4$$

$$(1-i)^4 = [(1-i)^2]^2 = (-2i)^2 = 4i^2 = -4$$

Step 4: Add the two individual results together as required by the expression:

$$(1+i)^4 + (1-i)^4 = -4 + (-4) = -8$$

Final Answer: -8

Answer: (B)

Go Back to Question 31

Q32.

Solution

Concept: The general equation of a circle is represented as $x^2 + y^2 + 2gx + 2fy + c = 0$. The radius r of this circle can be calculated from its coefficients using the standard formula $r = \sqrt{g^2 + f^2 - c}$.

Solution: Step 1: Match the given circle equation with the standard general form:

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

Step 2: Determine the parameters g, f, and c by equating coefficients:

$$2g = -4 \implies g = -2$$

$$2f = 6 \implies f = 3$$

$$c = -12$$

Step 3: Set up the radius calculation formula:

$$r = \sqrt{g^2 + f^2 - c}$$

Step 4: Substitute the values of g, f, and c into the formula:

$$r = \sqrt{(-2)^2 + 3^2 - (-12)} = \sqrt{4 + 9 + 12}$$

Step 5: Compute the sum inside the radical and find the square root:

$$r = \sqrt{25} = 5$$

Final Answer: $\boxed{5}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: Since the balls are drawn sequentially without replacement, the two selections are dependent events. We use the multiplication rule for dependent events: $P(B_1 \cap B_2) = P(B_1) \cdot P(B_2|B_1)$.

Solution: Step 1: Count the total number of balls in the bag initially:

$$\text{Total balls} = 4 \text{ white} + 6 \text{ black} = 10 \text{ balls}$$

Step 2: Find the probability of drawing a black ball on the first attempt ($P(B_1)$):

$$P(B_1) = \frac{\text{Number of black balls}}{\text{Total number of balls}} = \frac{6}{10}$$

Step 3: Update the counts for the second draw, accounting for the fact that one black ball was removed:

$$\text{Remaining black balls} = 5, \quad \text{Remaining total balls} = 9$$

Step 4: Find the conditional probability of drawing a black ball on the second attempt ($P(B_2|B_1)$):

$$P(B_2|B_1) = \frac{5}{9}$$

Step 5: Multiply the two probabilities together to find the joint probability:

$$\text{Total Probability} = \frac{6}{10} \cdot \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$$

Final Answer: $\boxed{\frac{1}{3}}$

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: The slope m of the tangent line to any differentiable curve $y = f(x)$ at a specific given point $x = a$ is equal to the value of its first derivative evaluated at that point, which means $m = \frac{dy}{dx}\Big|_{x=a}$.

Solution: Step 1: Write down the given equation of the curve:

$$y = x^3 - x$$

Step 2: Differentiate the function with respect to x using the power rule:

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 - x) = 3x^2 - 1$$

Step 3: Identify the given x-coordinate where the slope needs to be evaluated:

$$x = 2$$

Step 4: Substitute $x = 2$ into the derivative expression:

$$m = \frac{dy}{dx}\Big|_{x=2} = 3(2)^2 - 1$$

Step 5: Simplify the arithmetic expression to find the slope value:

$$m = 3(4) - 1 = 12 - 1 = 11$$

Final Answer: $\boxed{11}$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept: By definition, a matrix A is symmetric if $A^T = A$, and it is skew-symmetric if $A^T = -A$. We can combine these two definitions using substitution to determine the required properties of matrix A.

Solution: Step 1: Write down the mathematical definition for a symmetric matrix:

$$A^T = A \quad \text{— (1)}$$

Step 2: Write down the mathematical definition for a skew-symmetric matrix:

$$A^T = -A \quad \text{— (2)}$$

Step 3: Since the left-hand sides of both equations are identical (A^T), equate their right-hand sides:

$$A = -A$$

Step 4: Move $-A$ to the left side of the equation by adding A to both sides:

$$A + A = 0 \implies 2A = 0$$

Step 5: Divide the equation by 2 to solve for matrix A:

$$A = 0$$

This proves that A must be a zero matrix.

Final Answer: $\boxed{A \text{ is a zero matrix}}$

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept: We use the double-angle identity for the tangent function, $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$. By setting $\theta = \tan^{-1}(\frac{1}{3})$, we can simplify the expression because $\tan(\tan^{-1} x) = x$.

Solution: Step 1: Define the angle variable θ from the given expression:

$$\theta = \tan^{-1}\left(\frac{1}{3}\right) \implies \tan\theta = \frac{1}{3}$$

Step 2: Rewrite the original problem expression in terms of θ:

$$\tan\left(2\tan^{-1}\frac{1}{3}\right) = \tan(2\theta)$$

Step 3: Substitute the double-angle identity for $\tan(2\theta)$:

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Step 4: Substitute the value $\tan\theta = \frac{1}{3}$ into the identity:

$$\tan(2\theta) = \frac{2(\frac{1}{3})}{1-(\frac{1}{3})^2} = \frac{\frac{2}{3}}{1-\frac{1}{9}}$$

Step 5: Simplify the fractions in both the numerator and the denominator:

$$\tan(2\theta) = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{2}{3}\cdot\frac{9}{8} = \frac{18}{24} = \frac{3}{4}$$

Final Answer: $\boxed{\frac{3}{4}}$

Answer: (A)

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Q37.

Solution

Concept: The order of a differential equation is the order of the highest derivative present in the equation. The degree is the power to which this highest derivative is raised, after the equation has been cleared of any fractional exponents or radicals.

Solution: Step 1: Write down the given differential equation:

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$$

Step 2: Identify the highest order derivative present in the equation, which is $\frac{d^2y}{dx^2}$. This means the order of the differential equation is 2.

Step 3: Notice the fractional exponent $3/2$ on the left side. To find the degree, we must eliminate this fraction by squaring both sides of the equation:

$$\left(\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}\right)^2 = \left(\frac{d^2y}{dx^2}\right)^2$$

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3} = \left(\frac{d^2y}{dx^2}\right)^2$$

Step 4: Now that the equation is in polynomial form with respect to its derivatives, identify the exponent of the highest-order derivative:

$$\text{The power of } \frac{d^2y}{dx^2} \text{ is 2, so the degree is 2.}$$

Step 5: Combine the results: Order = 2, Degree = 2.

Final Answer: $\boxed{2, 2}$

Answer: (A)

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Q38.

Solution

Concept: The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in a Cartesian coordinate system is calculated using the standard distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. We plug in the coordinates and solve the equation for x.

Solution: Step 1: Write down the given coordinates and the distance value:

$$(x_1, y_1) = (x, 2), \quad (x_2, y_2) = (3, 4), \quad d = 2$$

Step 2: Substitute these values into the standard distance formula:

$$\sqrt{(3 - x)^2 + (4 - 2)^2} = 2$$

Step 3: Square both sides of the equation to eliminate the radical:

$$(3 - x)^2 + (2)^2 = 2^2$$

$$(3 - x)^2 + 4 = 4$$

Step 4: Subtract 4 from both sides to isolate the squared term:

$$(3 - x)^2 = 0$$

Step 5: Take the square root of both sides to find x:

$$3 - x = 0 \implies x = 3$$

Final Answer: 3

Answer: (B)

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Q39.

Solution

Concept: This problem requires evaluating a basic definite integral using the fundamental theorem of calculus. We use the standard integration rule $\int \frac{1}{x}\,dx = \ln|x|$ and evaluate it between the given upper and lower limits.

Solution: Step 1: Set up the given definite integral expression:

$$\int_1^2 \frac{1}{x}\,dx$$

Step 2: Find the antiderivative of the integrand function:

$$\int \frac{1}{x}\,dx = \ln|x|$$

Step 3: Apply the integration limits to the antiderivative:

$$[\ln|x|]_1^2 = \ln 2 - \ln 1$$

Step 4: Recall the log value for the base unit, which is $\ln 1 = 0$:

$$= \ln 2 - 0$$

Step 5: Simplify to find the final exact value:

$$= \ln 2$$

Final Answer: $\boxed{\ln 2}$

Answer: (B)

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Q40.

Solution

Concept: We use the addition rule for the probability of the union of two events: $P(K \cup C) = P(K) + P(C) - P(K \cap C)$, where K represents drawing a king and C represents drawing a club card.

Solution: Step 1: Identify the total number of cards in a standard deck:

$$n(S) = 52$$

Step 2: Find the probability of drawing a king card, $P(K)$. Since there are 4 kings in a deck:

$$P(K) = \frac{4}{52}$$

Step 3: Find the probability of drawing a club card, $P(C)$. Since there are 13 clubs in a deck:

$$P(C) = \frac{13}{52}$$

Step 4: Find the probability of drawing a card that is both a king and a club ($P(K \cap C)$). There is exactly 1 king of clubs in a standard deck:

$$P(K \cap C) = \frac{1}{52}$$

Step 5: Substitute these values into the probability addition formula:

$$P(K \cup C) = P(K) + P(C) - P(K \cap C)$$

$$P(K \cup C) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$$

Step 6: Simplify the fraction to its lowest terms by dividing the numerator and denominator by 4:

$$P(K \cup C) = \frac{4}{13}$$

Final Answer: $\boxed{\frac{4}{13}}$

Answer: (B)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	A	4	B	5	B
6	A	7	C	8	B	9	A	10	B
11	A	12	B	13	B	14	C	15	B
16	A	17	B	18	A	19	B	20	A
21	B	22	A	23	A	24	A	25	C
26	B	27	B	28	B	29	A	30	A
31	B	32	A	33	A	34	A	35	B
36	A	37	A	38	B	39	B	40	B