Rajasthan JET Mathematics Sample Paper-8

Duration: 40 Minutes **Maximum Marks: 160**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **−1 marks**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. If the matrix $A = \begin{bmatrix} 2x & 1 \\ 4 & x \end{bmatrix}$ is singular, then the possible real values of x are:

(A) $\pm\sqrt{2}$

(B) ± 2

(C) ± 1

(D) $\pm 2\sqrt{2}$

Q2. The value of $\lim_{x\to 0} \frac{1-\cos(6x)}{x^2}$ is:

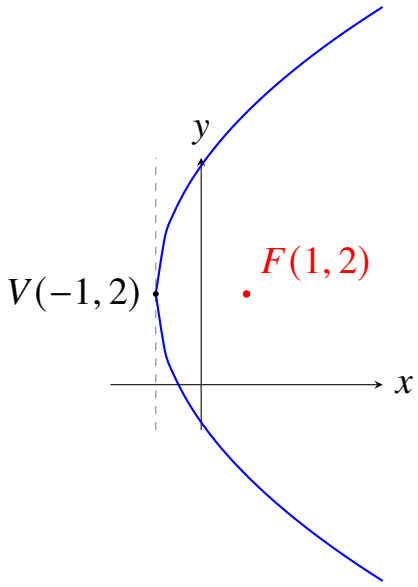
(A) 6

(B) 18

(C) 36

(D) 3

Q3. The focus of the parabola $y^2 - 4y - 8x - 4 = 0$ is:



(A) $(1, 2)$

(B) $(2, 1)$

(C) $(0, 2)$

(D) $(2, 2)$

Q4. Let A and B be two independent events such that $P(A) = 0.3$ and $P(B) = 0.4$. Then $P(A \cup B)$ is equal to:

(A) 0.70

(B) 0.12

(C) 0.58

(D) 0.82

Q5. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then the value of $\cos^{-1} x + \cos^{-1} y$ is:

(A) $\frac{\pi}{2}$

(B) π

(C) 0

(D) $\frac{\pi}{4}$

Q6. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = k\frac{d^2y}{dx^2}$ are respectively:

(A) $2, 3$

(B) $2, 2$

(C) $1, 2$

(D) $2, 1$

Q7. If the n-th term of an Arithmetic Progression is $3n + 5$, then the sum of its first 20 terms is:

(A) 610

(B) 730

(C) 740

(D) 1480

Q8. The derivative of $e^{\sin^2 x}$ with respect to x is:

(A) $e^{\sin^2 x} \cdot \cos^2 x$

(B) $2 \sin x \cdot e^{\sin^2 x}$

(C) $\sin(2x) \cdot e^{\sin^2 x}$

(D) $e^{\sin^2 x} \cdot \cos(2x)$

Q9. The mean of 5 observations is 4 and their variance is 5.2. If three of the observations are 1, 2, and 6, then the other two observations are:

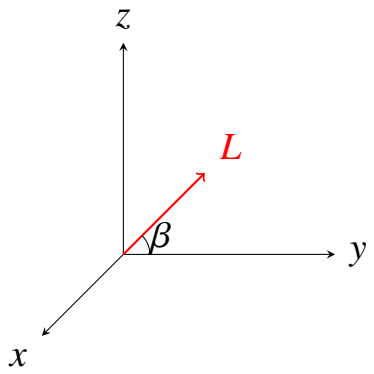
(A) $3, 8$

(B) $4, 7$

(C) $5, 6$

(D) $2, 9$

Q10. The direction cosines of a line equally inclined to the positive directions of the coordinate axes are:



(A) $(1, 1, 1)$

(B) $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

(C) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(D) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Q11. The value of $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\, dx$ is:

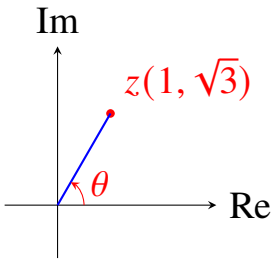
(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) π

(D) 0

Q12. If $z = 1 + i\sqrt{3}$, then the amplitude (argument) of z is:



(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Q13. Let R be a relation on the set of natural numbers $\mathbb{N}$ defined by aRb if $a+2b = 10$. The domain of R is:

(A) $\{2, 4, 6, 8\}$

(B) $\{1, 2, 3, 4\}$

(C) $\{2, 4, 6\}$

(D) $\{1, 3, 5, 7\}$

Q14. The vector equation of the line passing through the point $(2, -1, 4)$ and parallel to the vector $\hat{i} + 2\hat{j} - \hat{k}$ is:

(A) $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$

(B) $\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$

(C) $\vec{r} = (2\hat{i} + \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$

(D) $\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$

Q15. The slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 2 is:

(A) 9

(B) 3

(C) 6

(D) 12

Q16. If A is a square matrix of order 3 such that $|A| = 5$, then the value of $|\text{adj } A|$ is:

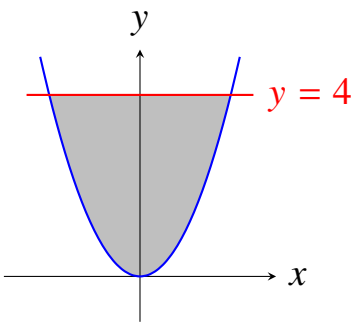
(A) 5

(B) 25

(C) 125

(D) 15

Q17. The area bounded by the curve $y = x^2$ and the line $y = 4$ is:



(A) $\frac{32}{3}$

(B) $\frac{16}{3}$

(C) $\frac{8}{3}$

(D) 16

Q18. Two dice are thrown simultaneously. The probability of getting a sum of 7 is:

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{5}{36}$

(D) $\frac{1}{9}$

Q19. If $\tan^{-1} x + \tan^{-1}(1) = \tan^{-1}(3)$, then the value of x is:

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) 2

(D) 1

Q20. The integrating factor of the differential equation $\frac{dy}{dx} + y \tan x = \sec x$ is:

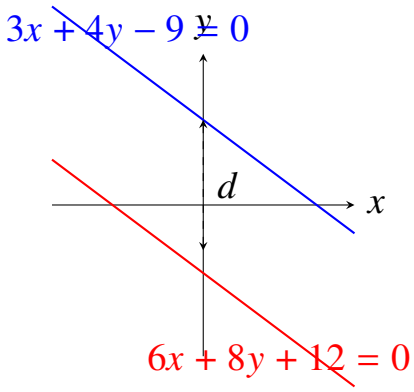
(A) $\log(\sec x)$

(B) $\sec x$

(C) $\tan x$

(D) $\log(\tan x)$

Q21. The distance between the parallel lines $3x + 4y - 9 = 0$ and $6x + 8y + 12 = 0$ is:



(A) 3 units

(B) $\frac{21}{10}$ units

(C) 6 units

(D) 3 units

Q22. If the first term of a G.P. is 7, the last term is 448 and the sum of the terms is 889, then the common ratio is:

(A) 2

(B) 3

(C) 4

(D) $\frac{1}{2}$

Q23. The value of $\int e^x(\cot x + \log|\sin x|)\,dx$ is:

(A) $e^x \cot x + C$

(B) $e^x \log|\sin x| + C$

(C) $-e^x \cot x + C$

(D) $e^x \csc x + C$

Q24. A bag contains 5 white and 7 red balls. If two balls are drawn at random one by one without replacement, the probability that both are red is:

(A) $\frac{7}{22}$

(B) $\frac{49}{144}$

(C) $\frac{21}{44}$

(D) $\frac{7}{12}$

Q25. If $f(x) = \frac{x-1}{x+1}$, then the value of $f(f(x))$ is:

(A) x

(B) $-x$

(C) $-\frac{1}{x}$

(D) $\frac{1}{x}$

Q26. If the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar, then the value of λ is:

(A) -2

(B) -4

(C) 2

(D) 4

Q27. The function $f(x) = 2x^3 - 9x^2 + 12x + 15$ is strictly decreasing in the interval:

(A) $(1, 2)$

(B) $(-\infty, 1)$

(C) $(2, \infty)$

(D) $(0, 3)$

Q28. The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is:

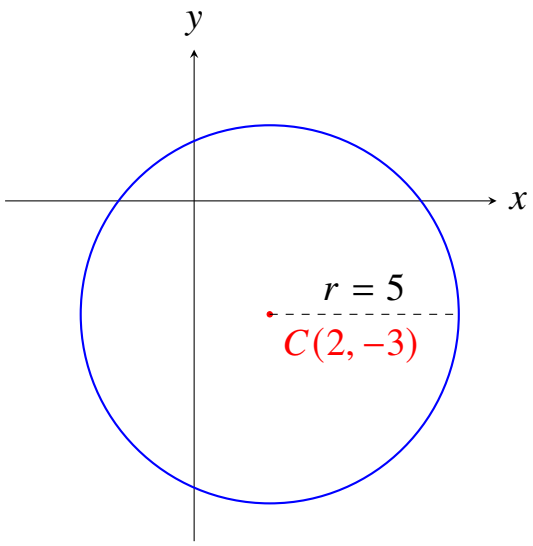
(A) $e^y = e^x + \frac{x^3}{3} + C$

(B) $e^{-y} = e^x + x^3 + C$

(C) $e^y = e^{-x} + \frac{x^3}{3} + C$

(D) $e^{-y} = e^{-x} + \frac{x^2}{2} + C$

Q29. The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is:



(A) 5

(B) $\sqrt{13}$

(C) 25

(D) 7

Q30. The value of $\cos(2\sin^{-1} x)$ at $x = \frac{1}{3}$ is:

(A) $\frac{7}{9}$

(B) $\frac{2\sqrt{2}}{3}$

(C) $\frac{1}{9}$

(D) $\frac{8}{9}$

Q31. If ω is an imaginary cube root of unity, then the value of $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$ is:

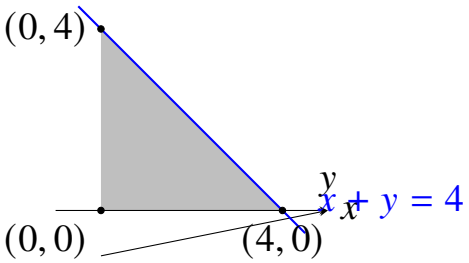
(A) 32

(B) -32

(C) 64

(D) 0

Q32. The maximum value of $z = 3x + 4y$ subject to constraints $x + y \leq 4$, $x \geq 0$, $y \geq 0$ is:



(A) 12

(B) 16

(C) 0

(D) 14

Q33. If the coefficient of variation of a distribution is 60% and its standard deviation is 12, then its arithmetic mean is:

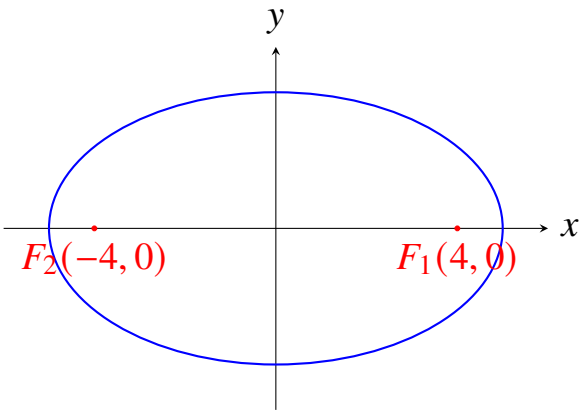
(A) 20

(B) 5

(C) 7.2

(D) 18

Q34. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is:



(A) $\frac{3}{5}$

(B) $\frac{4}{5}$

(C) $\frac{16}{25}$

(D) $\frac{2}{5}$

Q35. The angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ is:

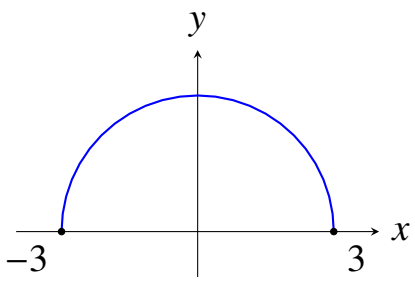
(A) $\cos^{-1}\left(\frac{1}{3}\right)$

(B) $\cos^{-1}\left(-\frac{1}{3}\right)$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Q36. The domain of the real-valued function $f(x) = \sqrt{9 - x^2}$ is:



(A) $(-3, 3)$

(B) $[-3, 3]$

(C) $(-\infty, -3] \cup [3, \infty)$

(D) $[0, 3]$

Q37. The values of x, y, z if $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ are respectively:

(A) $2, 4, 3$

(B) $4, 3, 2$

(C) $2, 3, 4$

(D) $3, 4, 2$

Q38. The value of $\int \frac{1}{x(1+\log x)}\, dx$ is:

(A) $\log|1 + \log x| + C$

(B) $\log|x| + \log|1 + \log x| + C$

(C) $\frac{1}{(1+\log x)^2} + C$

(D) $\tan^{-1}(\log x) + C$

Q39. A card is drawn from a well-shuffled pack of 52 cards. The probability that it is either a king or a spade is:

(A) $\frac{17}{52}$

(B) $\frac{4}{13}$

(C) $\frac{1}{13}$

(D) $\frac{2}{13}$

Q40. The value of $\lim_{x\to 2} \frac{x^4-16}{x-2}$ is:

(A) 8

(B) 16

(C) 32

(D) 64

Detailed Solutions

Q1.

Solution

Concept:

A square matrix A is said to be singular if its determinant is equal to zero, which means $|A| = 0$. For a given 2×2 matrix, the determinant is calculated by subtracting the product of the secondary diagonal elements from the product of the primary diagonal elements.

Solution:

Step 1: Write down the given matrix A:

$$A = \begin{bmatrix} 2x & 1 \\ 4 & x \end{bmatrix}$$

Step 2: Set the determinant of matrix A to zero because the matrix is singular:

$$|A| = 0$$

Step 3: Evaluate the determinant of the 2×2 matrix:

$$(2x)(x) - (1)(4) = 0$$

Step 4: Simplify the quadratic equation obtained:

$$2x^2 - 4 = 0$$

Step 5: Solve for x^2 by isolating the variable term:

$$2x^2 = 4$$

$$x^2 = 2$$

Step 6: Take the square root on both sides to find all possible real values of x:

$$x = \pm\sqrt{2}$$

Final Answer: $\pm\sqrt{2}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept:

To evaluate limits of trigonometric functions of the indeterminate form $0/0$, we can use standard limits such as $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$, or apply the trigonometric identity $1-\cos(2\theta)=2\sin^2\theta$ to simplify the expression before evaluating.

Solution:

Step 1: Identify the given limit expression:

$$\lim_{x\to 0}\frac{1-\cos(6x)}{x^2}$$

Substituting $x=0$ gives $\frac{1-\cos(0)}{0}=\frac{0}{0}$, which is an indeterminate form.

Step 2: Use the standard trigonometric half-angle identity $1-\cos\theta=2\sin^2\left(\frac{\theta}{2}\right)$. Replacing θ with $6x$:

$$1-\cos(6x)=2\sin^2(3x)$$

Step 3: Substitute this back into the limit expression:

$$\lim_{x\to 0}\frac{2\sin^2(3x)}{x^2}$$

Step 4: To utilize the standard limit $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$, multiply and divide the denominator by 9 to match the angle argument squared, $(3x)^2=9x^2$:

$$\lim_{x\to 0}2\cdot\frac{\sin^2(3x)}{x^2}\cdot\frac{9}{9}$$

$$=18\cdot\lim_{x\to 0}\left(\frac{\sin(3x)}{3x}\right)^2$$

Step 5: As $x\to 0$, $3x\to 0$. Therefore, the value of the limit inside the parenthesis becomes 1:

$$18\cdot(1)^2=18$$

Final Answer: 18

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept:

The standard equation of a parabola opening rightward is $(y-k)^2 = 4a(x-h)$, where (h,k) represents the vertex of the parabola. The focus of such a parabola is located at the position $(h+a,k)$. We must complete the square for the y-terms to convert the general equation into standard form.

Solution:

Step 1: Write down the given general equation of the parabola:

$$y^2 - 4y - 8x - 4 = 0$$

Step 2: Group the y-terms on one side and move the remaining terms to the right side:

$$y^2 - 4y = 8x + 4$$

Step 3: Complete the square on the left side by adding $\left(\frac{-4}{2}\right)^2 = 4$ to both sides of the equation:

$$y^2 - 4y + 4 = 8x + 4 + 4$$

$$(y-2)^2 = 8x + 8$$

Step 4: Factor out the coefficient of x on the right side to get the standard form:

$$(y-2)^2 = 8(x+1)$$

Step 5: Compare this equation with the standard form $(y-k)^2 = 4a(x-h)$:

$$h = -1, \quad k = 2, \quad 4a = 8 \implies a = 2$$

Step 6: Determine the coordinates of the focus using the formula $F(h+a,k)$:

$$F = (-1+2, 2) = (1,2)$$

Final Answer: $\boxed{(1,2)}$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept:

For any two events A and B, the probability of their union is given by the addition theorem of probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. If the two events are independent, the probability of their intersection satisfies the multiplication rule: $P(A \cap B) = P(A) \cdot P(B)$.

Solution:

Step 1: List the given probabilities of the individual independent events:

$$P(A) = 0.3$$

$$P(B) = 0.4$$

Step 2: Since events A and B are specified as independent, compute the probability of their simultaneous occurrence $P(A \cap B)$:

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B) = 0.3 \cdot 0.4 = 0.12$$

Step 3: Apply the addition theorem of probability to find the probability of the union $P(A \cup B)$:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Step 4: Substitute the values into the equation and perform the arithmetic:

$$P(A \cup B) = 0.3 + 0.4 - 0.12$$

$$P(A \cup B) = 0.7 - 0.12 = 0.58$$

Final Answer: 0.58

Answer: (C)

Go Back to Question 4

Q5.

Solution

Concept:

In inverse trigonometric functions, a fundamental identity connects the inverse sine function and the inverse cosine function for any valid input variable in the domain $[-1, 1]$. This relationship is stated as $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

Solution:

Step 1: Recall the standard complementary identity for inverse trigonometric functions:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \implies \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$\sin^{-1} y + \cos^{-1} y = \frac{\pi}{2} \implies \cos^{-1} y = \frac{\pi}{2} - \sin^{-1} y$$

Step 2: Express the required value of the expression $\cos^{-1} x + \cos^{-1} y$ in terms of inverse sine functions:

$$\cos^{-1} x + \cos^{-1} y = \left(\frac{\pi}{2} - \sin^{-1} x\right) + \left(\frac{\pi}{2} - \sin^{-1} y\right)$$

Step 3: Rearrange and group the terms algebraically:

$$\cos^{-1} x + \cos^{-1} y = \left(\frac{\pi}{2} + \frac{\pi}{2}\right) - (\sin^{-1} x + \sin^{-1} y)$$

$$\cos^{-1} x + \cos^{-1} y = \pi - (\sin^{-1} x + \sin^{-1} y)$$

Step 4: Substitute the value given in the problem statement, which is $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$:

$$\cos^{-1} x + \cos^{-1} y = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

Final Answer: $\boxed{\frac{\pi}{2}}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept:

The order of a differential equation is defined as the highest derivative present in the equation. The degree of a differential equation is the power of the highest order derivative after the differential equation has been cleared of fractional exponents and radicals relative to all derivatives.

Solution:

Step 1: Write down the given differential equation:

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = k\frac{d^2y}{dx^2}$$

Step 2: Identify the highest order derivative present. The term $\frac{d^2y}{dx^2}$ is a second-order derivative, while $\frac{dy}{dx}$ is a first-order derivative. Thus, the order of the differential equation is 2.

Step 3: Remove the fractional exponent $\frac{3}{2}$ on the left side by squaring both sides of the equation:

$$\left(\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}\right)^2 = \left(k\frac{d^2y}{dx^2}\right)^2$$

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = k^2\left(\frac{d^2y}{dx^2}\right)^2$$

Step 4: Observe the power raised on the highest order derivative $\frac{d^2y}{dx^2}$ in this radical-free form. The power is 2, meaning the degree of the equation is 2.

Step 5: Combine the values to get order and degree, which are 2 and 2 respectively.

Final Answer: $2, 2$

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept:

The sum of the first n terms of an Arithmetic Progression (A.P.) can be found using the formula $S_n = \frac{n}{2}[2a + (n-1)d]$, where a is the first term and d is the common difference. Alternatively, if the first term a and last term l are known, the formula is $S_n = \frac{n}{2}[a + l]$.

Solution:

Step 1: Identify the formula given for the n-th term of the sequence:

$$t_n = 3n + 5$$

Step 2: Find the first term (a) by setting $n = 1$:

$$a = t_1 = 3(1) + 5 = 8$$

Step 3: Find the second term (t_2) to determine the common difference, or directly compute the 20-th term (t_{20}) to act as the last term (l) for the sum calculation. Let us find t_{20}:

$$l = t_{20} = 3(20) + 5 = 60 + 5 = 65$$

Step 4: Use the finite sum formula $S_n = \frac{n}{2}[a + l]$ where $n = 20$:

$$S_{20} = \frac{20}{2}[8 + 65]$$

Step 5: Perform the remaining arithmetic operations:

$$S_{20} = 10 \cdot 73 = 730$$

Final Answer: 730

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept:

To find the derivative of a composite function of the form $f(g(h(x)))$, we apply the chain rule of differentiation sequentially from the outermost function to the innermost function. Specifically, $\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$ and $\frac{d}{dx}(v^n) = nv^{n-1}\frac{dv}{dx}$.

Solution:

Step 1: Let the given function be represented as $y = e^{\sin^2 x}$.

Step 2: Apply the chain rule with respect to x. First, differentiate the outer exponential function, leaving the exponent unchanged:

$$\frac{dy}{dx} = e^{\sin^2 x} \cdot \frac{d}{dx}(\sin^2 x)$$

Step 3: Differentiate the function in the exponent, which is a power function $(\sin x)^2$, by applying the power rule:

$$\frac{d}{dx}(\sin^2 x) = 2\sin x \cdot \frac{d}{dx}(\sin x)$$

Step 4: Differentiate the innermost trigonometric function, $\sin x$:

$$\frac{d}{dx}(\sin x) = \cos x$$

Step 5: Combine all differentiated parts together:

$$\frac{dy}{dx} = e^{\sin^2 x} \cdot 2\sin x \cos x$$

Step 6: Simplify the expression using the standard double-angle identity $2\sin x \cos x = \sin(2x)$:

$$\frac{dy}{dx} = \sin(2x) \cdot e^{\sin^2 x}$$

Final Answer: $\boxed{\sin(2x) \cdot e^{\sin^2 x}}$

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept:

The arithmetic mean ($\bar{x}$) of n values is $\frac{\sum x_i}{n}$ and their variance (σ^2) is calculated using the statistical formula $\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$. We can set up a system of two algebraic equations for two unknown observations using the values of the mean and variance.

Solution:

Step 1: Let the two missing observations be denoted as a and b. The complete set of 5 observations is $1, 2, 6, a, b$.

Step 2: Use the given mean ($\bar{x} = 4$) to find the sum of all observations:

$$\bar{x} = \frac{1 + 2 + 6 + a + b}{5} = 4$$

$$9 + a + b = 20 \implies a + b = 11 \quad \text{— (Equation 1)}$$

Step 3: Use the given variance ($\sigma^2 = 5.2$) to establish an equation for the sum of squares:

$$\sigma^2 = \frac{\sum x_i^2}{5} - (\bar{x})^2$$

$$5.2 = \frac{1^2 + 2^2 + 6^2 + a^2 + b^2}{5} - 4^2$$

$$5.2 = \frac{1 + 4 + 36 + a^2 + b^2}{5} - 16$$

Step 4: Clear the fractions and simplify the expression:

$$5.2 + 16 = \frac{41 + a^2 + b^2}{5}$$

$$21.2 \cdot 5 = 41 + a^2 + b^2 \implies 106 = 41 + a^2 + b^2$$

$$a^2 + b^2 = 65 \quad \text{— (Equation 2)}$$

Step 5: Use the identity $(a + b)^2 = a^2 + b^2 + 2ab$ to find ab:

$$11^2 = 65 + 2ab \implies 121 - 65 = 2ab \implies 2ab = 56 \implies ab = 28$$

Step 6: Find two numbers whose sum is 11 and product is 28. The factors are clearly 4 and 7.

Final Answer: $4, 7$

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept:

Direction cosines (l, m, n) of a straight line represent the cosines of the angles α, β, γ that the line makes with the positive x, y, z axes respectively. They always satisfy the fundamental geometric constraint identity $l^2 + m^2 + n^2 = 1$.

Solution:

Step 1: Let the line be equally inclined to the positive coordinate axes, which means:

$$\alpha = \beta = \gamma$$

Step 2: Take the cosine function on all sides of the equality:

$$\cos\alpha = \cos\beta = \cos\gamma \implies l = m = n$$

Step 3: Use the standard three-dimensional identity for direction cosines:

$$l^2 + m^2 + n^2 = 1$$

Step 4: Substitute $l = m = n$ into the identity:

$$l^2 + l^2 + l^2 = 1$$

$$3l^2 = 1 \implies l^2 = \frac{1}{3}$$

Step 5: Solve for l, keeping in mind the positive direction implies positive values:

$$l = \frac{1}{\sqrt{3}}$$

Since $l = m = n$, we have:

$$l = \frac{1}{\sqrt{3}}, \quad m = \frac{1}{\sqrt{3}}, \quad n = \frac{1}{\sqrt{3}}$$

Final Answer: $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Answer: (C)

Go Back to Question 10

Q11.

Solution

Concept:

A key definite integral property states that $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$. This property is especially helpful for evaluating trigonometric integrals with symmetric limits such as 0 to $\frac{\pi}{2}$.

Solution:

Step 1: Let the given integral be denoted as I:

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\,dx \quad \text{— (Equation 1)}$$

Step 2: Apply the integral reflection property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$:

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}\,dx$$

Step 3: Simplify using the complementary angle identities $\sin\left(\frac{\pi}{2}-x\right) = \cos x$ and $\cos\left(\frac{\pi}{2}-x\right) = \sin x$:

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}\,dx \quad \text{— (Equation 2)}$$

Step 4: Add Equation 1 and Equation 2 together:

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}\,dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x}\,dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx$$

Step 5: Simplify the integrand and evaluate the simple integral:

$$2I = \int_0^{\frac{\pi}{2}} 1\,dx = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Step 6: Isolate I by dividing by 2:

$$I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept:

The amplitude or principal argument θ of a complex number $z = x + iy$ located in the first quadrant ($x > 0, y > 0$) is given by the formula $\theta = \tan^{-1}\left(\frac{y}{x}\right)$, where x is the real part and y is the imaginary part.

Solution:

Step 1: Identify the real and imaginary parts of the given complex number $z = 1 + i\sqrt{3}$:

$$\text{Re}(z) = x = 1$$

$$\text{Im}(z) = y = \sqrt{3}$$

Step 2: Observe that since both $x > 0$ and $y > 0$, the complex number lies in the first quadrant of the Argand plane. Thus, the principal argument θ is directly calculated as:

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Step 3: Substitute the values of x and y into the formula:

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

Step 4: Recall the basic trigonometric value for which the tangent function equals $\sqrt{3}$:

$$\tan\left(\frac{\pi}{3}\right) = \sqrt{3} \implies \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept:

The domain of a relation R is the set of all first elements (or input values a) from the ordered pairs (a, b) that satisfy the relation's defining equation. Here, both a and b must belong to the set of natural numbers $\mathbb{N} = \{1, 2, 3, 4, \dots\}$.

Solution:

Step 1: Write down the given relation rule:

$$a + 2b = 10 \quad \text{where } a, b \in \mathbb{N}$$

Step 2: Isolate the variable a in terms of b:

$$a = 10 - 2b$$

Step 3: Substitute successive natural number values for b ($b = 1, 2, 3, \dots$) and find the corresponding values of a. Ensure that a also remains a positive integer ($a > 0$): For $b = 1 : a = 10 - 2(1) = 8 \in \mathbb{N}$ For $b = 2 : a = 10 - 2(2) = 6 \in \mathbb{N}$ For $b = 3 : a = 10 - 2(3) = 4 \in \mathbb{N}$ For $b = 4 : a = 10 - 2(4) = 2 \in \mathbb{N}$ For $b = 5 : a = 10 - 2(5) = 0 \notin \mathbb{N}$

Step 4: Collect all the valid obtained values of a to write the domain set:

$$\text{Domain}(R) = \{2, 4, 6, 8\}$$

Final Answer: $\{2, 4, 6, 8\}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept:

The vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to a given vector $\vec{b}$ is given by the standard parametric equation $\vec{r} = \vec{a} + \lambda\vec{b}$, where λ is a real scalar parameter.

Solution:

Step 1: Determine the position vector $\vec{a}$ of the point $(2, -1, 4)$ through which the line passes:

$$\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$$

Step 2: Identify the direction vector $\vec{b}$ that is parallel to the line:

$$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$$

Step 3: Substitute the vectors $\vec{a}$ and $\vec{b}$ into the standard vector line equation $\vec{r} = \vec{a} + \lambda\vec{b}$:

$$\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$$

Final Answer: $\boxed{\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})}$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept:

The slope of the tangent to any differentiable curve $y = f(x)$ at a specific point is equal to the value of its first derivative $\frac{dy}{dx}$ evaluated at that given point's coordinates.

Solution:

Step 1: Write down the equation of the given curve:

$$y = x^3 - 3x + 2$$

Step 2: Differentiate the function with respect to x using basic power rules of differentiation:

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(3x) + \frac{d}{dx}(2)$$

$$\frac{dy}{dx} = 3x^2 - 3$$

Step 3: Evaluate this derivative at the given value of the x-coordinate, which is $x = 2$:

$$\left.\frac{dy}{dx}\right|_{x=2} = 3(2)^2 - 3$$

Step 4: Simplify the numerical expression:

$$\left.\frac{dy}{dx}\right|_{x=2} = 3(4) - 3 = 12 - 3 = 9$$

Final Answer: 9

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept:

For any square matrix A of order n, the determinant of its adjoint matrix satisfies the matrix identity property $|\text{adj } A| = |A|^{n-1}$. This allows us to find the value without computing the actual adjoint matrix.

Solution:

Step 1: Identify the given information from the question statement: The order of the square matrix A is $n = 3$. The determinant value of the matrix is $|A| = 5$.

Step 2: Recall the formula relating the determinant of an adjoint matrix to the original determinant:

$$|\text{adj } A| = |A|^{n-1}$$

Step 3: Substitute the known values $n = 3$ and $|A| = 5$ into the formula:

$$|\text{adj } A| = 5^{3-1}$$

Step 4: Compute the final value:

$$|\text{adj } A| = 5^2 = 25$$

Final Answer: 25

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept:

The area bounded by a curve $y = f(x)$ and a horizontal line $y = c$ can be found by integrating with respect to y using the formula $\int_d^c (x_{\text{right}} - x_{\text{left}})\, dy$, or by integrating with respect to x using the boundaries determined by the intersection points.

Solution:

Step 1: Find the intersection points between the parabola $y = x^2$ and the straight line $y = 4$:

$$x^2 = 4 \implies x = -2 \text{ and } x = 2$$

Step 2: Set up the area integral with respect to x from $x = -2$ to $x = 2$. The line $y = 4$ lies above the parabola $y = x^2$ in this interval:

$$\text{Area} = \int_{-2}^{2} (4 - x^2)\, dx$$

Step 3: Use the even function property to simplify the integration limits:

$$\text{Area} = 2\int_{0}^{2} (4 - x^2)\, dx$$

Step 4: Integrate the terms term-by-term:

$$\text{Area} = 2\left[4x - \frac{x^3}{3}\right]_0^2$$

Step 5: Apply the upper limit of integration:

$$\text{Area} = 2\left[4(2) - \frac{2^3}{3}\right] = 2\left[8 - \frac{8}{3}\right] = 2\left[\frac{24-8}{3}\right] = 2\left[\frac{16}{3}\right] = \frac{32}{3}$$

Final Answer: $\boxed{\frac{32}{3}}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept:

When two independent unbiased six-sided dice are thrown simultaneously, the total number of elementary outcomes in the sample space S is $6 \times 6 = 36$. The probability of an event is the ratio of favorable outcomes to total outcomes.

Solution:

Step 1: Determine the total number of outcomes in the sample space:

$$n(S) = 6 \cdot 6 = 36$$

Step 2: List all the ordered pairs (d_1, d_2) that result in a sum of 7:

$$\text{Event } E = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

Step 3: Count the total number of successful favorable outcomes:

$$n(E) = 6$$

Step 4: Calculate the probability using the classical definition:

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Final Answer: $\boxed{\frac{1}{6}}$

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept:

The sum formula for two inverse tangent functions states that $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$. Alternatively, terms can be rearranged to apply the subtraction formula $\tan^{-1} A - \tan^{-1} B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$.

Solution:

Step 1: Write the given inverse trigonometric equation:

$$\tan^{-1} x + \tan^{-1}(1) = \tan^{-1}(3)$$

Step 2: Rearrange the equation to isolate the term containing x on the left-hand side:

$$\tan^{-1} x = \tan^{-1}(3) - \tan^{-1}(1)$$

Step 3: Apply the inverse tangent subtraction formula on the right side:

$$\tan^{-1} x = \tan^{-1}\left(\frac{3-1}{1+3\cdot 1}\right)$$

Step 4: Simplify the fraction inside the parentheses:

$$\tan^{-1} x = \tan^{-1}\left(\frac{2}{1+3}\right) = \tan^{-1}\left(\frac{2}{4}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

Step 5: Compare both sides to find the value of x:

$$x = \frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept:

A first-order linear differential equation written in the standard form is given by $\frac{dy}{dx} + P(x)y = Q(x)$. The integrating factor (I.F.) for such a differential equation is computed using the exponential formula I.F. $= e^{\int P(x)\,dx}$.

Solution:

Step 1: Write down the given differential equation:

$$\frac{dy}{dx} + y\tan x = \sec x$$

Step 2: Compare this with the standard linear form $\frac{dy}{dx} + P(x)y = Q(x)$ to identify $P(x)$:

$$P(x) = \tan x$$

Step 3: Set up the formula for evaluating the Integrating Factor (I.F.):

$$\text{I.F.} = e^{\int P(x)\,dx} = e^{\int \tan x\,dx}$$

Step 4: Recall the standard integration rule for the tangent function, which is $\int \tan x\,dx = \log|\sec x|$:

$$\text{I.F.} = e^{\log|\sec x|}$$

Step 5: Use the exponential-logarithmic cancellation identity $e^{\log f(x)} = f(x)$ to simplify the expression:

$$\text{I.F.} = \sec x$$

Final Answer: $\boxed{\sec x}$

Answer: (B)

Go Back to Question 20

Q21.

Solution

Concept:

The shortest distance between two parallel lines given in the general forms $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is calculated using the standard formula $d = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$. The coefficients of x and y must be made identical first.

Solution:

Step 1: Write down the equations of the two given parallel lines:

$$\text{Line 1: } 3x + 4y - 9 = 0$$

$$\text{Line 2: } 6x + 8y + 12 = 0$$

Step 2: Divide Line 2 by 2 so that its x and y coefficients match those of Line 1:

$$\frac{6x + 8y + 12}{2} = \frac{0}{2} \implies 3x + 4y + 6 = 0$$

Step 3: Identify the coefficients A, B and the constants C_1, C_2 from the modified equations:

$$A = 3, \quad B = 4, \quad C_1 = -9, \quad C_2 = 6$$

Step 4: Substitute these values into the parallel distance formula:

$$d = \frac{|-9 - 6|}{\sqrt{3^2 + 4^2}}$$

Step 5: Evaluate the values in the numerator and denominator:

$$d = \frac{|-15|}{\sqrt{9 + 16}} = \frac{15}{\sqrt{25}} = \frac{15}{5} = 3 \text{ units}$$

Final Answer: 3 units

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept:

In a Geometric Progression (G.P.), the sum of the first n terms is given by $S_n = \frac{a(r^n-1)}{r-1}$, and the n-th (last) term is given by $l = ar^{n-1}$. This can be rewritten as $S_n = \frac{lr-a}{r-1}$, allowing us to solve for r directly without knowing n.

Solution:

Step 1: Write down the given values for the Geometric Progression: First term $(a) = 7$ Last term $(l) = 448$ Sum of terms $(S_n) = 889$

Step 2: Recall the formula for the sum of a G.P. containing the last term explicitly:

$$S_n = \frac{lr - a}{r - 1}$$

Step 3: Substitute the known numerical values into this equation:

$$889 = \frac{448r - 7}{r - 1}$$

Step 4: Cross-multiply to clear the denominator and expand the terms:

$$889(r - 1) = 448r - 7$$

$$889r - 889 = 448r - 7$$

Step 5: Isolate the terms involving r on one side of the equation:

$$889r - 448r = 889 - 7$$

$$441r = 882$$

Step 6: Solve for the common ratio r:

$$r = \frac{882}{441} = 2$$

Final Answer: 2

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept:

A classic integral theorem states that $\int e^x [f(x) + f'(x)]\, dx = e^x f(x) + C$. To evaluate integrals of this specific exponential type, we identify which function represents $f(x)$ and verify if the other term matches its exact derivative $f'(x)$.

Solution:

Step 1: Analyze the given integration problem:

$$\int e^x (\cot x + \log|\sin x|)\, dx$$

Step 2: Let us choose one of the terms inside the parentheses to be $f(x)$. Let:

$$f(x) = \log|\sin x|$$

Step 3: Find the derivative of $f(x)$ with respect to x using the chain rule:

$$f'(x) = \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x) = \frac{1}{\sin x} \cdot \cos x = \cot x$$

Step 4: Notice that the other term present inside the parentheses is exactly $f'(x) = \cot x$. The integral is written precisely in the standard form:

$$\int e^x [f'(x) + f(x)]\, dx = e^x f(x) + C$$

Step 5: Substitute $f(x) = \log|\sin x|$ back into the result:

$$e^x \log|\sin x| + C$$

Final Answer: $\boxed{e^x \log|\sin x| + C}$

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept:

When objects are drawn one by one without replacement, the events are dependent. The probability of both events occurring is found by multiplying the probability of the first event by the conditional probability of the second event, $P(R_1 \cap R_2) = P(R_1) \cdot P(R_2|R_1)$.

Solution:

Step 1: Count the initial number of balls in the bag:

$$\text{White balls} = 5, \quad \text{Red balls} = 7$$

$$\text{Total number of balls} = 5 + 7 = 12$$

Step 2: Calculate the probability of drawing a red ball on the first attempt ($P(R_1)$):

$$P(R_1) = \frac{\text{Number of red balls}}{\text{Total number of balls}} = \frac{7}{12}$$

Step 3: Since the ball is not replaced, update the contents of the bag for the second draw:

$$\text{Remaining red balls} = 7 - 1 = 6$$

$$\text{Remaining total balls} = 12 - 1 = 11$$

Step 4: Calculate the probability of drawing a red ball on the second attempt ($P(R_2|R_1)$):

$$P(R_2|R_1) = \frac{6}{11}$$

Step 5: Multiply the two consecutive probabilities to find the joint probability:

$$P(\text{Both red}) = \frac{7}{12} \cdot \frac{6}{11} = \frac{7 \cdot 1}{2 \cdot 11} = \frac{7}{22}$$

Final Answer: $\boxed{\frac{7}{22}}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept:

The composition of a function with itself, denoted as $f(f(x))$, is obtained by substituting the entire expression of the function $f(x)$ as the input variable wherever x appears within the definition of $f(x)$, followed by algebraic simplification.

Solution:

Step 1: Write down the definition of the given function:

$$f(x) = \frac{x-1}{x+1}$$

Step 2: Express the composite function $f(f(x))$ by substituting $f(x)$ into itself:

$$f(f(x)) = \frac{f(x)-1}{f(x)+1}$$

Step 3: Substitute the fractional expression of $f(x)$ into this equation:

$$f(f(x)) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}$$

Step 4: Take the common denominator $(x+1)$ in both the numerator and the denominator expressions:

$$f(f(x)) = \frac{\frac{(x-1)-(x+1)}{x+1}}{\frac{(x-1)+(x+1)}{x+1}}$$

Step 5: Cancel out the common denominator $(x+1)$ and simplify the remaining algebraic terms:

$$f(f(x)) = \frac{x-1-x-1}{x-1+x+1} = \frac{-2}{2x} = -\frac{1}{x}$$

Final Answer: $\boxed{-\frac{1}{x}}$

Answer: (C)

Go Back to Question 25

Q26.

Solution

Concept:

Three vectors $\vec{a}, \vec{b}$, and $\vec{c}$ are coplanar if and only if their scalar triple product is equal to zero, which means $[\vec{a} \quad \vec{b} \quad \vec{c}] = 0$. This condition can be evaluated by setting the determinant formed by their rectangular components to zero.

Solution:

Step 1: Extract the components of the three given vectors to form a 3×3 determinant:

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k} \implies [2, -1, 1]$$

$$\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k} \implies [1, 2, -3]$$

$$\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k} \implies [3, \lambda, 5]$$

Step 2: Set up the coplanarity determinant equation:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

Step 3: Expand the determinant along the first row:

$$2(2(5) - (-3)(\lambda)) - (-1)(1(5) - (-3)(3)) + 1(1(\lambda) - 2(3)) = 0$$

$$2(10 + 3\lambda) + 1(5 + 9) + 1(\lambda - 6) = 0$$

Step 4: Expand the brackets and collect the like terms together:

$$20 + 6\lambda + 14 + \lambda - 6 = 0$$

$$7\lambda + 28 = 0$$

Step 5: Isolate and solve for the unknown parameter λ:

$$7\lambda = -28 \implies \lambda = -4$$

Final Answer: $\boxed{-4}$

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept:

A differentiable function $f(x)$ is strictly decreasing in an interval if its first derivative is strictly less than zero ($f'(x) < 0$) for all values of x belonging to that interval. We can find this interval by solving the resulting quadratic inequality.

Solution:

Step 1: Write down the given polynomial function:

$$f(x) = 2x^3 - 9x^2 + 12x + 15$$

Step 2: Find the first derivative of the function with respect to x:

$$f'(x) = 6x^2 - 18x + 12$$

Step 3: Set the condition for a strictly decreasing function, which requires $f'(x) < 0$:

$$6x^2 - 18x + 12 < 0$$

Step 4: Divide the entire inequality by the positive constant 6 to simplify it:

$$x^2 - 3x + 2 < 0$$

Step 5: Factor the quadratic expression into linear factors:

$$(x - 1)(x - 2) < 0$$

Step 6: Determine the interval satisfying this inequality using the wavy curve method. The product is negative between the roots, which means:

$$1 < x < 2 \implies x \in (1, 2)$$

Final Answer: $\boxed{(1, 2)}$

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept:

To solve a separable differential equation, we algebraically manipulate the expression to group all terms containing the variable y and dy on one side, and all terms containing x and dx on the opposite side, followed by integration of both sides.

Solution:

Step 1: Write down the given differential equation:

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

Step 2: Use the laws of exponents to factor out the common term e^{-y} on the right side:

$$\frac{dy}{dx} = e^{-y}(e^x + x^2)$$

Step 3: Separate the variables by multiplying both sides by e^y and moving dx to the right side:

$$e^y\, dy = (e^x + x^2)\, dx$$

Step 4: Integrate both sides of the equation independently:

$$\int e^y\, dy = \int (e^x + x^2)\, dx$$

Step 5: Compute the standard integrals and add the constant of integration C:

$$e^y = e^x + \frac{x^3}{3} + C$$

Final Answer: $\boxed{e^y = e^x + \frac{x^3}{3} + C}$

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept:

The general equation of a circle is represented as $x^2 + y^2 + 2gx + 2fy + c = 0$. The radius r of this circle can be computed using the standard algebraic formula $r = \sqrt{g^2 + f^2 - c}$.

Solution:

Step 1: Write down the given general equation of the circle:

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

Step 2: Compare it with the general form $x^2 + y^2 + 2gx + 2fy + c = 0$ to find the values of g, f, and c:

$$2g = -4 \implies g = -2$$

$$2f = 6 \implies f = 3$$

$$c = -12$$

Step 3: Write down the formula for the radius of a circle:

$$r = \sqrt{g^2 + f^2 - c}$$

Step 4: Substitute the identified values into the radius formula:

$$r = \sqrt{(-2)^2 + (3)^2 - (-12)}$$

Step 5: Simplify the terms inside the square root radical sign:

$$r = \sqrt{4 + 9 + 12} = \sqrt{25} = 5$$

Final Answer: $\boxed{5}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept:

Using the double-angle trigonometric identity for cosine, we know that $\cos(2\theta) = 1 - 2\sin^2\theta$. By setting $\theta = \sin^{-1} x$, the expression simplifies significantly because $\sin(\sin^{-1} x) = x$.

Solution:

Step 1: Identify the trigonometric expression to be evaluated:

$$\cos(2\sin^{-1} x)$$

Step 2: Let $\sin^{-1} x = \theta$, which implies that $\sin\theta = x$. The expression becomes:

$$\cos(2\theta)$$

Step 3: Use the standard double-angle identity for cosine in terms of sine:

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

Step 4: Replace $\sin\theta$ back with x to get a simplified algebraic form:

$$\cos(2\sin^{-1} x) = 1 - 2x^2$$

Step 5: Substitute the given value of $x = \frac{1}{3}$ into the simplified formula:

$$\text{Value} = 1 - 2\left(\frac{1}{3}\right)^2$$

Step 6: Perform the remaining fractional arithmetic:

$$\text{Value} = 1 - 2\left(\frac{1}{9}\right) = 1 - \frac{2}{9} = \frac{7}{9}$$

Final Answer: $\boxed{\frac{7}{9}}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept:

For the complex imaginary cube roots of unity, ω satisfies two fundamental properties: $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$. We use these substitution relations to simplify high powers of expressions involving ω.

Solution:

Step 1: Recall the core algebraic properties of the cube roots of unity:

$$1 + \omega^2 = -\omega \quad \text{and} \quad 1 + \omega = -\omega^2$$

Step 2: Take the first term $(1 - \omega + \omega^2)^5$ and substitute $1 + \omega^2 = -\omega$:

$$(1 + \omega^2 - \omega)^5 = (-\omega - \omega)^5 = (-2\omega)^5 = -32\omega^5$$

Step 3: Simplify ω^5 using the property $\omega^3 = 1$:

$$\omega^5 = \omega^3 \cdot \omega^2 = 1 \cdot \omega^2 = \omega^2 \implies -32\omega^2$$

Step 4: Take the second term $(1 + \omega - \omega^2)^5$ and substitute $1 + \omega = -\omega^2$:

$$(-\omega^2 - \omega^2)^5 = (-2\omega^2)^5 = -32\omega^{10}$$

Step 5: Simplify ω^{10} using the property $\omega^3 = 1$:

$$\omega^{10} = (\omega^3)^3 \cdot \omega = 1^3 \cdot \omega = \omega \implies -32\omega$$

Step 6: Add the two simplified terms together and factor out -32:

$$-32\omega^2 - 32\omega = -32(\omega^2 + \omega)$$

Step 7: Substitute $\omega^2 + \omega = -1$ from the identity $1 + \omega + \omega^2 = 0$:

$$-32(-1) = 32$$

Final Answer: 32

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept:

According to the fundamental theorem of Linear Programming, the maximum or minimum value of a linear objective function occurs at one of the corner points (vertices) of the bounded feasible region determined by the constraints.

Solution:

Step 1: Identify the objective function and the constraints:

$$\text{Maximize } z = 3x + 4y$$

$$\text{Subject to: } x + y \leq 4, \quad x \geq 0, \quad y \geq 0$$

Step 2: Determine the vertices of the feasible region formed by the inequalities. The boundary line $x + y = 4$ intersects the axes at $(4, 0)$ and $(0, 4)$. Combined with $x \geq 0, y \geq 0$, the corner points are:

$$O(0, 0), \quad A(4, 0), \quad B(0, 4)$$

Step 3: Evaluate the value of the objective function z at each of these three corner points: At $O(0, 0) : z = 3(0) + 4(0) = 0$ At $A(4, 0) : z = 3(4) + 4(0) = 12$ At $B(0, 4) : z = 3(0) + 4(4) = 16$
Step 4: Compare the calculated values to find the maximum value. The largest value obtained is 16, occurring at vertex $(0, 4)$.

Final Answer: 16

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept:

The coefficient of variation (C.V.) is a relative measure of dispersion defined as the percentage ratio of the standard deviation (σ) to the arithmetic mean (μ or $\bar{x}$). The algebraic formula is written as C.V. = $\left(\frac{\sigma}{\bar{x}}\right) \times 100$.

Solution:

Step 1: Write down the statistical values provided in the problem:

$$\text{Coefficient of Variation (C.V.)} = 60\%$$

$$\text{Standard Deviation } (\sigma) = 12$$

Step 2: State the standard formula relating these statistical measures:

$$\text{C.V.} = \left(\frac{\sigma}{\bar{x}}\right) \times 100$$

Step 3: Substitute the known numerical values into the equation:

$$60 = \left(\frac{12}{\bar{x}}\right) \times 100$$

Step 4: Isolate the variable for the arithmetic mean $\bar{x}$:

$$60 \cdot \bar{x} = 1200$$

$$\bar{x} = \frac{1200}{60}$$

Step 5: Calculate the final quotient:

$$\bar{x} = 20$$

Final Answer: 20

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept:

The standard equation of a horizontal ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$. The eccentricity e, which measures the flatness of the ellipse, is computed using the structural relationship formula $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution:

Step 1: Write down the given equation of the ellipse:

$$9x^2 + 25y^2 = 225$$

Step 2: Convert the equation into standard form by dividing both sides by 225:

$$\frac{9x^2}{225} + \frac{25y^2}{225} = \frac{225}{225}$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Step 3: Compare this with the standard equation to identify a^2 and b^2:

$$a^2 = 25, \quad b^2 = 9 \quad (\text{Since } 25 > 9, \text{ it is a horizontal ellipse})$$

Step 4: Write down the eccentricity formula and substitute the values:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$e = \sqrt{1 - \frac{9}{25}}$$

Step 5: Simplify the fraction inside the square root sign:

$$e = \sqrt{\frac{25 - 9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Final Answer: $\boxed{\frac{4}{5}}$

Answer: (B)

Go Back to Question 34

Q35.

Solution

Concept:

The angle θ between two non-zero vectors $\vec{a}$ and $\vec{b}$ is determined using the definition of the scalar dot product, which states that $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$. This can be rewritten as $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$.

Solution:

Step 1: Compute the scalar dot product of the two given vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$:

$$\vec{a} \cdot \vec{b} = (1)(1) + (1)(-1) + (-1)(1)$$

$$\vec{a} \cdot \vec{b} = 1 - 1 - 1 = -1$$

Step 2: Calculate the magnitude of vector $\vec{a}$:

$$|\vec{a}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$$

Step 3: Calculate the magnitude of vector $\vec{b}$:

$$|\vec{b}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$$

Step 4: Substitute these values into the cosine angle formula:

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{-1}{\sqrt{3} \cdot \sqrt{3}} = -\frac{1}{3}$$

Step 5: Express θ explicitly as an inverse cosine function:

$$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$$

Final Answer: $\boxed{\cos^{-1}\left(-\frac{1}{3}\right)}$

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept:

For a real-valued function involving a square root, $f(x) = \sqrt{g(x)}$, the domain consists of all real numbers x for which the expression inside the radical is non-negative, meaning $g(x) \geq 0$.

Solution:

Step 1: Write down the definition of the function:

$$f(x) = \sqrt{9 - x^2}$$

Step 2: Set the expression inside the square root to be greater than or equal to zero to ensure real values:

$$9 - x^2 \geq 0$$

Step 3: Rearrange the inequality by multiplying by -1 and reversing the inequality sign:

$$x^2 - 9 \leq 0$$

Step 4: Factor the algebraic expression using the difference of squares identity:

$$(x - 3)(x + 3) \leq 0$$

Step 5: Solve the inequality to find the valid range for x. The product is less than or equal to zero between its critical values:

$$-3 \leq x \leq 3 \implies x \in [-3, 3]$$

Final Answer: $\boxed{[-3, 3]}$

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept:

Two matrices are equal if and only if their corresponding elements are exactly equal. By equating corresponding rows of the given column matrices, we obtain a system of three linear equations that can be solved simultaneously.

Solution:

Step 1: Equate the corresponding entries of the two column vectors to form a system of equations:

$$x + y + z = 9 \quad \text{— (Equation 1)}$$

$$x + z = 5 \quad \text{— (Equation 2)}$$

$$y + z = 7 \quad \text{— (Equation 3)}$$

Step 2: Substitute Equation 3 ($y + z = 7$) directly into Equation 1:

$$x + (7) = 9 \implies x = 9 - 7 = 2$$

Step 3: Substitute the value of $x = 2$ into Equation 2 to find z:

$$2 + z = 5 \implies z = 5 - 2 = 3$$

Step 4: Substitute the value of $z = 3$ into Equation 3 to find y:

$$y + 3 = 7 \implies y = 7 - 3 = 4$$

Step 5: Collect the values of x, y, z in order:

$$x = 2, \quad y = 4, \quad z = 3$$

Final Answer: $2, 4, 3$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept:

To evaluate integrals by substitution, we look for a function $u = g(x)$ whose derivative $du = g'(x)dx$ is also present as a factor in the integrand, converting it into a simpler standard form.

Solution:

Step 1: Write down the given integration problem:

$$\int \frac{1}{x(1+\log x)}\,dx$$

Step 2: Choose a suitable substitution variable u. Let:

$$u = 1 + \log x$$

Step 3: Differentiate u with respect to x to find the differential relation:

$$dx \implies \frac{du}{dx} = \frac{1}{x} \implies du = \frac{1}{x}\,dx$$

Step 4: Rewrite the original integral in terms of the new variable u:

$$\int \frac{1}{1+\log x} \cdot \left(\frac{1}{x}\,dx\right) = \int \frac{1}{u}\,du$$

Step 5: Evaluate the standard integral, which results in a natural logarithm function:

$$\int \frac{1}{u}\,du = \log|u| + C$$

Step 6: Substitute back the original expression for u:

$$\log|1 + \log x| + C$$

Final Answer: $\boxed{\log|1+\log x| + C}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept:

According to the addition rule of probability for non-mutually exclusive events, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Here, the events are drawing a king and drawing a spade from a standard card deck.

Solution:

Step 1: Identify the total number of cards in a standard deck:

$$n(S) = 52$$

Step 2: Find the probability of drawing a king ($P(K)$). There are 4 kings in a deck:

$$P(K) = \frac{4}{52}$$

Step 3: Find the probability of drawing a spade ($P(S)$). There are 13 spade cards in a deck:

$$P(S) = \frac{13}{52}$$

Step 4: Find the probability of drawing a card that is both a king and a spade ($P(K \cap S)$). There is exactly 1 king of spades:

$$P(K \cap S) = \frac{1}{52}$$

Step 5: Apply the probability addition formula to find the combined probability:

$$P(K \cup S) = P(K) + P(S) - P(K \cap S)$$

$$P(K \cup S) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$$

Step 6: Reduce the fraction to its lowest terms by dividing by 4:

$$P(K \cup S) = \frac{4}{13}$$

Final Answer: $\boxed{\frac{4}{13}}$

Answer: (B)

Go Back to Question 39

Q40.

Solution

Concept:

An algebraic limit of the form $\lim_{x\to a} \frac{x^n - a^n}{x-a}$ can be evaluated using the standard algebraic limit formula $n \cdot a^{n-1}$. Alternatively, it can be solved by factoring the numerator completely.

Solution:

Step 1: State the given limit expression:

$$\lim_{x\to 2} \frac{x^4 - 16}{x - 2}$$

Step 2: Rewrite the constant 16 as a power with base 2 to match the standard formula format:

$$16 = 2^4 \implies \lim_{x\to 2} \frac{x^4 - 2^4}{x - 2}$$

Step 3: Recall the standard algebraic limit theorem formula:

$$\lim_{x\to a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$$

Step 4: Identify the corresponding parameter values from our limit expression:

$$a = 2, \quad n = 4$$

Step 5: Substitute these values into the formula to calculate the limit:

$$\text{Limit} = 4 \cdot 2^{4-1} = 4 \cdot 2^3 = 4 \cdot 8 = 32$$

Final Answer: 32

Answer: (C)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	A	4	C	5	A
6	B	7	B	8	C	9	B	10	C
11	B	12	C	13	A	14	B	15	A
16	B	17	A	18	B	19	A	20	B
21	A	22	A	23	B	24	A	25	C
26	B	27	A	28	A	29	A	30	A
31	A	32	B	33	A	34	B	35	B
36	B	37	A	38	A	39	B	40	C