Rajasthan JET Mathematics Sample Paper-11

Duration: 40 Minutes **Maximum Marks: 160**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **−1 marks**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. Let $A = \{x \in \mathbb{R} : x \geq 0\}$ and a function $f : A \to A$ be defined by $f(x) = x^2$. Then the function f is:

(A) One-one but not onto

(B) Onto but not one-one

(C) Both one-one and onto

(D) Neither one-one nor onto

Q2. If $z = \frac{1+i}{1-i}$, then the value of $z^{100} + z^{50}$ is equal to:

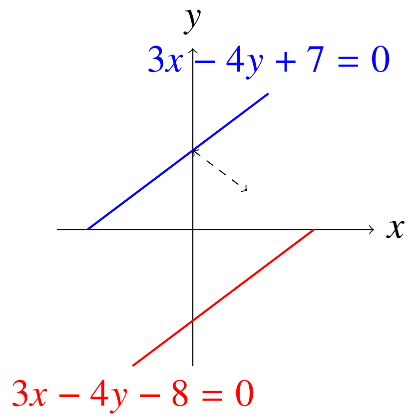
(A) 0

(B) 1

(C) 2

(D) -1

Q3. The distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y - 8 = 0$ is:



(A) 3 units

(B) 5 units

(C) 1 unit

(D) 15 units

Q4. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, then the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ is:

(A) $\frac{1}{\sqrt{75}}(\hat{i} + 7\hat{j} + 5\hat{k})$

(B) $\frac{1}{\sqrt{75}}(\hat{i} - 7\hat{j} - 5\hat{k})$

(C) $\frac{1}{\sqrt{75}}(-\hat{i} + 7\hat{j} + 5\hat{k})$

(D) $\frac{1}{\sqrt{75}}(\hat{i} + 7\hat{j} - 5\hat{k})$

Q5. The value of $\lim_{x \to 0} \frac{1 - \cos 4x}{x^2}$ is:

(A) 2

(B) 4

(C) 8

(D) 16

Q6. The contrapositive of the statement "If a triangle is equilateral, then it is isosceles" is:

(A) If a triangle is isosceles, then it is equilateral.

(B) If a triangle is not equilateral, then it is not isosceles.

(C) If a triangle is not isosceles, then it is not equilateral.

(D) A triangle is equilateral if and only if it is isosceles.

Q7. If the mean of five observations $x, x+2, x+4, x+6, x+8$ is 11, then the mean of the first three observations is:

(A) 9

(B) 11

(C) 13

(D) 7

Q8. The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is:

(A) $\frac{2\pi}{3}$

(B) $\frac{\pi}{3}$

(C) $-\frac{\pi}{3}$

(D) $\frac{4\pi}{3}$

Q9. Let R be a relation on the set of natural numbers $\mathbb{N}$ defined by aRb if $a+2b=10$. The domain of the relation R is:

(A) $\{2, 4, 6, 8\}$

(B) $\{1, 2, 3, 4\}$

(C) $\{1, 3, 5, 7\}$

(D) $\{2, 4, 6\}$

Q10. If the matrix $A = \begin{bmatrix} 2 & x \\ -1 & 3 \end{bmatrix}$ is such that $A^2 - 5A + 7I = O$, then the value of x is:

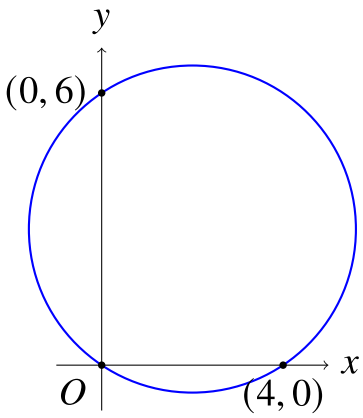
(A) 1

(B) -1

(C) 2

(D) -2

Q11. The equation of the circle passing through the origin and making intercepts 4 and 6 on the positive x and y axes respectively is:



(A) $x^2 + y^2 - 4x - 6y = 0$

(B) $x^2 + y^2 + 4x + 6y = 0$

(C) $x^2 + y^2 - 8x - 12y = 0$

(D) $x^2 + y^2 - 2x - 3y = 0$

Q12. The direction cosines of a line segment joining the points $A(1, 2, -3)$ and $B(-1, -2, 1)$ are:

(A) $\frac{-2}{6}, \frac{-4}{6}, \frac{4}{6}$

(B) $\frac{2}{6}, \frac{4}{6}, \frac{-4}{6}$

(C) $\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$

(D) $\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}$

Q13. The maximum value of the function $f(x) = x^3 - 3x$ on the interval $[0, 2]$ is achieved at x equal to:

(A) 0

(B) 1

(C) 2

(D) $\sqrt{3}$

Q14. Which of the following is logically equivalent to $\sim (p \vee q)$?

(A) $\sim p \vee \sim q$

(B) $\sim p \wedge \sim q$

(C) $\sim p \vee q$

(D) $p \wedge \sim q$

Q15. Two cards are drawn one by one without replacement from a well-shuffled pack of 52 cards. The probability that both cards are aces is:

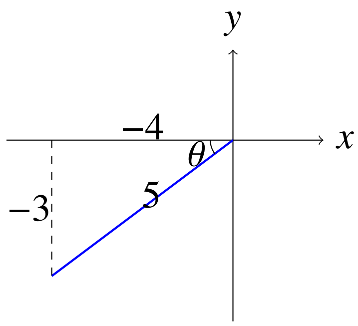
(A) $\frac{1}{221}$

(B) $\frac{1}{169}$

(C) $\frac{1}{26}$

(D) $\frac{3}{676}$

Q16. If $\tan\theta = \frac{3}{4}$ and θ lies in the third quadrant, then the value of $\cos\theta$ is:



(A) $\frac{4}{5}$

(B) $-\frac{4}{5}$

(C) $\frac{3}{5}$

(D) $-\frac{3}{5}$

Q17. The domain of the function $f(x) = \frac{1}{\sqrt{x^2-9}}$ is:

(A) $(-\infty, -3] \cup [3, \infty)$

(B) $(-3, 3)$

(C) $(-\infty, -3) \cup (3, \infty)$

(D) $\mathbb{R} \setminus \{-3, 3\}$

Q18. If the 3rd and 8th terms of an Arithmetic Progression (AP) are 7 and 22 respectively, then its 20th term is:

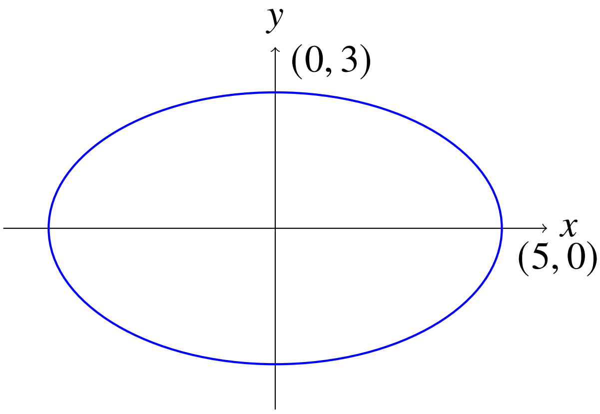
(A) 55

(B) 58

(C) 61

(D) 64

Q19. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is:



(A) $\frac{4}{5}$

(B) $\frac{3}{5}$

(C) $\frac{16}{25}$

(D) $\frac{2}{5}$

Q20. The angle between the lines $\frac{x-1}{2} = \frac{y+3}{3} = \frac{z-4}{-1}$ and $\frac{x+2}{1} = \frac{y-1}{1} = \frac{z+5}{5}$ is:

(A) $0°$

(B) $45°$

(C) $60°$

(D) $90°$

Q21. The value of the integral $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx$ is:

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) π

(D) 0

Q22. The statement $p \vee \sim p$ is a:

(A) Tautology

(B) Contradiction

(C) Contingency

(D) None of these

Q23. If a die is rolled and a coin is tossed simultaneously, the total number of sample points in the sample space is:

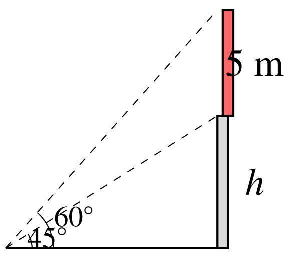
(A) 8

(B) 12

(C) 36

(D) 24

Q24. A flagstaff of height 5 meters stands on the top of a building. From a point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60° and 45° respectively. The height of the building is:



(A) $\frac{5}{\sqrt{3}-1}$ meters

(B) $5(\sqrt{3}+1)$ meters

(C) $\frac{5}{\sqrt{3}+1}$ meters

(D) $5(\sqrt{3}-1)$ meters

Q25. If $f(x) = \frac{x+1}{x-1}$, then the composite function $(f \circ f)(x)$ is equal to:

(A) x

(B) $\frac{1}{x}$

(C) $-x$

(D) x^2

Q26. If the value of the determinant $\begin{vmatrix} x & 2 \\ 4 & 3 \end{vmatrix} = 7$, then the value of x is:

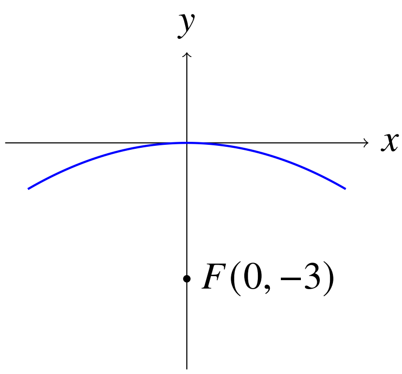
(A) 5

(B) -5

(C) 3

(D) 1

Q27. The equation of the parabola with vertex at the origin and focus at $(0, -3)$ is:



(A) $y^2 = -12x$

(B) $x^2 = -12y$

(C) $x^2 = 12y$

(D) $y^2 = 12x$

Q28. The shortest distance between the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$ can be evaluated by checking if they intersect. If these lines intersect, the shortest distance between them is:

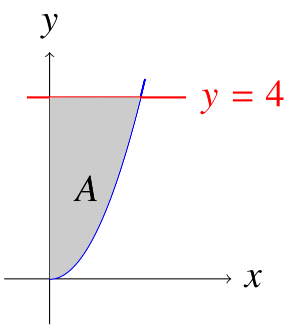
(A) 2 units

(B) 1 unit

(C) 0 units

(D) 4 units

Q29. The area of the region bounded by the curve $y = x^2$ and the line $y = 4$ in the first quadrant is:



(A) $\frac{16}{3}$ sq. units

(B) $\frac{8}{3}$ sq. units

(C) $\frac{4}{3}$ sq. units

(D) $\frac{32}{3}$ sq. units

Q30. The negation of the statement "All natural numbers are integers" is:

(A) No natural numbers are integers.

(B) Some natural numbers are not integers.

(C) All integers are natural numbers.

(D) There is no natural number which is not an integer.

Q31. If the variance of a data set is 64, then its standard deviation is:

(A) 64

(B) 32

(C) 8

(D) 4

Q32. The value of $\cos 15° - \sin 15°$ is equal to:

(A) $\frac{1}{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\sqrt{2}$

Q33. If A and B are symmetric matrices of the same order, then $AB - BA$ is a:

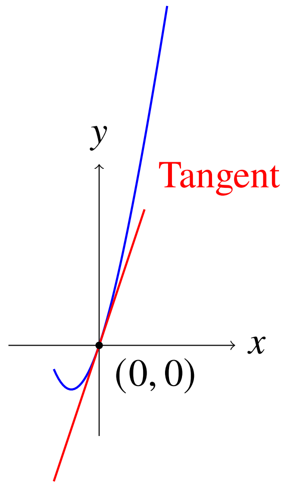
(A) Symmetric matrix

(B) Skew-symmetric matrix

(C) Identity matrix

(D) Zero matrix

Q34. The slope of the tangent to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:



(A) 3

(B) 2

(C) 0

(D) 5

Q35. The negation of the compound statement $p \wedge \sim q$ is given by:

(A) $\sim p \wedge q$

(B) $\sim p \vee q$

(C) $p \vee \sim q$

(D) $\sim p \vee \sim q$

Q36. Three coins are tossed simultaneously. The probability of getting at least two heads is:

(A) $\frac{1}{2}$

(B) $\frac{3}{8}$

(C) $\frac{1}{4}$

(D) $\frac{7}{8}$

Q37. If $\sin\alpha = \frac{3}{5}$ and $\cos\beta = \frac{5}{13}$ where α and β are acute angles, then the value of $\sin(\alpha + \beta)$ is:

(A) $\frac{56}{65}$

(B) $\frac{16}{65}$

(C) $\frac{63}{65}$

(D) $\frac{33}{65}$

Q38. If $\begin{bmatrix} x+y & 2 \\ 1 & x-y \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$, then the values of x and y are respectively:

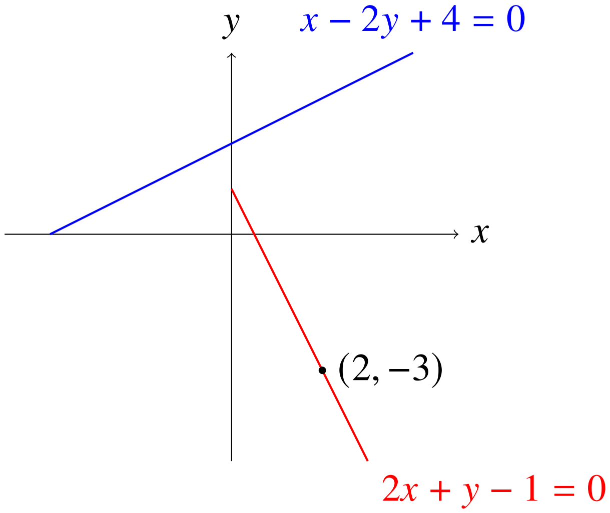
(A) $x = 4, y = 1$

(B) $x = 1, y = 4$

(C) $x = 3, y = 2$

(D) $x = 2, y = 3$

Q39. The equation of the line passing through $(2, -3)$ and perpendicular to the line $x - 2y + 4 = 0$ is:



(A) $2x + y - 1 = 0$

(B) $2x - y - 7 = 0$

(C) $x + 2y + 4 = 0$

(D) $2x + y + 1 = 0$

Q40. The projection of the vector $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$ on the vector $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$ is:

(A) $\frac{1}{3}$

(B) 1

(C) $\frac{5}{3}$

(D) 3

Detailed Solutions

Q1.

Solution

Concept: The definition of a function being one-one (injective) requires that distinct elements in the domain map to distinct elements in the codomain. A function is onto (surjective) if every element in the codomain has at least one pre-image in the domain.

Solution: Step 1: Consider the given function $f : A \to A$ where $A = \{x \in \mathbb{R} : x \geq 0\}$ and $f(x) = x^2$. Let us first test for the one-one property. Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$.

Step 2: This gives $x_1^2 = x_2^2$, which implies $x_1^2 - x_2^2 = 0$. Factoring the expression gives $(x_1 - x_2)(x_1 + x_2) = 0$. This means either $x_1 = x_2$ or $x_1 = -x_2$.

Step 3: Since the domain A contains only non-negative real numbers, both $x_1 \geq 0$ and $x_2 \geq 0$. Thus, $x_1 + x_2 = 0$ is only possible if $x_1 = x_2 = 0$. Therefore, we must have $x_1 = x_2$, which proves f is one-one.

Step 4: Now let us test for the onto property. Let y be any element in the codomain A, so $y \geq 0$. Set $f(x) = y$, which gives $x^2 = y$. Taking the square root gives $x = \sqrt{y}$.

Step 5: Since $y \geq 0$, the value $\sqrt{y}$ is always a well-defined real number and $\sqrt{y} \geq 0$, meaning $x \in A$. Thus, every element in the codomain has a valid pre-image in the domain, which proves f is onto.

Final Answer: $\boxed{\text{Both one-one and onto}}$

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept: The complex number expression can be simplified by rationalizing the denominator of the base term. Utilizing the properties of the imaginary unit i, where $i^2 = -1$, helps evaluate higher integer powers of i systematically.

Solution: Step 1: Analyze the base term $z = \frac{1+i}{1-i}$. To simplify this fraction, multiply both the numerator and the denominator by the complex conjugate of the denominator, which is $1 + i$.

Step 2: Perform the algebraic multiplication in the numerator and denominator:

$$z = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+2i+i^2}{1-i^2}$$

Step 3: Substitute $i^2 = -1$ into the expanded fraction expression:

$$z = \frac{1+2i-1}{1-(-1)} = \frac{2i}{2} = i$$

Step 4: Now substitute $z = i$ into the required expression $z^{100} + z^{50}$ to evaluate the powers:

$$z^{100} + z^{50} = i^{100} + i^{50}$$

Step 5: Simplify the powers of i by breaking them down into multiples of 4:

$$i^{100} = (i^4)^{25} = (1)^{25} = 1$$

$$i^{50} = (i^4)^{12} \cdot i^2 = (1)^{12} \cdot (-1) = -1$$

Step 6: Combine the calculated values to find the final scalar sum:

$$1 + (-1) = 0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept: The shortest distance d between two parallel straight lines given by the standard algebraic forms $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is determined using the coefficients formula $d = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$.

Solution: Step 1: Identify the coefficients from the given equations of the two parallel lines. The first line equation is $3x - 4y + 7 = 0$, giving $A = 3$, $B = -4$, and $C_1 = 7$.

Step 2: Identify the constants from the second parallel line equation, which is $3x - 4y - 8 = 0$, giving the constant term $C_2 = -8$.

Step 3: Substitute these values directly into the standard parallel line distance formula:

$$d = \frac{|7 - (-8)|}{\sqrt{3^2 + (-4)^2}}$$

Step 4: Simplify the absolute value expression in the numerator and the radical expression in the denominator:

$$d = \frac{|7 + 8|}{\sqrt{9 + 16}} = \frac{15}{\sqrt{25}}$$

Step 5: Compute the final numerical division to get the distance value:

$$d = \frac{15}{5} = 3 \text{ units}$$

Final Answer: $\boxed{\text{3 units}}$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept: A vector perpendicular to two given vectors $\vec{a}$ and $\vec{b}$ is obtained by calculating their vector cross product $\vec{a} \times \vec{b}$. The corresponding unit vector is found by dividing this cross product by its vector magnitude.

Solution: Step 1: Write down the given component vectors:

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$$

Step 2: Set up the matrix determinant form to compute the cross product $\vec{a} \times \vec{b}$:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 2 & -3 \end{vmatrix}$$

Step 3: Expand the determinant along the first row components:

$$\vec{a} \times \vec{b} = \hat{i}((-1)(-3) - (1)(2)) - \hat{j}((2)(-3) - (1)(1)) + \hat{k}((2)(2) - (-1)(1))$$

Step 4: Simplify the arithmetic scalar components:

$$\vec{a} \times \vec{b} = \hat{i}(3 - 2) - \hat{j}(-6 - 1) + \hat{k}(4 + 1) = \hat{i} + 7\hat{j} + 5\hat{k}$$

Step 5: Calculate the magnitude of the resulting cross product vector:

$$|\vec{a} \times \vec{b}| = \sqrt{1^2 + 7^2 + 5^2} = \sqrt{1 + 49 + 25} = \sqrt{75}$$

Step 6: Form the unit vector by dividing the vector by its magnitude:

$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{1}{\sqrt{75}}(\hat{i} + 7\hat{j} + 5\hat{k})$$

Final Answer: $\boxed{\frac{1}{\sqrt{75}}(\hat{i} + 7\hat{j} + 5\hat{k})}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: Standard trigonometric limits can be solved using the identity $1 - \cos\theta = 2\sin^2(\theta/2)$ or by applying L'Hopital's Rule when the limit presents an indeterminate form of type $0/0$.

Solution: Step 1: Check the form of the limit by substituting $x = 0$ into the expression $\lim_{x\to0} \frac{1-\cos 4x}{x^2}$. This gives $\frac{1-1}{0} = \frac{0}{0}$, which is indeterminate.

Step 2: Apply the trigonometric double angle identity where $\theta = 4x$. Thus, $1 - \cos 4x = 2\sin^2(2x)$. Substitute this back into the limit:

$$\lim_{x\to0} \frac{2\sin^2(2x)}{x^2}$$

Step 3: Rearrange the algebraic terms to match the standard fundamental limit form $\lim_{\theta\to0} \frac{\sin\theta}{\theta} = 1$:

$$\lim_{x\to0} 2 \cdot \left(\frac{\sin 2x}{x}\right)^2 = \lim_{x\to0} 2 \cdot \left(\frac{\sin 2x}{2x} \cdot 2\right)^2$$

Step 4: Bring the scalar constant factor outside the limit evaluation:

$$2 \cdot 4 \cdot \left(\lim_{x\to0} \frac{\sin 2x}{2x}\right)^2 = 8 \cdot (1)^2 = 8$$

Final Answer: $\boxed{8}$

Answer: (C)

Go Back to Question 5

Q6.

Solution

Concept: In mathematical logic, the contrapositive of a conditional statement of the form "If p, then q" (represented as $p \to q$) is logically defined as "If not q, then not p" (represented as $\sim q \to \sim p$).

Solution: Step 1: Identify the component propositions from the conditional statement. Let statement p be "a triangle is equilateral" and statement q be "it is isosceles".

Step 2: The given statement is structured in the formal logical connective template: "If p, then q".

Step 3: To construct the contrapositive statement, we must negate both statement components and reverse their conditional direction, forming the structure: "If not q, then not p".

Step 4: Translate the component negations back into natural language: "not q" translates to "a triangle is not isosceles", and "not p" translates to "it is not equilateral".

Step 5: Combine them into a single conditional statement: "If a triangle is not isosceles, then it is not equilateral."

Final Answer: If a triangle is not isosceles, then it is not equilateral.

Answer: (C)

Go Back to Question 6

Q7.

Solution

Concept: The arithmetic mean of a collection of numerical data observations is equal to the sum of all the observations divided by the total count of those observations.

Solution: Step 1: Write out the given five algebraic observations: $x, x+2, x+4, x+6, x+8$. The total number of data values is $n = 5$.

Step 2: Set up the statistical equation for the mean of these five values and equate it to 11:

$$\frac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5} = 11$$

Step 3: Combine the like terms in the numerator expression:

$$\frac{5x + 20}{5} = 11 \implies x + 4 = 11 \implies x = 7$$

Step 4: Find the values of the first three data observations by substituting $x = 7$:

$$x = 7, \quad x + 2 = 9, \quad x + 4 = 11$$

Step 5: Calculate the arithmetic mean of these first three numerical values:

$$\text{Mean} = \frac{7 + 9 + 11}{3} = \frac{27}{3} = 9$$

Final Answer: $\boxed{9}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: The principal value branch for the inverse sine function $\sin^{-1} x$ is strictly bounded within the closed interval $[-\pi/2, \pi/2]$. Any angle outside this range must be mapped back using trigonometric quadrant identities.

Solution: Step 1: Examine the given expression $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$. Notice that the angle $\frac{2\pi}{3}$ does not fall inside the principal value range because $\frac{2\pi}{3} > \frac{\pi}{2}$.

Step 2: Use the second quadrant sine reduction identity, $\sin(\pi - \theta) = \sin\theta$, to rewrite the inner trigonometric argument:

$$\sin\frac{2\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3}$$

Step 3: Substitute this equivalent value back into the original inverse expression:

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right)$$

Step 4: Since the angle $\frac{\pi}{3}$ lies within the valid principal interval $[-\pi/2, \pi/2]$, the inverse property applies directly:

$$\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept: The domain of a relation consists of all the first elements (input values) of the ordered pairs that satisfy the defining condition, restricted to the specified universal set (in this case, natural numbers $\mathbb{N}$).

Solution: Step 1: Consider the given relation condition equation $a + 2b = 10$, where both variables a and b must belong to the set of natural numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$.

Step 2: Rearrange the equation to isolate the variable a in terms of b:

$$a = 10 - 2b$$

Step 3: Substitute sequential positive integer values for b to find corresponding natural numbers for a:
If $b = 1$: $a = 10 - 2(1) = 8 \in \mathbb{N}$
If $b = 2$: $a = 10 - 2(2) = 6 \in \mathbb{N}$
If $b = 3$: $a = 10 - 2(3) = 4 \in \mathbb{N}$
If $b = 4$: $a = 10 - 2(4) = 2 \in \mathbb{N}$
If $b = 5$: $a = 10 - 2(5) = 0 \notin \mathbb{N}$

Step 4: For any values $b > 4$, the resulting value of a becomes negative, which is not allowed. Collect all valid values of a to form the domain set:

$$\text{Domain} = \{2, 4, 6, 8\}$$

Final Answer: $\boxed{\{2, 4, 6, 8\}}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: Every square matrix satisfies its own characteristic equation according to the Cayley-Hamilton Theorem. Alternatively, this can be solved directly by computing the matrix powers and substituting them into the polynomial matrix equation.

Solution: Step 1: Write down the matrix $A = \begin{bmatrix} 2 & x \\ -1 & 3 \end{bmatrix}$. Compute the matrix multiplication A^2:

$$A^2 = \begin{bmatrix} 2 & x \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 & x \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 4 - x & 2x + 3x \\ -2 - 3 & -x + 9 \end{bmatrix} = \begin{bmatrix} 4 - x & 5x \\ -5 & 9 - x \end{bmatrix}$$

Step 2: Substitute A^2, $5A$, and $7I$ into the matrix equation $A^2 - 5A + 7I = O$:

$$\begin{bmatrix} 4 - x & 5x \\ -5 & 9 - x \end{bmatrix} - \begin{bmatrix} 10 & 5x \\ -5 & 15 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Step 3: Combine corresponding entries to form simplified element matrix equations:

$$\begin{bmatrix} (4 - x) - 10 + 7 & 5x - 5x + 0 \\ -5 - (-5) + 0 & (9 - x) - 15 + 7 \end{bmatrix} = \begin{bmatrix} 1 - x & 0 \\ 0 & 1 - x \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Step 4: Equate the non-zero element expressions to zero:

$$1 - x = 0 \implies x = 1$$

Final Answer: $\boxed{1}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 10

Q11.

Solution

Concept: A circle passing through the origin $(0, 0)$ with x-intercept a and y-intercept b passes through the coordinates $(a, 0)$ and $(0, b)$. The line segment joining these intercept points forms a diameter of the circle.

Solution: Step 1: The circle cuts the positive axes at $(4, 0)$ and $(0, 6)$, and passes through $(0, 0)$. This creates a right-angled triangle inscribed in the circle.

Step 2: Since the angle in a semicircle is a right angle ($90°$ at the origin), the line segment connecting $(4, 0)$ and $(0, 6)$ is a diameter.

Step 3: Use the diameter form of a circle equation with endpoints $(x_1, y_1) = (4, 0)$ and $(x_2, y_2) = (0, 6)$:

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Step 4: Substitute the coordinate endpoint coordinates into the equation:

$$(x - 4)(x - 0) + (y - 0)(y - 6) = 0$$

Step 5: Expand the algebraic products to obtain the final standard circle form:

$$x^2 - 4x + y^2 - 6y = 0 \implies x^2 + y^2 - 4x - 6y = 0$$

Final Answer: $\boxed{x^2 + y^2 - 4x - 6y = 0}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: The direction cosines (l, m, n) of a line segment directed from point $A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$ are given by the formulas $l = \frac{x_2 - x_1}{d}$, $m = \frac{y_2 - y_1}{d}$, and $n = \frac{z_2 - z_1}{d}$, where d is the total distance between the points.

Solution: Step 1: Identify the given coordinates of the two points: $A(1, 2, -3)$ and $B(-1, -2, 1)$.

Step 2: Find the components of the directed displacement vector $\vec{AB}$:

$$\Delta x = -1 - 1 = -2$$

$$\Delta y = -2 - 2 = -4$$

$$\Delta z = 1 - (-3) = 4$$

Step 3: Calculate the total distance magnitude d using the three-dimensional distance formula:

$$d = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$$

Step 4: Compute the direction cosines by dividing each coordinate component by the distance d:

$$l = \frac{-2}{6} = -\frac{1}{3}, \quad m = \frac{-4}{6} = -\frac{2}{3}, \quad n = \frac{4}{6} = \frac{2}{3}$$

Final Answer: $\boxed{\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}}$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: To find the absolute maximum value of a continuous function on a closed interval, evaluate the function values at its critical numbers (where the first derivative vanishes) and at the boundary endpoints of the interval.

Solution: Step 1: Write down the function $f(x) = x^3 - 3x$ and differentiate it with respect to x to find the critical points:

$$f'(x) = 3x^2 - 3$$

Step 2: Set the derivative function equal to zero to solve for the critical values:

$$3x^2 - 3 = 0 \implies x^2 = 1 \implies x = 1 \text{ or } x = -1$$

Step 3: Check which critical numbers lie inside the specified interval $[0, 2]$. Only $x = 1$ belongs to the interval. Discard $x = -1$.

Step 4: Evaluate the original function $f(x)$ at the critical point $x = 1$ and at the boundary endpoints $x = 0$ and $x = 2$:

$$f(0) = 0^3 - 3(0) = 0$$

$$f(1) = 1^3 - 3(1) = -2$$

$$f(2) = 2^3 - 3(2) = 8 - 6 = 2$$

Step 5: Compare the values: $0, -2$, and 2. The absolute maximum value is 2, which occurs exactly at the boundary point $x = 2$.

Final Answer: $\boxed{2}$

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 13

Q14.

Solution

Concept: According to De Morgan's Laws in mathematical logic and set theory, the negation of a disjunction statement is logically equivalent to the conjunction of the individual negations.

Solution: Step 1: Consider the given logical statement expression $\sim (p \vee q)$, which represents the negation of "p or q".

Step 2: De Morgan's first law states that when distributing a negation operator over a logical OR ($\vee$) operator, the OR symbol flips to a logical AND ($\wedge$) operator, and both propositions are negated.

Step 3: Apply this rule step-by-step:

$$\sim (p \vee q) \equiv \sim p \wedge \sim q$$

Step 4: This shows that the negation of a disjunction is logically identical to the separate negations of both p and q joined by a conjunction operator.

Final Answer: $\boxed{\sim p \wedge \sim q}$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept: The probability of dependent compound events occurring in succession without replacement is calculated using conditional probability, multiplying the probability of the first event by the updated probability of the second event.

Solution: Step 1: A standard deck contains a total of 52 cards, out of which exactly 4 cards are aces.

Step 2: Calculate the probability of drawing an ace on the first selection:

$$P(A_1) = \frac{4}{52} = \frac{1}{13}$$

Step 3: Since the card is drawn without replacement, the deck composition changes. There are now 51 total cards remaining, and exactly 3 aces left.

Step 4: Calculate the conditional probability of drawing another ace on the second selection:

$$P(A_2|A_1) = \frac{3}{51} = \frac{1}{17}$$

Step 5: Multiply the two independent stage probabilities to find the total joint probability:

$$P(A_1 \cap A_2) = P(A_1) \cdot P(A_2|A_1) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{13} \cdot \frac{1}{17} = \frac{1}{221}$$

Final Answer: $\boxed{\frac{1}{221}}$

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept: Trigonometric values can be determined using reference right triangles with side ratios. The final algebraic sign must be calibrated based on the specific quadrant in which the angle θ resides.

Solution: Step 1: We are given that $\tan\theta = \frac{3}{4}$. From right-angle ratios, $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$. Let the perpendicular be 3 and the base be 4.

Step 2: Calculate the hypotenuse length using the Pythagorean theorem:

$$\text{Hypotenuse} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Step 3: Write down the standard magnitude ratio for the cosine function:

$$|\cos\theta| = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$$

Step 4: Identify the correct algebraic sign based on the quadrant. We are given that θ lies in the third quadrant. In the third quadrant, the cosine function is strictly negative.

Step 5: Combine the ratio with the proper sign:

$$\cos\theta = -\frac{4}{5}$$

Final Answer: $\boxed{-\frac{4}{5}}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: For a real-valued radical fraction function of the form $\frac{1}{\sqrt{g(x)}}$ to be defined, the mathematical expression inside the square root in the denominator must be strictly greater than zero.

Solution: Step 1: Analyze the condition required for the function $f(x) = \frac{1}{\sqrt{x^2-9}}$ to yield real values. The denominator cannot be zero, and the radicand must be non-negative. Combining these gives:

$$x^2 - 9 > 0$$

Step 2: Factor the quadratic difference of squares expression:

$$(x-3)(x+3) > 0$$

Step 3: Determine the critical boundary points, which are $x = 3$ and $x = -3$. Apply the sign-chart method (wavy curve method) across the real number line intervals:
For $x > 3$, the expression is positive.
For $-3 < x < 3$, the expression is negative.
For $x < -3$, the expression is positive.

Step 4: Select the open intervals where the expression evaluates to strictly positive values:

$$x \in (-\infty, -3) \cup (3, \infty)$$

Final Answer: $\boxed{(-\infty, -3) \cup (3, \infty)}$

Answer: (C)

Go Back to Question 17

Q18.

Solution

Concept: The general n^{th} term of an Arithmetic Progression is given by $a_n = a + (n-1)d$, where a is the first term and d is the common difference. Two terms provide a system of linear equations.

Solution: Step 1: Write down the algebraic expressions for the two given terms of the sequence:

$$a_3 = a + 2d = 7$$

$$a_8 = a + 7d = 22$$

Step 2: Solve this system of linear equations by subtracting the first equation from the second equation:

$$(a + 7d) - (a + 2d) = 22 - 7 \implies 5d = 15 \implies d = 3$$

Step 3: Substitute $d = 3$ back into the first equation to find the value of the first term a:

$$a + 2(3) = 7 \implies a + 6 = 7 \implies a = 1$$

Step 4: Use the calculated values of $a = 1$ and $d = 3$ to evaluate the 20^{th} term expression:

$$a_{20} = a + 19d = 1 + 19(3)$$

Step 5: Simplify the arithmetic computation:

$$a_{20} = 1 + 57 = 58$$

Final Answer: $\boxed{58}$

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept: The standard form of a horizontally oriented ellipse equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The eccentricity is calculated using the relation formula $e = \sqrt{1 - \frac{b^2}{a^2}}$ when $a > b$.

Solution: Step 1: Convert the given equation $9x^2 + 25y^2 = 225$ into standard form by dividing both sides by 225:

$$\frac{9x^2}{225} + \frac{25y^2}{225} = 1 \implies \frac{x^2}{25} + \frac{y^2}{9} = 1$$

Step 2: Compare this with the standard form equation to determine the squared parameters:

$$a^2 = 25 \implies a = 5$$

$$b^2 = 9 \implies b = 3$$

Step 3: Since $a^2 > b^2$, this is a horizontal ellipse. Write out the eccentricity calculation formula:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Step 4: Substitute the values of a^2 and b^2 into the radical expression:

$$e = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{25 - 9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Final Answer: $\boxed{\frac{4}{5}}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: The angle θ between two lines in three-dimensional space is equal to the angle between their respective direction vector lines $\vec{b}_1$ and $\vec{b}_2$, calculated using the dot product formula $\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$.

Solution: Step 1: Extract the direction ratios from the denominators of the first line equation: $a_1 = 2, b_1 = 3, c_1 = -1$. This gives the direction vector $\vec{b}_1 = 2\hat{i} + 3\hat{j} - \hat{k}$.

Step 2: Extract the direction ratios from the second line equation denominators: $a_2 = 1, b_2 = 1, c_2 = 5$. This gives the direction vector $\vec{b}_2 = \hat{i} + \hat{j} + 5\hat{k}$.

Step 3: Compute the scalar dot product of the two direction vectors:

$$\vec{b}_1 \cdot \vec{b}_2 = (2)(1) + (3)(1) + (-1)(5)$$

$$\vec{b}_1 \cdot \vec{b}_2 = 2 + 3 - 5 = 0$$

Step 4: Since the vector dot product evaluates to exactly zero, the two direction vectors are perpendicular to each other.

Step 5: Therefore, the angle θ between the two lines is 90°.

Final Answer: $\boxed{90^\circ}$

Answer: (D)

Go Back to Question 20

Q21.

Solution

Concept: The definite integral property $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ (often called the King's Property) is useful for simplifying fractional integrals containing symmetric trigonometric functions.

Solution: Step 1: Let the given definite integral be represented by the variable equation:

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx \quad \text{— (Equation 1)}$$

Step 2: Apply the integration property by replacing the variable x with $(\frac{\pi}{2} - x)$ throughout the expression:

$$I = \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2} - x)}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)}\,dx$$

Step 3: Use the co-function trigonometric identities $\sin(\frac{\pi}{2} - x) = \cos x$ and $\cos(\frac{\pi}{2} - x) = \sin x$:

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\,dx \quad \text{— (Equation 2)}$$

Step 4: Add Equation 1 and Equation 2 together. Since their denominators are identical, combine the numerators:

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx = \int_0^{\pi/2} 1\,dx$$

Step 5: Integrate the constant function and evaluate across the upper and lower limits:

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2} \implies I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept: A compound logical statement is classified as a tautology if it evaluates to True (T) under all possible truth value assignments for its component propositions.

Solution: Step 1: Consider the logical disjunction statement $p \vee \sim p$, where the symbol $\vee$ represents the logical OR connective, and $\sim$ represents the negation operator.

Step 2: Let us construct a truth table analysis to evaluate the statement for all possible cases of proposition p.

Step 3: Case 1: Suppose the truth value of p is True (T). Then its negation $\sim p$ is False (F). The disjunction becomes $T \vee F = T$.

Step 4: Case 2: Suppose the truth value of p is False (F). Then its negation $\sim p$ is True (T). The disjunction becomes $F \vee T = T$.

Step 5: Since the truth value of the compound expression is always True (T) in every possible scenario, it forms a tautology by definition.

Final Answer: $\boxed{\text{Tautology}}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 22

Q23.

Solution

Concept: According to the Fundamental Counting Principle, if one event can occur in m independent ways and a second event can occur in n independent ways, then both events can occur together in $m \times n$ total ways.

Solution: Step 1: Analyze the first random experiment component. A standard fair die has six distinct faces numbered from 1 to 6. Thus, the number of outcomes for the die is $m = 6$.

Step 2: Analyze the second random experiment component. A standard fair coin has two distinct faces, namely Heads (H) and Tails (T). Thus, the number of outcomes for the coin is $n = 2$.

Step 3: Since the rolling of the die and the tossing of the coin are performed simultaneously, the two events are independent.

Step 4: Apply the fundamental multiplication counting rule to calculate the total size of the joint sample space:

$$\text{Total sample points} = m \times n = 6 \times 2 = 12$$

Final Answer: $\boxed{12}$

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept: Height and distance problems can be modeled using right-angle triangle trigonometry. The tangent function ($\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$) relates the heights to the horizontal ground distance.

Solution: Step 1: Let the height of the building be h meters, and let the horizontal distance from the observation point on the ground to the base of the building be x meters.

Step 2: A flagstaff of height 5 meters is on top of the building, making the total height from the ground to the top of the flagstaff equal to $(h + 5)$ meters.

Step 3: From the smaller right triangle representing the building, use the 45° angle of elevation:

$$\tan 45^\circ = \frac{h}{x} \implies 1 = \frac{h}{x} \implies x = h$$

Step 4: From the larger right triangle representing the building and flagstaff together, use the 60° angle of elevation:

$$\tan 60^\circ = \frac{h+5}{x} \implies \sqrt{3} = \frac{h+5}{h} \quad (\text{since } x = h)$$

Step 5: Solve this algebraic equation for h by cross-multiplying and isolating terms:

$$\sqrt{3}h = h + 5 \implies \sqrt{3}h - h = 5 \implies h(\sqrt{3} - 1) = 5$$

$$h = \frac{5}{\sqrt{3} - 1} \text{ meters}$$

Final Answer: $\boxed{\frac{5}{\sqrt{3}-1} \text{ meters}}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: A composite function $(f \circ f)(x)$ is evaluated by substituting the entire definition of the function $f(x)$ as the input variable back into the function f itself, followed by algebraic simplification.

Solution: Step 1: Write down the definition of the function: $f(x) = \frac{x+1}{x-1}$. The composition is defined as:

$$(f \circ f)(x) = f(f(x))$$

Step 2: Substitute the expression for $f(x)$ into the input positions of the function formula:

$$f(f(x)) = \frac{f(x)+1}{f(x)-1} = \frac{\left(\frac{x+1}{x-1}\right)+1}{\left(\frac{x+1}{x-1}\right)-1}$$

Step 3: Find a common denominator for both the numerator expression and the denominator expression:

$$f(f(x)) = \frac{\frac{(x+1)+(x-1)}{x-1}}{\frac{(x+1)-(x-1)}{x-1}}$$

Step 4: Cancel out the common $(x-1)$ terms in the denominators:

$$f(f(x)) = \frac{(x+1)+(x-1)}{(x+1)-(x-1)}$$

Step 5: Simplify the remaining algebraic terms in both parts:

$$f(f(x)) = \frac{2x}{2} = x$$

Final Answer: $\boxed{\text{x}}$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept: The value of a second-order determinant is evaluated by calculating the product of the principal diagonal elements and subtracting the product of the secondary diagonal elements: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Solution: Step 1: Write down the given determinant equation expression:

$$\begin{vmatrix} x & 2 \\ 4 & 3 \end{vmatrix} = 7$$

Step 2: Expand the determinant on the left-hand side using diagonal cross-multiplication:

$$(x \cdot 3) - (2 \cdot 4) = 7$$

Step 3: Simplify the products to form a linear equation:

$$3x - 8 = 7$$

Step 4: Isolate the term containing the variable x by adding 8 to both sides:

$$3x = 7 + 8 \implies 3x = 15$$

Step 5: Divide by 3 to find the value of x:

$$x = \frac{15}{3} = 5$$

Final Answer: $\boxed{5}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 26

Q27.

Solution

Concept: A parabola with its vertex at the origin $(0, 0)$ that has its focus on the negative y-axis at $(0, -a)$ is vertically oriented and opens downward. Its standard geometric equation is $x^2 = -4ay$.

Solution: Step 1: Identify the coordinates of the focus point provided: $F(0, -3)$. Notice that the focus lies on the y-axis because the x-coordinate is zero, and it is specifically on the negative side.

Step 2: Since the focus is at $(0, -3)$ and the vertex is at $(0, 0)$, the distance parameter value is $a = 3$.

Step 3: Recall the standard equation for a downward-opening vertical parabola:

$$x^2 = -4ay$$

Step 4: Substitute the value $a = 3$ directly into the standard conic equation:

$$x^2 = -4(3)y \implies x^2 = -12y$$

Final Answer: $\boxed{x^2 = -12y}$

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept: If two lines in three-dimensional space intersect each other at a common point, they share a physical location, which means the shortest perpendicular distance between them is exactly zero.

Solution: Step 1: The problem states that the lines intersect each other. Let us verify this conceptually. The shortest distance between two vector lines is measured along a mutual perpendicular line.

Step 2: If two lines are skewed, the shortest distance is a positive value. If they are parallel, it is also positive. However, if they actively intersect, they cross at a point.

Step 3: At the point of intersection, the distance between the two lines reduces to nothing.

Step 4: Therefore, without performing any extensive vector determinant calculations, the geometric definition of intersecting lines confirms that their shortest distance must be 0.

Final Answer: $\boxed{0 \text{ units}}$

Answer: (C)

Go Back to Question 28

Q29.

Solution

Concept: The area bounded by a curve along the y-axis from $y = c$ to $y = d$ is calculated using the definite integral $\int_c^d x\,dy$, where x is expressed as a function of y.

Solution: Step 1: Identify the bounding regions. The curve is given by $y = x^2$, which means $x = \sqrt{y}$ for the region in the first quadrant. The horizontal line boundary is $y = 4$.

Step 2: The region is bounded by the y-axis ($x = 0$), the curve, and the line $y = 4$ in the first quadrant. Thus, the integration limits along the y-axis run from $y = 0$ to $y = 4$.

Step 3: Set up the area integration formula with respect to the variable y:

$$\text{Area} = \int_0^4 x\,dy = \int_0^4 \sqrt{y}\,dy$$

Step 4: Compute the anti-derivative of $y^{1/2}$ using the power integration rule:

$$\int y^{1/2}\,dy = \frac{y^{3/2}}{3/2} = \frac{2}{3}y^{3/2}$$

Step 5: Evaluate this anti-derivative at the boundary upper and lower limits:

$$\text{Area} = \left[\frac{2}{3}y^{3/2}\right]_0^4 = \frac{2}{3}(4^{3/2} - 0) = \frac{2}{3}(8) = \frac{16}{3}\text{ sq. units}$$

Final Answer: $\boxed{\frac{16}{3}\text{ sq. units}}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: In predicate logic, the negation of a universally quantified statement of the form "All S are P" is logically equivalent to the existentially quantified statement "Some S are not P".

Solution: Step 1: Analyze the given statement structure: "All natural numbers are integers". This is a universal statement asserting a property for every single element in the set of natural numbers.

Step 2: To falsify or negate a universal "All" statement, you do not need to prove that it applies to no elements at all. You only need to show that there exists at least one exception.

Step 3: Therefore, the mathematical negation must state that there is at least one natural number that does not belong to the set of integers.

Step 4: Express this existential exception using standard logical terms: "Some natural numbers are not integers."

Final Answer: $\boxed{\text{Some natural numbers are not integers.}}$

Answer: (B)

Go Back to Question 30

Q31.

Solution

Concept: In statistics, the standard deviation (σ) of a statistical population or data distribution is defined as the positive square root of its mathematical variance (σ^2).

Solution: Step 1: Identify the statistical relationship formula connecting variance and standard deviation:

$$\text{Standard Deviation} = \sqrt{\text{Variance}}$$

Step 2: We are given that the variance value for the data set is equal to 64. Let $\sigma^2 = 64$.

Step 3: Take the principal positive square root of this variance number:

$$\sigma = \sqrt{64}$$

Step 4: Simplify the radical to find the final integer value:

$$\sigma = 8$$

Note that standard deviation is a measure of dispersion magnitude, so it is always a non-negative value.

Final Answer: $\boxed{8}$

Answer: (C)

Go Back to Question 31

Q32.

Solution

Concept: Trigonometric expressions containing non-standard angles like 15° can be evaluated by converting them using linear combinations like $\cos A - \sin A = \sqrt{2}\cos(A + 45^\circ)$ or by using subtraction formulas.

Solution: Step 1: Let us rewrite the expression $\cos 15^\circ - \sin 15^\circ$ by multiplying and dividing the entire expression by the scalar value $\sqrt{2}$:

$$\cos 15^\circ - \sin 15^\circ = \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos 15^\circ - \frac{1}{\sqrt{2}}\sin 15^\circ\right)$$

Step 2: Substitute standard values for the constants: $\frac{1}{\sqrt{2}} = \sin 45^\circ$ and $\frac{1}{\sqrt{2}} = \cos 45^\circ$:

$$\sqrt{2}(\sin 45^\circ \cos 15^\circ - \cos 45^\circ \sin 15^\circ)$$

Step 3: Apply the standard sine subtraction angle formula, $\sin(A - B) = \sin A \cos B - \cos A \sin B$:

$$\sqrt{2} \cdot \sin(45^\circ - 15^\circ)$$

Step 4: Simplify the angle argument inside the sine function:

$$\sqrt{2} \cdot \sin(30^\circ) = \sqrt{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

Final Answer: $\boxed{\frac{1}{\sqrt{2}}}$

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept: A matrix M is symmetric if $M^T = M$, and it is skew-symmetric if $M^T = -M$. Matrix transpose operations distribute across addition and reverse the order of multiplication: $(AB)^T = B^T A^T$.

Solution: Step 1: We are given that A and B are symmetric matrices of the same order. This properties mean:

$$A^T = A \quad \text{and} \quad B^T = B$$

Step 2: Let us define a new matrix variable $M = AB - BA$. To determine its symmetry type, apply the transpose operation to the entire expression:

$$M^T = (AB - BA)^T$$

Step 3: Distribute the transpose operation across the matrix subtraction:

$$M^T = (AB)^T - (BA)^T$$

Step 4: Apply the reversal rule for the transpose of matrix products:

$$M^T = (B^T A^T) - (A^T B^T)$$

Step 5: Substitute the given symmetry conditions $A^T = A$ and $B^T = B$ into the expression:

$$M^T = BA - AB$$

Step 6: Factor out a negative sign from the resulting expression:

$$M^T = -(AB - BA) = -M$$

Since $M^T = -M$, the matrix $AB - BA$ is a skew-symmetric matrix.

Final Answer: $\boxed{\text{Skew-symmetric matrix}}$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 33

Q34.

Solution

Concept: The slope of the tangent line to a curve $y = f(x)$ at any specific point is equal to the numerical value of its first derivative $\frac{dy}{dx}$ evaluated at that point's x-coordinate.

Solution: Step 1: Write down the given curve equation: $y = 2x^2 + 3\sin x$. Differentiate this function with respect to x term-by-term:

$$\frac{dy}{dx} = \frac{d}{dx}(2x^2) + \frac{d}{dx}(3\sin x)$$

Step 2: Apply the power rule and standard trigonometric derivative rules:

$$\frac{dy}{dx} = 4x + 3\cos x$$

Step 3: We need to evaluate the slope specifically at the point where $x = 0$. Substitute $x = 0$ into the derivative expression:

$$\text{Slope } m = \left.\frac{dy}{dx}\right|_{x=0} = 4(0) + 3\cos(0)$$

Step 4: Substitute the trigonometric constant value $\cos(0) = 1$:

$$m = 0 + 3(1) = 3$$

Final Answer: $\boxed{3}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 34

Q35.

Solution

Concept: The negation of a logical conjunction statement $\sim (p \wedge q)$ is equivalent to $\sim p \vee \sim q$ according to De Morgan's Laws. This rule can be applied to statements containing existing internal negations.

Solution: Step 1: We are tasked with finding the negation of the compound logical statement $p \wedge \sim q$. Write this out formally with a negation operator outside:

$$\sim (p \wedge \sim q)$$

Step 2: Apply De Morgan's Law for conjunctions, which states that the negation of an AND statement becomes the OR statement of the separate component negations:

$$\sim (p \wedge \sim q) \equiv \sim p \vee \sim (\sim q)$$

Step 3: Apply the Double Negation Law, which states that the negation of a negation returns the original component proposition: $\sim (\sim q) \equiv q$.

Step 4: Substitute this back into the expression to obtain the final simplified logical form:

$$\sim p \vee q$$

Final Answer: $\boxed{\sim p \vee q}$

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept: The probability of an event is the ratio of the number of favorable outcomes to the total number of equally likely outcomes in the sample space. "At least two" means two or more.

Solution: Step 1: Write down the total sample space S for tossing three coins simultaneously. The total number of outcomes is $2^3 = 8$:

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Step 2: Identify the condition for the favorable event E, which requires getting "at least two heads". This includes outcomes with exactly two heads or exactly three heads.

Step 3: List out the sample points that satisfy this condition from the sample space:

$$E = \{HHT, HTH, THH, HHH\}$$

Step 4: Count the number of favorable outcomes: $n(E) = 4$. The total number of points is $n(S) = 8$.

Step 5: Calculate the probability fraction and reduce it to lowest terms:

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept: The trigonometric sine addition identity is $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$. Missing cosine or sine values are found using the identity $\sin^2\theta + \cos^2\theta = 1$.

Solution: Step 1: We are given $\sin\alpha = \frac{3}{5}$. Since α is an acute angle (first quadrant), its cosine value must be positive. Compute $\cos\alpha$:

$$\cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Step 2: We are given $\cos\beta = \frac{5}{13}$. Since β is also an acute angle, its sine value is positive. Compute $\sin\beta$:

$$\sin\beta = \sqrt{1 - \cos^2\beta} = \sqrt{1 - \left(\frac{5}{13}\right)^2} = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$$

Step 3: Write out the trigonometric addition identity formula for $\sin(\alpha + \beta)$:

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

Step 4: Substitute all four known fraction values into the identity:

$$\sin(\alpha + \beta) = \left(\frac{3}{5}\right)\left(\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right)$$

Step 5: Perform the fraction multiplications and add them together over their common denominator:

$$\sin(\alpha + \beta) = \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$$

Final Answer: $\boxed{\frac{63}{65}}$

Answer: (C)

Go Back to Question 37

Q38.

Solution

Concept: Two matrices are equal if and only if they share the same dimensions and every pair of corresponding entries located at the same row and column positions are exactly equal.

Solution: Step 1: Equate the corresponding entries from the given matrix equality statement:

$$\begin{bmatrix} x+y & 2 \\ 1 & x-y \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$$

Step 2: Extract the two independent linear algebraic equations involving variables x and y:

$$x + y = 5 \quad \text{— (Equation 1)}$$

$$x - y = 3 \quad \text{— (Equation 2)}$$

Step 3: Solve the linear system by adding Equation 1 and Equation 2 together:

$$(x + y) + (x - y) = 5 + 3 \implies 2x = 8 \implies x = 4$$

Step 4: Substitute $x = 4$ back into Equation 1 to solve for the remaining variable y:

$$4 + y = 5 \implies y = 5 - 4 = 1$$

Step 5: Collect the values together: $x = 4$ and $y = 1$.

Final Answer: $\boxed{x = 4, y = 1}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: The slope m_1 of a line $Ax + By + C = 0$ is $-\frac{A}{B}$. A line perpendicular to it must have a slope m_2 satisfying $m_1 \cdot m_2 = -1$. The point-slope formula is then used to find its equation.

Solution: Step 1: Find the slope of the given line $x - 2y + 4 = 0$. Rearrange it to slope-intercept form $2y = x + 4 \implies y = \frac{1}{2}x + 2$. Thus, the base slope is $m_1 = \frac{1}{2}$.

Step 2: Let m_2 be the slope of the required perpendicular line. Use the perpendicular condition:

$$m_1 \cdot m_2 = -1 \implies \frac{1}{2} \cdot m_2 = -1 \implies m_2 = -2$$

Step 3: Use the point-slope straight line formula with slope $m_2 = -2$ and passing point coordinates $(x_1, y_1) = (2, -3)$:

$$y - y_1 = m_2(x - x_1)$$

Step 4: Substitute the values into the line formula:

$$y - (-3) = -2(x - 2) \implies y + 3 = -2x + 4$$

Step 5: Move all terms to the left side to write the final answer in general linear form:

$$2x + y + 3 - 4 = 0 \implies 2x + y - 1 = 0$$

Final Answer: $\boxed{2x + y - 1 = 0}$

Answer: (A)

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Q40.

Solution

Concept: The scalar projection of a vector $\vec{a}$ onto another vector $\vec{b}$ represents the length of the orthogonal projection segment, calculated using the formula $\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Solution: Step 1: Write down the component details for the two given vectors:

$$\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$$

Step 2: Calculate the vector dot product $\vec{a}\cdot\vec{b}$ by multiplying matching components:

$$\vec{a}\cdot\vec{b} = (1)(2) + (3)(-1) + (1)(2)$$

$$\vec{a}\cdot\vec{b} = 2 - 3 + 2 = 1$$

Step 3: Calculate the magnitude length of the target vector $\vec{b}$ using the square root formula:

$$|\vec{b}| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{4+1+4} = \sqrt{9} = 3$$

Step 4: Substitute the dot product value and the magnitude into the scalar projection formula:

$$\text{Projection} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{1}{3}$$

Final Answer: $\boxed{\frac{1}{3}}$

Answer: (A)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	A	4	A	5	C
6	C	7	A	8	B	9	A	10	A
11	A	12	C	13	C	14	B	15	A
16	B	17	C	18	B	19	A	20	D
21	B	22	A	23	B	24	A	25	A
26	A	27	B	28	C	29	A	30	B
31	C	32	B	33	B	34	A	35	B
36	A	37	C	38	A	39	A	40	A