Rajasthan JET Mathematics Sample Paper-12

Duration: 40 Minutes **Maximum Marks: 160**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries: **−1 marks**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. Let $A = \{x \in \mathbb{R} : |x - 2| \leq 3\}$ and $B = \{x \in \mathbb{R} : |x - 5| \leq 2\}$. Then, the number of integers in the set $A \cap B$ is

(A) 2

(B) 3

(C) 4

(D) 5

Q2. If z is a complex number such that $\left|\frac{z-3i}{z+3i}\right| = 1$, then the locus of z in the complex plane is

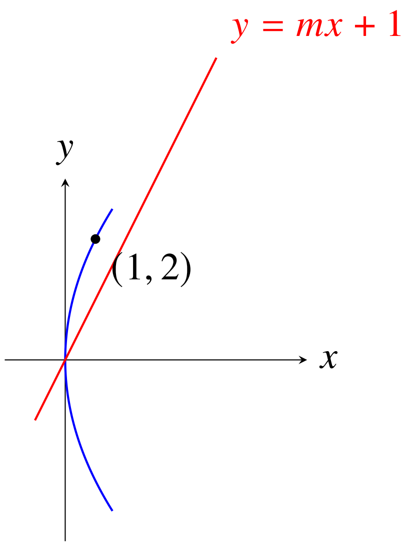
(A) A circle with radius 3

(B) The x-axis

(C) The y-axis

(D) A line parallel to the x-axis passing through $(0, 3)$

Q3. The line $y = mx + 1$ is a tangent to the parabola $y^2 = 8x$ if the value of m is



(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) 4

Q4. If $\vec{a}$ and $\vec{b}$ are unit vectors such that the angle between them is $\frac{\pi}{3}$, then the value of $|2\vec{a} + 3\vec{b}|$ is

(A) $\sqrt{7}$

(B) $\sqrt{13}$

(C) $\sqrt{19}$

(D) 5

Q5. The value of $\lim_{x\to 0} \frac{1-\cos(4x)}{x\tan(3x)}$ is

(A) $\frac{4}{3}$

(B) $\frac{8}{3}$

(C) $\frac{16}{3}$

(D) 0

Q6. Consider the statement: "If it rains, then the match will be cancelled." The contrapositive of this statement is

(A) If the match is cancelled, then it rains.

(B) If it does not rain, then the match will not be cancelled.

(C) If the match is not cancelled, then it does not rain.

(D) It rains if and only if the match is cancelled.

Q7. If the mean of five observations $x, x+2, x+4, x+6, x+8$ is 11, then the variance of these observations is

(A) 4

(B) 8

(C) 5

(D) 10

Q8. The principal value of $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$ is

(A) $\frac{4\pi}{3}$

(B) $\frac{\pi}{3}$

(C) $-\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Q9. Let a relation R be defined on the set of natural numbers $\mathbb{N}$ by aRb if and only if $a + 2b = 10$. The range of the relation R is

(A) $\{1, 2, 3, 4\}$

(B) $\{2, 4, 6, 8\}$

(C) $\{1, 2, 3, 4, 5, 6, 7, 8\}$

(D) $\{1, 3, 5, 7\}$

Q10. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then the matrix A^{10} is equal to

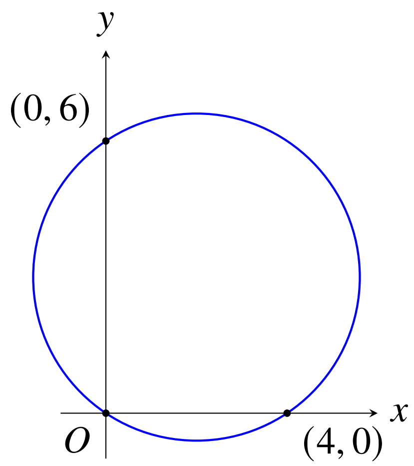
(A) $\begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1024 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2^{10} \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 10 & 20 \\ 0 & 10 \end{bmatrix}$

Q11. The equation of the circle passing through the origin and making intercepts 4 and 6 on the positive x and y-axes respectively is



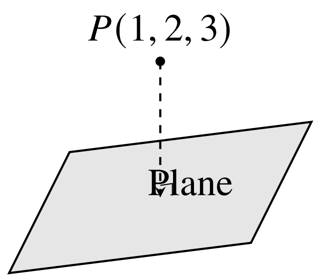
(A) $x^2 + y^2 - 4x - 6y = 0$

(B) $x^2 + y^2 + 4x + 6y = 0$

(C) $x^2 + y^2 - 8x - 12y = 0$

(D) $x^2 + y^2 - 2x - 3y = 0$

Q12. The distance of the point $(1, 2, 3)$ from the plane $2x - 2y + z + 5 = 0$ is



(A) 1 unit

(B) 2 units

(C) 3 units

(D) 4 units

Q13. The maximum value of the function $f(x) = x(12-2x)^2$ in the interval $[0,6]$ occurs at x equal to

(A) 2

(B) 4

(C) 1

(D) 3

Q14. Which of the following compound statements is a contradiction?

(A) $p \vee \sim p$

(B) $p \wedge \sim p$

(C) $p \longrightarrow (p \vee q)$

(D) $\sim (\sim p) \leftrightarrow p$

Q15. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is a prime number?

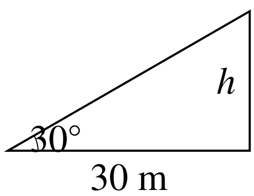
(A) $\frac{5}{12}$

(B) $\frac{7}{18}$

(C) $\frac{1}{2}$

(D) $\frac{11}{36}$

Q16. A tower stands vertically on the ground. From a point on the ground, which is 30 meters away from the foot of the tower, the angle of elevation of the top of the tower is found to be 30°. The height of the tower (in meters) is



(A) $30\sqrt{3}$

(B) $10\sqrt{3}$

(C) 15

(D) 20

Q17. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x - 5$. If $g(x) = f^{-1}(x)$, then the value of $g(4)$ is

(A) 7

(B) 3

(C) 1

(D) -1

Q18. If the third term of a geometric progression (G.P.) is 4, then the product of its first 5 terms is

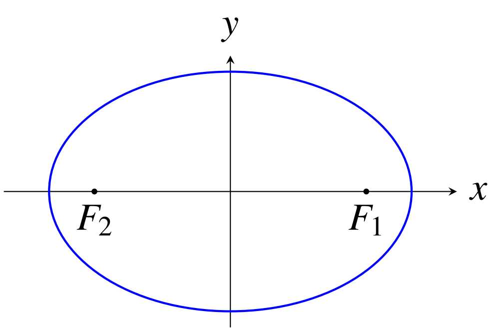
(A) 4^3

(B) 4^4

(C) 4^5

(D) Cannot be determined without the common ratio

Q19. The eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$ is



(A) $\frac{3}{4}$

(B) $\frac{\sqrt{7}}{4}$

(C) $\frac{9}{16}$

(D) $\frac{1}{2}$

Q20. If a line makes angles α, β, γ with the coordinate axes, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is

(A) 1

(B) 2

(C) 3

(D) 0

Q21. The value of the definite integral $\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}\, dx$ is

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) π

(D) 0

Q22. The negation of the statement "All natural numbers are integers" is

(A) No natural numbers are integers.

(B) There exists a natural number which is not an integer.

(C) All integers are natural numbers.

(D) If a number is an integer, it is a natural number.

Q23. A bag contains 4 white and 6 black balls. Two balls are drawn at random one after another without replacement. The probability that both balls are white is

(A) $\frac{4}{25}$

(B) $\frac{2}{15}$

(C) $\frac{1}{6}$

(D) $\frac{4}{15}$

Q24. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^4\theta + \cot^4\theta$ is

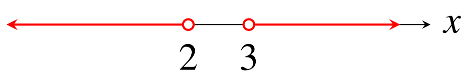
(A) 2

(B) 4

(C) 16

(D) 8

Q25. The domain of the function $f(x) = \frac{1}{\sqrt{x^2-5x+6}}$ is



(A) $(2, 3)$

(B) $(-\infty, 2) \cup (3, \infty)$

(C) $(-\infty, 2] \cup [3, \infty)$

(D) $\mathbb{R} \setminus \{2, 3\}$

Q26. If the system of linear equations $x + y + z = 2$, $2x + 3y + 2z = 5$, and $2x + 3y + (\alpha^2 - 1)z = \alpha + 1$ has infinitely many solutions, then the value of α is

(A) $\sqrt{3}$

(B) $-\sqrt{3}$

(C) 2

(D) 1

Q27. The equation of the straight line passing through the intersection of the lines $x - y - 1 = 0$ and $2x - 3y + 1 = 0$ and parallel to the x-axis is

(A) $y = 3$

(B) $y = 2$

(C) $y = 4$

(D) $y = 1$

Q28. The vector equation of the line passing through the point $(2, -1, 4)$ and parallel to the vector $\hat{i} + 2\hat{j} - \hat{k}$ is

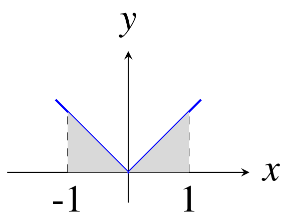
(A) $\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$

(B) $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$

(C) $\vec{r} = (2\hat{i} + \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$

(D) $\vec{r} = (2\hat{i} - \hat{j} - 4\hat{k}) + \lambda(\hat{i} - 2\hat{j} + \hat{k})$

Q29. The value of $\int_{-1}^{1} |x|\, dx$ is



(A) 0

(B) 1

(C) 2

(D) $\frac{1}{2}$

Q30. Let p and q be two propositions. The logical statement $p \rightarrow q$ is equivalent to which of the following?

(A) $\sim p \vee q$

(B) $\sim p \wedge q$

(C) $p \vee \sim q$

(D) $\sim q \rightarrow p$

Q31. If the standard deviation of a set of data is 4.5, and each observation is multiplied by 2, then the new variance of the data set will be

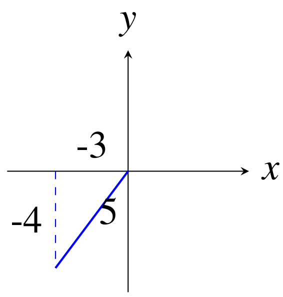
(A) 9

(B) 18

(C) 20.25

(D) 81

Q32. If $\cos\theta = -\frac{3}{5}$ and θ lies in the third quadrant, then the value of $\sin(2\theta)$ is



(A) $\frac{24}{25}$

(B) $-\frac{24}{25}$

(C) $\frac{12}{25}$

(D) $-\frac{12}{25}$

Q33. If $f(x) = \log(x^2 + 1)$ and $g(x) = e^x$, then the composite function $(f \circ g)(x)$ is

(A) $\log(e^{2x} + 1)$

(B) $e^{\log(x^2+1)}$

(C) $x^2 + 1$

(D) $\log(e^x + 1)$

Q34. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is

(A) $a + b + c$

(B) abc

(C) 0

(D) 1

Q35. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is

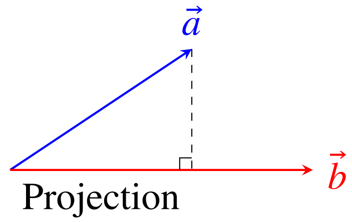
(A) 3

(B) -3

(C) $\frac{1}{3}$

(D) $-\frac{1}{3}$

Q36. The projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is



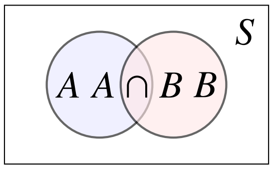
(A) $\sqrt{6}$

(B) $\frac{10}{\sqrt{6}}$

(C) $\frac{10}{3}$

(D) $2\sqrt{6}$

Q37. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.2$, then the conditional probability $P(\sim A \mid B)$ is



(A) 0.4

(B) 0.5

(C) 0.6

(D) 0.3

Q38. The value of $\tan\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)$ is

(A) $\frac{3}{4}$

(B) $\frac{4}{3}$

(C) $\frac{3}{5}$

(D) $\frac{1}{2}$

Q39. If the line passing through the points $(2, 5)$ and $(4, 9)$ is perpendicular to the line passing through $(a, 3)$ and $(6, 2)$, then the value of a is

(A) 4

(B) 8

(C) 5

(D) 2

Q40. Let p and q be two statements. The proposition $\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to

(A) p

(B) q

(C) $\sim p$

(D) $\sim q$

Detailed Solutions

Q1.

Solution

Concept: The problem requires finding the number of integers in the intersection of two sets defined by absolute value inequalities. We can find the interval for each set separately by clearing the absolute values, find the overlapping region, and then count the integers contained within that region.

Solution: Step 1: Simplify the inequality for set A:

$$|x - 2| \leq 3$$

This can be rewritten as:

$$-3 \leq x - 2 \leq 3$$

Adding 2 to all parts of the inequality gives:

$$-1 \leq x \leq 5$$

Thus, set $A = [-1, 5]$.
Step 2: Simplify the inequality for set B:

$$|x - 5| \leq 2$$

This can be rewritten as:

$$-2 \leq x - 5 \leq 2$$

Adding 5 to all parts of the inequality gives:

$$3 \leq x \leq 7$$

Thus, set $B = [3, 7]$. Step 3: Find the intersection of set A and set B:

$$A \cap B = [-1, 5] \cap [3, 7] = [3, 5]$$

Step 4: Identify the integers that lie within the closed interval $[3, 5]$. These integers are $3, 4$, and 5. Step 5: Count the number of elements obtained in the intersection. There are exactly 3 integers in total.

Final Answer: $\boxed{3}$

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept: The equation involves the modulus of a complex fraction set equal to unity. We can use the property of the modulus of a quotient, $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, to clear the fraction, substitute $z = x + iy$, and find the relationship between x and y to determine the geometric locus.

Solution: Step 1: Rewrite the given equation using modulus properties:

$$\left|\frac{z-3i}{z+3i}\right| = 1 \implies |z-3i| = |z+3i|$$

Step 2: Substitute $z = x + iy$, where x and y are real numbers, into the equation:

$$|x + iy - 3i| = |x + iy + 3i|$$

$$|x + i(y-3)| = |x + i(y+3)|$$

Step 3: Square both sides to remove the square root from the definition of the modulus:

$$x^2 + (y-3)^2 = x^2 + (y+3)^2$$

Step 4: Cancel x^2 from both sides and expand the remaining quadratic terms:

$$y^2 - 6y + 9 = y^2 + 6y + 9$$

Step 5: Simplify the equation by gathering like terms:

$$-6y = 6y \implies 12y = 0 \implies y = 0$$

Since $y = 0$ represents the equation of the x-axis, the locus of z is the real axis.

Final Answer: $\boxed{\text{The x-axis}}$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 2

Q3.

Solution

Concept: For a straight line $y = mx + c$ to be a tangent to the standard parabola $y^2 = 4ax$, it must satisfy the specific condition of tangency, which is given by the formula $c = \frac{a}{m}$. By identifying the parameters from the given line and parabola, we can solve directly for the slope m.

Solution: Step 1: Compare the given parabola $y^2 = 8x$ with the standard form $y^2 = 4ax$ to find the value of a:

$$4a = 8 \implies a = 2$$

Step 2: Identify the parameters of the given line $y = mx + 1$. Comparing with $y = mx + c$, we get the y-intercept:

$$c = 1$$

Step 3: Apply the necessary condition for tangency to a parabola, which states:

$$c = \frac{a}{m}$$

Step 4: Substitute the known values of a and c into the tangency condition formula:

$$1 = \frac{2}{m}$$

Step 5: Solve the resulting algebraic linear equation for the unknown slope parameter m:

$$m = 2$$

Final Answer: $\boxed{2}$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 3

Q4.

Solution

Concept: To find the magnitude of a linear combination of vectors, we use the vector identity $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$. Expanding this expression allows us to utilize the magnitudes of individual unit vectors and their mutual dot product, which depends on the cosine of the angle between them.

Solution: Step 1: Let $V = |2\vec{a} + 3\vec{b}|$. Square both sides of the expression to expand it using the vector dot product:

$$V^2 = (2\vec{a} + 3\vec{b}) \cdot (2\vec{a} + 3\vec{b})$$

Step 2: Use the distributive law of dot products to fully expand the right-hand side:

$$V^2 = 4(\vec{a} \cdot \vec{a}) + 12(\vec{a} \cdot \vec{b}) + 9(\vec{b} \cdot \vec{b})$$

$$V^2 = 4|\vec{a}|^2 + 12|\vec{a}||\vec{b}| \cos\theta + 9|\vec{b}|^2$$

Step 3: Substitute the properties of unit vectors, which state that $|\vec{a}| = 1$ and $|\vec{b}| = 1$:

$$V^2 = 4(1)^2 + 12(1)(1)\cos\theta + 9(1)^2$$

$$V^2 = 4 + 12\cos\theta + 9 = 13 + 12\cos\theta$$

Step 4: Substitute the given angle $\theta = \frac{\pi}{3}$ into the equation:

$$V^2 = 13 + 12\cos\left(\frac{\pi}{3}\right) = 13 + 12\left(\frac{1}{2}\right)$$

$$V^2 = 13 + 6 = 19$$

Step 5: Take the positive square root to find the actual magnitude of the combined vector:

$$V = \sqrt{19}$$

Final Answer: $\boxed{\sqrt{19}}$

Answer: (C)

Go Back to Question 4

Q5.

Solution

Concept: The limit problem presents a $\frac{0}{0}$ indeterminate form. We can solve it by applying standard trigonometric limits, specifically $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ and $\lim_{\theta\to 0}\frac{\tan\theta}{\theta} = 1$, along with the identity $1 - \cos(2\theta) = 2\sin^2(\theta)$.

Solution: Step 1: Rewrite the numerator using the double-angle identity $1 - \cos(4x) = 2\sin^2(2x)$:

$$\lim_{x\to 0}\frac{2\sin^2(2x)}{x\tan(3x)}$$

Step 2: Multiply and divide the expression by appropriate factors of x to group them into standard limits:

$$\lim_{x\to 0}\frac{2\left(\frac{\sin(2x)}{2x}\right)^2\cdot 4x^2}{x\cdot\left(\frac{\tan(3x)}{3x}\right)\cdot 3x}$$

Step 3: Simplify the algebraic terms involving x in the numerator and denominator:

$$\lim_{x\to 0}\frac{8x^2\cdot\left(\frac{\sin(2x)}{2x}\right)^2}{3x^2\cdot\left(\frac{\tan(3x)}{3x}\right)} = \lim_{x\to 0}\frac{8}{3}\cdot\frac{\left(\frac{\sin(2x)}{2x}\right)^2}{\left(\frac{\tan(3x)}{3x}\right)}$$

Step 4: Apply the limits individual parts as $x \to 0$, knowing that $\frac{\sin(2x)}{2x} \to 1$ and $\frac{\tan(3x)}{3x} \to 1$:

$$\frac{8}{3}\cdot\frac{(1)^2}{1} = \frac{8}{3}$$

Final Answer: $\boxed{8_{\overline{3}}}$

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept: In mathematical logic, the conditional statement is represented as $p \rightarrow q$, which reads "If p, then q". The contrapositive of this conditional statement is logically equivalent and is defined as $\sim q \rightarrow \sim p$, which reads "If not q, then not p".

Solution: Step 1: Identify the component propositions in the given conditional statement. Let p be the statement "It rains" and q be the statement "The match will be cancelled".

Step 2: Express the original given sentence in symbolic logical notation:

$$p \rightarrow q$$

Step 3: Determine the symbolic form of the contrapositive statement using logical rules:

$$\sim q \rightarrow \sim p$$

Step 4: Translate the components of the contrapositive back into English statements. The negation of q ($\sim q$) is "The match is not cancelled", and the negation of p ($\sim p$) is "It does not rain".

Step 5: Combine these components into a single conditional structure: "If the match is not cancelled, then it does not rain."

Final Answer: If the match is not cancelled, then it does not rain.

Answer: (C)

Go Back to Question 6

Q7.

Solution

Concept: The problem requires computing the variance of an arithmetic progression sequence. First, we use the given mean value to find the baseline variable x. Then, we apply the standard variance formula $\sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$ to find the measure of dispersion.

Solution: Step 1: Write the equation for the arithmetic mean of the five given observations:

$$\bar{x} = \frac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5} = 11$$

Step 2: Simplify the numerator and solve for x:

$$\frac{5x + 20}{5} = 11 \implies x + 4 = 11 \implies x = 7$$

Step 3: List the numerical values of the five observations using $x = 7$:

$$7, 9, 11, 13, 15$$

The mean $\bar{x}$ is 11.

Step 4: Calculate the deviation of each observation from the mean, $(x_i - \bar{x})$:

$$(7-11) = -4, \quad (9-11) = -2, \quad (11-11) = 0, \quad (13-11) = 2, \quad (15-11) = 4$$

Step 5: Square each deviation and sum them up to find the total sum of squared deviations:

$$(-4)^2 + (-2)^2 + (0)^2 + (2)^2 + (4)^2 = 16 + 4 + 0 + 4 + 16 = 40$$

Step 6: Divide the sum by the total number of observations ($n = 5$) to get the variance:

$$\sigma^2 = \frac{40}{5} = 8$$

Final Answer: $\boxed{8}$

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept: The principal value range of the inverse sine function, $\sin^{-1}(x)$, is restricted to the closed interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. If the given angle lies outside this domain, we must use trigonometric periodicity and reduction formulas to bring the angle into the proper quadrant.

Solution: Step 1: Check if the angle $\frac{4\pi}{3}$ falls within the principal range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Since $\frac{4\pi}{3} > \frac{\pi}{2}$, we cannot directly cancel the functions.

Step 2: Rewrite the inner angle $\frac{4\pi}{3}$ using a standard quadrant reduction form:

$$\frac{4\pi}{3} = \pi + \frac{\pi}{3}$$

Step 3: Apply the trigonometric identity for the third quadrant, $\sin(\pi + \theta) = -\sin\theta$:

$$\sin\left(\frac{4\pi}{3}\right) = \sin\left(\pi + \frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$$

Step 4: Substitute this back into the original inverse sine expression:

$$\sin^{-1}\left(-\sin\left(\frac{\pi}{3}\right)\right)$$

Step 5: Use the property of inverse trigonometric functions, $\sin^{-1}(-x) = -\sin^{-1}(x)$:

$$-\sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right) = -\frac{\pi}{3}$$

Since $-\frac{\pi}{3}$ falls within $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, this is the valid principal value.

Final Answer: -π 3

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept: The range of a relation consists of all the second elements (the values of b) in the ordered pairs (a, b) that satisfy the given condition. Both variables a and b must belong to the set of natural numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$.

Solution: Step 1: Isolate variable a in terms of variable b from the given constraint equation:

$$a + 2b = 10 \implies a = 10 - 2b$$

Step 2: Since a must be a natural number ($a \in \mathbb{N}$), it must be strictly greater than zero:

$$10 - 2b > 0 \implies 10 > 2b \implies b < 5$$

Step 3: List all possible values of b that are natural numbers and satisfy the condition $b < 5$:

$$b \in \{1, 2, 3, 4\}$$

Step 4: Verify that each value of b yields a corresponding natural number for a:

$$\text{For } b = 1 : a = 10 - 2(1) = 8 \in \mathbb{N}$$

$$\text{For } b = 2 : a = 10 - 2(2) = 6 \in \mathbb{N}$$

$$\text{For } b = 3 : a = 10 - 2(3) = 4 \in \mathbb{N}$$

$$\text{For } b = 4 : a = 10 - 2(4) = 2 \in \mathbb{N}$$

Step 5: Collect the valid values of b to form the range set of the relation:

$$\text{Range} = \{1, 2, 3, 4\}$$

Final Answer: $\boxed{\{1, 2, 3, 4\}}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: To find a higher power of a matrix, we compute its lower powers such as A^2 and A^3 to identify a recurring mathematical pattern or general formula for A^n. This method avoids performing full matrix multiplication ten separate times.

Solution: Step 1: Write down the given matrix A:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Step 2: Compute the matrix square $A^2 = A \cdot A$ using standard matrix row-by-column multiplication rules:

$$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 2 \cdot 0 & 1 \cdot 2 + 2 \cdot 1 \\ 0 \cdot 1 + 1 \cdot 0 & 0 \cdot 2 + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2(2) \\ 0 & 1 \end{bmatrix}$$

Step 3: Compute the matrix cube $A^3 = A^2 \cdot A$ to confirm the inductive pattern:

$$A^3 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 4 \cdot 0 & 1 \cdot 2 + 4 \cdot 1 \\ 0 \cdot 1 + 1 \cdot 0 & 0 \cdot 2 + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2(3) \\ 0 & 1 \end{bmatrix}$$

Step 4: Generalize the pattern for the n-th power of matrix A:

$$A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$$

Step 5: Substitute $n = 10$ into the generalized pattern formula to find the final matrix:

$$A^{10} = \begin{bmatrix} 1 & 2(10) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix}$$

Final Answer: $\boxed{\begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix}}$

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: A circle passing through the origin $(0, 0)$ with positive intercepts g' and f' on the coordinate axes passes through the specific points $(g', 0)$ and $(0, f')$. The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$. We substitute these three points to find the constants.

Solution: Step 1: Use the fact that the circle passes through the origin $(0, 0)$. Substituting $x = 0, y = 0$ into the general equation gives:

$$0^2 + 0^2 + 2g(0) + 2f(0) + c = 0 \implies c = 0$$

Step 2: The circle makes an intercept of 4 on the positive x-axis, meaning it passes through $(4, 0)$. Substitute $x = 4, y = 0, c = 0$:

$$4^2 + 0^2 + 2g(4) + 2f(0) = 0 \implies 16 + 8g = 0 \implies g = -2$$

Step 3: The circle makes an intercept of 6 on the positive y-axis, meaning it passes through $(0, 6)$. Substitute $x = 0, y = 6, c = 0$:

$$0^2 + 6^2 + 2g(0) + 2f(6) = 0 \implies 36 + 12f = 0 \implies f = -3$$

Step 4: Substitute the computed values of $g = -2$, $f = -3$, and $c = 0$ back into the general circle equation:

$$x^2 + y^2 + 2(-2)x + 2(-3)y + 0 = 0$$

Step 5: Simplify the terms to get the final circle equation:

$$x^2 + y^2 - 4x - 6y = 0$$

Final Answer: $\boxed{x^2 + y^2 - 4x - 6y = 0}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: The shortest perpendicular distance d from a given point (x_1, y_1, z_1) to a 3D plane described by the linear algebraic equation $Ax + By + Cz + D = 0$ is determined by applying the standard formula: $d = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$.

Solution: Step 1: Identify the coordinates of the given point and coefficients of the plane:

$$(x_1, y_1, z_1) = (1, 2, 3)$$

$$A = 2, \quad B = -2, \quad C = 1, \quad D = 5$$

Step 2: Write down the standard formula for the perpendicular distance from a point to a plane:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Step 3: Substitute the numerical values into the absolute value expression in the numerator:

$$\text{Numerator} = |2(1) - 2(2) + 1(3) + 5| = |2 - 4 + 3 + 5| = |6| = 6$$

Step 4: Substitute the plane coefficients into the square root expression in the denominator:

$$\text{Denominator} = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{4 + 4 + 1} = \sqrt{9} = 3$$

Step 5: Divide the numerator value by the denominator value to find the final distance:

$$d = \frac{6}{3} = 2 \text{ units}$$

Final Answer: $\boxed{\text{2 units}}$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 12

Q13.

Solution

Concept: To find the maximum value of a function over a closed interval, we must identify its critical points by setting its first derivative equal to zero ($f'(x) = 0$). We then compare the function values at these critical points and at the boundary endpoints of the interval.

Solution: Step 1: Expand or prepare the function $f(x) = x(12 - 2x)^2$ for differentiation. We can pull out a constant factor to simplify calculations:

$$f(x) = x[2(6 - x)]^2 = 4x(6 - x)^2 = 4x(36 - 12x + x^2) = 4(36x - 12x^2 + x^3)$$

Step 2: Differentiate the function with respect to x to find $f'(x)$:

$$f'(x) = 4(36 - 24x + 3x^2) = 12(12 - 8x + x^2)$$

Step 3: Set the first derivative equal to zero to find the critical points:

$$12(x^2 - 8x + 12) = 0 \implies (x - 2)(x - 6) = 0$$

This gives critical points at $x = 2$ and $x = 6$. Both values lie within the given interval $[0, 6]$.

Step 4: Evaluate the original function $f(x)$ at the critical points and boundary endpoints ($x = 0, 2, 6$):

$$\text{For } x = 0 : f(0) = 4(0)(6 - 0)^2 = 0$$

$$\text{For } x = 6 : f(6) = 4(6)(6 - 6)^2 = 0$$

$$\text{For } x = 2 : f(2) = 4(2)(6 - 2)^2 = 8(4)^2 = 8 \cdot 16 = 128$$

Step 5: Compare the values. The maximum value is 128, which occurs exactly at $x = 2$.

Final Answer: $\boxed{2}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 13

Q14.

Solution

Concept: A compound statement is classified as a contradiction (or fallacy) if its logical truth value is false (F) for all possible truth values of its constituent statements. We can verify this by analyzing basic logical connectives.

Solution: Step 1: Analyze option (A): $p \vee \sim p$. This statement represents a proposition or its negation. Since one of them must always be true, their disjunction is always true (T), making it a tautology.

Step 2: Analyze option (B): $p \wedge \sim p$. This statement asserts that both a proposition and its exact negation are true at the same time. Since a proposition and its negation cannot be true simultaneously, this conjunction is always false (F), which defines a contradiction.

Step 3: Analyze option (C): $p \to (p \vee q)$. If p is true, then $p \vee q$ is true, making the implication true. If p is false, the implication is automatically true by definition. Thus, it is a tautology.

Step 4: Analyze option (D): $\sim (\sim p) \leftrightarrow p$. The double negation of p is logically identical to p. Therefore, $p \leftrightarrow p$ is always true, making it a tautology.

Step 5: Conclude that only option (B) consistently yields a false value across all scenarios.

Final Answer: $\boxed{\text{p} \wedge \sim p}$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept: When rolling two unbiased six-sided dice simultaneously, the total number of independent outcomes in the sample space is $6 \times 6 = 36$. We need to identify all pairs whose sum forms a prime number and divide that count by 36.

Solution: Step 1: Determine the range of possible sums when throwing two dice. The minimum sum is $1 + 1 = 2$, and the maximum sum is $6 + 6 = 12$.

Step 2: List the prime numbers that fall within the range from 2 to 12. These are 2, 3, 5, 7, and 11.

Step 3: List all favorable sample pairs that sum up to each of these prime values:

$$\text{Sum} = 2: \ (1, 1) \implies 1 \text{ outcome}$$

$$\text{Sum} = 3: \ (1, 2), (2, 1) \implies 2 \text{ outcomes}$$

$$\text{Sum} = 5: \ (1, 4), (2, 3), (3, 2), (4, 1) \implies 4 \text{ outcomes}$$

$$\text{Sum} = 7: \ (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) \implies 6 \text{ outcomes}$$

$$\text{Sum} = 11: \ (5, 6), (6, 5) \implies 2 \text{ outcomes}$$

Step 4: Sum the favorable outcomes to find the total number of favorable outcomes:

$$\text{Total Favorable} = 1 + 2 + 4 + 6 + 2 = 15$$

Step 5: Compute the probability by dividing the favorable outcomes by the total sample space size:

$$P = \frac{15}{36} = \frac{5}{12}$$

Final Answer: $\boxed{5_{\overline{12}}}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 15

Q16.

Solution

Concept: This problem can be modeled as a right-angled triangle where the tower represents the vertical perpendicular side (h), the distance along the ground is the horizontal base ($b = 30$ m), and the given angle of elevation is $\theta = 30°$. We can relate these quantities using the tangent trigonometric ratio: $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$.

Solution: Step 1: Set up the trigonometric relationship using the tangent function for the right triangle:

$$\tan(30°) = \frac{\text{Height of the tower } (h)}{\text{Distance from the foot } (b)}$$

Step 2: Substitute the known values into the equation:

$$\tan(30°) = \frac{h}{30}$$

Step 3: Recall the standard exact numerical value for $\tan(30°)$, which is $\frac{1}{\sqrt{3}}$:

$$\frac{1}{\sqrt{3}} = \frac{h}{30}$$

Step 4: Isolate the height variable h by cross-multiplying:

$$h = \frac{30}{\sqrt{3}}$$

Step 5: Rationalize the denominator to simplify the radical expression into standard form:

$$h = \frac{30 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3} \text{ meters}$$

Final Answer: $\boxed{10\sqrt{3}}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: The inverse function $g(x) = f^{-1}(x)$ undoes the mapping performed by $f(x)$. By definition, if $g(4) = k$, then it must be true that $f(k) = 4$. This relationship allows us to solve for k directly using the original function expression without explicitly deriving the inverse formula.

Solution: Step 1: Set up the definition of the inverse function evaluation. Let:

$$g(4) = f^{-1}(4) = k$$

Step 2: Apply the function f to both sides of the equation to rewrite it in terms of $f(x)$:

$$f(k) = 4$$

Step 3: Use the given algebraic rule for the function, $f(x) = 3x - 5`, to express$f(k):$3k - 5 = 4$
Step 4: Solve the resulting linear equation for k by isolating the variable:

$$3k = 4 + 5$$

$$3k = 9$$

$$k = 3$$

Step 5: Conclude that since $k = 3$, the value of the inverse function at the specified point is $g(4) = 3$.

Final Answer: $\boxed{3}$

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept: The general terms of a geometric progression (G.P.) with first term a and common ratio r are given by $t_n = ar^{n-1}$. Writing out the product of the first five terms allows us to simplify the exponents of r and express the entire product as a power of the third term, t_3.

Solution: Step 1: Express the first five individual terms of the geometric progression symbolically:

$$t_1 = a, \quad t_2 = ar, \quad t_3 = ar^2, \quad t_4 = ar^3, \quad t_5 = ar^4$$

Step 2: Write out the mathematical expression for the product P of these five terms:

$$P = a \cdot (ar) \cdot (ar^2) \cdot (ar^3) \cdot (ar^4)$$

Step 3: Combine the base variables by adding their corresponding exponents together:

$$P = a^{1+1+1+1+1} \cdot r^{0+1+2+3+4} = a^5 \cdot r^{10}$$

Step 4: Factor the combined exponential expression to match the form of the third term:

$$P = (ar^2)^5$$

Step 5: We are given that the third term is $t_3 = ar^2 = 4$. Substitute this value into our product formula:

$$P = (4)^5$$

Final Answer: $\boxed{4^5}$

Answer: (C)

Go Back to Question 18

Q19.

Solution

Concept: The standard equation of a horizontal ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$. The eccentricity e, which measures how much the ellipse deviates from being a perfect circle, is computed using the formula: $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution: Step 1: Identify the values of a^2 and b^2 by comparing the given ellipse equation with the standard form:

$$\frac{x^2}{16} + \frac{y^2}{7} = 1 \implies a^2 = 16, \quad b^2 = 7$$

Step 2: Verify that $a^2 > b^2$ ($16 > 7$), which confirms that the major axis of this ellipse lies along the horizontal x-axis.

Step 3: Write down the standard algebraic formula used to compute eccentricity:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Step 4: Substitute the numerical values of a^2 and b^2 into the eccentricity formula:

$$e = \sqrt{1 - \frac{7}{16}}$$

Step 5: Simplify the fraction under the square root sign:

$$e = \sqrt{\frac{16 - 7}{16}} = \sqrt{\frac{9}{16}}$$

Step 6: Take the square root of the numerator and denominator to get the final eccentricity value:

$$e = \frac{3}{4}$$

Final Answer: $\boxed{3_{4}}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: The direction cosines of a line making angles α, β, γ with the coordinate axes are given by $l = \cos\alpha$, $m = \cos\beta$, and $n = \cos\gamma$. They satisfy the fundamental vector identity $l^2 + m^2 + n^2 = 1$. We can convert this identity to sine functions using $\cos^2\theta = 1 - \sin^2\theta$.

Solution: Step 1: Write down the fundamental identity for the direction cosines of any line in 3D space:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Step 2: Substitute the basic trigonometric identity $\cos^2\theta = 1 - \sin^2\theta$ for each of the three terms:

$$(1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$$

Step 3: Combine the constant integer terms on the left side of the equation:

$$3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$$

Step 4: Isolate the sum of the squared sine terms by moving them to the right-hand side:

$$3 - 1 = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$$

Step 5: Simplify the subtraction to obtain the final numerical value:

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 20

Q21.

Solution

Concept: This problem can be efficiently solved by applying a key definite integral property known as King's Property: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$. This transforms the integrand by replacing x with $\left(\frac{\pi}{2}-x\right)$, allowing us to eliminate the trigonometric terms by adding the two integrals together.

Solution: Step 1: Define the given definite integral expression as equation (1):

$$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}\,dx \quad \text{— (1)}$$

Step 2: Apply the integral property by replacing the variable x with $\left(0+\frac{\pi}{2}-x\right) = \frac{\pi}{2}-x$:

$$I = \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2}-x\right)}{\sin^3\left(\frac{\pi}{2}-x\right) + \cos^3\left(\frac{\pi}{2}-x\right)}\,dx$$

Step 3: Apply the standard co-function identities, $\sin\left(\frac{\pi}{2}-x\right) = \cos x$ and $\cos\left(\frac{\pi}{2}-x\right) = \sin x$:

$$I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x}\,dx \quad \text{— (2)}$$

Step 4: Add equation (1) and equation (2) together. Since they share identical integration limits and denominators, we can directly sum their numerators:

$$2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x}\,dx$$

Step 5: Simplify the integrand fraction, which reduces to 1, and evaluate the simple integral:

$$2I = \int_0^{\pi/2} 1\,dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Step 6: Divide both sides by 2 to solve for the final value of the single integral I:

$$I = \frac{\pi}{4}$$

Final Answer: $\boxed{\pi_4}$

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept: The given statement is a universally quantified statement of the form "All P are Q". In formal mathematical logic, the negation of a universally quantified statement is an existentially quantified statement of the form "There exists at least one P that is not Q".

Solution: Step 1: Identify the logical structure of the original statement. It states that every single member belonging to the set of natural numbers is also included within the set of integers.

Step 2: Apply the rule for negating a universal statement. The logical negation of "All X satisfy condition Y" is written as "Some X do not satisfy condition Y" or "There is at least one X that does not satisfy Y".

Step 3: Formulate the negation in words based on this logical structure: "There exists a natural number which is not an integer."

Step 4: Analyze why other options are incorrect. Option (A) is too strong because it changes the quantifier to "No", which is not the exact logical negation. Options (C) and (D) rearrange the statement into conditionals rather than negating it.

Final Answer: $\boxed{\text{There exists a natural number which is not an integer.}}$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 22

Q23.

Solution

Concept: The problem involves two consecutive dependent events because the sampling is done without replacement. We use the multiplication rule for dependent probability: $P(W_1 \cap W_2) = P(W_1) \times P(W_2 \mid W_1)$, adjusting the remaining number of white balls and total balls for the second draw.

Solution: Step 1: Calculate the total initial number of balls contained within the bag:

$$\text{Total balls} = 4 \text{ white} + 6 \text{ black} = 10 \text{ balls}$$

Step 2: Find the probability of drawing a white ball on the first attempt ($P(W_1)$):

$$P(W_1) = \frac{\text{Number of white balls}}{\text{Total number of balls}} = \frac{4}{10}$$

Step 3: Update the quantities in the bag for the second draw, keeping in mind that the first ball was white and was not replaced:

$$\text{Remaining white balls} = 4 - 1 = 3$$

$$\text{Remaining total balls} = 10 - 1 = 9$$

Step 4: Find the conditional probability of drawing a second white ball ($P(W_2 \mid W_1)$):

$$P(W_2 \mid W_1) = \frac{3}{9}$$

Step 5: Multiply these two probabilities together to find the joint probability of both events occurring:

$$P(W_1 \cap W_2) = \frac{4}{10} \times \frac{3}{9} = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$$

Final Answer: $\boxed{2_{\overline{15}}}$

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept: Given the sum of two reciprocal trigonometric functions $\tan\theta + \cot\theta = 2$, we can determine the value of $\tan\theta$. By converting the expression into a quadratic equation, we find $\tan\theta$, which can then be raised to the fourth power to evaluate $\tan^4\theta + \cot^4\theta$.

Solution: Step 1: Use the reciprocal identity $\cot\theta = \frac{1}{\tan\theta}$ to rewrite the given equation:

$$\tan\theta + \frac{1}{\tan\theta} = 2$$

Step 2: Let $x = \tan\theta$. Substitute x into the equation to form a standard algebraic equation:

$$x + \frac{1}{x} = 2 \implies x^2 + 1 = 2x \implies x^2 - 2x + 1 = 0$$

Step 3: Factor the quadratic equation, which is a perfect square trinomial:

$$(x-1)^2 = 0 \implies x = 1$$

This means that $\tan\theta = 1$.

Step 4: Find the value of $\cot\theta$ using its reciprocal relationship with $\tan\theta$:

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{1} = 1$$

Step 5: Substitute these values into the target expression $\tan^4\theta + \cot^4\theta$:

$$\tan^4\theta + \cot^4\theta = (1)^4 + (1)^4 = 1 + 1 = 2$$

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: For a function of the form $f(x) = \frac{1}{\sqrt{P(x)}}$ to be properly defined for real values, the expression inside the square root in the denominator must be strictly greater than zero ($P(x) > 0$). It cannot be negative because of the square root, and it cannot equal zero because it sits in the denominator.

Solution: Step 1: Set up the governing condition for the domain of the function based on its algebraic structure:

$$x^2 - 5x + 6 > 0$$

Step 2: Factor the quadratic expression by splitting the middle linear term:

$$x^2 - 2x - 3x + 6 > 0 \implies x(x-2) - 3(x-2) > 0$$

$$(x-2)(x-3) > 0$$

Step 3: Identify the critical roots of the quadratic equation, which occur at $x = 2$ and $x = 3$.

Step 4: Use the sign-wavy curve method to analyze the intervals produced by these critical roots:

$$\text{For } x > 3, \text{ the product is } (+)\cdot(+) = (+)$$

$$\text{For } 2 < x < 3, \text{ the product is } (+)\cdot(-) = (-)$$

$$\text{For } x < 2, \text{ the product is } (-)\cdot(-) = (+)$$

Step 5: Select the regions where the expression evaluates to a strictly positive value:

$$x \in (-\infty, 2) \cup (3, \infty)$$

Final Answer: $\boxed{(\text{-}\infty, 2) \cup (3, \infty) \quad (\text{-}\infty, 2) \cup (3, \infty) \quad (\text{-}\infty, 2) \cup (3, \infty)}$

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept: A system of linear equations has infinitely many solutions when the determinant of its coefficient matrix (Δ) as well as the auxiliary determinants ($\Delta_x, \Delta_y, \Delta_z$) are all simultaneously equal to zero. Alternatively, we can use row reduction operations to find when the equations become identical.

Solution: Step 1: Write down the first two independent linear equations from the system:

$$1) \quad x + y + z = 2$$

$$2) \quad 2x + 3y + 2z = 5$$

Step 2: Perform row operations to eliminate x from the second equation. Multiply the first equation by 2 and subtract it from the second equation:

$$(2x + 3y + 2z) - 2(x + y + z) = 5 - 2(2)$$

$$2x + 3y + 2z - 2x - 2y - 2z = 5 - 4 \implies y = 1$$

Step 3: Substitute $y = 1$ back into the first equation to find a simplified relationship between x and z:

$$x + 1 + z = 2 \implies x + z = 1$$

Step 4: Examine the third given linear equation from the system:

$$2x + 3y + (\alpha^2 - 1)z = \alpha + 1$$

Substitute $y = 1$ into this third equation:

$$2x + 3(1) + (\alpha^2 - 1)z = \alpha + 1 \implies 2x + (\alpha^2 - 1)z = \alpha - 2$$

Step 5: For the system to have infinitely many solutions, this modified third equation must be a direct multiple of the simplified equation $x + z = 1$. Multiplying $x + z = 1$ by 2 gives:

$$2x + 2z = 2$$

Step 6: Equate the corresponding coefficients of z and the constant terms from both sides:

$$\alpha^2 - 1 = 2 \implies \alpha^2 = 3 \implies \alpha = \pm\sqrt{3}$$

$$\alpha - 2 = 2 \implies \alpha = 4$$

Let us re-evaluate using the determinant method ($\Delta = 0$):

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & \alpha^2 - 1 \end{vmatrix} = 0$$

Subtracting Row 2 from Row 3:

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 0 & \alpha^2 - 3 \end{vmatrix} = 0 \implies 1(3(\text{[illegible]}-2(0))\text{[illegible]}(\alpha^2-3)) = 0 \implies (\alpha^2-3) = 0 \implies \alpha = \pm\sqrt{3}$$

Q27.

Solution

Concept: To find the equation of a line passing through the intersection of two given lines, we first solve their equations simultaneously to find the coordinates of their intersection point. Since the target line is parallel to the x-axis, its equation will be of the simple form y = constant.

Solution: Step 1: Write down the system of two intersecting lines given in the problem:

$$1) \quad x - y = 1 \implies x = y + 1$$

$$2) \quad 2x - 3y = -1$$

Step 2: Substitute the expression for x from the first equation into the second equation:

$$2(y + 1) - 3y = -1$$

Step 3: Expand the brackets and solve the resulting linear equation for y:

$$2y + 2 - 3y = -1$$

$$-y + 2 = -1 \implies y = 3$$

Step 4: Substitute $y = 3$ back into the first equation to find the corresponding value of x:

$$x = 3 + 1 = 4$$

The point of intersection is $(4, 3)$.

Step 5: A line parallel to the x-axis has a slope of $m = 0$. Using the point-slope form $y - y_1 = m(x - x_1)$ with our point $(4, 3)$:

$$y - 3 = 0(x - 4) \implies y = 3$$

Final Answer: $\boxed{y = 3}$

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: The vector equation of a straight line in 3D space passing through a point with position vector $\vec{a}$ and running parallel to a direction vector $\vec{b}$ is given by the standard parametric vector formula: $\vec{r} = \vec{a} + \lambda\vec{b}$, where λ is a scalar parameter.

Solution: Step 1: Find the position vector $\vec{a}$ of the given point $(2, -1, 4)$ through which the line passes:

$$\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$$

Step 2: Identify the direction vector $\vec{b}$ that the line runs parallel to from the problem text:

$$\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$$

Step 3: Recall the standard parametric vector equation format for a straight line:

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Step 4: Substitute the specific vectors $\vec{a}$ and $\vec{b}$ into the standard format line equation:

$$\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$$

Step 5: Verify that this vector expression matches option (A).

Final Answer: $\boxed{\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})}$

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept: The absolute value function $|x|$ changes its algebraic definition at the critical origin point $x = 0$. To evaluate a definite integral across this boundary, we must split the integration interval $[-1, 1]$ into two separate sub-intervals: $[-1, 0]$ where $|x| = -x$, and $[0, 1]$ where $|x| = x$.

Solution: Step 1: Split the given definite integral into two parts at the critical inflection point $x = 0$:

$$\int_{-1}^{1} |x|\,dx = \int_{-1}^{0} |x|\,dx + \int_{0}^{1} |x|\,dx$$

Step 2: Replace the absolute value expression $|x|$ with its specific definition for each interval:

$$\int_{-1}^{1} |x|\,dx = \int_{-1}^{0} (-x)\,dx + \int_{0}^{1} (x)\,dx$$

Step 3: Integrate each component term separately using the standard power rule of integration:

$$\left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$$

Step 4: Substitute the upper and lower integration limits into the first integrated term:

$$\left(-\frac{0^2}{2}\right) - \left(-\frac{(-1)^2}{2}\right) = 0 - \left(-\frac{1}{2}\right) = \frac{1}{2}$$

Step 5: Substitute the upper and lower integration limits into the second integrated term:

$$\left(\frac{1^2}{2}\right) - \left(\frac{0^2}{2}\right) = \frac{1}{2} - 0 = \frac{1}{2}$$

Step 6: Sum the values of the two components together to get the final result:

$$\text{Total Value} = \frac{1}{2} + \frac{1}{2} = 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept: In mathematical logic, an implication statement $p \to q$ (read as "p implies q") can be rewritten as an equivalent logical disjunction statement. By checking its standard truth table definition, we can show that $p \to q$ is logically identical to $\sim p \vee q$.

Solution: Step 1: Recall the conditional truth table for the implication operator $p \to q$. The expression evaluates to False (F) only when the premise p is True (T) and the conclusion q is False (F); it is True (T) in all other scenarios.

Step 2: Examine the logical behavior of option (A), which is $\sim p \vee q$. Let us construct its truth value behavior:

$$\text{If } p = \text{T}, q = \text{T} :\sim p = \text{F} \implies \text{F} \vee \text{T} = \text{T}$$

$$\text{If } p = \text{T}, q = \text{F} :\sim p = \text{F} \implies \text{F} \vee \text{F} = \text{F}$$

$$\text{If } p = \text{F}, q = \text{T} :\sim p = \text{T} \implies \text{T} \vee \text{T} = \text{T}$$

$$\text{If } p = \text{F}, q = \text{F} :\sim p = \text{T} \implies \text{T} \vee \text{F} = \text{T}$$

Step 3: Compare the truth value columns of $p \to q$ and $\sim p \vee q$. Since they are identical under all possible combinations of truth values, they are logically equivalent expressions.

Final Answer: $\boxed{\sim p \vee q}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: Variance and standard deviation measure data dispersion. If every single observation in a data set is multiplied by a constant scalar factor k, the standard deviation is multiplied by $|k|$, and the variance is multiplied by k^2. This property helps us compute the new values directly.

Solution: Step 1: Identify the initial standard deviation (σ_{old}) given in the problem statement:

$$\sigma_{\text{old}} = 4.5$$

Step 2: Determine the new standard deviation (σ_{new}) after multiplying every data point by the constant scaling factor $k = 2$:

$$\sigma_{\text{new}} = k \cdot \sigma_{\text{old}} = 2 \cdot 4.5 = 9$$

Step 3: Recall the mathematical relationship connecting variance and standard deviation, which states that variance is the square of standard deviation:

$$\text{Variance} = \sigma^2$$

Step 4: Square the newly calculated standard deviation to find the new variance value (Var_{new}):

$$\text{Var}_{\text{new}} = (\sigma_{\text{new}})^2 = 9^2 = 81$$

Final Answer: 81

Answer: (D)

Go Back to Question 31

Q32.

Solution

Concept: The problem requires computing a double-angle trigonometric value using the identity $\sin(2\theta) = 2\sin\theta\cos\theta$. We use the fundamental identity $\sin^2\theta + \cos^2\theta = 1$ to determine $\sin\theta$, taking care to pick the correct negative sign since the angle lies in the third quadrant.

Solution: Step 1: Determine the value of $\sin\theta$ using the identity $\sin^2\theta = 1 - \cos^2\theta$:

$$\sin^2\theta = 1 - \left(-\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$$

Step 2: Since the angle θ lies in the third quadrant ($180^\circ < \theta < 270^\circ$), both the sine and cosine functions must take negative values. Therefore, we select the negative square root:

$$\sin\theta = -\frac{4}{5}$$

Step 3: Write down the standard double-angle expansion formula for the sine function:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

Step 4: Substitute the values of $\sin\theta = -\frac{4}{5}$ and $\cos\theta = -\frac{3}{5}$ into the expansion formula:

$$\sin(2\theta) = 2 \cdot \left(-\frac{4}{5}\right) \cdot \left(-\frac{3}{5}\right)$$

Step 5: Multiply the fractions together, noting that the product of two negative signs results in a positive sign:

$$\sin(2\theta) = \frac{2 \cdot 4 \cdot 3}{25} = \frac{24}{25}$$

Final Answer: $\boxed{24_{\overline{25}}}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: A composite function $(f \circ g)(x)$ is formed by substituting the entire output expression of the inner function $g(x)$ into the input variable of the outer function $f(x)$. This can be written mathematically as $f(g(x))$.

Solution: Step 1: Write down the expressions for the two individual component functions given in the problem:

$$f(x) = \log(x^2 + 1), \quad g(x) = e^x$$

Step 2: Set up the structure for the composite function according to its operational definition:

$$(f \circ g)(x) = f(g(x))$$

Step 3: Substitute the explicit function expression of $g(x) = e^x$ inside the outer function bracket:

$$f(g(x)) = f(e^x)$$

Step 4: Replace every instance of the variable x in the definition of $f(x)$ with the new input term (e^x):

$$f(e^x) = \log((e^x)^2 + 1)$$

Step 5: Apply the standard laws of exponents to simplify the inner squared term, $(e^x)^2 = e^{2x}$:

$$(f \circ g)(x) = \log(e^{2x} + 1)$$

Final Answer: $\boxed{\log(e^{2x} + 1)}$

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: The value of a determinant can be easily found by using row or column operations to simplify its rows. If a row operation creates a common factor across a column, or makes two columns identical, the determinant reduces directly to zero.

Solution: Step 1: Write down the given initial 3×3 matrix determinant:

$$\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

Step 2: Apply the column addition operation $C_3 \to C_3 + C_2$. This replaces each element in the third column with the sum of itself and the corresponding element from the second column:

$$\Delta = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

Step 3: Factor out the common algebraic scalar term $(a + b + c)$ from the third column:

$$\Delta = (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

Step 4: Examine the columns of the remaining simplified determinant. Notice that Column 1 (C_1) and Column 3 (C_3) contain identical elements:

$$C_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Step 5: Use the property of determinants which states that if any two rows or columns are identical, the value of the determinant is zero:

$$\Delta = (a + b + c) \cdot 0 = 0$$

Final Answer: $\boxed{0}$

Answer: (C)

Go Back to Question 34

Q35.

Solution

Concept: The derivative $\frac{dy}{dx}$ gives the slope of the tangent line to a curve at a given point. The normal line is perpendicular to the tangent line, so its slope (m_n) is the negative reciprocal of the tangent slope (m_t), given by the formula $m_n = -\frac{1}{m_t}$.

Solution: Step 1: Differentiate the given curve function $y = 2x^2 + 3\sin x$ with respect to x to find the general formula for the tangent slope:

$$\frac{dy}{dx} = \frac{d}{dx}(2x^2) + \frac{d}{dx}(3\sin x) = 4x + 3\cos x$$

Step 2: Evaluate this derivative at the specified point $x = 0$ to find the exact slope of the tangent line (m_t):

$$m_t = \left.\frac{dy}{dx}\right|_{x=0} = 4(0) + 3\cos(0)$$

Step 3: Substitute $\cos(0) = 1$ into the equation to simplify the numerical value:

$$m_t = 0 + 3(1) = 3$$

Step 4: Use the perpendicular geometric relationship to find the slope of the normal line (m_n):

$$m_n = -\frac{1}{m_t}$$

Step 5: Substitute the tangent slope value $m_t = 3$ into the reciprocal formula:

$$m_n = -\frac{1}{3}$$

Final Answer: -1 3

Answer: (D)

Go Back to Question 35

Q36.

Solution

Concept: The scalar projection of a vector $\vec{a}$ onto another vector $\vec{b}$ measures the length of the shadow that $\vec{a}$ casts along the direction of $\vec{b}$. It is computed using the dot product formula: Projection = $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Solution: Step 1: Write down the components of the two vectors given in the problem:

$$\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$$

Step 2: Compute the scalar dot product $\vec{a} \cdot \vec{b}$ by multiplying corresponding components:

$$\vec{a} \cdot \vec{b} = (2 \cdot 1) + (3 \cdot 2) + (2 \cdot 1) = 2 + 6 + 2 = 10$$

Step 3: Calculate the magnitude of vector $\vec{b}$ using the square root of the sum of its squared components:

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$$

Step 4: Substitute the calculated dot product and magnitude values into the projection formula:

$$\text{Projection} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{10}{\sqrt{6}}$$

Final Answer: $\boxed{10\frac{}{\sqrt{6}miniature}}$

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept: By conditional probability rules, $P(\sim A \mid B) = \frac{P(\sim A \cap B)}{P(B)}$. The set $\sim A \cap B$ represents elements that belong to event B but do not belong to event A, which can be computed using the probability formula $P(B) - P(A \cap B)$.

Solution: Step 1: Write down the standard formula for evaluating conditional probability:

$$P(\sim A \mid B) = \frac{P(\sim A \cap B)}{P(B)}$$

Step 2: Use set theory to express the numerator term $P(\sim A \cap B)$ in terms of known probabilities:

$$P(\sim A \cap B) = P(B) - P(A \cap B)$$

Step 3: Substitute the numerical values given in the problem statement into this expression:

$$P(\sim A \cap B) = 0.5 - 0.2 = 0.3$$

Step 4: Substitute the calculated numerator and the given denominator $P(B) = 0.5$ back into the conditional probability equation:

$$P(\sim A \mid B) = \frac{0.3}{0.5}$$

Step 5: Simplify the fraction to find the final probability value:

$$P(\sim A \mid B) = \frac{3}{5} = 0.6$$

Final Answer: $\boxed{0.6}$

Answer: (C)

Go Back to Question 37

Q38.

Solution

Concept: To find the value of $\tan(2\theta)$ where $\theta = \tan^{-1}\left(\frac{1}{3}\right)$, we use the standard double-angle identity for the tangent function: $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$. This allows us to clear the inverse trigonometric functions.

Solution: Step 1: Let $\theta = \tan^{-1}\left(\frac{1}{3}\right)$. Taking the tangent of both sides gives:

$$\tan\theta = \frac{1}{3}$$

Step 2: Express the target expression in terms of the variable θ:

$$\tan\left(2\tan^{-1}\left(\frac{1}{3}\right)\right) = \tan(2\theta)$$

Step 3: Write down the trigonometric identity for the double-angle tangent function:

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Step 4: Substitute the value $\tan\theta = \frac{1}{3}$ into the numerator and denominator of the identity:

$$\tan(2\theta) = \frac{2\cdot\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2} = \frac{\frac{2}{3}}{1-\frac{1}{9}}$$

Step 5: Simplify the fractional terms in the denominator:

$$1-\frac{1}{9} = \frac{8}{9}$$

$$\tan(2\theta) = \frac{\frac{2}{3}}{\frac{8}{9}}$$

Step 6: Perform the fraction division by multiplying by the reciprocal of the denominator:

$$\tan(2\theta) = \frac{2}{3}\times\frac{9}{8} = \frac{18}{24} = \frac{3}{4}$$

Final Answer: $\boxed{3_{4}}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: The slope of a line passing through points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$. Two lines are perpendicular if and only if the product of their slopes equals negative one $(m_1 \cdot m_2 = -1)$. We use this relationship to solve for a.

Solution: Step 1: Compute the slope m_1 of the first line passing through the points $(2, 5)$ and $(4, 9)$:

$$m_1 = \frac{9-5}{4-2} = \frac{4}{2} = 2$$

Step 2: Express the slope m_2 of the second line passing through the points $(a, 3)$ and $(6, 2)$:

$$m_2 = \frac{2-3}{6-a} = \frac{-1}{6-a} = \frac{1}{a-6}$$

Step 3: Use the geometric condition for perpendicular lines, which states that the product of their slopes must equal -1:

$$m_1 \cdot m_2 = -1$$

Step 4: Substitute the calculated expressions for m_1 and m_2 into the perpendicular condition equation:

$$2 \cdot \left(\frac{1}{a-6}\right) = -1$$

$$\frac{2}{a-6} = -1$$

Step 5: Solve the equation for the unknown coordinate parameter a:

$$2 = -1(a-6) \implies 2 = -a + 6$$

$$a = 6 - 2 \implies a = 4$$

Final Answer: $\boxed{4}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 39

Q40.

Solution

Concept: To find the logical equivalent of a complex proposition, we can apply De Morgan's Laws to simplify the negated terms, and then use the distributive and complement laws of Boolean logic to simplify the expression.

Solution: Step 1: Write down the initial logical statement given in the problem:

$$\text{Expression} =\sim (p \vee q) \vee (\sim p \wedge q)$$

Step 2: Apply De Morgan's Law to expand the first term, $\sim (p \vee q) =\sim p\wedge \sim q$:

$$\text{Expression} = (\sim p\wedge \sim q) \vee (\sim p \wedge q)$$

Step 3: Use the distributive law to factor out the common term $(\sim p\wedge)$ from both parts:

$$\text{Expression} =\sim p \wedge (\sim q \vee q)$$

Step 4: Apply the basic complement law to simplify the inner disjunction term, $(\sim q \vee q) = \text{T}$ (always True):

$$\text{Expression} =\sim p \wedge \text{T}$$

Step 5: Apply the identity law of logical conjunction. Any proposition conjoined with a True value reduces to the proposition itself:

$$\text{Expression} =\sim p$$

Final Answer: $\boxed{\sim p}$

Answer: (C)

Go Back to Question 40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	B	4	C	5	B
6	C	7	B	8	C	9	A	10	A
11	A	12	B	13	A	14	B	15	A
16	B	17	B	18	C	19	A	20	B
21	B	22	B	23	B	24	A	25	B
26	A	27	A	28	A	29	B	30	A
31	D	32	A	33	A	34	C	35	D
36	B	37	C	38	A	39	A	40	C