

# SAAT Mathematics

## Sample Paper – 3

Duration: 40 Minutes

Maximum Marks: 40

### Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of the **SAAT** (Siksha 'O' Anusandhan Admission Test).
- Each correct answer carries **+1 mark**. There is **no negative marking** for incorrect or unattempted answers.
- Only **one** option is correct. Attempt every question, since wrong answers are not penalised.
- Use of mobile phones, calculators, or other electronic gadgets is strictly prohibited.

**Q1.** If a set  $A$  has 5 elements, then the number of subsets of  $A$  is

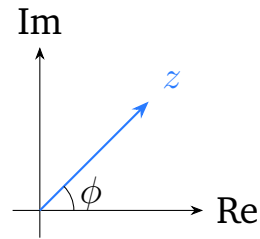
- (A) 25
- (B) 32
- (C) 10
- (D) 64

**Q2.** The range of the function  $f(x) = x^2 + 1$ , where  $x \in \mathbb{R}$ , is

- (A)  $[1, \infty)$
- (B)  $(-\infty, \infty)$
- (C)  $(0, \infty)$
- (D)  $[0, \infty)$

**Q3.** The argument (amplitude) of the complex number  $z = 1 + i$ , plotted below, is





- (A)  $\frac{\pi}{3}$
- (B)  $\frac{\pi}{6}$
- (C)  $\frac{\pi}{4}$
- (D)  $\frac{\pi}{2}$

**Q4.** The value of  $i^{10}$  is

- (A) 1
- (B)  $i$
- (C)  $-i$
- (D)  $-1$

**Q5.** The nature of the roots of the quadratic equation  $x^2 - 4x + 4 = 0$  is

- (A) real and equal
- (B) real and distinct
- (C) complex (non-real)
- (D) one real, one complex

**Q6.** A square matrix  $A$  is called *symmetric* if

- (A)  $A^T = -A$
- (B)  $A^T = A$
- (C)  $A^2 = A$
- (D)  $A^T = A^{-1}$



- Q7. The value of the determinant  $\begin{vmatrix} 2 & 5 & 7 \\ 3 & 1 & 4 \\ 2 & 5 & 7 \end{vmatrix}$ , in which two rows are identical, is
- (A) 14  
(B) 7  
(C) 0  
(D) 1
- Q8. A square matrix  $A$  is invertible if and only if
- (A)  $A = A^T$   
(B)  $|A| = 1$   
(C)  $A$  is of even order  
(D)  $|A| \neq 0$
- Q9. The number of distinct arrangements of the letters of the word **LEVEL** is
- (A) 30  
(B) 120  
(C) 60  
(D) 20
- Q10. The number of ways of selecting a committee of 3 members from 6 persons is
- (A) 18  
(B) 20  
(C) 120  
(D) 216
- Q11. The middle term in the binomial expansion of  $(1 + x)^6$  is



- (A)  $15x^2$
- (B)  $6x^3$
- (C)  $15x^4$
- (D)  $20x^3$

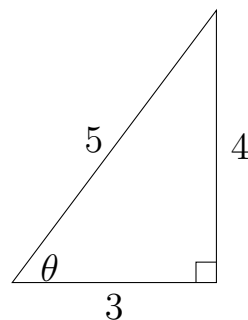
**Q12.** The arithmetic mean between 7 and 19 is

- (A) 26
- (B) 12
- (C) 13
- (D) 11

**Q13.** The sum of the first 5 terms of the geometric progression 3, 6, 12, ... is

- (A) 93
- (B) 96
- (C) 48
- (D) 189

**Q14.** If  $\cos \theta = \frac{3}{5}$  for the angle shown, then  $\cos 2\theta$  equals



- (A)  $\frac{9}{25}$
- (B)  $-\frac{7}{25}$
- (C)  $\frac{7}{25}$



(D)  $\frac{24}{25}$

**Q15.** The general solution of  $\tan \theta = 1$  is ( $n \in \mathbb{Z}$ )

(A)  $2n\pi + \frac{\pi}{4}$

(B)  $n\pi + \frac{\pi}{3}$

(C)  $n\pi + \frac{\pi}{4}$

(D)  $2n\pi + \frac{\pi}{3}$

**Q16.** The principal value of  $\tan^{-1}(1)$  is

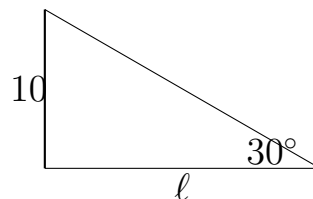
(A)  $\frac{\pi}{6}$

(B)  $\frac{\pi}{3}$

(C)  $\frac{\pi}{2}$

(D)  $\frac{\pi}{4}$

**Q17.** The Sun is at an elevation of  $30^\circ$ . The length of the shadow cast by a vertical pole of height 10 m is



(A) 10 m

(B)  $10\sqrt{3}$  m

(C)  $\frac{10}{\sqrt{3}}$  m

(D) 5 m

**Q18.** The value of  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$  is

(A) 1



- (B) 0
- (C)  $e$
- (D)  $\infty$

**Q19.** The function  $f(x) = \frac{x^2 - 9}{x - 3}$  ( $x \neq 3$ ) has, at  $x = 3$ , a

- (A) jump discontinuity
- (B) infinite discontinuity
- (C) point of continuity
- (D) removable discontinuity

**Q20.** Using the first principle, the derivative of  $f(x) = x^2$  is

- (A)  $x$
- (B)  $x^2$
- (C)  $2x$
- (D) 2

**Q21.** The derivative of  $\frac{x}{x+1}$  with respect to  $x$  is

- (A)  $\frac{1}{(x+1)^2}$
- (B)  $\frac{1}{x+1}$
- (C)  $\frac{-1}{(x+1)^2}$
- (D)  $\frac{2x+1}{(x+1)^2}$

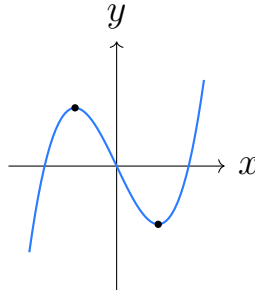
**Q22.** The radius of a circle increases at 2 cm/s. When the radius is 5 cm, the rate of change of its area is

- (A)  $10\pi$  cm<sup>2</sup>/s
- (B)  $20\pi$  cm<sup>2</sup>/s
- (C)  $25\pi$  cm<sup>2</sup>/s



(D)  $4\pi \text{ cm}^2/\text{s}$

**Q23.** The number of critical points of the function  $f(x) = x^3 - 3x$ , whose graph is shown, is



(A) 0

(B) 1

(C) 2

(D) 3

**Q24.** The value of  $\int e^x dx$  is

(A)  $x e^x + C$

(B)  $\frac{e^x}{x} + C$

(C)  $e^{x-1} + C$

(D)  $e^x + C$

**Q25.** The value of  $\int 2x(x^2 + 1) dx$  is

(A)  $\frac{(x^2 + 1)^2}{2} + C$

(B)  $(x^2 + 1)^2 + C$

(C)  $2(x^2 + 1) + C$

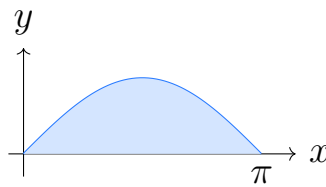
(D)  $\frac{x^4}{2} + C$

**Q26.** The value of  $\int_0^1 x^2 dx$  is



- (A) 1
- (B)  $\frac{1}{3}$
- (C)  $\frac{1}{2}$
- (D)  $\frac{2}{3}$

**Q27.** The area bounded by the curve  $y = \sin x$  and the  $x$ -axis from  $x = 0$  to  $x = \pi$ , shown below, is



- (A) 0
- (B) 1
- (C) 2
- (D)  $\pi$

**Q28.** The order of the differential equation  $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$  is

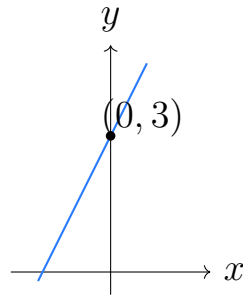
- (A) 0
- (B) 1
- (C) 2
- (D) 3

**Q29.** The solution of the differential equation  $\frac{dy}{dx} = \frac{x}{y}$  is

- (A)  $y^2 - x^2 = C$
- (B)  $y^2 + x^2 = C$
- (C)  $y - x = C$
- (D)  $xy = C$



**Q30.** The  $y$ -intercept of the straight line  $y = 2x + 3$ , shown below, is



- (A) 2
- (B) 3
- (C)  $-\frac{3}{2}$
- (D) 0

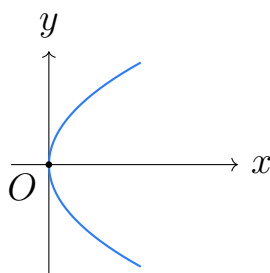
**Q31.** The angle between the lines  $y = x$  and  $y = -x$  is

- (A)  $30^\circ$
- (B)  $45^\circ$
- (C)  $90^\circ$
- (D)  $60^\circ$

**Q32.** The equation of the circle with centre  $(1, -2)$  and radius 3 is

- (A)  $(x - 1)^2 + (y - 2)^2 = 9$
- (B)  $(x + 1)^2 + (y - 2)^2 = 3$
- (C)  $(x - 1)^2 + (y + 2)^2 = 3$
- (D)  $(x - 1)^2 + (y + 2)^2 = 9$

**Q33.** The vertex of the parabola  $y^2 = 12x$ , shown below, is



- (A) (3, 0)
- (B) (0, 0)
- (C) (0, 3)
- (D) (12, 0)

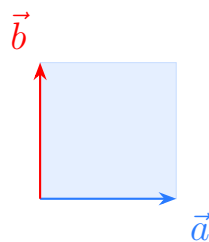
**Q34.** The conic represented by the equation  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  is

- (A) a circle
- (B) a parabola
- (C) an ellipse
- (D) a hyperbola

**Q35.** The scalar projection of  $\vec{a} = 3\hat{i} + 4\hat{j}$  on  $\vec{b} = \hat{i}$  is

- (A) 3
- (B) 4
- (C) 5
- (D) 7

**Q36.** For the vectors  $\vec{a} = 3\hat{i}$  and  $\vec{b} = 4\hat{j}$ , the area of the parallelogram with  $\vec{a}$  and  $\vec{b}$  as adjacent sides, shown below, is



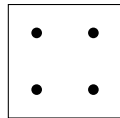
- (A) 7
- (B) 0
- (C) 5
- (D) 12



- Q37.** If  $l, m, n$  are the direction cosines of a line, then  $l^2 + m^2 + n^2$  equals
- (A) 0
  - (B) 1
  - (C) 3
  - (D) depends on the line

- Q38.** The mode of the data 4, 5, 5, 6, 7, 5, 8, 9 is
- (A) 5
  - (B) 6
  - (C) 7
  - (D) 4

- Q39.** A fair die is rolled once. The probability of getting the number 4 is



- (A)  $\frac{1}{3}$
  - (B)  $\frac{1}{2}$
  - (C)  $\frac{1}{6}$
  - (D)  $\frac{2}{3}$
- Q40.** Two identical bags are given. Bag I has 3 red and 2 black balls; Bag II has 1 red and 4 black balls. A bag is chosen at random and a red ball is drawn. The probability that it came from Bag I is
- (A)  $\frac{1}{2}$
  - (B)  $\frac{3}{5}$
  - (C)  $\frac{1}{4}$
  - (D)  $\frac{3}{4}$



## Detailed Solutions

Q1.

## Solution

**Concept — Number of subsets:** A set with  $n$  elements has exactly  $2^n$  subsets, because each element can either be included or left out (2 choices each).

**Step 1 — Read off  $n$ :** The set  $A$  has 5 elements, so  $n = 5$ .

**Step 2 — Write the formula:** Number of subsets =  $2^n$ .

**Step 3 — Substitute  $n = 5$ :** Number of subsets =  $2^5$ .

**Step 4 — Expand the power:**  $2^5 = 2 \times 2 \times 2 \times 2 \times 2$ .

**Step 5 — Multiply step by step:**  $2 \times 2 = 4$ ;  $4 \times 2 = 8$ ;  $8 \times 2 = 16$ ;  $16 \times 2 = 32$ .

**Step 6 — State the count:** Number of subsets = 32.

**Why other options are wrong:**  $25 = 5^2$  swaps base and exponent;  $10 = 2 \times 5$  multiplies instead of raising to a power;  $64 = 2^6$  uses the wrong exponent.

**Final Answer:**  $2^5 = 32 \Rightarrow$

[Go Back to Q1](#)

Q2.

## Solution

**Concept — Range of  $x^2 + 1$ :** The range is the set of all output values  $f(x)$  as  $x$  runs over the reals. We find the smallest possible output and check it can be reached.

**Step 1 — Bound the squared term:** For every real  $x$ , a square is never negative, so  $x^2 \geq 0$ .

**Step 2 — Add 1 to both sides:**  $x^2 + 1 \geq 0 + 1$ .

**Step 3 — Simplify the inequality:**  $f(x) = x^2 + 1 \geq 1$ .

**Step 4 — Check the minimum is attained:** Put  $x = 0$ , then  $f(0) = 0^2 + 1 = 1$ , so the value 1 is actually reached.

**Step 5 — Check large values:** As  $x$  grows,  $x^2$  grows without bound, so  $f(x)$  takes every value above 1 up to  $\infty$ .

**Step 6 — Write the range:** Outputs run from 1 (included) to  $\infty$ , so the range is



$[1, \infty)$ .

**Why other options are wrong:**  $(-\infty, \infty)$  ignores the lower bound;  $(0, \infty)$  wrongly starts at 0 and excludes it;  $[0, \infty)$  uses minimum 0 instead of 1.

**Final Answer:** Range =  $[1, \infty) \Rightarrow \boxed{A}$

**Answer: (A)** [Go Back to Q2](#)

**Q3.**

### Solution

**Concept — Argument of a complex number:** For  $z = a + bi$  lying in the first quadrant, the argument is  $\arg z = \tan^{-1}\left(\frac{b}{a}\right)$ , the angle the line to  $z$  makes with the positive real axis.

**Step 1 — Identify the real part:** In  $z = 1 + i$ , the real part is  $a = 1$ .

**Step 2 — Identify the imaginary part:** The imaginary part is  $b = 1$ .

**Step 3 — Form the ratio:**  $\frac{b}{a} = \frac{1}{1} = 1$ .

**Step 4 — Apply the formula:**  $\arg z = \tan^{-1}(1)$ .

**Step 5 — Evaluate the inverse tangent:** The angle whose tangent is 1 is  $\frac{\pi}{4}$ , so  $\arg z = \frac{\pi}{4}$ .

**Why other options are wrong:**  $\frac{\pi}{3}$  comes from  $\tan^{-1}\sqrt{3}$  and  $\frac{\pi}{6}$  from  $\tan^{-1}\frac{1}{\sqrt{3}}$ ;  $\frac{\pi}{2}$  is the argument of a purely imaginary number ( $a = 0$ ).

**Final Answer:**  $\arg z = \frac{\pi}{4} \Rightarrow \boxed{C}$

**Answer: (C)** [Go Back to Q3](#)

**Q4.**

### Solution

**Concept — Powers of  $i$  cycle with period 4:** The powers repeat in blocks of four:  $i^1 = i$ ,  $i^2 = -1$ ,  $i^3 = -i$ ,  $i^4 = 1$ , then start again. So  $i^n$  depends only on the remainder of  $n$  when divided by 4.

**Step 1 — Divide the exponent by 4:**  $10 \div 4 = 2$  with remainder 2.

**Step 2 — Write 10 as  $4 \times$  quotient + remainder:**  $10 = 4 \times 2 + 2$ .

**Step 3 — Split the power:**  $i^{10} = i^{4 \times 2 + 2} = (i^4)^2 \cdot i^2$ .



**Step 4 — Use  $i^4 = 1$ :**  $(i^4)^2 = 1^2 = 1$ , so  $i^{10} = 1 \cdot i^2 = i^2$ .

**Step 5 — Evaluate  $i^2$ :**  $i^2 = -1$ , so  $i^{10} = -1$ .

**Why other options are wrong:** 1,  $i$  and  $-i$  correspond to remainders 0, 1 and 3; here the remainder is 2.

**Final Answer:**  $i^{10} = -1 \Rightarrow$   D

Answer: (D) [Go Back to Q4](#)

Q5.

### Solution

**Concept — Discriminant:** For  $ax^2 + bx + c = 0$ , the discriminant  $D = b^2 - 4ac$  decides the nature of the roots:  $D > 0$  gives real distinct roots,  $D = 0$  gives real equal roots,  $D < 0$  gives complex roots.

**Step 1 — Read the coefficients:** Comparing  $x^2 - 4x + 4 = 0$  with  $ax^2 + bx + c = 0$  gives  $a = 1$ ,  $b = -4$ ,  $c = 4$ .

**Step 2 — Write the discriminant:**  $D = b^2 - 4ac$ .

**Step 3 — Substitute the values:**  $D = (-4)^2 - 4(1)(4)$ .

**Step 4 — Square the  $b$  term:**  $(-4)^2 = 16$ .

**Step 5 — Multiply the  $4ac$  term:**  $4(1)(4) = 16$ .

**Step 6 — Subtract:**  $D = 16 - 16 = 0$ .

**Step 7 — Interpret:** Since  $D = 0$ , the roots are real and equal.

**Why other options are wrong:** Real-and-distinct needs  $D > 0$ ; complex needs  $D < 0$ ; one-real-one-complex is impossible for real coefficients (complex roots come in pairs).

**Final Answer:** Real and equal  $\Rightarrow$   A

Answer: (A) [Go Back to Q5](#)



Q6.

**Solution**

**Concept — Symmetric matrix:** The transpose  $A^T$  is obtained by interchanging rows and columns of  $A$ . A matrix is symmetric when this interchange leaves it unchanged.

**Step 1 — State the entry condition:** Symmetry means every entry equals its mirror across the diagonal:  $a_{ij} = a_{ji}$  for all  $i, j$ .

**Step 2 — Rewrite in matrix form:** Saying  $a_{ij} = a_{ji}$  for all  $i, j$  is the same as saying  $A$  equals its transpose.

**Step 3 — Write the definition:** Therefore  $A$  is symmetric  $\iff A^T = A$ .

**Why other options are wrong:**  $A^T = -A$  defines a skew-symmetric matrix;  $A^2 = A$  is the idempotent condition;  $A^T = A^{-1}$  defines an orthogonal matrix.

**Final Answer:**  $A^T = A \Rightarrow$   B

Answer: (B) [Go Back to Q6](#)

Q7.

**Solution**

**Concept — Equal rows:** A standard property of determinants states that if two rows (or two columns) are identical, the determinant equals 0.

**Step 1 — Write out row 1:** Row 1 = (2, 5, 7).

**Step 2 — Write out row 3:** Row 3 = (2, 5, 7).

**Step 3 — Compare the rows:** Row 1 and row 3 are entry-by-entry identical.

**Step 4 — Apply the property:** Because two rows are equal, the determinant is 0.

**Why other options are wrong:** 14, 7 and 1 would require expanding the determinant and ignore the equal-row property that forces the value to 0.

**Final Answer:** Determinant = 0  $\Rightarrow$   C

Answer: (C) [Go Back to Q7](#)



Q8.

**Solution**

**Concept — Existence of inverse:** A square matrix  $A$  has an inverse exactly when it is non-singular, that is when its determinant is non-zero.

**Step 1 — Recall the inverse formula:**  $A^{-1} = \frac{1}{|A|} \text{adj } A$ .

**Step 2 — Spot the danger:** The formula divides by  $|A|$ , and division is only allowed when  $|A| \neq 0$ .

**Step 3 — Conclude the condition:** Hence  $A^{-1}$  exists if and only if  $|A| \neq 0$ .

**Why other options are wrong:** Being symmetric, having  $|A| = 1$ , or being of even order are none of them required; a matrix can be invertible without any of these and can fail to be invertible while having them.

**Final Answer:**  $|A| \neq 0 \Rightarrow \boxed{\text{D}}$

**Answer: (D)** [Go Back to Q8](#)

Q9.

**Solution**

**Concept — Arrangements with repetition:** If  $n$  letters contain repeats, the number of distinct arrangements is  $\frac{n!}{(\text{factorials of each repeat count})}$ , because swapping identical letters does not create a new word.

**Step 1 — Count the total letters:** LEVEL has 5 letters, so  $n = 5$ .

**Step 2 — Count each repeated letter:** L appears 2 times and E appears 2 times; V appears once.

**Step 3 — Write the formula:** Arrangements =  $\frac{5!}{2! \times 2!}$ .

**Step 4 — Compute 5!:**  $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ .

**Step 5 — Compute the denominator:**  $2! = 2$ , so  $2! \times 2! = 2 \times 2 = 4$ .

**Step 6 — Divide:**  $\frac{120}{4} = 30$ .

**Why other options are wrong:** 120 ignores the repeats; 60 divides by only one 2!; 20 is unrelated.

**Final Answer:** 30 arrangements  $\Rightarrow \boxed{\text{A}}$

**Answer: (A)** [Go Back to Q9](#)



Q10.

**Solution**

**Concept — Selection (combination):** The number of ways to choose  $r$  people from  $n$ , where order does not matter, is  ${}^nC_r = \frac{n!}{r!(n-r)!}$ .

**Step 1 — Identify  $n$  and  $r$ :** Choosing 3 members from 6 persons gives  $n = 6$ ,  $r = 3$ .

**Step 2 — Write the formula:**  ${}^6C_3 = \frac{6!}{3!(6-3)!} = \frac{6!}{3!3!}$ .

**Step 3 — Cancel to a short form:** Writing  $6! = 6 \times 5 \times 4 \times 3!$  and cancelling one  $3!$  gives  ${}^6C_3 = \frac{6 \times 5 \times 4}{3!}$ .

**Step 4 — Expand the denominator:**  $3! = 3 \times 2 \times 1 = 6$ .

**Step 5 — Multiply the numerator:**  $6 \times 5 = 30$ , then  $30 \times 4 = 120$ .

**Step 6 — Divide:**  $\frac{120}{6} = 20$ .

**Why other options are wrong:** 18 comes from a wrong multiplication;  $120 = {}^6P_3$  counts ordered selections;  $216 = 6^3$  allows repetition.

**Final Answer:**  ${}^6C_3 = 20 \Rightarrow$  **B**

**Answer: (B)** [Go Back to Q10](#)

Q11.

**Solution**

**Concept — Middle term:** In  $(1+x)^n$  there are  $n+1$  terms. When  $n$  is even there is exactly one middle term, namely the  $(\frac{n}{2} + 1)$ th term, and the general term is  $T_{r+1} = {}^nC_r x^r$ .

**Step 1 — Read off  $n$ :** Here  $n = 6$ .

**Step 2 — Locate the middle term:** The middle term is the  $(\frac{6}{2} + 1) = (3+1) = 4$ th term,  $T_4$ .

**Step 3 — Find  $r$  for  $T_4$ :** Since  $T_{r+1} = T_4$ , we have  $r+1 = 4$ , so  $r = 3$ .

**Step 4 — Write the term:**  $T_4 = {}^6C_3 x^3$ .

**Step 5 — Evaluate the coefficient:**  ${}^6C_3 = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = \frac{120}{6} = 20$ .

**Step 6 — Assemble:**  $T_4 = 20x^3$ .



**Why other options are wrong:**  $15x^2$  and  $15x^4$  are the neighbouring terms  $T_3$  and  $T_5$ ;  $6x^3$  uses an incorrect coefficient.

**Final Answer:** Middle term =  $20x^3 \Rightarrow$   D

**Answer: (D)** [Go Back to Q11](#)

Q12.

### Solution

**Concept — Arithmetic mean:** The arithmetic mean (AM) of two numbers  $a$  and  $b$  is their sum divided by 2:  $AM = \frac{a+b}{2}$ .

**Step 1 — Identify the numbers:** Here  $a = 7$  and  $b = 19$ .

**Step 2 — Write the formula:**  $AM = \frac{a+b}{2} = \frac{7+19}{2}$ .

**Step 3 — Add the numerator:**  $7 + 19 = 26$ , so  $AM = \frac{26}{2}$ .

**Step 4 — Divide:**  $\frac{26}{2} = 13$ .

**Why other options are wrong:** 26 forgets to divide by 2; 12 and 11 are arithmetic slips.

**Final Answer:**  $AM = 13 \Rightarrow$   C

**Answer: (C)** [Go Back to Q12](#)

Q13.

### Solution

**Concept — Sum of  $n$  terms of a GP:** For a geometric progression with first term  $a$  and common ratio  $r > 1$ , the sum of the first  $n$  terms is  $S_n = \frac{a(r^n - 1)}{r - 1}$ .

**Step 1 — First term:** The first term is  $a = 3$ .

**Step 2 — Common ratio:** Dividing a term by the previous one,  $r = \frac{6}{3} = 2$  (check:  $\frac{12}{6} = 2$ ).

**Step 3 — Number of terms:** We need the first 5 terms, so  $n = 5$ .

**Step 4 — Substitute into the formula:**  $S_5 = \frac{3(2^5 - 1)}{2 - 1}$ .

**Step 5 — Evaluate the power:**  $2^5 = 32$ .



**Step 6 — Simplify inside the bracket:**  $32 - 1 = 31$ .

**Step 7 — Simplify the denominator:**  $2 - 1 = 1$ .

**Step 8 — Finish:**  $S_5 = \frac{3 \times 31}{1} = 93$ .

**Why other options are wrong:** 96 is  $3 \times 32$  (forgets the  $-1$ ); 48 is the 5th term alone; 189 uses a wrong ratio.

**Final Answer:**  $S_5 = 93 \Rightarrow \boxed{\text{A}}$

**Answer: (A)** [Go Back to Q13](#)

**Q14.**

### Solution

**Concept — Double angle:** The double-angle identity  $\cos 2\theta = 2 \cos^2 \theta - 1$  lets us find  $\cos 2\theta$  from  $\cos \theta$  alone.

**Step 1 — Write the identity:**  $\cos 2\theta = 2 \cos^2 \theta - 1$ .

**Step 2 — Substitute  $\cos \theta = \frac{3}{5}$ :**  $\cos 2\theta = 2 \left(\frac{3}{5}\right)^2 - 1$ .

**Step 3 — Square the fraction:**  $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$ .

**Step 4 — Multiply by 2:**  $2 \times \frac{9}{25} = \frac{18}{25}$ .

**Step 5 — Write 1 over 25:**  $1 = \frac{25}{25}$ .

**Step 6 — Subtract:**  $\frac{18}{25} - \frac{25}{25} = \frac{18 - 25}{25} = -\frac{7}{25}$ .

**Why other options are wrong:**  $\frac{9}{25}$  is just  $\cos^2 \theta$ ;  $\frac{7}{25}$  drops the negative sign;  $\frac{24}{25}$  is  $\sin 2\theta$ .

**Final Answer:**  $\cos 2\theta = -\frac{7}{25} \Rightarrow \boxed{\text{B}}$

**Answer: (B)** [Go Back to Q14](#)



Q15.

**Solution**

**Concept — General solution of  $\tan \theta = k$ :** Since  $\tan$  repeats every  $\pi$ , if  $\tan \alpha = k$  then all solutions are  $\theta = n\pi + \alpha, n \in \mathbb{Z}$ .

**Step 1 — Write the equation:** We are given  $\tan \theta = 1$ , so here  $k = 1$ .

**Step 2 — Find the principal angle:** We need  $\alpha$  with  $\tan \alpha = 1$ ; since  $\tan \frac{\pi}{4} = 1$ , we take  $\alpha = \frac{\pi}{4}$ .

**Step 3 — Apply the general form:** Substitute  $\alpha = \frac{\pi}{4}$  into  $\theta = n\pi + \alpha$ .

**Step 4 — Write the solution:**  $\theta = n\pi + \frac{\pi}{4}, n \in \mathbb{Z}$ .

**Why other options are wrong:** The tangent has period  $\pi$  (not  $2\pi$ ), so the  $2n\pi$  forms are wrong;  $\frac{\pi}{3}$  is not the angle whose tangent is 1.

**Final Answer:**  $\theta = n\pi + \frac{\pi}{4} \Rightarrow \boxed{\text{C}}$

**Answer: (C)** [Go Back to Q15](#)

Q16.

**Solution**

**Concept — Principal value of  $\tan^{-1}$ :** The principal value of  $\tan^{-1} x$  is the unique angle in the open interval  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  whose tangent is  $x$ .

**Step 1 — Set up the requirement:** We want the angle  $\theta$  in  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  with  $\tan \theta = 1$ .

**Step 2 — Test a known angle:**  $\tan \frac{\pi}{4} = 1$ .

**Step 3 — Check it is in range:**  $\frac{\pi}{4}$  lies inside  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , so it is the principal value.

**Step 4 — Conclude:**  $\tan^{-1}(1) = \frac{\pi}{4}$ .

**Why other options are wrong:**  $\frac{\pi}{6}$  gives  $\tan = \frac{1}{\sqrt{3}}$ ;  $\frac{\pi}{3}$  gives  $\tan = \sqrt{3}$ ;  $\frac{\pi}{2}$  is outside the principal range and  $\tan$  is undefined there.

**Final Answer:**  $\tan^{-1}(1) = \frac{\pi}{4} \Rightarrow \boxed{\text{D}}$

**Answer: (D)** [Go Back to Q16](#)



Q17.

**Solution**

**Concept — Elevation and shadow:** In the right triangle formed by the pole and its shadow,  $\tan(\text{angle of elevation}) = \frac{\text{height (opposite)}}{\text{shadow (adjacent)}}$ .

**Step 1 — Label the triangle:** Height (opposite the  $30^\circ$  angle) = 10 m; shadow (adjacent) =  $\ell$ .

**Step 2 — Write the relation:**  $\tan 30^\circ = \frac{10}{\ell}$ .

**Step 3 — Insert the value of  $\tan 30^\circ$ :**  $\frac{1}{\sqrt{3}} = \frac{10}{\ell}$ .

**Step 4 — Cross-multiply:**  $\ell \times 1 = 10 \times \sqrt{3}$ .

**Step 5 — Read off the shadow:**  $\ell = 10\sqrt{3}$  m.

**Why other options are wrong:** 10 assumes a  $45^\circ$  elevation;  $\frac{10}{\sqrt{3}}$  inverts the ratio (puts shadow over height); 5 is unrelated.

**Final Answer:** Shadow =  $10\sqrt{3}$  m  $\Rightarrow$  **B**

**Answer: (B)** [Go Back to Q17](#)

Q18.

**Solution**

**Concept — Standard limit:** This is a standard limit whose value is  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$ . We verify it using the series for  $e^x$ .

**Step 1 — Write the series for  $e^x$ :**  $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

**Step 2 — Subtract 1:**  $e^x - 1 = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

**Step 3 — Divide by  $x$ :**  $\frac{e^x - 1}{x} = 1 + \frac{x}{2!} + \frac{x^2}{3!} + \dots$

**Step 4 — Let  $x \rightarrow 0$ :** Every term containing  $x$  vanishes, leaving 1.

**Step 5 — State the limit:**  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$ .

**Why other options are wrong:** The limit exists and equals 1, so 0,  $e$  and  $\infty$  are all incorrect.

**Final Answer:** The limit is 1  $\Rightarrow$  **A**



**Answer: (A)** [Go Back to Q18](#)

Q19.

### Solution

**Concept — Removable discontinuity:** A discontinuity at a point is called removable if the two-sided limit exists there but the function is either undefined or has the wrong value; the gap can be patched by redefining a single point.

**Step 1 — Factor the numerator:**  $x^2 - 9$  is a difference of squares, so  $x^2 - 9 = (x - 3)(x + 3)$ .

**Step 2 — Rewrite the function:**  $\frac{x^2 - 9}{x - 3} = \frac{(x - 3)(x + 3)}{x - 3}$ .

**Step 3 — Cancel the common factor:** For  $x \neq 3$ ,  $\frac{(x - 3)(x + 3)}{x - 3} = x + 3$ .

**Step 4 — Take the limit at  $x = 3$ :**  $\lim_{x \rightarrow 3} (x + 3) = 3 + 3 = 6$ , which is a finite number.

**Step 5 — Compare with  $f(3)$ :**  $f(3)$  is not defined (denominator is 0), yet the limit exists, so the single missing point can be filled in.

**Step 6 — Classify:** A finite limit at an undefined point means the discontinuity is removable.

**Why other options are wrong:** There is no sudden jump and no infinite blow-up, and the function is not defined at 3, so it is not a point of continuity either.

**Final Answer:** Removable discontinuity  $\Rightarrow$  **D**

**Answer: (D)** [Go Back to Q19](#)

Q20.

### Solution

**Concept — First principle:** The derivative from first principles is  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$ .

**Step 1 — Write  $f(x + h)$ :** With  $f(x) = x^2$ ,  $f(x + h) = (x + h)^2$ .

**Step 2 — Form the difference quotient:**  $\frac{f(x + h) - f(x)}{h} = \frac{(x + h)^2 - x^2}{h}$ .

**Step 3 — Expand  $(x + h)^2$ :**  $(x + h)^2 = x^2 + 2xh + h^2$ .

**Step 4 — Subtract  $x^2$  in the numerator:**  $x^2 + 2xh + h^2 - x^2 = 2xh + h^2$ .



**Step 5 — Divide by  $h$ :**  $\frac{2xh + h^2}{h} = 2x + h$ .

**Step 6 — Take the limit  $h \rightarrow 0$ :**  $2x + h \rightarrow 2x + 0 = 2x$ .

**Step 7 — State the derivative:**  $f'(x) = 2x$ .

**Why other options are wrong:**  $x$  and  $2$  drop terms during differentiation;  $x^2$  is the original function, not its derivative.

**Final Answer:**  $f'(x) = 2x \Rightarrow$   C

**Answer: (C)** [Go Back to Q20](#)

**Q21.**

### Solution

**Concept — Quotient rule:** To differentiate a fraction  $\frac{u}{v}$ , use  $\frac{d}{dx} \frac{u}{v} = \frac{u'v - uv'}{v^2}$ .

**Step 1 — Identify  $u$  and  $v$ :**  $u = x$  and  $v = x + 1$ .

**Step 2 — Differentiate  $u$ :**  $u' = \frac{d}{dx}(x) = 1$ .

**Step 3 — Differentiate  $v$ :**  $v' = \frac{d}{dx}(x + 1) = 1$ .

**Step 4 — Substitute into the rule:**  $\frac{u'v - uv'}{v^2} = \frac{(1)(x + 1) - (x)(1)}{(x + 1)^2}$ .

**Step 5 — Simplify the numerator:**  $(x + 1) - x = 1$ .

**Step 6 — Write the result:**  $\frac{1}{(x + 1)^2}$ .

**Why other options are wrong:**  $\frac{1}{x+1}$  forgets to square  $v$ ; the negative sign and  $\frac{2x+1}{(x+1)^2}$  come from sign or algebra slips.

**Final Answer:**  $\frac{1}{(x + 1)^2} \Rightarrow$   A

**Answer: (A)** [Go Back to Q21](#)



Q22.

**Solution**

**Concept — Related rates:** The area of a circle is  $A = \pi r^2$ ; differentiating both sides with respect to time links the rate of change of area to the rate of change of radius.

**Step 1 — Write the area:**  $A = \pi r^2$ .

**Step 2 — Differentiate with respect to  $t$ :** Using the chain rule,  $\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt} = 2\pi r \frac{dr}{dt}$ .

**Step 3 — List the given data:**  $r = 5$  cm and  $\frac{dr}{dt} = 2$  cm/s.

**Step 4 — Substitute:**  $\frac{dA}{dt} = 2\pi(5)(2)$ .

**Step 5 — Multiply the numbers:**  $2 \times 5 = 10$ , then  $10 \times 2 = 20$ .

**Step 6 — State the rate:**  $\frac{dA}{dt} = 20\pi$  cm<sup>2</sup>/s.

**Why other options are wrong:**  $10\pi$  uses  $\frac{dr}{dt} = 1$ ;  $25\pi$  is the area  $\pi r^2$  itself;  $4\pi$  drops the factor  $r$ .

**Final Answer:**  $\frac{dA}{dt} = 20\pi$  cm<sup>2</sup>/s  $\Rightarrow$  **B**

**Answer: (B)** [Go Back to Q22](#)

Q23.

**Solution**

**Concept — Critical points:** Critical points are the  $x$ -values where the derivative  $f'(x)$  is zero (or undefined). Counting them tells us how many turning points the curve can have.

**Step 1 — Differentiate term by term:** For  $f(x) = x^3 - 3x$ ,  $\frac{d}{dx}(x^3) = 3x^2$  and  $\frac{d}{dx}(-3x) = -3$ , so  $f'(x) = 3x^2 - 3$ .

**Step 2 — Set the derivative to zero:**  $3x^2 - 3 = 0$ .

**Step 3 — Factor out 3:**  $3(x^2 - 1) = 0$ .

**Step 4 — Divide by 3:**  $x^2 - 1 = 0$ .

**Step 5 — Solve for  $x$ :**  $x^2 = 1$ , so  $x = +1$  or  $x = -1$ .



**Step 6 — Count the solutions:** There are 2 values of  $x$ , hence 2 critical points (one local max and one local min, matching the graph).

**Why other options are wrong:** 0 and 1 undercount the solutions of  $x^2 = 1$ ; 3 over-counts.

**Final Answer:** 2 critical points  $\Rightarrow$   C

Answer: (C) [Go Back to Q23](#)

Q24.

### Solution

**Concept — Integral of  $e^x$ :** Integration reverses differentiation, so the integral of  $e^x$  is the function whose derivative is  $e^x$ .

**Step 1 — Recall the derivative:**  $\frac{d}{dx}(e^x) = e^x$ .

**Step 2 — Reverse it:** Since differentiating  $e^x$  returns  $e^x$ , the antiderivative of  $e^x$  is  $e^x$ .

**Step 3 — Add the constant:** An indefinite integral always carries an arbitrary constant, so  $\int e^x dx = e^x + C$ .

**Step 4 — Check by differentiating:**  $\frac{d}{dx}(e^x + C) = e^x + 0 = e^x$ , confirming the answer.

**Why other options are wrong:** Differentiating  $x e^x$ ,  $\frac{e^x}{x}$  or  $e^{x-1}$  does not give back  $e^x$ , so none is the correct antiderivative.

**Final Answer:**  $e^x + C \Rightarrow$   D

Answer: (D) [Go Back to Q24](#)

Q25.

### Solution

**Concept —  $u$ -substitution:** When an integrand contains a function and its derivative, substitute  $u$  for the inner function to simplify.

**Step 1 — Choose the substitution:** Let  $u = x^2 + 1$ .

**Step 2 — Differentiate  $u$ :**  $\frac{du}{dx} = 2x$ , so  $du = 2x dx$ .

**Step 3 — Rewrite the integral:** The integrand is  $2x(x^2 + 1) dx = (x^2 + 1)(2x dx) = u du$ .



**Step 4 — Integrate in  $u$ :**  $\int u \, du = \frac{u^2}{2} + C$ .

**Step 5 — Substitute back  $u = x^2 + 1$ :**  $\frac{(x^2 + 1)^2}{2} + C$ .

**Why other options are wrong:**  $(x^2 + 1)^2$  omits the  $\frac{1}{2}$ ;  $2(x^2 + 1)$  is a derivative, not the integral;  $\frac{x^4}{2}$  ignores the inner function  $+1$ .

**Final Answer:**  $\frac{(x^2 + 1)^2}{2} + C \Rightarrow \boxed{\text{A}}$

**Answer: (A)** [Go Back to Q25](#)

**Q26.**

### Solution

**Concept — Definite integral of a power:** Use the power rule  $\int x^n \, dx = \frac{x^{n+1}}{n+1}$ , then substitute the upper and lower limits and subtract.

**Step 1 — Apply the power rule:** With  $n = 2$ ,  $\int x^2 \, dx = \frac{x^3}{3}$ .

**Step 2 — Write with limits:**  $\int_0^1 x^2 \, dx = \left[ \frac{x^3}{3} \right]_0^1$ .

**Step 3 — Substitute the upper limit  $x = 1$ :**  $\frac{1^3}{3} = \frac{1}{3}$ .

**Step 4 — Substitute the lower limit  $x = 0$ :**  $\frac{0^3}{3} = 0$ .

**Step 5 — Subtract:**  $\frac{1}{3} - 0 = \frac{1}{3}$ .

**Why other options are wrong:** 1 forgets the  $\frac{1}{3}$ ;  $\frac{1}{2}$  integrates  $x$  instead of  $x^2$ ;  $\frac{2}{3}$  doubles wrongly.

**Final Answer:** The integral  $= \frac{1}{3} \Rightarrow \boxed{\text{B}}$

**Answer: (B)** [Go Back to Q26](#)



Q27.

**Solution**

**Concept — Area under a curve:** Since  $\sin x \geq 0$  on  $[0, \pi]$ , the area between the curve and the  $x$ -axis is  $\int_0^\pi \sin x \, dx$ .

**Step 1 — Recall the antiderivative:**  $\int \sin x \, dx = -\cos x$ .

**Step 2 — Write with limits:** Area =  $[-\cos x]_0^\pi$ .

**Step 3 — Substitute the upper limit  $x = \pi$ :**  $-\cos \pi = -(-1) = 1$ .

**Step 4 — Substitute the lower limit  $x = 0$ :**  $-\cos 0 = -(1) = -1$ .

**Step 5 — Subtract (upper minus lower):**  $1 - (-1) = 1 + 1$ .

**Step 6 — Compute:**  $= 2$ .

**Why other options are wrong:** 0 is the value of  $\int_0^{2\pi} \sin x \, dx$  (positive and negative areas cancel); 1 and  $\pi$  are miscomputations.

**Final Answer:** Area = 2  $\Rightarrow$   C

**Answer: (C)** [Go Back to Q27](#)

Q28.

**Solution**

**Concept — Order of a differential equation:** The order is the order of the highest derivative that appears in the equation.

**Step 1 — List the derivatives present:** The equation contains  $\frac{d^3y}{dx^3}$ ,  $\frac{d^2y}{dx^2}$  and  $\frac{dy}{dx}$ , of orders 3, 2 and 1.

**Step 2 — Pick the highest:** The largest of 3, 2, 1 is 3, coming from  $\frac{d^3y}{dx^3}$ .

**Step 3 — State the order:** Therefore the order = 3.

**Why other options are wrong:** 0, 1 and 2 ignore the third derivative, which is the highest present.

**Final Answer:** Order = 3  $\Rightarrow$   D

**Answer: (D)** [Go Back to Q28](#)



Q29.

**Solution**

**Concept — Variable separable:** Move all  $y$  terms to one side and all  $x$  terms to the other, then integrate both sides.

**Step 1 — Write the equation:**  $\frac{dy}{dx} = \frac{x}{y}$ .

**Step 2 — Cross-multiply to separate:** Multiply both sides by  $y dx$  to get  $y dy = x dx$ .

**Step 3 — Integrate both sides:**  $\int y dy = \int x dx$ .

**Step 4 — Apply the power rule:**  $\frac{y^2}{2} = \frac{x^2}{2} + c$ .

**Step 5 — Multiply through by 2:**  $y^2 = x^2 + 2c$ .

**Step 6 — Rearrange and rename the constant:**  $y^2 - x^2 = 2c = C$ .

**Why other options are wrong:**  $y^2 + x^2 = C$  comes from  $\frac{dy}{dx} = -\frac{x}{y}$ ;  $y - x = C$  and  $xy = C$  do not satisfy the given equation when differentiated.

**Final Answer:**  $y^2 - x^2 = C \Rightarrow \boxed{\text{A}}$

**Answer: (A)** [Go Back to Q29](#)

Q30.

**Solution**

**Concept — Slope–intercept form:** A line written as  $y = mx + c$  has slope  $m$  and  $y$ -intercept  $c$  (the value of  $y$  where  $x = 0$ ).

**Step 1 — Compare with the standard form:** Matching  $y = 2x + 3$  with  $y = mx + c$  gives  $m = 2$  and  $c = 3$ .

**Step 2 — Set  $x = 0$  to find the intercept:**  $y = 2(0) + 3 = 3$ .

**Step 3 — Conclude:** The line meets the  $y$ -axis at  $(0, 3)$ , so the  $y$ -intercept is 3.

**Why other options are wrong:** 2 is the slope  $m$ ;  $-\frac{3}{2}$  is the  $x$ -intercept (where  $y = 0$ ); 0 would mean the line passes through the origin.

**Final Answer:**  $y$ -intercept = 3  $\Rightarrow \boxed{\text{B}}$

**Answer: (B)** [Go Back to Q30](#)



Q31.

**Solution**

**Concept — Angle between lines:** For two lines with slopes  $m_1$  and  $m_2$ , the angle  $\phi$  between them satisfies  $\tan \phi = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$ .

**Step 1 — Slope of the first line:**  $y = x$  has  $m_1 = 1$ .

**Step 2 — Slope of the second line:**  $y = -x$  has  $m_2 = -1$ .

**Step 3 — Compute the denominator:**  $1 + m_1 m_2 = 1 + (1)(-1) = 1 - 1 = 0$ .

**Step 4 — Interpret a zero denominator:** Dividing by 0 makes  $\tan \phi$  infinitely large, which happens at  $\phi = 90^\circ$ .

**Step 5 — Conclude:** The lines are perpendicular, so the angle is  $90^\circ$ .

**Why other options are wrong:**  $30^\circ$ ,  $45^\circ$  and  $60^\circ$  do not satisfy the perpendicularity condition  $m_1 m_2 = -1$  that holds here.

**Final Answer:** Angle =  $90^\circ \Rightarrow$   C

Answer: (C) [Go Back to Q31](#)

Q32.

**Solution**

**Concept — Circle equation:** A circle with centre  $(h, k)$  and radius  $r$  has equation  $(x - h)^2 + (y - k)^2 = r^2$ .

**Step 1 — Read the centre:**  $(h, k) = (1, -2)$ , so  $h = 1$  and  $k = -2$ .

**Step 2 — Read the radius:**  $r = 3$ .

**Step 3 — Substitute into the  $x$  term:**  $(x - h)^2 = (x - 1)^2$ .

**Step 4 — Substitute into the  $y$  term:**  $(y - k)^2 = (y - (-2))^2 = (y + 2)^2$ .

**Step 5 — Square the radius:**  $r^2 = 3^2 = 9$ .

**Step 6 — Assemble the equation:**  $(x - 1)^2 + (y + 2)^2 = 9$ .

**Why other options are wrong:**  $(x - 1)^2 + (y - 2)^2 = 9$  has the wrong sign on  $k$ ; the other two use  $r = 3$  instead of  $r^2 = 9$  or wrong signs.

**Final Answer:**  $(x - 1)^2 + (y + 2)^2 = 9 \Rightarrow$   D

Answer: (D) [Go Back to Q32](#)



Q33.

**Solution**

**Concept — Standard parabola  $y^2 = 4ax$ :** A parabola of the form  $y^2 = 4ax$  opens rightward with its vertex fixed at the origin  $(0, 0)$  and focus at  $(a, 0)$ .

**Step 1 — Match the equation:**  $y^2 = 12x$  matches  $y^2 = 4ax$ , so  $4a = 12$ .

**Step 2 — Solve for  $a$ :**  $a = \frac{12}{4} = 3$ .

**Step 3 — Identify the vertex:** For  $y^2 = 4ax$  the vertex is always at the origin, regardless of  $a$ , so the vertex is  $(0, 0)$ .

**Why other options are wrong:**  $(3, 0)$  is the focus  $(a, 0)$ ;  $(0, 3)$  swaps the coordinates;  $(12, 0)$  misreads  $4a$  as the  $x$ -coordinate.

**Final Answer:** Vertex =  $(0, 0) \Rightarrow$  **B**

**Answer: (B)** [Go Back to Q33](#)

Q34.

**Solution**

**Concept — Identifying a conic:** An equation  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  with both terms positive ( $+$  sign) and unequal denominators represents an ellipse.

**Step 1 — Read the denominators:** Comparing with the standard form,  $a^2 = 9$  and  $b^2 = 4$ .

**Step 2 — Check the sign:** Both squared terms are added ( $+$  sign), ruling out a hyperbola.

**Step 3 — Compare the denominators:**  $9 \neq 4$ , so  $a^2 \neq b^2$ , which rules out a circle.

**Step 4 — Conclude:** Two positive, unequal denominators with a  $+$  sign means the conic is an ellipse.

**Why other options are wrong:** A circle needs  $a^2 = b^2$ ; a parabola has only one squared variable; a hyperbola has a minus sign between the terms.

**Final Answer:** Ellipse  $\Rightarrow$  **C**

**Answer: (C)** [Go Back to Q34](#)



Q35.

**Solution**

**Concept — Scalar projection:** The scalar projection of  $\vec{a}$  onto  $\vec{b}$  is  $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$ .

**Step 1 — Write the components:**  $\vec{a} = 3\hat{i} + 4\hat{j}$  and  $\vec{b} = \hat{i} = 1\hat{i} + 0\hat{j}$ .

**Step 2 — Form the dot product:**  $\vec{a} \cdot \vec{b} = (3)(1) + (4)(0)$ .

**Step 3 — Evaluate the dot product:**  $= 3 + 0 = 3$ .

**Step 4 — Find  $|\vec{b}|$ :**  $|\vec{b}| = \sqrt{1^2 + 0^2} = 1$ .

**Step 5 — Divide:** Projection  $= \frac{3}{1} = 3$ .

**Why other options are wrong:** 4 is the projection on  $\hat{j}$ ;  $5 = |\vec{a}|$  is the length of  $\vec{a}$ ; 7 wrongly adds the two components.

**Final Answer:** Projection  $= 3 \Rightarrow$

**Answer: (A)** [Go Back to Q35](#)

Q36.

**Solution**

**Concept — Area of a parallelogram:** When  $\vec{a}$  and  $\vec{b}$  are adjacent sides, the area equals the magnitude of their cross product,  $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta$ , where  $\theta$  is the angle between them.

**Step 1 — Find  $|\vec{a}|$ :**  $\vec{a} = 3\hat{i}$ , so  $|\vec{a}| = 3$ .

**Step 2 — Find  $|\vec{b}|$ :**  $\vec{b} = 4\hat{j}$ , so  $|\vec{b}| = 4$ .

**Step 3 — Find the angle:**  $\hat{i}$  and  $\hat{j}$  are perpendicular, so  $\theta = 90^\circ$  and  $\sin 90^\circ = 1$ .

**Step 4 — Substitute into the area formula:** Area  $= |\vec{a}||\vec{b}| \sin \theta = 3 \times 4 \times 1$ .

**Step 5 — Multiply:**  $3 \times 4 = 12$ , then  $12 \times 1 = 12$ .

**Why other options are wrong:** 7 adds the magnitudes; 0 is the area for parallel vectors ( $\sin 0^\circ = 0$ ); 5 is unrelated.

**Final Answer:** Area  $= 12 \Rightarrow$

**Answer: (D)** [Go Back to Q36](#)



Q37.

**Solution**

**Concept — Direction cosines:** The direction cosines  $l, m, n$  of a line are the cosines of the angles it makes with the  $x, y, z$  axes, and they are the components of a unit vector along the line.

**Step 1 — Write the unit vector:** A unit vector along the line is  $\hat{u} = l\hat{i} + m\hat{j} + n\hat{k}$ .

**Step 2 — Use the unit-length condition:** A unit vector has magnitude 1, so  $|\hat{u}| = 1$ .

**Step 3 — Write magnitude in components:**  $|\hat{u}| = \sqrt{l^2 + m^2 + n^2}$ .

**Step 4 — Set equal to 1 and square:**  $\sqrt{l^2 + m^2 + n^2} = 1 \Rightarrow l^2 + m^2 + n^2 = 1$ .

**Why other options are wrong:** 0 and 3 are wrong constants; the sum is always exactly 1 and never depends on the particular line.

**Final Answer:**  $l^2 + m^2 + n^2 = 1 \Rightarrow$  **B**

**Answer: (B)** [Go Back to Q37](#)

Q38.

**Solution**

**Concept — Mode:** The mode of a data set is the value that occurs most often.

**Step 1 — List the data:** 4, 5, 5, 6, 7, 5, 8, 9.

**Step 2 — Count each value:** 4 once; 5 three times; 6 once; 7 once; 8 once; 9 once.

**Step 3 — Pick the highest frequency:** The largest count is 3, belonging to the value 5.

**Step 4 — State the mode:** The mode is 5.

**Why other options are wrong:** 6, 7 and 4 each occur only once, far fewer times than 5.

**Final Answer:** Mode = 5  $\Rightarrow$  **A**

**Answer: (A)** [Go Back to Q38](#)



Q39.

**Solution**

**Concept — Equally likely outcomes:** For equally likely outcomes,  $P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ .

**Step 1 — Count total outcomes:** A fair die has 6 faces, so total outcomes = 6.

**Step 2 — Count favourable outcomes:** Exactly one face shows the number 4, so favourable = 1.

**Step 3 — Form the ratio:**  $P(4) = \frac{\text{favourable}}{\text{total}} = \frac{1}{6}$ .

**Why other options are wrong:**  $\frac{1}{3}$ ,  $\frac{1}{2}$  and  $\frac{2}{3}$  would require more than one favourable face out of six.

**Final Answer:**  $P(4) = \frac{1}{6} \Rightarrow \boxed{\text{C}}$

**Answer: (C)** [Go Back to Q39](#)

Q40.

**Solution**

**Concept — Bayes' theorem:** Given that a red ball was drawn, the probability it came from Bag I is  $P(B_1 | R) = \frac{P(B_1)P(R | B_1)}{P(B_1)P(R | B_1) + P(B_2)P(R | B_2)}$ .

**Step 1 — Probability of choosing each bag:** A bag is chosen at random, so  $P(B_1) = P(B_2) = \frac{1}{2}$ .

**Step 2 — Red probability from Bag I:** Bag I has 3 red of 5 balls, so  $P(R | B_1) = \frac{3}{5}$ .

**Step 3 — Red probability from Bag II:** Bag II has 1 red of 5 balls, so  $P(R | B_2) = \frac{1}{5}$ .

**Step 4 — Numerator (Bag I path):**  $P(B_1)P(R | B_1) = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$ .

**Step 5 — Second term (Bag II path):**  $P(B_2)P(R | B_2) = \frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$ .

**Step 6 — Denominator (total red):**  $\frac{3}{10} + \frac{1}{10} = \frac{4}{10}$ .

**Step 7 — Divide numerator by denominator:**  $P(B_1 | R) = \frac{3/10}{4/10} = \frac{3}{4}$ .

**Why other options are wrong:**  $\frac{1}{2}$  ignores the red-ball evidence;  $\frac{3}{5}$  is the conditional  $P(R | B_1)$ ;  $\frac{1}{4}$  is the complement  $P(B_2 | R)$ .



**Final Answer:**  $P(B_1 | R) = \frac{3}{4} \Rightarrow \boxed{D}$

**Answer: (D)** [Go Back to Q40](#)



## Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	C	4	D	5	A
6	B	7	C	8	D	9	A	10	B
11	D	12	C	13	A	14	B	15	C
16	D	17	B	18	A	19	D	20	C
21	A	22	B	23	C	24	D	25	A
26	B	27	C	28	D	29	A	30	B
31	C	32	D	33	B	34	C	35	A
36	D	37	B	38	A	39	C	40	D

