SAAT Mathematics

Sample Paper – 4

Duration: 40 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of the **SAAT** (Siksha 'O' Anusandhan Admission Test).
- Each correct answer carries **+1 mark**. There is **no negative marking** for incorrect or unattempted answers.
- Only **one** option is correct. Attempt every question, since wrong answers are not penalised.
- Use of mobile phones, calculators, or other electronic gadgets is strictly prohibited.

Q1. If a set A has 5 elements, then the number of elements in its power set $P(A)$ is

(A) 32

(B) 25

(C) 10

(D) 5

Q2. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3$ is

(A) neither one-one nor onto

(B) one-one and onto (bijective)

(C) many-one but onto

(D) one-one but not onto

Q3. The complex conjugate of $z = 5 - 2i$ is

(A) $-5 - 2i$

(B) $-5 + 2i$

(C) $5 + 2i$

(D) $2 - 5i$

Q4. If ω is a non-real cube root of unity, then the value of $\omega + \omega^2$ is

(A) 0

(B) 1

(C) ω^2

(D) -1

Q5. One root of the equation $x^2 - 7x + 12 = 0$ is 3. The other root is

(A) 4

(B) -4

(C) 7

(D) 12

Q6. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, then the $(2, 1)$ element of the matrix $3A$ is

(A) 6

(B) 9

(C) 12

(D) -3

Q7. If A is a 3×3 matrix with $|A| = 4$, then the value of $|2A|$ is

(A) 8

(B) 16

(C) 24

(D) 32

Q8. A square matrix A possesses an inverse A^{-1} if and only if

(A) A is symmetric

(B) A is of order 2

(C) $|A| \neq 0$

(D) $|A| = 0$

Q9. The number of distinct arrangements of the letters of the word "LEVEL" is

(A) 30

(B) 120

(C) 60

(D) 20

Q10. The value of ${}^{8}C_{6}$ is

(A) 48

(B) 28

(C) 56

(D) 14

Q11. The coefficient of x^2 in the binomial expansion of $(1 + x)^6$ is

(A) 6

(B) 20

(C) 15

(D) 12

Q12. If the 4th term of an arithmetic progression is 11 and the 7th term is 20, then the common difference d is

(A) 2

(B) 9

(C) $\frac{9}{2}$

(D) 3

Q13. The geometric mean of the two numbers 4 and 9 is

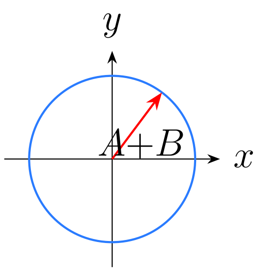
(A) 13

(B) 6

(C) $\frac{13}{2}$

(D) 36

Q14. Using the identity illustrated on the unit circle below, the value of $\tan(A+B)$ when $\tan A = 1$ and $\tan B = \frac{1}{2}$ is



(A) 3

(B) -3

(C) $\frac{3}{2}$

(D) $\frac{1}{2}$

Q15. The general solution of $\sin\theta = \frac{1}{2}$ is $(n \in \mathbb{Z})$

(A) $2n\pi \pm \frac{\pi}{6}$

(B) $n\pi + \frac{\pi}{6}$

(C) $2n\pi + \frac{\pi}{3}$

(D) $n\pi + (-1)^n\frac{\pi}{6}$

Q16. The principal value of $\tan^{-1}(1)$ is

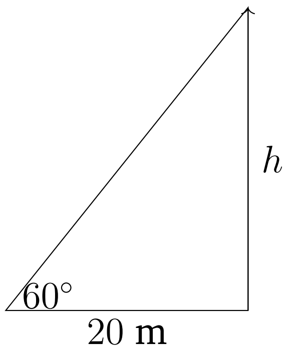
(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

Q17. The angle of elevation of the top of a tower from a point 20 m from its base is $60°$, as shown. The height of the tower is



(A) $20\sqrt{3}$ m

(B) 20 m

(C) $\frac{20}{\sqrt{3}}$ m

(D) 40 m

Q18. The value of $\lim_{x\to 0} \dfrac{1-\cos x}{x^2}$ is

(A) 1

(B) $\frac{1}{2}$

(C) 0

(D) 2

Q19. If $f(x) = \begin{cases} \dfrac{x^2-9}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$ is continuous at $x = 3$, then k equals

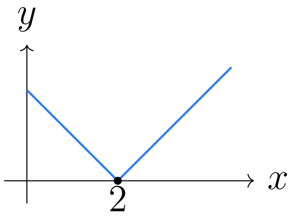
(A) 0

(B) 3

(C) 6

(D) 9

Q20. At which point is the function $f(x) = |x - 2|$, whose graph is shown, not differentiable?



(A) $x = 0$

(B) $x = -2$

(C) $x = 1$

(D) $x = 2$

Q21. The derivative of $\sin(x^2)$ with respect to x is

(A) $2x\cos(x^2)$

(B) $\cos(x^2)$

(C) $2x\sin(x^2)$

(D) $-2x\cos(x^2)$

Q22. The function $f(x) = x^2 - 6x + 5$ is strictly increasing on the interval

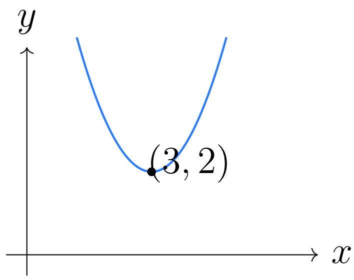
(A) $(-\infty, 3)$

(B) $(3, \infty)$

(C) $(-\infty, \infty)$

(D) $(0, 3)$

Q23. The minimum value of $f(x) = x^2 - 6x + 11$, whose graph is shown, is



(A) 11

(B) 3

(C) 2

(D) 0

Q24. The value of $\int \cos x \, dx$ is

(A) $-\sin x + C$

(B) $\cos x + C$

(C) $-\cos x + C$

(D) $\sin x + C$

Q25. The value of $\int 2x\,(x^2+1)^3\, dx$ is

(A) $\dfrac{(x^2+1)^4}{4} + C$

(B) $(x^2+1)^4 + C$

(C) $\dfrac{(x^2+1)^3}{3} + C$

(D) $2(x^2+1)^4 + C$

Q26. The value of $\int_0^{\pi} \sin x \, dx$ is

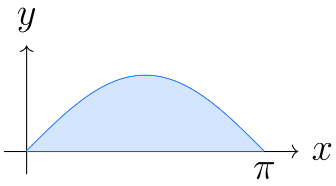
(A) 0

(B) 2

(C) 1

(D) -2

Q27. The area of the shaded region bounded by $y = \sin x$ and the x-axis from $x = 0$ to $x = \pi$, shown below, is



(A) 0

(B) 1

(C) 2

(D) π

Q28. The differential equation obtained by eliminating the arbitrary constant C from $y = Cx^2$ is

(A) $x\dfrac{dy}{dx} = y$

(B) $\dfrac{dy}{dx} = 2x$

(C) $\dfrac{dy}{dx} = 2y$

(D) $x\dfrac{dy}{dx} = 2y$

Q29. The general solution of the differential equation $\dfrac{dy}{dx} = x$ is

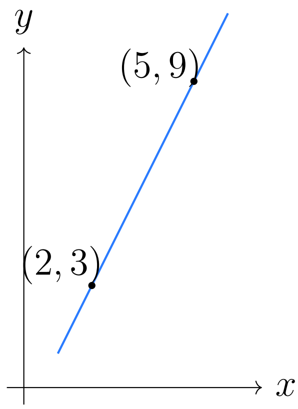
(A) $y = x + C$

(B) $y = \dfrac{x^2}{2} + C$

(C) $y = x^2 + C$

(D) $y = Ce^x$

Q30. The slope of the straight line passing through the points $(2, 3)$ and $(5, 9)$, shown below, is



(A) $\frac{1}{2}$

(B) 3

(C) 2

(D) -2

Q31. The distance between the parallel lines $3x+4y-5=0$ and $3x+4y+5=0$ is

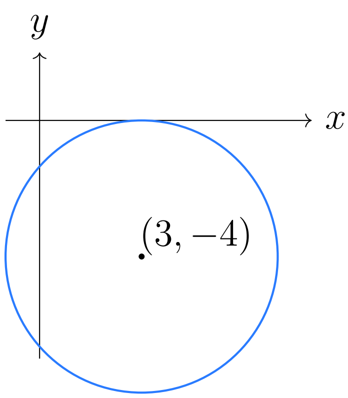
(A) 2

(B) 10

(C) 1

(D) $\frac{1}{2}$

Q32. The coordinates of the centre of the circle $x^2 + y^2 - 6x + 8y + 9 = 0$, shown below, are



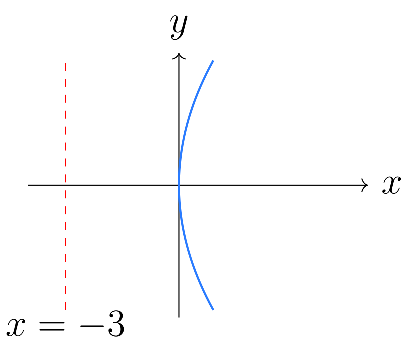
(A) $(6, -8)$

(B) $(-3, 4)$

(C) $(3, 4)$

(D) $(3, -4)$

Q33. The equation of the directrix of the parabola $y^2 = 12x$, shown below, is



(A) $x = 3$

(B) $x = -3$

(C) $y = -3$

(D) $x = -12$

Q34. The equations of the asymptotes of the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ are

(A) $y = \pm\frac{2}{3}x$

(B) $y = \pm\frac{3}{2}x$

(C) $y = \pm\frac{4}{9}x$

(D) $y = \pm x$

Q35. The scalar projection of $\vec{a} = 3\hat{i} + 4\hat{j}$ on $\vec{b} = \hat{i}$ is

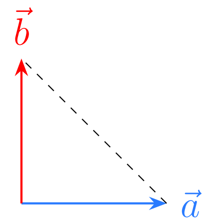
(A) 5

(B) 4

(C) 3

(D) 7

Q36. The area of the triangle whose two adjacent sides are $\vec{a} = 2\hat{i}$ and $\vec{b} = 3\hat{j}$ is



(A) 6

(B) 5

(C) $\frac{5}{2}$

(D) 3

Q37. The midpoint of the line segment joining the points $(2, 4, 6)$ and $(6, 8, 2)$ is

(A) $(8, 12, 8)$

(B) $(4, 6, 4)$

(C) $(2, 2, 2)$

(D) $(4, 4, 4)$

Q38. The median of the data $3, 7, 8, 5, 12, 14, 21, 13, 18$ is

(A) 12

(B) 8

(C) 13

(D) 11

Q39. A fair die is rolled once. The probability of getting the number 4 is

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

Q40. If $P(A \cap B) = 0.2$ and $P(B) = 0.5$, then the conditional probability $P(A \mid B)$ is

(A) 0.1

(B) 0.7

(C) 0.5

(D) 0.4

Detailed Solutions

Q1.

Solution

Concept — Power set: The power set $P(A)$ is the set of all subsets of A. If $|A| = n$, then the number of subsets is $|P(A)| = 2^n$.

Step 1 — Note the given size: The set A has 5 elements, so $n = 5$.

Step 2 — Write the formula with this value: $|P(A)| = 2^n = 2^5$.

Step 3 — Expand the power: $2^5 = 2 \times 2 \times 2 \times 2 \times 2$.

Step 4 — Multiply step by step: $2 \times 2 = 4$; then $4 \times 2 = 8$; then $8 \times 2 = 16$; then $16 \times 2 = 32$.

Step 5 — State the result: $|P(A)| = 32$.

Why other options are wrong: 25 computes 5^2; 10 computes 2×5; 5 is just n.

Final Answer: $|P(A)| = 32 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q1

Q2.

Solution

Concept — One-one and onto: A function is one-one (injective) if different inputs give different outputs, and onto (surjective) if every value in the codomain is attained. If both hold, the function is bijective.

Step 1 — Test one-one, assume equal outputs: Suppose $f(x_1) = f(x_2)$, that is $x_1^3 = x_2^3$.

Step 2 — Take cube roots: Taking the cube root of both sides gives $x_1 = x_2$.

Step 3 — Conclude injective: Equal outputs force equal inputs, so f is one-one.

Step 4 — Test onto, pick any target value: Let y be any real number in the codomain $\mathbb{R}$.

Step 5 — Find an input that hits it: Choose $x = y^{1/3}$; then $f(x) = (y^{1/3})^3 = y$.

Step 6 — Conclude surjective: Every $y \in \mathbb{R}$ is attained, so f is onto.

Step 7 — Combine: Since f is both one-one and onto, it is bijective.

Why other options are wrong: It is both one-one and onto, so the other descriptions fail.

Final Answer: f is one-one and onto $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept — Conjugate: The complex conjugate of $a + bi$ is $a - bi$. The real part stays the same and only the sign of the imaginary part flips.

Step 1 — Identify the real and imaginary parts: For $z = 5 - 2i$, the real part is $a = 5$ and the imaginary part is $b = -2$.

Step 2 — Flip the sign of the imaginary part: The imaginary part -2 becomes $+2$.

Step 3 — Keep the real part unchanged: The real part remains 5.

Step 4 — Write the conjugate: $\bar{z} = 5 + 2i$.

Why other options are wrong: $-5 - 2i$ and $-5 + 2i$ wrongly flip the real part; $2 - 5i$ swaps the parts.

Final Answer: $\bar{z} = 5 + 2i \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q3

Q4.

Solution

Concept — Cube roots of unity: The three cube roots of unity are 1, ω, ω^2, and their sum is zero: $1 + \omega + \omega^2 = 0$.

Step 1 — Write the sum identity: $1 + \omega + \omega^2 = 0$.

Step 2 — Subtract 1 from both sides: $\omega + \omega^2 = 0 - 1$.

Step 3 — Simplify: $\omega + \omega^2 = -1$.

Why other options are wrong: 0 is the full sum including 1; 1 and ω^2 contradict the identity.

Final Answer: $\omega + \omega^2 = -1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Product of roots: For a quadratic $ax^2 + bx + c = 0$ with roots α and β, the product of the roots is $\alpha\beta = \frac{c}{a}$.

Step 1 — Read off the coefficients: For $x^2 - 7x + 12 = 0$, we have $a = 1$, $b = -7$, $c = 12$.

Step 2 — Compute the product of roots: $\alpha\beta = \frac{c}{a} = \frac{12}{1} = 12$.

Step 3 — Insert the known root: One root is $\alpha = 3$, so $3\beta = 12$.

Step 4 — Solve for the other root: $\beta = \frac{12}{3} = 4$.

Step 5 — Verify using the sum of roots: The sum should equal $-\frac{b}{a} = -\frac{-7}{1} = 7$, and indeed $3 + 4 = 7$, so the answer is consistent.

Why other options are wrong: -4 gives a wrong product; 7 is the sum; 12 is the product.

Final Answer: Other root $= 4 \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q5

Q6.

Solution

Concept — Scalar multiple: When a matrix is multiplied by a scalar k, every entry is multiplied by k. So each entry of $3A$ is 3 times the corresponding entry of A.

Step 1 — Understand the index $(2, 1)$: The notation $(2, 1)$ means row 2, column 1.

Step 2 — Locate that entry in A: In $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, the row-2, column-1 entry is 3.

Step 3 — Multiply that entry by the scalar: $(3A)_{21} = 3 \times 3$.

Step 4 — Compute: $3 \times 3 = 9$.

Why other options are wrong: 6 uses the $(1, 1)$ entry; 12 uses the $(2, 2)$ entry; -3 uses $3 \times (-1)$.

Final Answer: $(3A)_{21} = 9 \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q6

Q7.

Solution

Concept — Scaling a determinant: If every entry of an $n \times n$ matrix is multiplied by k, the determinant is multiplied by k^n: $|kA| = k^n|A|$.

Step 1 — Identify n and k: The matrix is 3×3, so $n = 3$, and the scalar is $k = 2$.

Step 2 — Substitute into the formula: $|2A| = 2^3\,|A|$.

Step 3 — Evaluate the power: $2^3 = 8$, so $|2A| = 8\,|A|$.

Step 4 — Insert the given determinant: Since $|A| = 4$, $|2A| = 8 \times 4$.

Step 5 — Multiply: $8 \times 4 = 32$.

Why other options are wrong: 8 uses $|A| = 1$; 16 uses k^2; 24 uses $k^1 \times 6$ incorrectly.

Final Answer: $|2A| = 32 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q7

Q8.

Solution

Concept — Invertibility: A square matrix is invertible if and only if it is non-singular, which means its determinant is non-zero.

Step 1 — Write the inverse formula: The inverse is given by $A^{-1} = \dfrac{1}{|A|}\,\text{adj}\,A$.

Step 2 — Spot the requirement in the formula: This expression divides by $|A|$, so it is only defined when $|A| \neq 0$.

Step 3 — State the condition: Therefore A^{-1} exists exactly when $|A| \neq 0$.

Why other options are wrong: Symmetry and order are irrelevant; $|A| = 0$ makes A singular (no inverse).

Final Answer: A^{-1} exists iff $|A| \neq 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q8

Q9.

Solution

Concept — Permutations with repetition: The number of distinct arrangements of n objects, where one item repeats p times, another q times, and so on, is $\dfrac{n!}{p!\,q!\cdots}$.

Step 1 — Count the total letters: "LEVEL" has 5 letters, so $n = 5$.

Step 2 — Count the repeats: The letter L appears twice ($p = 2$) and the letter E appears twice ($q = 2$); V appears once.

Step 3 — Write the formula with these values: Arrangements $= \dfrac{5!}{2!\,2!}$.

Step 4 — Expand the factorials: $5! = 120$, $2! = 2$, and $2! = 2$.

Step 5 — Substitute: Arrangements $= \dfrac{120}{2 \times 2} = \dfrac{120}{4}$.

Step 6 — Divide: $\dfrac{120}{4} = 30$.

Why other options are wrong: 120 ignores repeats; 60 divides by one $2!$; 20 is unrelated.

Final Answer: Arrangements $= 30 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Symmetry of combinations: Choosing r items is the same as leaving out $n - r$ items, so ${}^nC_r = {}^nC_{n-r}$.

Step 1 — Apply the symmetry to reduce work: Here $n = 8$ and $r = 6$, so $n - r = 8 - 6 = 2$, giving ${}^8C_6 = {}^8C_2$.

Step 2 — Write the combination formula: ${}^8C_2 = \dfrac{8!}{2!\,6!} = \dfrac{8 \times 7}{2!}$.

Step 3 — Expand the denominator: $2! = 2 \times 1 = 2$.

Step 4 — Multiply the numerator: $8 \times 7 = 56$.

Step 5 — Divide: $\dfrac{56}{2} = 28$.

Why other options are wrong: 48 forgets to divide by $2!$; 56 is 8×7; 14 is half of 28.

Final Answer: ${}^8C_6 = 28 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

Q11.

Solution

Concept — General term: In the expansion of $(1+x)^n$, the general term is ${}^nC_r\, x^r$, so the coefficient of x^r is ${}^nC_r$.

Step 1 — Identify n and r: Here the power is $n = 6$ and we want the x^2 term, so $r = 2$.

Step 2 — Write the coefficient: Coefficient of $x^2 = {}^6C_2$.

Step 3 — Write the combination formula: ${}^6C_2 = \dfrac{6!}{2!\,4!} = \dfrac{6 \times 5}{2!}$.

Step 4 — Expand the denominator: $2! = 2$.

Step 5 — Multiply the numerator: $6 \times 5 = 30$.

Step 6 — Divide: $\dfrac{30}{2} = 15$.

Why other options are wrong: 6 is ${}^6C_1$; 20 is ${}^6C_3$; 12 is unrelated.

Final Answer: Coefficient of $x^2 = 15 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q11

Q12.

Solution

Concept — Common difference: In an AP, the difference between the mth and nth terms is $a_m - a_n = (m-n)d$, where d is the common difference.

Step 1 — Write the difference of the given terms: $a_7 - a_4 = (7-4)d$.

Step 2 — Simplify the index gap: $7 - 4 = 3$, so $a_7 - a_4 = 3d$.

Step 3 — Substitute the given term values: $a_7 = 20$ and $a_4 = 11$, so $20 - 11 = 3d$.

Step 4 — Compute the left side: $20 - 11 = 9$, giving $9 = 3d$.

Step 5 — Solve for d: $d = \dfrac{9}{3} = 3$.

Why other options are wrong: 2 and $\frac{9}{2}$ use a wrong term gap; 9 is the difference, not d.

Final Answer: $d = 3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Geometric mean: The geometric mean (GM) of two positive numbers a and b is $\sqrt{ab}$.

Step 1 — Identify the two numbers: Here $a = 4$ and $b = 9$.

Step 2 — Write the formula with these values: $\text{GM} = \sqrt{4 \times 9}$.

Step 3 — Multiply inside the root: $4 \times 9 = 36$, so $\text{GM} = \sqrt{36}$.

Step 4 — Take the square root: $\sqrt{36} = 6$.

Why other options are wrong: 13 and $\frac{13}{2}$ are arithmetic-mean style; 36 forgets the square root.

Final Answer: $\text{GM} = 6 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q13

Q14.

Solution

Concept — Tangent addition: The tangent of a sum is $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.

Step 1 — Note the given values: $\tan A = 1$ and $\tan B = \frac{1}{2}$.

Step 2 — Form the numerator: $\tan A + \tan B = 1 + \frac{1}{2}$.

Step 3 — Add the numerator: $1 + \frac{1}{2} = \frac{3}{2}$.

Step 4 — Form the denominator: $1 - \tan A \tan B = 1 - (1)\left(\frac{1}{2}\right)$.

Step 5 — Simplify the denominator: $1 - \frac{1}{2} = \frac{1}{2}$.

Step 6 — Divide numerator by denominator: $\tan(A + B) = \dfrac{3/2}{1/2}$.

Step 7 — Simplify the fraction: $\dfrac{3/2}{1/2} = \frac{3}{2} \times \frac{2}{1} = 3$.

Why other options are wrong: -3 flips a sign; $\frac{3}{2}$ is only the numerator; $\frac{1}{2}$ is the denominator.

Final Answer: $\tan(A+B)=3 \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q14

Q15.

Solution

Concept — General solution of $\sin\theta = \sin\alpha$: The complete set of solutions is $\theta = n\pi + (-1)^n\alpha$, where n is any integer.

Step 1 — Rewrite the right side as a sine: We need an angle α with $\sin\alpha = \frac{1}{2}$.

Step 2 — Find the principal angle: Since $\sin\frac{\pi}{6} = \frac{1}{2}$, take $\alpha = \frac{\pi}{6}$.

Step 3 — Match to the equation: The equation becomes $\sin\theta = \sin\frac{\pi}{6}$.

Step 4 — Apply the general-solution formula: $\theta = n\pi + (-1)^n\alpha = n\pi + (-1)^n\frac{\pi}{6}$.

Why other options are wrong: The $2n\pi\pm$ and $2n\pi+$ forms belong to cosine; $n\pi + \frac{\pi}{6}$ misses the $(-1)^n$.

Final Answer: $\theta = n\pi + (-1)^n\frac{\pi}{6} \Rightarrow \boxed{D}$

Answer: (D) Go Back to Q15

Q16.

Solution

Concept — Principal value of $\tan^{-1}$: The principal value of $\tan^{-1}$ is the unique angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent equals the given number.

Step 1 — State what we are looking for: We need an angle θ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with $\tan\theta = 1$.

Step 2 — Recall the standard value: $\tan\frac{\pi}{4} = 1$.

Step 3 — Check the angle lies in range: $\frac{\pi}{4}$ is indeed inside $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Step 4 — Conclude: $\tan^{-1}(1) = \frac{\pi}{4}$.

Why other options are wrong: $\frac{\pi}{6}$ gives $\frac{1}{\sqrt{3}}$; $\frac{\pi}{3}$ gives $\sqrt{3}$; $\frac{\pi}{2}$ is undefined for tangent.

Final Answer: $\tan^{-1}(1) = \frac{\pi}{4} \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q16

Q17.

Solution

Concept — Angle of elevation: In the right triangle formed by the tower and the ground, the tangent of the elevation angle equals the opposite side (height) over the adjacent side (base): $\tan(\text{angle}) = \dfrac{\text{height}}{\text{base}}$.

Step 1 — Write the relation for this triangle: $\tan 60^\circ = \dfrac{h}{20}$.

Step 2 — Make h the subject: Multiply both sides by 20: $h = 20 \tan 60^\circ$.

Step 3 — Substitute the known value: $\tan 60^\circ = \sqrt{3}$, so $h = 20\sqrt{3}$.

Step 4 — State with units: $h = 20\sqrt{3}$ m.

Why other options are wrong: 20 uses 45°; $\frac{20}{\sqrt{3}}$ uses 30°; 40 is double.

Final Answer: Height $= 20\sqrt{3}$ m $\Rightarrow$ $\boxed{\text{A}}$

Answer: (A) Go Back to Q17

Q18.

Solution

Concept — Standard limit: We use the half-angle identity $1 - \cos x = 2\sin^2 \frac{x}{2}$ together with the standard limit $\lim_{t \to 0} \dfrac{\sin t}{t} = 1$.

Step 1 — Replace the numerator: $\dfrac{1 - \cos x}{x^2} = \dfrac{2\sin^2(x/2)}{x^2}$.

Step 2 — Rewrite x^2 to match the half-angle: Write $x^2 = 4\left(\dfrac{x}{2}\right)^2$, giving $\dfrac{2\sin^2(x/2)}{4(x/2)^2}$.

Step 3 — Pull out the constant: $= \dfrac{1}{2} \cdot \dfrac{\sin^2(x/2)}{(x/2)^2} = \dfrac{1}{2}\left(\dfrac{\sin(x/2)}{x/2}\right)^2$.

Step 4 — Apply the standard limit: As $x \to 0$, $\dfrac{x}{2} \to 0$, so $\dfrac{\sin(x/2)}{x/2} \to 1$.

Step 5 — Substitute the limit: $\dfrac{1}{2} \times (1)^2 = \dfrac{1}{2}$.

Why other options are wrong: 1, 0 and 2 ignore the $\frac{1}{2}$ factor.

Final Answer: The limit is $\frac{1}{2} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q18

Q19.

Solution

Concept — Removable discontinuity: A piecewise function is continuous at $x = 3$ when the value defined there equals the limit, that is $k = \lim_{x \to 3} f(x)$.

Step 1 — Factor the numerator: $x^2 - 9$ is a difference of squares, so $x^2 - 9 = (x-3)(x+3)$.

Step 2 — Write the quotient with the factor: $\frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3}$.

Step 3 — Cancel the common factor: For $x \neq 3$ the $(x-3)$ terms cancel, leaving $x + 3$.

Step 4 — Take the limit as $x \to 3$: $\lim_{x \to 3}(x+3) = 3 + 3 = 6$.

Step 5 — Set k equal to the limit: For continuity, $k = 6$.

Why other options are wrong: 0, 3 and 9 do not equal the limit.

Final Answer: $k = 6 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q19

Q20.

Solution

Concept — Corner point: An absolute-value function $|x-a|$ makes a sharp corner where the inside is zero, and at a corner the left and right derivatives differ, so the function is not differentiable there.

Step 1 — Find where the inside is zero: Set $x - 2 = 0$, which gives $x = 2$; this is the corner.

Step 2 — Write the function near the corner: For $x > 2$, $f(x) = x - 2$; for $x < 2$, $f(x) = -(x-2)$.

Step 3 — Right derivative: For $x > 2$, $\frac{d}{dx}(x-2) = +1$.

Step 4 — Left derivative: For $x < 2$, $\frac{d}{dx}\big(-(x-2)\big) = -1$.

Step 5 — Compare: Left derivative $-1 \neq$ right derivative $+1$, so f is not differentiable at $x = 2$.

Why other options are wrong: At $x = 0, -2, 1$ the function is smooth (locally linear).

Final Answer: Not differentiable at $x = 2 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

Q21.

Solution

Concept — Chain rule: To differentiate a composite function, differentiate the outer function and multiply by the derivative of the inner function: $\frac{d}{dx}\sin(u) = \cos(u) \cdot \frac{du}{dx}$.

Step 1 — Name the inner function: Let $u = x^2$, so the function is $\sin u$.

Step 2 — Differentiate the inner function: $\frac{du}{dx} = 2x$.

Step 3 — Differentiate the outer function: $\frac{d}{du}\sin u = \cos u = \cos(x^2)$.

Step 4 — Multiply the two pieces: $\frac{d}{dx}\sin(x^2) = \cos(x^2) \cdot 2x$.

Step 5 — Write neatly: $= 2x\cos(x^2)$.

Why other options are wrong: $\cos(x^2)$ drops the $2x$; $2x\sin(x^2)$ uses the wrong outer derivative; the last has a wrong sign.

Final Answer: $2x\cos(x^2) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept — Increasing function: A differentiable function is strictly increasing on an interval where its derivative is positive, $f'(x) > 0$.

Step 1 — Differentiate term by term: $f(x) = x^2 - 6x + 5$, so $f'(x) = 2x - 6$.

Step 2 — Set up the increasing condition: Require $f'(x) > 0$, that is $2x - 6 > 0$.

Step 3 — Add 6 to both sides: $2x > 6$.

Step 4 — Divide both sides by 2: $x > 3$.

Step 5 — Write as an interval: The function is strictly increasing on $(3, \infty)$.

Why other options are wrong: $(-\infty, 3)$ is where it decreases; the others do not match $f' > 0$.

Final Answer: Increasing on $(3, \infty) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q22

Q23.

Solution

Concept — Minimum of a quadratic: An upward parabola attains its minimum at its vertex. We find the vertex by completing the square.

Step 1 — Group the x-terms: $x^2 - 6x + 11 = (x^2 - 6x) + 11$.

Step 2 — Find the completing term: Half of the x-coefficient is $\frac{-6}{2} = -3$, and $(-3)^2 = 9$.

Step 3 — Add and subtract 9: $(x^2 - 6x + 9) - 9 + 11$.

Step 4 — Write as a perfect square: $(x-3)^2 - 9 + 11 = (x-3)^2 + 2$.

Step 5 — Find the minimum: The smallest value of $(x-3)^2$ is 0 (at $x = 3$), so the minimum of f is $0 + 2 = 2$.

Why other options are wrong: 11 is $f(0)$; 3 is the location, not the value; 0 is the square term only.

Final Answer: Minimum value $= 2 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q23

Q24.

Solution

Concept — Standard integral: Integration reverses differentiation. We look for a function whose derivative is $\cos x$.

Step 1 — Recall the matching derivative: $\dfrac{d}{dx}(\sin x) = \cos x$.

Step 2 — Reverse it to integrate: Therefore $\displaystyle\int \cos x \, dx = \sin x$.

Step 3 — Add the constant of integration: $\displaystyle\int \cos x \, dx = \sin x + C$.

Why other options are wrong: $-\sin x$ and $\pm\cos x$ are antiderivatives of other functions.

Final Answer: $\sin x + C \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q24

Q25.

Solution

Concept — Substitution: When the integrand contains a function and its derivative, substitute u for the inner function to simplify the integral.

Step 1 — Choose the substitution: Let $u = x^2 + 1$.

Step 2 — Differentiate to find du: $\frac{du}{dx} = 2x$, so $du = 2x\,dx$.

Step 3 — Replace in the integral: The $2x\,dx$ becomes du and $(x^2+1)^3$ becomes u^3, so $\int 2x(x^2+1)^3\,dx = \int u^3\,du$.

Step 4 — Integrate the power: $\int u^3\,du = \frac{u^4}{4} + C$.

Step 5 — Substitute back $u = x^2 + 1$: $= \frac{(x^2+1)^4}{4} + C$.

Why other options are wrong: They drop the $\frac{1}{4}$, keep power 3, or double the result.

Final Answer: $\frac{(x^2+1)^4}{4} + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q25

Q26.

Solution

Concept — Definite integral: First find an antiderivative, then apply the limits using $\int_a^b f = F(b) - F(a)$.

Step 1 — Find the antiderivative: The antiderivative of $\sin x$ is $-\cos x$, so $\int_0^{\pi} \sin x\,dx = [-\cos x]_0^{\pi}$.

Step 2 — Substitute the upper limit: At $x = \pi$, $-\cos \pi$.

Step 3 — Substitute the lower limit: At $x = 0$, $-\cos 0$.

Step 4 — Form upper minus lower: $\big(-\cos \pi\big) - \big(-\cos 0\big)$.

Step 5 — Insert the cosine values: $\cos \pi = -1$ and $\cos 0 = 1$, giving $-(-1) - (-1)$.

Step 6 — Simplify: $1 + 1 = 2$.

Why other options are wrong: 0 ignores the sign change; 1 halves; -2 flips the sign.

Final Answer: The integral $= 2 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q26

Q27.

Solution

Concept — Area under a curve: Since $\sin x \geq 0$ on $[0, \pi]$, the curve stays above the x-axis, so the area equals the definite integral $\int_0^{\pi} \sin x \, dx$.

Step 1 — Find the antiderivative: The antiderivative of $\sin x$ is $-\cos x$, so Area $= [-\cos x]_0^{\pi}$.

Step 2 — Substitute the limits: $= \big(-\cos \pi\big) - \big(-\cos 0\big) = -\cos \pi + \cos 0$.

Step 3 — Insert the cosine values: $\cos \pi = -1$ and $\cos 0 = 1$, giving $-(-1) + 1$.

Step 4 — Simplify: $1 + 1 = 2$.

Why other options are wrong: 0 is the signed value over $[0, 2\pi]$; 1 and π are incorrect.

Final Answer: Area $= 2 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q27

Q28.

Solution

Concept — Forming a DE: To form a differential equation, differentiate the given relation and then use the original equation to eliminate the arbitrary constant.

Step 1 — Differentiate the given relation: From $y = Cx^2$, differentiate both sides: $\frac{dy}{dx} = 2Cx$.

Step 2 — Solve the original for C: From $y = Cx^2$, divide by x^2 to get $C = \frac{y}{x^2}$.

Step 3 — Substitute C into the derivative: $\frac{dy}{dx} = 2\left(\frac{y}{x^2}\right)x$.

Step 4 — Simplify the right side: $2 \cdot \frac{y}{x^2} \cdot x = \frac{2y}{x}$, so $\frac{dy}{dx} = \frac{2y}{x}$.

Step 5 — Clear the denominator: Multiply both sides by x: $x\frac{dy}{dx} = 2y$.

Why other options are wrong: The others still contain C implicitly or use a wrong factor.

Final Answer: $x\frac{dy}{dx} = 2y \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q28

Q29.

Solution

Concept — Direct integration: When $\frac{dy}{dx}$ is given purely in terms of x, integrate both sides with respect to x to recover y.

Step 1 — Separate and integrate: From $\frac{dy}{dx} = x$, we have $y = \int x\,dx$.

Step 2 — Apply the power rule of integration: $\int x^n\,dx = \frac{x^{n+1}}{n+1}$, with $n = 1$ here.

Step 3 — Substitute $n = 1$: $\int x\,dx = \frac{x^2}{2}$.

Step 4 — Add the constant: $y = \frac{x^2}{2} + C$.

Why other options are wrong: $x + C$ integrates a constant; $x^2 + C$ omits $\frac{1}{2}$; Ce^x solves $y' = y$.

Final Answer: $y = \frac{x^2}{2} + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q29

Q30.

Solution

Concept — Slope from two points: The slope of the line through (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Step 1 — Label the points: Take $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (5, 9)$.

Step 2 — Substitute into the formula: $m = \frac{9 - 3}{5 - 2}$.

Step 3 — Compute the numerator: $9 - 3 = 6$.

Step 4 — Compute the denominator: $5 - 2 = 3$.

Step 5 — Divide: $m = \dfrac{6}{3} = 2.$

Why other options are wrong: $\frac{1}{2}$ inverts; 3 uses the wrong difference; -2 has the wrong sign.

Final Answer: Slope $= 2 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q30

Q31.

Solution

Concept — Distance between parallel lines: For two parallel lines $ax+by+c_1 = 0$ and $ax + by + c_2 = 0$, the distance is $d = \dfrac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$.

Step 1 — Read off the coefficients: Both lines have $a = 3$ and $b = 4$; the constants are $c_1 = -5$ and $c_2 = 5$.

Step 2 — Form the numerator: $|c_1 - c_2| = | - 5 - 5| = | - 10| = 10.$

Step 3 — Form the denominator: $\sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25}.$

Step 4 — Simplify the root: $\sqrt{25} = 5.$

Step 5 — Divide: $d = \dfrac{10}{5} = 2.$

Why other options are wrong: 10 forgets the denominator; 1 and $\frac{1}{2}$ miscompute.

Final Answer: Distance $= 2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q31

Q32.

Solution

Concept — Centre of a circle: For the general circle $x^2 + y^2 + 2gx + 2fy + c = 0$, the centre is $(-g, -f)$.

Step 1 — Match the x-term: Comparing $-6x$ with $2gx$ gives $2g = -6$, so $g = -3$.

Step 2 — Match the y-term: Comparing $8y$ with $2fy$ gives $2f = 8$, so $f = 4$.

Step 3 — Apply the centre formula: Centre $= (-g, -f) = (-(-3), -(4)).$

Step 4 — Simplify: $= (3, -4).$

Why other options are wrong: $(6, -8)$ uses the raw coefficients; $(-3, 4)$ forgets the sign flip; $(3,4)$ has the wrong y-sign.

Final Answer: Centre $= (3, -4) \Rightarrow \boxed{D}$

Answer: (D) Go Back to Q32

Q33.

Solution

Concept — Directrix of $y^2 = 4ax$: For the standard right-opening parabola $y^2 = 4ax$, the directrix is the vertical line $x = -a$.

Step 1 — Compare with the standard form: Matching $y^2 = 12x$ with $y^2 = 4ax$ gives $4a = 12$.

Step 2 — Solve for a: $a = \frac{12}{4} = 3$.

Step 3 — Apply the directrix formula: $x = -a = -3$.

Why other options are wrong: $x = 3$ is the wrong sign; $y = -3$ is a horizontal line; $x = -12$ uses $4a$.

Final Answer: Directrix $x = -3 \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q33

Q34.

Solution

Concept — Asymptotes of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$: The asymptotes of this hyperbola are the lines $y = \pm\frac{b}{a}x$.

Step 1 — Read off a^2 and b^2: Comparing with the standard form, $a^2 = 9$ and $b^2 = 4$.

Step 2 — Take square roots: $a = \sqrt{9} = 3$ and $b = \sqrt{4} = 2$.

Step 3 — Form the slope $\frac{b}{a}$: $\frac{b}{a} = \frac{2}{3}$.

Step 4 — Write the asymptotes: $y = \pm\frac{2}{3}x$.

Why other options are wrong: $\pm\frac{3}{2}x$ inverts; $\pm\frac{4}{9}x$ skips the square roots; $\pm x$ ignores a, b.

Final Answer: $y = \pm\frac{2}{3}x \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q34

Q35.

Solution

Concept — Scalar projection: The scalar projection of $\vec{a}$ onto $\vec{b}$ is $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Step 1 — Write the components: $\vec{a} = 3\hat{\imath} + 4\hat{\jmath} = (3, 4)$ and $\vec{b} = \hat{\imath} = (1, 0)$.

Step 2 — Multiply matching components: The $\hat{\imath}$ parts give $(3)(1) = 3$; the $\hat{\jmath}$ parts give $(4)(0) = 0$.

Step 3 — Add for the dot product: $\vec{a}\cdot\vec{b} = 3 + 0 = 3$.

Step 4 — Find the length of $\vec{b}$: $|\vec{b}| = \sqrt{1^2 + 0^2} = \sqrt{1} = 1$.

Step 5 — Divide: Projection $= \dfrac{3}{1} = 3$.

Why other options are wrong: 5 is $|\vec{a}|$; 4 is the $\hat{\jmath}$ component; 7 adds wrongly.

Final Answer: Projection $= 3 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q35

Q36.

Solution

Concept — Area of triangle: The area of a triangle with two adjacent sides $\vec{a}$ and $\vec{b}$ is half the magnitude of their cross product: Area $= \frac{1}{2}|\vec{a} \times \vec{b}|$.

Step 1 — Use the unit-vector rule: $\hat{\imath} \times \hat{\jmath} = \hat{k}$.

Step 2 — Compute the cross product: $\vec{a} \times \vec{b} = (2\hat{\imath}) \times (3\hat{\jmath}) = (2)(3)(\hat{\imath} \times \hat{\jmath}) = 6\hat{k}$.

Step 3 — Find its magnitude: $|\vec{a} \times \vec{b}| = |6\hat{k}| = 6$.

Step 4 — Take half: Area $= \frac{1}{2} \times 6 = 3$.

Why other options are wrong: 6 is the parallelogram area; 5 and $\frac{5}{2}$ are unrelated.

Final Answer: Area $= 3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q36

Q37.

Solution

Concept — Midpoint in 3D: The midpoint of the segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is the average of the coordinates: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

Step 1 — Substitute the points: With $(2, 4, 6)$ and $(6, 8, 2)$, $M = \left(\frac{2+6}{2}, \frac{4+8}{2}, \frac{6+2}{2}\right)$.

Step 2 — Compute the x-coordinate: $\frac{2+6}{2} = \frac{8}{2} = 4$.

Step 3 — Compute the y-coordinate: $\frac{4+8}{2} = \frac{12}{2} = 6$.

Step 4 — Compute the z-coordinate: $\frac{6+2}{2} = \frac{8}{2} = 4$.

Step 5 — Assemble: $M = (4, 6, 4)$.

Why other options are wrong: $(8, 12, 8)$ forgets to halve; $(2, 2, 2)$ and $(4, 4, 4)$ miscompute.

Final Answer: Midpoint $= (4, 6, 4) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q37

Q38.

Solution

Concept — Median: Arrange the data in increasing order; for an odd number n of values the median is the $\frac{n+1}{2}$th value.

Step 1 — Sort the data: $3, 5, 7, 8, 12, 13, 14, 18, 21$.

Step 2 — Count the values: There are $n = 9$ values, which is odd.

Step 3 — Find the position of the median: The median is the $\frac{9+1}{2} = \frac{10}{2} = 5$th value.

Step 4 — Read the 5th value: Counting in, the 5th value is 12.

Why other options are wrong: 8 is the 4th value; 13 is the 6th; 11 is not in the list.

Final Answer: Median $= 12 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q38

Q39.

Solution

Concept — Classical probability: For equally likely outcomes, $P = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$.

Step 1 — Count the total outcomes: A die has faces $1, 2, 3, 4, 5, 6$, so there are 6 possible outcomes.

Step 2 — Count the favourable outcomes: Only one face shows 4, so there is 1 favourable outcome.

Step 3 — Form the ratio: $P = \dfrac{1}{6}$.

Why other options are wrong: $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{1}{3}$ count too many outcomes.

Final Answer: $P = \dfrac{1}{6} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q39

Q40.

Solution

Concept — Conditional probability: The probability of A given B is $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

Step 1 — Note the given values: $P(A \cap B) = 0.2$ and $P(B) = 0.5$.

Step 2 — Substitute into the formula: $P(A \mid B) = \dfrac{0.2}{0.5}$.

Step 3 — Clear the decimals: Multiply numerator and denominator by 10: $\dfrac{2}{5}$.

Step 4 — Divide: $\dfrac{2}{5} = 0.4$.

Why other options are wrong: 0.1 multiplies; 0.7 adds; 0.5 is $P(B)$ itself.

Final Answer: $P(A \mid B) = 0.4 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	A
6	B	7	D	8	C	9	A	10	B
11	C	12	D	13	B	14	A	15	D
16	C	17	A	18	B	19	C	20	D
21	A	22	B	23	C	24	D	25	A
26	B	27	C	28	D	29	B	30	C
31	A	32	D	33	B	34	A	35	C
36	D	37	B	38	A	39	C	40	D