SAAT Mathematics

Sample Paper – 5

Duration: 40 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of the **SAAT** (Siksha 'O' Anusandhan Admission Test).
- Each correct answer carries **+1 mark**. There is **no negative marking** for incorrect or unattempted answers.
- Only **one** option is correct. Attempt every question, since wrong answers are not penalised.
- Use of mobile phones, calculators, or other electronic gadgets is strictly prohibited.

Q1. If the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $A = \{2, 4, 6, 8\}$, then the complement A' is

(A) $\{1, 3, 5, 7\}$

(B) $\{2, 4, 6, 8\}$

(C) $\{1, 2, 3, 4\}$

(D) $\varnothing$

Q2. If $f(x) = x^2$ and $g(x) = x + 1$, then $(f \circ g)(x)$ equals

(A) $x^2 + 1$

(B) $(x + 1)^2$

(C) $x^2 + x$

(D) $x^3 + 1$

Q3. The real part of the complex number $z = \dfrac{1}{2 + 3i}$ is

(A) $\frac{3}{13}$

(B) $-\frac{3}{13}$

(C) $\frac{1}{2}$

(D) $\frac{2}{13}$

Q4. Using De Moivre's theorem, $(\cos\theta + i\sin\theta)^5$ equals

(A) $\cos\theta + i\sin\theta$

(B) $5(\cos\theta + i\sin\theta)$

(C) $\cos 5\theta + i\sin 5\theta$

(D) $\cos\theta^5 + i\sin\theta^5$

Q5. The quadratic equation whose roots are 3 and -5 is

(A) $x^2 + 2x - 15 = 0$

(B) $x^2 - 2x - 15 = 0$

(C) $x^2 + 2x + 15 = 0$

(D) $x^2 - 8x - 15 = 0$

Q6. If I is the 3×3 identity matrix and A is any 3×3 matrix, then AI equals

(A) I

(B) A

(C) A^2

(D) the zero matrix

Q7. If A and B are square matrices of the same order with $|A| = 3$ and $|B| = 4$, then $|AB|$ is

(A) 7

(B) $\frac{3}{4}$

(C) 12

(D) 1

Q8. For a square matrix A of order n, the product $A \cdot (\text{adj } A)$ equals

(A) A

(B) $\text{adj } A$

(C) I

(D) $|A|\, I$

Q9. In how many ways can 5 people be seated in a row if two particular people must sit together (treated as one block)?

(A) 24

(B) 48

(C) 120

(D) 12

Q10. At a party every pair of 8 guests shakes hands exactly once. The total number of handshakes is

(A) 28

(B) 56

(C) 64

(D) 16

Q11. The sum of all the coefficients in the binomial expansion of $(2x + 3)^4$ is

(A) 16

(B) 81

(C) 256

(D) 625

Q12. The sum of the first 20 terms of the arithmetic progression $3, 7, 11, \ldots$ is

(A) 620

(B) 1640

(C) 820

(D) 400

Q13. The sum to infinity of the geometric progression $9 + 3 + 1 + \cdots$ is

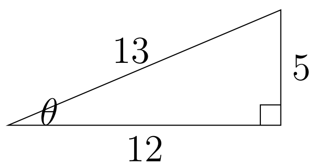
(A) $\frac{27}{2}$

(B) 12

(C) $\frac{3}{2}$

(D) ∞

Q14. In the right triangle shown, $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$. The value of $\sin 2\theta$ is



(A) $\frac{60}{169}$

(B) $\frac{120}{169}$

(C) $\frac{119}{169}$

(D) $\frac{10}{13}$

Q15. The number of solutions of $\sin\theta = \frac{1}{2}$ in the interval $[0, 2\pi]$ is

(A) 1

(B) 3

(C) 2

(D) 4

Q16. The range (principal-value branch) of $\cos^{-1} x$ is

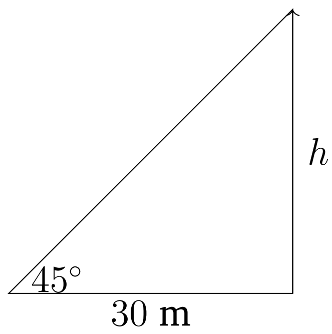
(A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(B) $(0, \pi)$

(C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(D) $[0, \pi]$

Q17. From a point 30 m from the foot of a vertical pole the angle of elevation of its top is $45°$, as shown. The height of the pole is



(A) 30 m

(B) $30\sqrt{3}$ m

(C) 15 m

(D) 60 m

Q18. The value of $\lim_{x\to 0} \frac{e^x - 1}{x}$ is

(A) 0

(B) 1

(C) e

(D) does not exist

Q19. If $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$ is continuous at $x = 3$, then k equals

(A) 0

(B) 3

(C) 6

(D) 9

Q20. At $x = 2$, the function $f(x) = |x - 2|$ is

(A) discontinuous

(B) differentiable

(C) neither continuous nor differentiable

(D) continuous but not differentiable

Q21. The derivative of $\ln x$ with respect to x (for $x > 0$) is

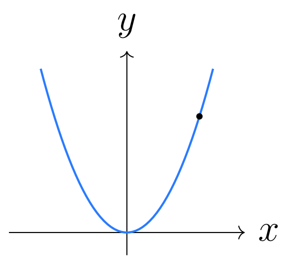
(A) $\frac{1}{x}$

(B) $\ln x$

(C) $x \ln x$

(D) $-\frac{1}{x^2}$

Q22. The slope of the tangent to the curve $y = x^2$ at the point where $x = 3$, indicated below, is



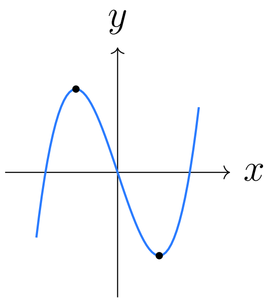
(A) 3

(B) 6

(C) 9

(D) $\frac{1}{6}$

Q23. The number of critical points (points where $f'(x) = 0$) of $f(x) = x^3 - 3x$, whose graph is shown, is



(A) 0

(B) 1

(C) 2

(D) 3

Q24. The value of $\int \cos x\, dx$ is

(A) $-\sin x + C$

(B) $-\cos x + C$

(C) $\cos x + C$

(D) $\sin x + C$

Q25. The value of $\int 2x\,(x^2+1)\,dx$ is

(A) $\dfrac{(x^2+1)^2}{2} + C$

(B) $(x^2+1)^2 + C$

(C) $2(x^2+1) + C$

(D) $\dfrac{x^2+1}{2} + C$

Q26. The value of $\int_0^{\pi} \sin x\, dx$ is

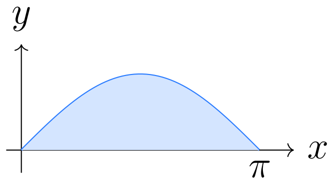
(A) 0

(B) 2

(C) 1

(D) -2

Q27. The area of the shaded region bounded by $y = \sin x$ and the x-axis from $x = 0$ to $x = \pi$, shown below, is



(A) 0

(B) 1

(C) 2

(D) π

Q28. The differential equation obtained by eliminating the arbitrary constant C from $y = Cx$ is

(A) $\frac{dy}{dx} = x$

(B) $\frac{dy}{dx} = C$

(C) $x\frac{dy}{dx} = y^2$

(D) $x\frac{dy}{dx} = y$

Q29. The general solution of the differential equation $\frac{dy}{dx} = x$ is

(A) $y = x + C$

(B) $y = \frac{x^2}{2} + C$

(C) $y = Ce^x$

(D) $y = x^2 + C$

Q30. The slope of the line passing through the points $(1, 2)$ and $(4, 11)$ is

(A) 3

(B) $\frac{1}{3}$

(C) -3

(D) 9

Q31. The distance between the parallel lines $3x+4y-5=0$ and $3x+4y+15=0$ is

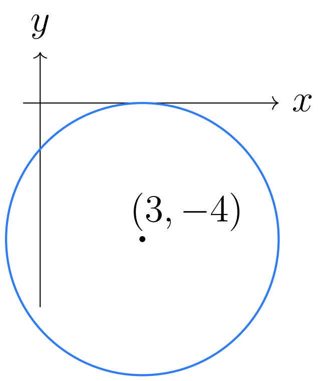
(A) 20

(B) 5

(C) 4

(D) $\frac{1}{5}$

Q32. The coordinates of the centre of the circle $x^2+y^2-6x+8y+9=0$, shown below, are



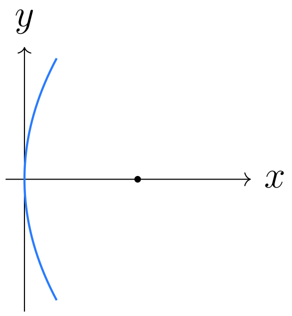
(A) $(6,-8)$

(B) $(-3,4)$

(C) $(3,4)$

(D) $(3,-4)$

Q33. The focus of the parabola $y^2=12x$, shown below, is



(A) $(3, 0)$

(B) $(0, 3)$

(C) $(6, 0)$

(D) $(-3, 0)$

Q34. The equations of the asymptotes of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ are

(A) $y = \pm\frac{3}{2}x$

(B) $y = \pm\frac{2}{3}x$

(C) $y = \pm x$

(D) $y = \pm\frac{9}{4}x$

Q35. The scalar projection of $\vec{a} = 3\hat{i} + 4\hat{j}$ on $\vec{b} = \hat{i}$ is

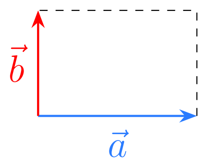
(A) 5

(B) 4

(C) 3

(D) 7

Q36. For $\vec{a} = 3\hat{i}$ and $\vec{b} = 2\hat{j}$, the area of the parallelogram with adjacent sides $\vec{a}$ and $\vec{b}$, shown below, is



(A) 5

(B) $\sqrt{13}$

(C) 3

(D) 6

Q37. The midpoint of the line segment joining $A(2, 4, 6)$ and $B(6, 8, 4)$ is

(A) $(4, 6, 5)$

(B) $(8, 12, 10)$

(C) $(2, 2, 1)$

(D) $(4, 4, 5)$

Q38. The median of the observations $7, 3, 9, 5, 11$ is

(A) 9

(B) 7

(C) 5

(D) 11

Q39. A fair die is rolled once. The probability of getting the number 3 is

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{6}$

(D) $\frac{3}{6}$

Q40. For two events with $P(A \cap B) = \frac{1}{4}$ and $P(B) = \frac{1}{2}$, the conditional probability $P(A \mid B)$ is

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{3}{4}$

(D) $\frac{1}{2}$

Detailed Solutions

Q1.

Solution

Concept — Complement of a set: $A' = U \setminus A$ contains every element of U that is not in A.

Step 1 — Write down U: $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

Step 2 — Write down A: $A = \{2, 4, 6, 8\}$, which are the even numbers in U.

Step 3 — Cross off each element of A from U: Remove 2, then 4, then 6, then 8 from the list of U.

Step 4 — Collect the survivors: The elements of U that remain are $1, 3, 5, 7$.

Step 5 — State the complement: $A' = \{1, 3, 5, 7\}$.

Why other options are wrong: $\{2, 4, 6, 8\}$ is A itself; $\{1, 2, 3, 4\}$ mixes in elements of A; $\varnothing$ would mean $A = U$.

Final Answer: $A' = \{1, 3, 5, 7\} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q1

Q2.

Solution

Concept — Composite function: $(f \circ g)(x) = f(g(x))$ means apply g first, then feed the result into f.

Step 1 — Write the inner function: $g(x) = x + 1$.

Step 2 — Substitute $g(x)$ into f: $(f \circ g)(x) = f(g(x)) = f(x + 1)$.

Step 3 — Use the rule for f: Since $f(\text{input}) = (\text{input})^2$, replace the input by $x+1$.

Step 4 — Square the inner function: $f(x + 1) = (x + 1)^2$.

Why other options are wrong: $x^2 + 1$ is $g(f(x))$ reversed wrongly; $x^2 + x$ and $x^3 + 1$ misuse the rule.

Final Answer: $(f \circ g)(x) = (x + 1)^2 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept — Real part of a reciprocal: To split $\frac{1}{2+3i}$ into real and imaginary parts, multiply top and bottom by the conjugate of the denominator.

Step 1 — Identify the conjugate: The denominator is $2+3i$, so its conjugate is $2-3i$.

Step 2 — Multiply by the conjugate over itself: $z = \frac{1}{2+3i} \cdot \frac{2-3i}{2-3i}$.

Step 3 — Multiply out the numerator: $1 \cdot (2-3i) = 2-3i$.

Step 4 — Multiply out the denominator: $(2+3i)(2-3i) = 2^2 - (3i)^2 = 4 - 9i^2$.

Step 5 — Use $i^2 = -1$: $4 - 9(-1) = 4 + 9 = 13$.

Step 6 — Write z in standard form: $z = \frac{2-3i}{13} = \frac{2}{13} - \frac{3}{13}i$.

Step 7 — Read off the real part: $\text{Re}(z) = \frac{2}{13}$.

Why other options are wrong: $\pm\frac{3}{13}$ is the imaginary part (up to sign); $\frac{1}{2}$ ignores the imaginary term.

Final Answer: $\text{Re}(z) = \frac{2}{13} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q3

Q4.

Solution

Concept — De Moivre's theorem: For any integer n, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

Step 1 — Identify the power: Here the exponent is $n = 5$.

Step 2 — Apply the theorem: $(\cos\theta + i\sin\theta)^5 = \cos(5\theta) + i\sin(5\theta)$.

Step 3 — Write the result: $\cos 5\theta + i\sin 5\theta$.

Why other options are wrong: $\cos\theta + i\sin\theta$ uses $n = 1$; the multiple $5(\cdots)$ is wrong; $\cos\theta^5$ raises the angle, not the multiple.

Final Answer: $\cos 5\theta + i\sin 5\theta \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q4

Q5.

Solution

Concept — Equation from roots: A quadratic with roots α and β is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

Step 1 — Note the roots: $\alpha = 3$ and $\beta = -5$.

Step 2 — Compute the sum of roots: $\alpha + \beta = 3 + (-5) = -2$.

Step 3 — Compute the product of roots: $\alpha\beta = 3 \times (-5) = -15$.

Step 4 — Plug into the formula: $x^2 - (-2)x + (-15) = 0$.

Step 5 — Simplify the signs: $x^2 + 2x - 15 = 0$.

Why other options are wrong: $x^2 - 2x - 15$ has the wrong sum sign; $x^2 + 2x + 15$ has wrong product; $x^2 - 8x - 15$ uses wrong sum.

Final Answer: $x^2 + 2x - 15 = 0 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q5

Q6.

Solution

Concept — Identity matrix: I is the multiplicative identity for matrices, just as 1 is for numbers.

Step 1 — Recall the defining property: For any conformable matrix A, multiplying by I does not change it.

Step 2 — Write the two-sided form: $AI = IA = A$.

Step 3 — Read off the required product: $AI = A$.

Why other options are wrong: I would need $A = I$; A^2 requires multiplying by A; the zero matrix requires a zero factor.

Final Answer: $AI = A \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q6

Q7.

Solution

Concept — Determinant of a product: The determinant of a product equals the product of the determinants, $|AB| = |A|\,|B|$.

Step 1 — Note the given values: $|A| = 3$ and $|B| = 4$.

Step 2 — Write the formula: $|AB| = |A|\,|B|$.

Step 3 — Substitute the values: $|AB| = 3 \times 4$.

Step 4 — Multiply: $3 \times 4 = 12$.

Why other options are wrong: 7 adds; $\frac{3}{4}$ divides; 1 is unrelated.

Final Answer: $|AB| = 12 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q7

Q8.

Solution

Concept — Adjoint identity: For a square matrix, the matrix times its adjoint gives a scalar multiple of the identity.

Step 1 — State the standard result: $A \cdot (\text{adj } A) = (\text{adj } A) \cdot A = |A|\, I$.

Step 2 — Note it holds for every square matrix: The scalar factor is the determinant $|A|$.

Step 3 — Read off the required product: $A \cdot (\text{adj } A) = |A|\, I$.

Why other options are wrong: A, $\text{adj } A$ and I omit the scalar factor $|A|$.

Final Answer: $A \cdot \text{adj } A = |A|\, I \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — Arrangement with a block: Tie the two people who must sit together into a single block, arrange all units, then multiply by the internal arrangements of the block.

Step 1 — Count the units: The block counts as 1 unit, and there are 3 other people, giving $1 + 3 = 4$ units.

Step 2 — Arrange the units: 4 units arrange in $4!$ ways.

Step 3 — Evaluate $4!$: $4! = 4 \times 3 \times 2 \times 1 = 24$.

Step 4 — Arrange inside the block: The 2 people in the block can swap in $2!$ ways.

Step 5 — Evaluate $2!$: $2! = 2 \times 1 = 2$.

Step 6 — Multiply the two counts: Total $= 24 \times 2 = 48$.

Why other options are wrong: 24 forgets the internal $2!$; 120 is $5!$ with no restriction; 12 is too small.

Final Answer: 48 ways $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q9

Q10.

Solution

Concept — Handshakes: Each handshake involves a pair of guests, so the number of handshakes is the number of ways to choose 2 guests from n, namely ${}^{n}C_2$.

Step 1 — Set $n = 8$: There are 8 guests, so we need ${}^{8}C_2$.

Step 2 — Write the combination formula: ${}^{8}C_2 = \dfrac{8!}{2!\,(8-2)!} = \dfrac{8 \times 7}{2 \times 1}$.

Step 3 — Multiply the numerator: $8 \times 7 = 56$.

Step 4 — Divide by 2: $\dfrac{56}{2} = 28$.

Why other options are wrong: 56 forgets to divide by 2; 64 is 8^2; 16 is 2×8.

Final Answer: 28 handshakes $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q10

Q11.

Solution

Concept — Sum of coefficients: The sum of all coefficients of a polynomial expansion is found by substituting $x = 1$.

Step 1 — Substitute $x = 1$ into $2x + 3$: $2(1) + 3$.

Step 2 — Simplify the base: $2 + 3 = 5$.

Step 3 — Raise to the power 4**:** $(2x+3)^4$ at $x=1$ becomes 5^4.

Step 4 — Evaluate 5^4 **step by step:** $5^2 = 25$, then $25 \times 25 = 625$.

Why other options are wrong: $16 = 2^4$ uses only the $2x$ part; $81 = 3^4$ uses only the constant; $256 = 4^4$ is unrelated.

Final Answer: Sum of coefficients $= 625 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q11

Q12.

Solution

Concept — Sum of an AP: $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, where a is the first term and d the common difference.

Step 1 — Identify the first term: $a = 3$.

Step 2 — Identify the common difference: $d = 7 - 3 = 4$.

Step 3 — Identify the number of terms: $n = 20$.

Step 4 — Substitute into the formula: $S_{20} = \frac{20}{2}\left[2(3) + (20-1)(4)\right]$.

Step 5 — Simplify $\frac{20}{2}$ **and** $(20-1)$**:** $S_{20} = 10\left[2(3) + 19(4)\right]$.

Step 6 — Evaluate inside the bracket: $2(3) = 6$ and $19(4) = 76$, so the bracket is $6 + 76 = 82$.

Step 7 — Multiply: $S_{20} = 10 \times 82 = 820$.

Why other options are wrong: 1640 forgets the factor $\frac{1}{2}$; 400 and 620 use a wrong bracket value.

Final Answer: $S_{20} = 820 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q12

Q13.

Solution

Concept — Sum to infinity of a GP: $S_\infty = \frac{a}{1-r}$, valid when $|r| < 1$.

Step 1 — Identify the first term: $a = 9$.

Step 2 — Find the common ratio: $r = \dfrac{\text{second term}}{\text{first term}} = \dfrac{3}{9} = \dfrac{1}{3}$.

Step 3 — Check convergence: $|r| = \frac{1}{3} < 1$, so the formula applies.

Step 4 — Substitute into the formula: $S_\infty = \dfrac{9}{1 - \frac{1}{3}}$.

Step 5 — Simplify the denominator: $1 - \frac{1}{3} = \frac{2}{3}$.

Step 6 — Divide by the fraction: $S_\infty = \dfrac{9}{\frac{2}{3}} = 9 \times \dfrac{3}{2} = \dfrac{27}{2}$.

Why other options are wrong: 12 and $\frac{3}{2}$ use a wrong ratio; the series converges, so not ∞.

Final Answer: $S_\infty = \dfrac{27}{2} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept — Double-angle identity: $\sin 2\theta = 2 \sin\theta \cos\theta$.

Step 1 — Note the given ratios: $\sin\theta = \dfrac{5}{13}$ and $\cos\theta = \dfrac{12}{13}$.

Step 2 — Write the identity: $\sin 2\theta = 2 \sin\theta \cos\theta$.

Step 3 — Substitute the values: $\sin 2\theta = 2 \cdot \dfrac{5}{13} \cdot \dfrac{12}{13}$.

Step 4 — Multiply the numerators: $2 \times 5 \times 12 = 120$.

Step 5 — Multiply the denominators: $13 \times 13 = 169$.

Step 6 — Write the result: $\sin 2\theta = \dfrac{120}{169}$.

Why other options are wrong: $\frac{60}{169}$ drops the factor 2; $\frac{119}{169}$ is $\cos 2\theta$; $\frac{10}{13}$ misuses the identity.

Final Answer: $\sin 2\theta = \dfrac{120}{169} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q14

Q15.

Solution

Concept — Counting solutions: $\sin\theta$ is positive in the first and second quadrants, so a positive value of $\sin\theta$ gives one solution in each.

Step 1 — Find the reference angle: $\sin\theta = \frac{1}{2}$ gives reference angle $\frac{\pi}{6}$.

Step 2 — First-quadrant solution: $\theta = \frac{\pi}{6}$.

Step 3 — Second-quadrant solution: $\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

Step 4 — Check both lie in $[0, 2\pi]$: Both $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ are in the interval.

Step 5 — Count the solutions: There are 2 solutions.

Why other options are wrong: 1 misses the second-quadrant solution; 3 and 4 overcount.

Final Answer: 2 solutions $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q15

Q16.

Solution

Concept — Principal-value range of $\cos^{-1}$: The inverse cosine takes inputs in $[-1, 1]$ and is defined to return angles in a fixed principal branch.

Step 1 — Value at $x = 1$: $\cos^{-1}(1) = 0$, the smallest output.

Step 2 — Value at $x = -1$: $\cos^{-1}(-1) = \pi$, the largest output.

Step 3 — Behaviour in between: As x goes from 1 to -1, the output rises continuously from 0 to π, attaining every value between.

Step 4 — State the range: The range is the closed interval $[0, \pi]$.

Why other options are wrong: $[-\frac{\pi}{2}, \frac{\pi}{2}]$ is the range of $\sin^{-1}$; the open intervals exclude the attained endpoints.

Final Answer: Range $= [0, \pi] \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — Angle of elevation: In the right triangle, $\tan(\text{angle of elevation}) = \dfrac{\text{height (opposite)}}{\text{base (adjacent)}}$.

Step 1 — Set up the relation: $\tan 45^\circ = \dfrac{h}{30}$.

Step 2 — Use the known value: $\tan 45^\circ = 1$.

Step 3 — Substitute: $1 = \dfrac{h}{30}$.

Step 4 — Solve for h: Multiply both sides by 30 to get $h = 30$ m.

Why other options are wrong: $30\sqrt{3}$ uses 60°; 15 halves; 60 doubles.

Final Answer: Height $= 30$ m $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q17

Q18.

Solution

Concept — Standard limit: The limit $\lim_{x \to 0} \dfrac{e^x - 1}{x}$ is a fundamental exponential limit equal to 1.

Step 1 — Expand e^x as a series: $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$.

Step 2 — Subtract 1: $e^x - 1 = x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$.

Step 3 — Divide by x: $\dfrac{e^x - 1}{x} = 1 + \dfrac{x}{2!} + \dfrac{x^2}{3!} + \cdots$.

Step 4 — Let $x \to 0$: Every term with x vanishes, leaving 1.

Why other options are wrong: The limit exists and equals 1, so 0, e and "does not exist" are incorrect.

Final Answer: The limit is $1 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q18

Q19.

Solution

Concept — Removable discontinuity: For f to be continuous at $x = 3$, the value $k = f(3)$ must equal $\lim_{x\to 3} f(x)$.

Step 1 — Factor the numerator: $x^2 - 9 = (x-3)(x+3)$.

Step 2 — Write the quotient: $\dfrac{x^2-9}{x-3} = \dfrac{(x-3)(x+3)}{x-3}$.

Step 3 — Cancel the common factor: For $x \neq 3$, this simplifies to $x + 3$.

Step 4 — Take the limit as $x \to 3$: $\lim_{x\to 3}(x+3) = 3+3 = 6$.

Step 5 — Set k equal to the limit: $k = 6$.

Why other options are wrong: 0, 3 and 9 do not match the limit value 6.

Final Answer: $k = 6 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q19

Q20.

Solution

Concept — Corner point: A function is continuous if its limit equals its value, and differentiable only if the left and right derivatives agree.

Step 1 — Value at the point: $f(2) = |2-2| = 0$.

Step 2 — Limit at the point: $\lim_{x\to 2}|x-2| = 0$.

Step 3 — Conclude continuity: Since the limit 0 equals $f(2) = 0$, f is continuous at $x = 2$.

Step 4 — Left derivative: For $x < 2$, $f(x) = -(x-2)$, so the slope is -1.

Step 5 — Right derivative: For $x > 2$, $f(x) = (x-2)$, so the slope is $+1$.

Step 6 — Compare: Left derivative $-1 \neq$ right derivative $+1$, so f is not differentiable at $x = 2$.

Why other options are wrong: It is neither discontinuous nor differentiable there.

Final Answer: Continuous but not differentiable $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

Q21.

Solution

Concept — Derivative of $\ln x$: The natural logarithm has the standard derivative $\frac{d}{dx}\ln x = \frac{1}{x}$.

Step 1 — State the rule: $\frac{d}{dx}\ln x = \frac{1}{x}$.

Step 2 — Note the domain: This result is valid for $x > 0$, which matches the question.

Why other options are wrong: $\ln x$ is the function itself; $x \ln x$ is unrelated; $-\frac{1}{x^2}$ is the derivative of $\frac{1}{x}$.

Final Answer: $\frac{d}{dx}\ln x = \frac{1}{x} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept — Slope of tangent: The slope of the tangent to a curve at a point is the value of $\frac{dy}{dx}$ there.

Step 1 — Write the function: $y = x^2$.

Step 2 — Apply the power rule: $\frac{d}{dx}(x^2) = 2x^{2-1} = 2x$.

Step 3 — Substitute $x = 3$: Slope $= 2(3)$.

Step 4 — Compute: $2 \times 3 = 6$.

Why other options are wrong: 3 drops the factor 2; 9 is x^2; $\frac{1}{6}$ inverts.

Final Answer: Slope $= 6 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q22

Q23.

Solution

Concept — Critical points: Critical points occur where $f'(x) = 0$; count the number of such x-values.

Step 1 — Write the function: $f(x) = x^3 - 3x$.

Step 2 — Differentiate term by term: $\frac{d}{dx}(x^3) = 3x^2$ and $\frac{d}{dx}(3x) = 3$, so $f'(x) = 3x^2 - 3$.

Step 3 — Set the derivative to zero: $3x^2 - 3 = 0$.

Step 4 — Divide by 3**:** $x^2 - 1 = 0$.

Step 5 — Solve for x**:** $x^2 = 1$, so $x = \pm 1$.

Step 6 — Count the roots: The two values $x = 1$ and $x = -1$ give 2 critical points.

Why other options are wrong: 0 and 1 undercount the two roots; 3 overcounts.

Final Answer: 2 critical points $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q23

Q24.

Solution

Concept — Standard integral: Integration reverses differentiation, so $\int \cos x \, dx$ is the function whose derivative is $\cos x$.

Step 1 — Recall the matching derivative: $\frac{d}{dx}(\sin x) = \cos x$.

Step 2 — Reverse the operation: Therefore $\int \cos x \, dx = \sin x$.

Step 3 — Add the constant of integration: $\int \cos x \, dx = \sin x + C$.

Why other options are wrong: $-\sin x$ and $-\cos x$ have wrong signs; $\cos x$ is the integrand, not its integral.

Final Answer: $\sin x + C \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q24

Q25.

Solution

Concept — Substitution: Choose a substitution that turns the integral into a simple power of u.

Step 1 — Choose u**:** Let $u = x^2 + 1$.

Step 2 — Differentiate u**:** $\frac{du}{dx} = 2x$, so $du = 2x \, dx$.

Step 3 — Rewrite the integral in u: $\displaystyle\int (x^2+1)\,(2x\,dx) = \int u\,du.$

Step 4 — Integrate the power: $\int u\,du = \dfrac{u^2}{2} + C.$

Step 5 — Substitute back $u = x^2+1$: $\dfrac{(x^2+1)^2}{2} + C.$

Why other options are wrong: $(x^2+1)^2$ omits the $\frac{1}{2}$; the other two mis-integrate.

Final Answer: $\dfrac{(x^2+1)^2}{2} + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q25

Q26.

Solution

Concept — Definite integral: First find the antiderivative, then substitute the upper and lower limits and subtract.

Step 1 — Find the antiderivative: $\int \sin x\,dx = -\cos x.$

Step 2 — Write with limits: $\displaystyle\int_0^{\pi} \sin x\,dx = \big[-\cos x\big]_0^{\pi}.$

Step 3 — Substitute the upper limit: At $x=\pi$, $-\cos\pi = -(-1) = 1.$

Step 4 — Substitute the lower limit: At $x=0$, $-\cos 0 = -(1) = -1.$

Step 5 — Subtract lower from upper: $1-(-1) = 1+1 = 2.$

Why other options are wrong: 0 ignores the sign; 1 uses half the interval; -2 flips the sign.

Final Answer: The integral $= 2 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q26

Q27.

Solution

Concept — Area under a curve: Since $\sin x \geq 0$ on $[0,\pi]$, the area equals $\displaystyle\int_0^{\pi} \sin x\,dx.$

Step 1 — Find the antiderivative: $\int \sin x\,dx = -\cos x.$

Step 2 — Write with limits: Area $= \big[-\cos x\big]_0^{\pi}.$

Step 3 — Substitute the upper limit: At $x = \pi$, $-\cos\pi = -(-1) = 1$.

Step 4 — Substitute the lower limit: At $x = 0$, $-\cos 0 = -(1) = -1$.

Step 5 — Subtract lower from upper: $1 - (-1) = 2$.

Why other options are wrong: 0 and 1 use wrong limits; π confuses area with interval length.

Final Answer: Area $= 2 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q27

Q28.

Solution

Concept — Eliminating a constant: Differentiate the given relation to introduce $\frac{dy}{dx}$, then use the original relation to remove the constant C.

Step 1 — Write the relation: $y = Cx$.

Step 2 — Differentiate with respect to x: $\frac{dy}{dx} = C$.

Step 3 — Solve the original relation for C: From $y = Cx$, $C = \frac{y}{x}$.

Step 4 — Substitute C into the derivative: $\frac{dy}{dx} = \frac{y}{x}$.

Step 5 — Clear the fraction: Multiply both sides by x to get $x\frac{dy}{dx} = y$.

Why other options are wrong: $\frac{dy}{dx} = x$ is wrong; $\frac{dy}{dx} = C$ still contains C; $x\frac{dy}{dx} = y^2$ has the wrong power.

Final Answer: $x\frac{dy}{dx} = y \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q28

Q29.

Solution

Concept — Direct integration: When $\frac{dy}{dx}$ is given explicitly in x, integrate both sides with respect to x to find y.

Step 1 — Write the equation: $\frac{dy}{dx} = x$.

Step 2 — Integrate both sides: $y = \int x\,dx$.

Step 3 — Apply the power rule for integration: $\int x\,dx = \frac{x^{1+1}}{1+1} = \frac{x^2}{2}$.

Step 4 — Add the constant: $y = \frac{x^2}{2} + C$.

Why other options are wrong: $x + C$ integrates a constant; Ce^x solves $y' = y$; $x^2 + C$ omits the $\frac{1}{2}$.

Final Answer: $y = \frac{x^2}{2} + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q29

Q30.

Solution

Concept — Slope from two points: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Step 1 — Label the points: $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (4, 11)$.

Step 2 — Substitute into the formula: $m = \frac{11 - 2}{4 - 1}$.

Step 3 — Subtract in the numerator: $11 - 2 = 9$.

Step 4 — Subtract in the denominator: $4 - 1 = 3$.

Step 5 — Divide: $m = \frac{9}{3} = 3$.

Why other options are wrong: $\frac{1}{3}$ inverts; -3 flips the sign; 9 forgets the denominator.

Final Answer: Slope $= 3 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q30

Q31.

Solution

Concept — Distance between parallel lines: For lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$, the distance is $d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$.

Step 1 — Identify the coefficients: $a = 3$, $b = 4$, $c_1 = -5$, $c_2 = 15$.

Step 2 — Substitute into the formula: $d = \dfrac{|-5-15|}{\sqrt{3^2+4^2}}$.

Step 3 — Numerator: $|-5-15| = |-20| = 20$.

Step 4 — Denominator: $\sqrt{9+16} = \sqrt{25} = 5$.

Step 5 — Divide: $d = \dfrac{20}{5} = 4$.

Why other options are wrong: 20 forgets the denominator; 5 is the denominator; $\frac{1}{5}$ inverts.

Final Answer: Distance $= 4 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q31

Q32.

Solution

Concept — Centre of a circle: For $x^2 + y^2 + 2gx + 2fy + c = 0$, the centre is $(-g, -f)$.

Step 1 — Match the x-coefficient: $2g = -6$.

Step 2 — Solve for g: $g = \dfrac{-6}{2} = -3$.

Step 3 — Match the y-coefficient: $2f = 8$.

Step 4 — Solve for f: $f = \dfrac{8}{2} = 4$.

Step 5 — Form the centre: $(-g, -f) = (-(-3), -(4)) = (3, -4)$.

Why other options are wrong: $(6, -8)$ uses $2g, 2f$; $(-3, 4)$ uses (g, f); $(3, 4)$ has the wrong y-sign.

Final Answer: Centre $= (3, -4) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q32

Q33.

Solution

Concept — Focus of $y^2 = 4ax$: A right-opening parabola in this standard form has its focus at $(a, 0)$.

Step 1 — Compare with the standard form: $y^2 = 12x$ matches $y^2 = 4ax$ with $4a = 12$.

Step 2 — Solve for a: $a = \frac{12}{4} = 3$.

Step 3 — Write the focus: Focus $= (a, 0) = (3, 0)$.

Why other options are wrong: $(0, 3)$ swaps axes; $(6, 0)$ uses $4a/2$ wrongly; $(-3, 0)$ has the wrong sign.

Final Answer: Focus $= (3, 0) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q33

Q34.

Solution

Concept — Asymptotes of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$: The asymptotes are the lines $y = \pm\frac{b}{a}x$.

Step 1 — Read a^2 and find a: $a^2 = 9$, so $a = \sqrt{9} = 3$.

Step 2 — Read b^2 and find b: $b^2 = 4$, so $b = \sqrt{4} = 2$.

Step 3 — Form the ratio $\frac{b}{a}$: $\frac{b}{a} = \frac{2}{3}$.

Step 4 — Write the asymptotes: $y = \pm\frac{2}{3}x$.

Why other options are wrong: $\pm\frac{3}{2}x$ inverts b/a; $\pm x$ ignores the ratio; $\pm\frac{9}{4}x$ uses a^2, b^2.

Final Answer: $y = \pm\frac{2}{3}x \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q34

Q35.

Solution

Concept — Scalar projection: The scalar projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Step 1 — Write the components: $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = \hat{i} = 1\hat{i} + 0\hat{j}$.

Step 2 — Form the dot product: $\vec{a}\cdot\vec{b} = (3)(1) + (4)(0)$.

Step 3 — Compute the dot product: $3 + 0 = 3$.

Step 4 — Find $|\vec{b}|$: $|\hat{i}| = \sqrt{1^2 + 0^2} = 1$.

Step 5 — Divide: Projection $= \frac{3}{1} = 3$.

Why other options are wrong: 5 is $|\vec{a}|$; 4 is the $\hat{j}$-component; 7 adds the components.

Final Answer: Scalar projection $= 3 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q35

Q36.

Solution

Concept — Area of parallelogram: The area of the parallelogram on adjacent sides $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$.

Step 1 — Write the vectors: $\vec{a} = 3\hat{\imath}$ and $\vec{b} = 2\hat{\jmath}$.

Step 2 — Set up the cross product: $\vec{a} \times \vec{b} = (3\hat{\imath}) \times (2\hat{\jmath}) = (3)(2)\,(\hat{\imath} \times \hat{\jmath})$.

Step 3 — Use $\hat{\imath} \times \hat{\jmath} = \hat{k}$: $\vec{a} \times \vec{b} = 6\hat{k}$.

Step 4 — Take the magnitude: $|6\hat{k}| = 6$.

Why other options are wrong: 5 and $\sqrt{13}$ treat sides as a diagonal; 3 uses only $|\vec{a}|$.

Final Answer: Area $= 6 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q36

Q37.

Solution

Concept — Midpoint in 3D: The midpoint is the average of corresponding coordinates, $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.

Step 1 — Label the points: $A(2, 4, 6)$ and $B(6, 8, 4)$.

Step 2 — Average the x-coordinates: $\frac{2+6}{2} = \frac{8}{2} = 4$.

Step 3 — Average the y-coordinates: $\frac{4+8}{2} = \frac{12}{2} = 6$.

Step 4 — Average the z-coordinates: $\frac{6+4}{2} = \frac{10}{2} = 5$.

Step 5 — Collect the midpoint: $(4, 6, 5)$.

Why other options are wrong: $(8, 12, 10)$ forgets to halve; $(2, 2, 1)$ uses differences; $(4, 4, 5)$ mis-adds the y-values.

Final Answer: Midpoint $= (4, 6, 5) \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q37

Q38.

Solution

Concept — Median: Arrange the data in increasing order; for an odd count the median is the middle value.

Step 1 — Write the data: $7, 3, 9, 5, 11$.

Step 2 — Sort in ascending order: $3, 5, 7, 9, 11$.

Step 3 — Count the observations: There are 5 values (an odd number).

Step 4 — Locate the middle position: The middle is position $\dfrac{5+1}{2} = 3$.

Step 5 — Read the middle value: The 3rd term is 7.

Why other options are wrong: 9 and 5 are off-centre; 11 is the maximum.

Final Answer: Median $= 7 \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q38

Q39.

Solution

Concept — Classical probability: $P = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$.

Step 1 — Count the total outcomes: A die has 6 faces, so total $= 6$.

Step 2 — Count the favourable outcomes: Only one face shows 3, so favourable $= 1$.

Step 3 — Form the ratio: $P = \dfrac{1}{6}$.

Why other options are wrong: $\frac{1}{2}$ and $\frac{1}{3}$ overcount; $\frac{3}{6}$ wrongly counts three faces.

Final Answer: $P = \dfrac{1}{6} \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q39

Q40.

Solution

Concept — Conditional probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

Step 1 — Note the given values: $P(A \cap B) = \dfrac{1}{4}$ and $P(B) = \dfrac{1}{2}$.

Step 2 — Substitute into the formula: $P(A \mid B) = \frac{\frac{1}{4}}{\frac{1}{2}}$.

Step 3 — Divide by inverting and multiplying: $\dfrac{1}{4} \div \dfrac{1}{2} = \dfrac{1}{4} \times \dfrac{2}{1}$.

Step 4 — Multiply: $\dfrac{1 \times 2}{4 \times 1} = \dfrac{2}{4} = \dfrac{1}{2}$.

Why other options are wrong: $\frac{1}{8}$ multiplies instead of dividing; $\frac{1}{4}$ is $P(A \cap B)$; $\frac{3}{4}$ is unrelated.

Final Answer: $P(A \mid B) = \dfrac{1}{2} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	D	4	C	5	A
6	B	7	C	8	D	9	B	10	A
11	D	12	C	13	A	14	B	15	C
16	D	17	A	18	B	19	C	20	D
21	A	22	B	23	C	24	D	25	A
26	B	27	C	28	D	29	B	30	A
31	C	32	D	33	A	34	B	35	C
36	D	37	A	38	B	39	C	40	D