SAAT Mathematics

Sample Paper – 8

Duration: 40 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of the **SAAT** (Siksha 'O' Anusandhan Admission Test).
- Each correct answer carries **+1 mark**. There is **no negative marking** for incorrect or unattempted answers.
- Only **one** option is correct. Attempt every question, since wrong answers are not penalised.
- Use of mobile phones, calculators, or other electronic gadgets is strictly prohibited.

Q1. For two subsets A and B of a universal set, De Morgan's law states that $(A \cup B)'$ equals

(A) $A' \cap B'$

(B) $A' \cup B'$

(C) $A \cap B$

(D) $A' \cap B$

Q2. The period of the function $f(x) = \sin 2x$ is

(A) 2π

(B) π

(C) $\frac{\pi}{2}$

(D) 4π

Q3. The modulus of the complex number $z = \frac{1}{i}$ is

(A) i

(B) $-i$

(C) 1

(D) 0

Q4. If ω is a non-real cube root of unity, then a value of ω is

(A) 1

(B) -1

(C) i

(D) $\dfrac{-1+i\sqrt{3}}{2}$

Q5. If α and β are the roots of $x^2 - 5x + 6 = 0$, then $\alpha^2 + \beta^2$ equals

(A) 13

(B) 25

(C) 12

(D) 37

Q6. The trace of the matrix $A = \begin{pmatrix} 2 & 5 \\ 1 & 7 \end{pmatrix}$ is

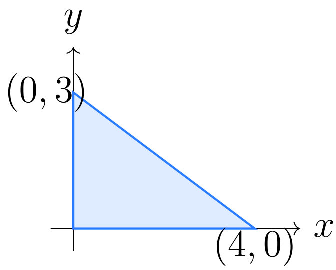
(A) 14

(B) 9

(C) 6

(D) 3

Q7. The area of the triangle with vertices $(0, 0)$, $(4, 0)$ and $(0, 3)$, shown below, is



(A) 12

(B) 7

(C) 6

(D) 24

Q8. A square matrix A is said to be orthogonal if

(A) $A = A^T$

(B) $A^2 = A$

(C) $A = -A^T$

(D) $AA^T = I$

Q9. The number of ways in which 5 distinct books can be arranged on a shelf is

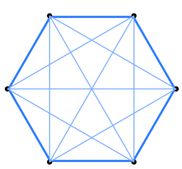
(A) 120

(B) 60

(C) 25

(D) 24

Q10. The number of diagonals of a regular hexagon, shown below, is



(A) 6

(B) 9

(C) 12

(D) 15

Q11. In the expansion of $(1 + x)^5$, the ratio of the coefficient of x^2 to the coefficient of x^1 is

(A) $1 : 2$

(B) $1 : 1$

(C) $2 : 1$

(D) $5 : 2$

Q12. If 3, x, 11 are in arithmetic progression, then the value of x is

(A) 5

(B) 8

(C) 6

(D) 7

Q13. The value of the sum $1^2 + 2^2 + 3^2 + \cdots + 10^2$ is

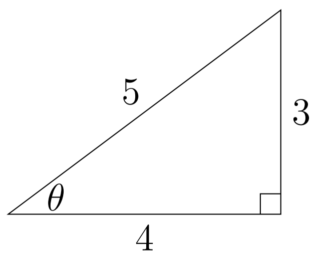
(A) 285

(B) 385

(C) 440

(D) 100

Q14. In the right-angled triangle shown, $\cos\theta = \frac{4}{5}$. Using the half-angle relation $\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$, the value of $\sin^2\frac{\theta}{2}$ is



(A) $\frac{9}{10}$

(B) $\frac{2}{5}$

(C) $\frac{1}{10}$

(D) $\frac{4}{5}$

Q15. The general solution of the equation $\sin^2\theta = 1$ is $(n \in \mathbb{Z})$

(A) $(2n+1)\dfrac{\pi}{2}$

(B) $n\pi$

(C) $2n\pi$

(D) $\dfrac{n\pi}{2}$

Q16. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is

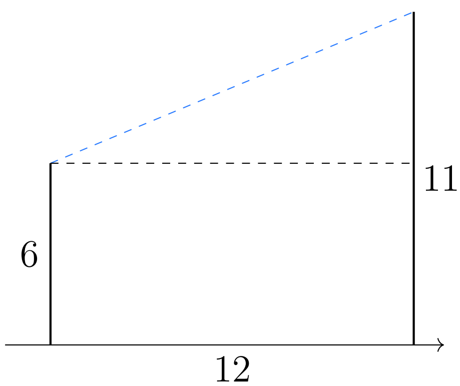
(A) $\dfrac{\pi}{3}$

(B) $\dfrac{\pi}{6}$

(C) $-\dfrac{\pi}{3}$

(D) $\dfrac{2\pi}{3}$

Q17. Two vertical poles of heights 6 m and 11 m stand on level ground 12 m apart, as shown. The distance between their tops is



(A) 12 m

(B) 13 m

(C) 17 m

(D) $\sqrt{120}$ m

Q18. The value of $\lim_{x\to 0} \dfrac{\tan x}{x}$ is

(A) 1

(B) 0

(C) ∞

(D) does not exist

Q19. If f and g are both continuous at $x = a$, then the composite function $f \circ g$ at $x = a$ is

(A) always discontinuous

(B) continuous only if $f = g$

(C) continuous at $x = a$

(D) undefined

Q20. At the point $x = 0$, the function $f(x) = \sqrt{x}$ (defined for $x \geq 0$) is

(A) differentiable with derivative 0

(B) differentiable with derivative 1

(C) discontinuous

(D) continuous but not differentiable

Q21. If $x = t^2$ and $y = t^3$, then $\dfrac{dy}{dx}$ equals

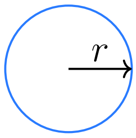
(A) $\dfrac{2}{3t}$

(B) $\dfrac{3t}{2}$

(C) $\dfrac{3t^2}{2}$

(D) t

Q22. The volume of a sphere is $V = \frac{4}{3}\pi r^3$. The rate of change of volume with respect to the radius r is



(A) $4\pi r^2$

(B) $\frac{4}{3}\pi r^2$

(C) $4\pi r^3$

(D) $2\pi r$

Q23. The absolute maximum value of $f(x) = x^2$ on the closed interval $[-3, 2]$ is

(A) 0

(B) 4

(C) 9

(D) 6

Q24. The value of $\int 2^x \, dx$ is

(A) $2^x \ln 2 + C$

(B) $x\, 2^{x-1} + C$

(C) $2^x + C$

(D) $\frac{2^x}{\ln 2} + C$

Q25. The value of $\int \frac{2x}{x^2+1} \, dx$ is

(A) $\ln(x^2 + 1) + C$

(B) $\tan^{-1} x + C$

(C) $\frac{1}{x^2+1} + C$

(D) $2 \ln x + C$

Q26. The value of $\int_{-2}^{2} x^2\,dx$ is

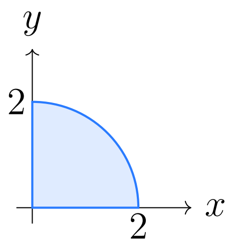
(A) 0

(B) $\frac{16}{3}$

(C) $\frac{8}{3}$

(D) 8

Q27. The area enclosed by the curve $y = \sqrt{4 - x^2}$, the x-axis and the y-axis (a quarter circle of radius 2), shown below, is



(A) 2π

(B) 4π

(C) π

(D) $\frac{\pi}{2}$

Q28. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \frac{dy}{dx} + y = 0$ is

(A) 0

(B) 4

(C) 1

(D) 2

Q29. The general solution of the differential equation $\frac{dy}{dx} = e^x$ is

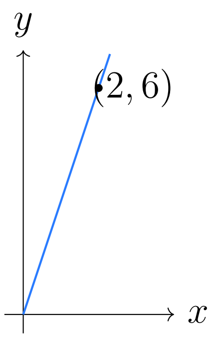
(A) $y = e^x + C$

(B) $y = e^{-x} + C$

(C) $y = xe^x + C$

(D) $y = \dfrac{e^x}{x} + C$

Q30. The slope of the straight line passing through the origin and the point $(2, 6)$, shown below, is



(A) 2

(B) 3

(C) $\dfrac{1}{3}$

(D) 6

Q31. The line $4x + 6y + 5 = 0$ is parallel to which of the following lines?

(A) $6x - 4y + 1 = 0$

(B) $4x - 6y + 1 = 0$

(C) $x + y + 1 = 0$

(D) $2x + 3y + 7 = 0$

Q32. The equation of the circle concentric with $x^2 + y^2 - 4x - 6y + 9 = 0$ and having radius 5 is

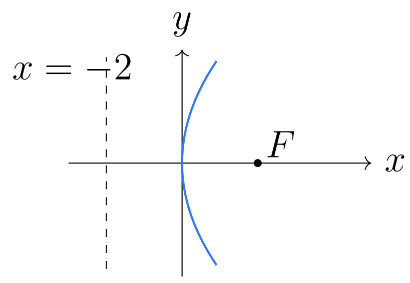
(A) $(x + 2)^2 + (y + 3)^2 = 25$

(B) $(x - 2)^2 + (y - 3)^2 = 5$

(C) $(x - 2)^2 + (y - 3)^2 = 25$

(D) $(x - 2)^2 + (y - 3)^2 = 4$

Q33. The equation of the parabola with focus $(2, 0)$ and directrix $x = -2$, shown below, is



(A) $y^2 = 8x$

(B) $y^2 = 4x$

(C) $x^2 = 8y$

(D) $y^2 = -8x$

Q34. For a non-degenerate conic, which of the following statements about the eccentricity e is correct?

(A) $e > 1$ for an ellipse

(B) $e < 1$ for an ellipse and $e > 1$ for a hyperbola

(C) $e = 1$ for an ellipse

(D) $e < 1$ for a hyperbola

Q35. A unit vector in the direction of $\vec{a} = 3\hat{\imath} + 4\hat{\jmath}$ is

(A) $3\hat{\imath} + 4\hat{\jmath}$

(B) $\dfrac{3\hat{\imath} + 4\hat{\jmath}}{7}$

(C) $\dfrac{3\hat{\imath} + 4\hat{\jmath}}{5}$

(D) $\dfrac{3\hat{\imath} + 4\hat{\jmath}}{25}$

Q36. The volume of the parallelepiped whose edges are along $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ with lengths 2, 3 and 4 respectively (scalar triple product) is

(A) 9

(B) 0

(C) 12

(D) 24

Q37. Three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar if and only if their scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}]$ equals

(A) 1

(B) 0

(C) $|\vec{a}||\vec{b}||\vec{c}|$

(D) -1

Q38. The mean of 10 observations is 4 and the mean of another 5 observations is 7. The combined mean of all 15 observations is

(A) 5

(B) 5.5

(C) 4.5

(D) 11

Q39. A bag contains 3 red and 5 green balls. One ball is drawn at random. The probability that it is red is

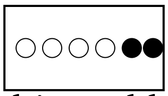
(A) $\frac{5}{8}$

(B) $\frac{1}{3}$

(C) $\frac{3}{5}$

(D) $\frac{3}{8}$

Q40. A box contains 4 white and 2 black balls. Two balls are drawn one after another without replacement. The probability that both are white is



4 white, 2 black

(A) $\frac{4}{9}$

(B) $\frac{1}{3}$

(C) $\frac{2}{5}$

(D) $\frac{1}{2}$

Detailed Solutions

Q1.

Solution

Concept — De Morgan's laws: The complement of a union of two sets equals the intersection of their individual complements.

Step 1 — Write the law: The first De Morgan law states

$$(A \cup B)' = A' \cap B'.$$

Step 2 — Take a typical element: Let $x \in (A \cup B)'$.

Step 3 — Read what this means: Then $x \notin (A \cup B)$.

Step 4 — Break the union: If x is not in $A \cup B$, then x is not in A and x is not in B.

Step 5 — Rewrite with complements: So $x \in A'$ and $x \in B'$.

Step 6 — Combine: "$x \in A'$ and $x \in B'$" means $x \in A' \cap B'$.

Step 7 — Conclude: Hence $(A \cup B)' = A' \cap B'$, which is option (A).

Why other options are wrong: $A' \cup B'$ equals $(A \cap B)'$ (the other De Morgan law), not $(A \cup B)'$; $A \cap B$ is the intersection of the original sets, not their complements; $A' \cap B$ mixes one complement with one original set.

Final Answer: $(A \cup B)' = A' \cap B' \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q1

Q2.

Solution

Concept — Period of $\sin kx$: The function $\sin kx$ completes one full cycle when kx increases by 2π. Its period is therefore $\frac{2\pi}{|k|}$.

Step 1 — Compare with the standard form: Write the given function as $f(x) = \sin(kx)$ with the unknown k.

Step 2 — Identify k: Matching $\sin 2x$ with $\sin kx$ gives $k = 2$.

Step 3 — Write the period formula: Period $= \frac{2\pi}{|k|}$.

Step 4 — Substitute $k = 2$: Period $= \frac{2\pi}{|2|} = \frac{2\pi}{2}$.

Step 5 — Simplify: $\frac{2\pi}{2} = \pi$.

Why other options are wrong: 2π is the period of $\sin x$ (the case $k = 1$); $\frac{\pi}{2}$ would come from $k = 4$; 4π comes from $k = \frac{1}{2}$, not $k = 2$.

Final Answer: Period $= \pi \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept — Modulus of a quotient: For complex numbers, $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$. The modulus of $a + bi$ is $\sqrt{a^2 + b^2}$.

Step 1 — Multiply top and bottom by $-i$: $\frac{1}{i} = \frac{1}{i} \cdot \frac{-i}{-i}$.

Step 2 — Simplify the numerator and denominator: $= \frac{-i}{-i^2}$.

Step 3 — Use $i^2 = -1$: The denominator becomes $-i^2 = -(-1) = 1$, so $\frac{-i}{1} = -i$.

Step 4 — Write in $a + bi$ form: $-i = 0 + (-1)i$, so $a = 0$ and $b = -1$.

Step 5 — Apply the modulus formula: $|-i| = \sqrt{0^2 + (-1)^2}$.

Step 6 — Compute: $= \sqrt{0 + 1} = \sqrt{1} = 1$.

Why other options are wrong: i and $-i$ are complex numbers, not a (real) modulus; the modulus of a nonzero number cannot be 0.

Final Answer: $\left|\frac{1}{i}\right| = 1 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q3

Q4.

Solution

Concept — Cube roots of unity: The cube roots of unity are the three solutions of $x^3 = 1$. One is real and two are non-real (complex).

Step 1 — Set up the equation: A cube root of unity satisfies $x^3 = 1$.

Step 2 — Bring to one side: $x^3 - 1 = 0$.

Step 3 — Factorise: $(x-1)(x^2+x+1) = 0$.

Step 4 — First factor: $x - 1 = 0 \Rightarrow x = 1$ (the real root).

Step 5 — Second factor by the quadratic formula: For $x^2 + x + 1 = 0$, $a = 1$, $b = 1$, $c = 1$, so $x = \dfrac{-1 \pm \sqrt{1-4}}{2}$.

Step 6 — Simplify the discriminant: $\sqrt{1-4} = \sqrt{-3} = i\sqrt{3}$, so $x = \dfrac{-1 \pm i\sqrt{3}}{2}$.

Step 7 — Pick a non-real root: Taking the $+$ sign gives $\omega = \dfrac{-1 + i\sqrt{3}}{2}$, which is option (D).

Why other options are wrong: 1 is the real cube root, not a non-real one; -1 satisfies $x^3 = -1$, not $x^3 = 1$; i gives $i^3 = -i \neq 1$.

Final Answer: $\omega = \dfrac{-1 + i\sqrt{3}}{2} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Sum of squares of roots: Use the identity $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$, together with the sum and product of roots from the coefficients.

Step 1 — Recall Vieta's formulas: For $x^2 + bx + c = 0$, the sum of roots is $\alpha + \beta = -b$ and the product is $\alpha\beta = c$.

Step 2 — Read the coefficients: Here $x^2 - 5x + 6 = 0$, so $b = -5$ and $c = 6$.

Step 3 — Sum of roots: $\alpha + \beta = -(-5) = 5$.

Step 4 — Product of roots: $\alpha\beta = 6$.

Step 5 — Write the identity: $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$.

Step 6 — Substitute: $= (5)^2 - 2(6)$.

Step 7 — Square and multiply: $= 25 - 12$.

Step 8 — Subtract: $= 13$.

Why other options are wrong: 25 forgets the $-2\alpha\beta$ term; 12 is just $2\alpha\beta$; 37 comes from adding $(25 + 12)$ instead of subtracting.

Final Answer: $\alpha^2 + \beta^2 = 13 \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q5

Q6.

Solution

Concept — Trace of a matrix: The trace of a square matrix is the sum of the entries on its leading (top-left to bottom-right) diagonal.

Step 1 — Write the matrix: $A = \begin{pmatrix} 2 & 5 \\ 1 & 7 \end{pmatrix}$.

Step 2 — Locate the leading diagonal: The diagonal runs from the top-left entry to the bottom-right entry: these are 2 (row 1, column 1) and 7 (row 2, column 2).

Step 3 — Write the trace as a sum: Trace $= 2 + 7$.

Step 4 — Add: $= 9$.

Why other options are wrong: 14 comes from multiplying 2×7; 6 and 3 use the off-diagonal entries 5 and 1, which do not belong to the trace.

Final Answer: Trace $= 9 \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q6

Q7.

Solution

Concept — Area via determinant: The area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is $\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$.

Step 1 — Label the vertices: $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (4, 0)$, $(x_3, y_3) = (0, 3)$.

Step 2 — Substitute into the formula: Area $= \frac{1}{2}\Big|\,0(0-3) + 4(3-0) + 0(0-0)\,\Big|$.

Step 3 — Evaluate each bracket: $0 - 3 = -3,\ \ 3 - 0 = 3,\ \ 0 - 0 = 0$.

Step 4 — Multiply each term: $0 \times (-3) = 0,\ \ 4 \times 3 = 12,\ \ 0 \times 0 = 0$.

Step 5 — Add inside the bars: $0 + 12 + 0 = 12$.

Step 6 — Take the absolute value and halve: Area $= \frac{1}{2} \times |12| = \frac{1}{2} \times 12 = 6$.

Step 7 — Check with base $\times$ height: Base $= 4$, height $= 3$, so $\frac{1}{2} \times 4 \times 3 = 6$, which agrees.

Why other options are wrong: 12 forgets the factor $\frac{1}{2}$; 24 is the full product $4 \times 3 \times 2$; 7 is unrelated.

Final Answer: Area $= 6 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q7

Q8.

Solution

Concept — Orthogonal matrix: A square matrix A is orthogonal when its transpose equals its inverse.

Step 1 — Write the inverse condition: $A^T = A^{-1}$.

Step 2 — Multiply both sides by A on the right: $AA^T = AA^{-1}$.

Step 3 — Use $AA^{-1} = I$: $AA^T = I$.

Step 4 — State the definition: Therefore A is orthogonal exactly when $AA^T = I$ (equivalently $A^T A = I$), which is option (D).

Why other options are wrong: $A = A^T$ defines a symmetric matrix; $A^2 = A$ defines an idempotent matrix; $A = -A^T$ defines a skew-symmetric matrix; none of these is the orthogonal condition.

Final Answer: $AA^T = I \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — Arrangement of distinct objects: The number of ways to arrange n distinct objects in a row is $n!$ (the factorial of n).

Step 1 — Identify n: There are 5 distinct books, so $n = 5$.

Step 2 — Write the formula: Number of arrangements $= n! = 5!$.

Step 3 — Expand the factorial: $5! = 5 \times 4 \times 3 \times 2 \times 1$.

Step 4 — Multiply step by step: $5 \times 4 = 20$; $20 \times 3 = 60$; $60 \times 2 = 120$; $120 \times 1 = 120$.

Why other options are wrong: 60 is $\frac{5!}{2}$; 25 is 5^2; 24 is $4!$ (arranging only four books).

Final Answer: Number of arrangements $= 120 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Diagonals of a polygon: A convex polygon with n sides has $\frac{n(n-3)}{2}$ diagonals.

Step 1 — Identify n: A hexagon has 6 sides, so $n = 6$.

Step 2 — Write the formula: Number of diagonals $= \frac{n(n-3)}{2}$.

Step 3 — Substitute $n = 6$: $= \frac{6(6-3)}{2}$.

Step 4 — Simplify the bracket: $6 - 3 = 3$, so $= \frac{6 \times 3}{2}$.

Step 5 — Multiply the numerator: $6 \times 3 = 18$, so $= \frac{18}{2}$.

Step 6 — Divide: $= 9$.

Why other options are wrong: 6 is the number of sides; 12 is double the answer; 15 is ${}^6C_2$, which counts all joining segments (sides plus diagonals).

Final Answer: Number of diagonals $= 9 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

Q11.

Solution

Concept — Binomial coefficients: In $(1+x)^n$, the coefficient of x^r is ${}^nC_r = \frac{n!}{r!\,(n-r)!}$.

Step 1 — Coefficient of x^2: With $n = 5$, $r = 2$: ${}^5C_2 = \frac{5!}{2!\,3!} = \frac{5 \times 4}{2 \times 1} = \frac{20}{2} = 10$.

Step 2 — Coefficient of x^1: With $n = 5$, $r = 1$: ${}^5C_1 = \frac{5!}{1!\,4!} = \frac{5}{1} = 5$.

Step 3 — Form the ratio: $\frac{\text{coeff of } x^2}{\text{coeff of } x^1} = \frac{10}{5}$.

Step 4 — Simplify: $\frac{10}{5} = \frac{2}{1}$, i.e. $2:1$.

Why other options are wrong: $1:2$ inverts the ratio; $1:1$ and $5:2$ come from miscomputing the two coefficients.

Final Answer: Ratio $= 2:1 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q11

Q12.

Solution

Concept — Middle term of an AP: In an arithmetic progression, the middle term is the average (arithmetic mean) of its neighbours. If a, x, b are in AP, then $x = \frac{a+b}{2}$.

Step 1 — Use the AP condition: The common difference is the same on both sides, so $x - 3 = 11 - x$.

Step 2 — Collect x terms: $x + x = 11 + 3$, i.e. $2x = 14$.

Step 3 — Solve for x: $x = \frac{14}{2}$.

Step 4 — Compute: $x = 7$.

Why other options are wrong: 5, 8 and 6 are not the arithmetic mean of 3 and 11, so they break the equal-spacing condition.

Final Answer: $x = 7 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Sum of squares: The sum of the first n square numbers is $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

Step 1 — Identify n: The last term is 10^2, so $n = 10$.

Step 2 — Write the formula: Sum $= \frac{n(n+1)(2n+1)}{6}$.

Step 3 — Substitute $n = 10$: $= \frac{10 \times (10+1) \times (2 \times 10 + 1)}{6}$.

Step 4 — Simplify each bracket: $10 + 1 = 11$ and $2 \times 10 + 1 = 21$, so $= \frac{10 \times 11 \times 21}{6}$.

Step 5 — Multiply the numerator: $10 \times 11 = 110;\ \ 110 \times 21 = 2310$, so $= \dfrac{2310}{6}$.

Step 6 — Divide: $= 385$.

Why other options are wrong: 285 is the sum up to 9^2; 440 and 100 do not match the formula.

Final Answer: Sum $= 385 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q13

Q14.

Solution

Concept — Half-angle relation: The identity $\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$ links $\cos\theta$ to $\sin^2\frac{\theta}{2}$.

Step 1 — Read $\cos\theta$**:** From the 3-4-5 triangle, $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{4}{5}$.

Step 2 — Substitute into the identity: $\dfrac{4}{5} = 1 - 2\sin^2\frac{\theta}{2}$.

Step 3 — Move the $2\sin^2$ **term across:** $2\sin^2\frac{\theta}{2} = 1 - \dfrac{4}{5}$.

Step 4 — Subtract on the right: $1 - \dfrac{4}{5} = \dfrac{5}{5} - \dfrac{4}{5} = \dfrac{1}{5}$, so $2\sin^2\frac{\theta}{2} = \dfrac{1}{5}$.

Step 5 — Divide both sides by 2**:** $\sin^2\frac{\theta}{2} = \dfrac{1}{5} \times \dfrac{1}{2} = \dfrac{1}{10}$.

Why other options are wrong: $\frac{9}{10}$ is $\cos^2\frac{\theta}{2}$ (the complement); $\frac{2}{5}$ forgets to divide by 2; $\frac{4}{5}$ just repeats $\cos\theta$.

Final Answer: $\sin^2\frac{\theta}{2} = \frac{1}{10} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q14

Q15.

Solution

Concept — Solving $\sin^2\theta = 1$**:** Take square roots and find all angles where the sine reaches its extreme values.

Step 1 — Take square roots: $\sin^2\theta = 1 \Rightarrow \sin\theta = \pm 1$.

Step 2 — Where $\sin\theta = +1$**:** This happens at $\theta = \frac{\pi}{2}, \frac{\pi}{2} + 2\pi, \ldots$

Step 3 — Where $\sin\theta = -1$**:** This happens at $\theta = \frac{3\pi}{2}, \frac{3\pi}{2} + 2\pi, \ldots$

Step 4 — List both together: The solutions are $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$, i.e. the odd multiples of $\frac{\pi}{2}$.

Step 5 — Write the general form: An odd multiple of $\frac{\pi}{2}$ is $(2n+1)\frac{\pi}{2}$ for $n \in \mathbb{Z}$.

Why other options are wrong: $n\pi$ and $2n\pi$ give $\sin\theta = 0$; $\frac{n\pi}{2}$ also includes the even multiples where $\sin\theta = 0$.

Final Answer: $\theta = (2n+1)\frac{\pi}{2} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q15

Q16.

Solution

Concept — Range of $\cos^{-1}$: The principal value of $\cos^{-1}$ always lies in $[0, \pi]$. We seek the angle in this range whose cosine is $-\frac{1}{2}$.

Step 1 — Find the reference angle: The angle with $\cos = +\frac{1}{2}$ is $\frac{\pi}{3}$.

Step 2 — Account for the negative sign: Cosine is negative in the second quadrant, so the required angle is $\pi - \frac{\pi}{3}$.

Step 3 — Subtract: $\pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$.

Step 4 — Check the range: $\frac{2\pi}{3}$ lies in $[0, \pi]$, and $\cos\frac{2\pi}{3} = -\frac{1}{2}$, so it is valid.

Why other options are wrong: $\frac{\pi}{3}$ gives $+\frac{1}{2}$; $\frac{\pi}{6}$ gives $\frac{\sqrt{3}}{2}$; $-\frac{\pi}{3}$ is outside the principal range $[0, \pi]$.

Final Answer: $\cos^{-1}\left(-\frac{1}{2}\right) = \dfrac{2\pi}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — Distance between two tops: Drop a horizontal line from the shorter top; this forms a right triangle whose horizontal leg is the ground gap and whose vertical leg is the height difference. The line joining the tops is the hypotenuse.

Step 1 — Horizontal leg: The poles are 12 m apart, so the horizontal leg $= 12$ m.

Step 2 — Vertical leg: The height difference $= 11 - 6 = 5$ m.

Step 3 — Apply Pythagoras: Distance $= \sqrt{(\text{horizontal})^2 + (\text{vertical})^2} = \sqrt{12^2 + 5^2}$.

Step 4 — Square each leg: $12^2 = 144$ and $5^2 = 25$, so $= \sqrt{144 + 25}$.

Step 5 — Add: $= \sqrt{169}$.

Step 6 — Take the square root: $= 13$ m.

Why other options are wrong: 12 ignores the height difference; 17 adds the legs $(12 + 5)$ instead of using Pythagoras; $\sqrt{120}$ comes from a wrong subtraction.

Final Answer: Distance $= 13$ m $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q17

Q18.

Solution

Concept — Standard limit: We use the known result $\lim_{x\to 0} \frac{\sin x}{x} = 1$ and continuity of $\cos x$.

Step 1 — Write $\tan x$ in terms of sine and cosine: $\tan x = \frac{\sin x}{\cos x}$.

Step 2 — Substitute into the expression: $\frac{\tan x}{x} = \frac{\sin x}{x \cos x}$.

Step 3 — Split into two factors: $= \frac{\sin x}{x} \cdot \frac{1}{\cos x}$.

Step 4 — Take the limit of each factor: $\lim_{x\to 0} \frac{\sin x}{x} = 1$ and $\lim_{x\to 0} \frac{1}{\cos x} = \frac{1}{\cos 0} = \frac{1}{1} = 1$.

Step 5 — Multiply the limits: $1 \times 1 = 1$.

Why other options are wrong: The limit clearly exists and equals 1, so 0, ∞, and "does not exist" are all incorrect.

Final Answer: The limit is $1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q18

Q19.

Solution

Concept — Continuity of a composite: The composition theorem says: if g is continuous at a and f is continuous at $g(a)$, then the composite $f \circ g$ is continuous at a.

Step 1 — Inner function: g is given to be continuous at $x = a$.

Step 2 — Outer function: f is given to be continuous (in particular at the point $g(a)$).

Step 3 — Apply the theorem: Both hypotheses of the composition theorem hold, so $f \circ g$ is continuous at $x = a$.

Why other options are wrong: A composite of continuous functions is never automatically discontinuous; continuity does not require $f = g$; the composite is well defined wherever g is.

Final Answer: $f \circ g$ is continuous at $x = a \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q19

Q20.

Solution

Concept — $\sqrt{x}$ at the origin: Check continuity (does the limit equal the value?) and differentiability (does the derivative stay finite?) separately.

Step 1 — Value at 0: $f(0) = \sqrt{0} = 0$.

Step 2 — Limit at 0: $\lim_{x \to 0^+} \sqrt{x} = 0$.

Step 3 — Conclude continuity: Since the limit 0 equals the value $f(0) = 0$, f is continuous at 0.

Step 4 — Differentiate using the power rule: $f(x) = x^{1/2}$, so $f'(x) = \frac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}}$.

Step 5 — Behaviour of the derivative at 0: As $x \to 0^+$, $\sqrt{x} \to 0$, so $\dfrac{1}{2\sqrt{x}} \to \infty$.

Step 6 — Conclude: The derivative is unbounded (vertical tangent), so f is not differentiable at 0, although it is continuous there.

Why other options are wrong: The derivative is not the finite value 0 or 1; the function is not discontinuous (Step 3 shows it is continuous).

Final Answer: Continuous but not differentiable $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

Q21.

Solution

Concept — Parametric differentiation: When x and y are both given in terms of a parameter t, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

Step 1 — Differentiate x with respect to t: $x = t^2 \Rightarrow \frac{dx}{dt} = 2t$ (power rule).

Step 2 — Differentiate y with respect to t: $y = t^3 \Rightarrow \frac{dy}{dt} = 3t^2$ (power rule).

Step 3 — Form the quotient: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t}$.

Step 4 — Cancel one factor of t: $\frac{3t^2}{2t} = \frac{3t}{2}$.

Why other options are wrong: $\frac{2}{3t}$ inverts the quotient; $\frac{3t^2}{2}$ forgets to cancel a t in the denominator; t wrongly drops the constants 3 and 2.

Final Answer: $\frac{dy}{dx} = \frac{3t}{2} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q21

Q22.

Solution

Concept — Rate of change: The rate of change of V with respect to r is the derivative $\frac{dV}{dr}$. Constants stay put; differentiate r^3 by the power rule.

Step 1 — Write the volume: $V = \frac{4}{3}\pi r^3$.

Step 2 — Apply the power rule to r^3: $\frac{d}{dr}(r^3) = 3r^2$.

Step 3 — Keep the constant factor: $\frac{dV}{dr} = \frac{4}{3}\pi \cdot 3r^2$.

Step 4 — Cancel the 3: $\frac{4}{3} \times 3 = 4$, so $\frac{dV}{dr} = 4\pi r^2$ (which is the sphere's surface area).

Why other options are wrong: $\frac{4}{3}\pi r^2$ forgets to multiply by the 3 from the power rule; $4\pi r^3$ wrongly keeps the cube; $2\pi r$ is the derivative of a circle's area, not a sphere's volume.

Final Answer: $\frac{dV}{dr} = 4\pi r^2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q22

Q23.

Solution

Concept — Absolute extremum on a closed interval: On $[a, b]$, the absolute maximum occurs either at an interior critical point or at an endpoint. So find the critical points, then compare all candidate values.

Step 1 — Differentiate: $f(x) = x^2 \Rightarrow f'(x) = 2x$.

Step 2 — Set the derivative to zero: $2x = 0$.

Step 3 — Solve for the critical point: $x = 0$ (which lies inside $[-3, 2]$).

Step 4 — Value at the critical point: $f(0) = 0^2 = 0$.

Step 5 — Value at the left endpoint: $f(-3) = (-3)^2 = 9$.

Step 6 — Value at the right endpoint: $f(2) = 2^2 = 4$.

Step 7 — Compare: Among 0, 9 and 4, the largest is 9.

Why other options are wrong: 0 is the absolute minimum (at the critical point); 4 is the value at $x = 2$, not the maximum; 6 is never attained.

Final Answer: Absolute maximum $= 9 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q23

Q24.

Solution

Concept — Integral of an exponential: For a constant base $a > 0$, $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + C$ (this reverses $\frac{d}{dx}a^x = a^x \ln a$).

Step 1 — Identify the base: The integrand is 2^x, so $a = 2$.

Step 2 — Write the formula: $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + C$.

Step 3 — Substitute $a = 2$: $\displaystyle\int 2^x\,dx = \frac{2^x}{\ln 2} + C$.

Step 4 — Verify by differentiating: $\displaystyle\frac{d}{dx}\left(\frac{2^x}{\ln 2}\right) = \frac{2^x \ln 2}{\ln 2} = 2^x$, confirming the result.

Why other options are wrong: $2^x \ln 2$ is the derivative, not the integral; $x\,2^{x-1}$ wrongly applies the power rule (which is for x^n, not a^x); 2^x omits the $\ln 2$ factor.

Final Answer: $\dfrac{2^x}{\ln 2} + C \Rightarrow \boxed{D}$

Answer: (D) Go Back to Q24

Q25.

Solution

Concept — Integral of f'/f: Whenever the numerator is the derivative of the denominator, $\displaystyle\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$.

Step 1 — Name the denominator: Let $f(x) = x^2 + 1$.

Step 2 — Differentiate it: $f'(x) = 2x$.

Step 3 — Compare with the numerator: The numerator is exactly $2x = f'(x)$, so the integrand is $\dfrac{f'(x)}{f(x)}$.

Step 4 — Apply the rule: $\displaystyle\int \frac{2x}{x^2+1}\,dx = \ln|x^2+1| + C$.

Step 5 — Drop the bars: Since $x^2 + 1 > 0$ always, $\ln|x^2+1| = \ln(x^2+1)$, giving $\ln(x^2+1) + C$.

Why other options are wrong: $\tan^{-1} x$ is the integral of $\frac{1}{1+x^2}$ (no $2x$ on top); $\frac{1}{x^2+1}$ is unrelated; $2\ln x$ ignores the f'/f form.

Final Answer: $\ln(x^2+1) + C \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q25

Q26.

Solution

Concept — Even-function property: If f is even (i.e. $f(-x) = f(x)$), then $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

Step 1 — Check evenness: $f(x) = x^2$ gives $f(-x) = (-x)^2 = x^2 = f(x)$, so x^2 is even.

Step 2 — Use the property: $\displaystyle\int_{-2}^{2} x^2\,dx = 2\int_0^2 x^2\,dx$.

Step 3 — Find the antiderivative: $\int x^2\,dx = \frac{x^3}{3}$.

Step 4 — Apply the limits 0 **to** 2**:** $\left[\frac{x^3}{3}\right]_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3} - 0 = \frac{8}{3}$.

Step 5 — Multiply by the factor 2**:** $2 \times \frac{8}{3} = \frac{16}{3}$.

Why other options are wrong: 0 would be the answer for an odd function; $\frac{8}{3}$ forgets the factor 2; 8 does not match the integration.

Final Answer: The integral $= \frac{16}{3} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q26

Q27.

Solution

Concept — Quarter-circle area: A full circle of radius r has area πr^2; one quarter of it is $\frac{1}{4}\pi r^2$.

Step 1 — Recognise the curve: Squaring $y = \sqrt{4 - x^2}$ gives $y^2 = 4 - x^2$, i.e. $x^2 + y^2 = 4$, a circle centred at the origin.

Step 2 — Read the radius: $x^2 + y^2 = 4 = 2^2$, so the radius is $r = 2$.

Step 3 — Identify the region: With $y \geq 0$ and bounded by both axes, the region is the first-quadrant quarter of this circle.

Step 4 — Write the quarter-circle area: Area $= \frac{1}{4}\pi r^2$.

Step 5 — Substitute $r = 2$**:** $= \frac{1}{4}\pi(2)^2$.

Step 6 — Compute: $(2)^2 = 4$, so $= \frac{1}{4}\pi \times 4 = \pi$.

Why other options are wrong: 2π is the half-circle area; 4π is the full-circle area; $\frac{\pi}{2}$ uses radius 1 wrongly.

Final Answer: Area $= \pi \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q27

Q28.

Solution

Concept — Degree of a differential equation: Once the equation is written as a polynomial in the derivatives (no fractional or radical powers), the degree is the highest power of the highest-order derivative.

Step 1 — Check it is polynomial in derivatives: The equation $\left(\frac{d^2y}{dx^2}\right)^2 + \frac{dy}{dx} + y = 0$ is already polynomial in the derivatives (no roots or fractions of them).

Step 2 — Find the highest-order derivative: The highest order present is the second derivative $\frac{d^2y}{dx^2}$.

Step 3 — Read its power: This highest-order derivative appears raised to the power 2.

Step 4 — Conclude: Therefore the degree is 2.

Why other options are wrong: 1 ignores the square on the second derivative; 4 confuses degree with $(\text{order})^2$; 0 is meaningless here.

Final Answer: Degree $= 2 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q28

Q29.

Solution

Concept — Direct integration: If $\frac{dy}{dx} = g(x)$, integrate both sides with respect to x to get $y = \int g(x)\,dx$.

Step 1 — Integrate both sides: $\int \frac{dy}{dx}\,dx = \int e^x\,dx$.

Step 2 — Left side simplifies: $\int \frac{dy}{dx}\,dx = y$.

Step 3 — Integrate the right side: $\int e^x\,dx = e^x + C$ (since $\frac{d}{dx}e^x = e^x$).

Step 4 — Combine: $y = e^x + C$.

Why other options are wrong: $e^{-x} + C$ has derivative $-e^{-x}$, not e^x; the remaining options also fail to differentiate back to e^x.

Final Answer: $y = e^x + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q29

Q30.

Solution

Concept — Slope through two points: The slope of the line through (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$. Through the origin this reduces to $\frac{y_1}{x_1}$.

Step 1 — Identify the two points: The line passes through $(0, 0)$ and $(2, 6)$.

Step 2 — Write the slope formula: Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{2 - 0}$.

Step 3 — Simplify the differences: $= \frac{6}{2}$.

Step 4 — Divide: $= 3$.

Why other options are wrong: 2 inverts the ratio (uses $\frac{x}{y}$); $\frac{1}{3}$ is the reciprocal of the slope; 6 ignores the x-coordinate 2.

Final Answer: Slope $= 3 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q30

Q31.

Solution

Concept — Parallel lines: For a line $ax + by + c = 0$, the slope is $-\frac{a}{b}$. Two lines are parallel when their slopes are equal.

Step 1 — Slope of the given line: For $4x + 6y + 5 = 0$, $a = 4$, $b = 6$, so slope $= -\frac{4}{6} = -\frac{2}{3}$.

Step 2 — Slope of option (A): $6x - 4y + 1 = 0$ has slope $-\frac{6}{-4} = \frac{3}{2}$ (not a match).

Step 3 — Slope of option (B): $4x - 6y + 1 = 0$ has slope $-\frac{4}{-6} = \frac{2}{3}$ (not a match).

Step 4 — Slope of option (C): $x + y + 1 = 0$ has slope $-\frac{1}{1} = -1$ (not a match).

Step 5 — Slope of option (D): $2x + 3y + 7 = 0$ has slope $-\frac{2}{3}$, which equals the given line's slope.

Step 6 — Conclude: Equal slopes $\Rightarrow$ option (D) is parallel to the given line.

Why other options are wrong: (A) and (B) have positive slopes $\frac{3}{2}$ and $\frac{2}{3}$; (C) has slope -1; none equals $-\frac{2}{3}$.

Final Answer: $2x + 3y + 7 = 0$ is parallel $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q31

Q32.

Solution

Concept — Concentric circles: Concentric circles share the same centre but have different radii. For $x^2 + y^2 + 2gx + 2fy + c = 0$, the centre is $(-g, -f)$.

Step 1 — Read the coefficients: For $x^2 + y^2 - 4x - 6y + 9 = 0$, $2g = -4$ and $2f = -6$.

Step 2 — Solve for g and f: $g = -2$ and $f = -3$.

Step 3 — Find the centre: Centre $= (-g, -f) = (2, 3)$.

Step 4 — Keep the same centre: The new circle is also centred at $(2, 3)$.

Step 5 — Use the given radius: With radius 5, the standard form $(x - h)^2 + (y - k)^2 = r^2$ gives $(x - 2)^2 + (y - 3)^2 = 5^2$.

Step 6 — Square the radius: $5^2 = 25$, so $(x - 2)^2 + (y - 3)^2 = 25$.

Why other options are wrong: $(x+2)^2 + (y+3)^2$ uses centre $(-2, -3)$, the wrong sign; radius 5 must be squared to 25, so 5 and 4 on the right are wrong.

Final Answer: $(x - 2)^2 + (y - 3)^2 = 25 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q32

Q33.

Solution

Concept — Parabola from focus and directrix: A parabola opening rightward with focus $(a, 0)$ and directrix $x = -a$ has equation $y^2 = 4ax$.

Step 1 — Read the focus: The focus is $(2, 0)$, which is of the form $(a, 0)$ with $a = 2$.

Step 2 — Check with the directrix: The directrix $x = -2$ matches $x = -a$ with $a = 2$, confirming $a = 2$.

Step 3 — Write the standard equation: $y^2 = 4ax$.

Step 4 — Substitute $a = 2$: $y^2 = 4(2)x$.

Step 5 — Simplify: $4 \times 2 = 8$, so $y^2 = 8x$.

Why other options are wrong: $y^2 = 4x$ uses $a = 1$; $x^2 = 8y$ describes a parabola opening upward; $y^2 = -8x$ opens leftward (focus on the negative x-axis).

Final Answer: $y^2 = 8x \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q33

Q34.

Solution

Concept — Eccentricity classification: The eccentricity e classifies a conic: $e < 1$ is an ellipse, $e = 1$ a parabola, and $e > 1$ a hyperbola.

Step 1 — Recall the ellipse case: An ellipse has $e < 1$.

Step 2 — Recall the hyperbola case: A hyperbola has $e > 1$.

Step 3 — Match the options: Option (B), "$e < 1$ for an ellipse and $e > 1$ for a hyperbola," matches both facts.

Why other options are wrong: "$e > 1$ for an ellipse" is false ($e > 1$ is a hyperbola); "$e = 1$ for an ellipse" is the parabola condition; "$e < 1$ for a hyperbola" contradicts Step 2.

Final Answer: $e < 1$ (ellipse), $e > 1$ (hyperbola) $\Rightarrow \boxed{B}$

Answer: (B) Go Back to Q34

Q35.

Solution

Concept — Unit vector: A unit vector in the direction of $\vec{a}$ is $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$, where $|\vec{a}|$ is the magnitude.

Step 1 — Read the components: $\vec{a} = 3\hat{i} + 4\hat{j}$, so the components are 3 and 4.

Step 2 — Write the magnitude formula: $|\vec{a}| = \sqrt{3^2 + 4^2}$.

Step 3 — Square the components: $3^2 = 9$ and $4^2 = 16$, so $|\vec{a}| = \sqrt{9 + 16}$.

Step 4 — Add and take the root: $= \sqrt{25} = 5$.

Step 5 — Divide the vector by its magnitude: $\hat{a} = \dfrac{3\hat{i} + 4\hat{j}}{5}$.

Why other options are wrong: $3\hat{i} + 4\hat{j}$ has magnitude 5, not 1, so it is not a unit vector; dividing by 7 or 25 uses a wrong magnitude.

Final Answer: $\hat{a} = \dfrac{3\hat{i} + 4\hat{j}}{5} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q35

Q36.

Solution

Concept — Scalar triple product as volume: The volume of a parallelepiped is $|[\vec{a}\,\vec{b}\,\vec{c}]|$. When the edges are mutually perpendicular (along $\hat{i}, \hat{j}, \hat{k}$), this reduces to the product of the edge lengths.

Step 1 — Write the edge vectors: $\vec{a} = 2\hat{i}$, $\vec{b} = 3\hat{j}$, $\vec{c} = 4\hat{k}$.

Step 2 — Form the scalar triple product determinant: $[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{vmatrix}$.

Step 3 — Diagonal determinant: For a diagonal matrix this equals the product of the diagonal entries: $2 \times 3 \times 4$.

Step 4 — Multiply step by step: $2 \times 3 = 6;\ \ 6 \times 4 = 24$.

Step 5 — Take the magnitude: Volume $= |24| = 24$.

Why other options are wrong: 9 and 12 drop one of the factors; 0 would require the edges to be coplanar (linearly dependent), which they are not.

Final Answer: Volume $= 24 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q36

Q37.

Solution

Concept — Coplanarity test: Three vectors lie in one plane exactly when the parallelepiped they span is flat, i.e. has zero volume.

Step 1 — Volume in terms of the triple product: The volume of the parallelepiped is $|[\vec{a}\,\vec{b}\,\vec{c}]|$.

Step 2 — Flat box means zero volume: If the three vectors are coplanar, the box collapses, so its volume is 0.

Step 3 — Translate to the triple product: Zero volume means $|[\vec{a}\,\vec{b}\,\vec{c}]| = 0$, hence $[\vec{a}\,\vec{b}\,\vec{c}] = 0$.

Step 4 — Conclude: Therefore the vectors are coplanar if and only if $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.

Why other options are wrong: 1 and -1 are nonzero values, indicating a genuine 3-D box; $|\vec{a}||\vec{b}||\vec{c}|$ is generally nonzero and is not the coplanarity condition.

Final Answer: $[\vec{a}\ \vec{b}\ \vec{c}] = 0 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q37

Q38.

Solution

Concept — Combined mean: The combined mean weights each group's mean by its size: $\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$.

Step 1 — List the data: $n_1 = 10, \bar{x}_1 = 4;\ \ n_2 = 5, \bar{x}_2 = 7$.

Step 2 — Total of the first group: $n_1\bar{x}_1 = 10 \times 4 = 40$.

Step 3 — Total of the second group: $n_2\bar{x}_2 = 5 \times 7 = 35$.

Step 4 — Add the totals: $40 + 35 = 75$.

Step 5 — Add the counts: $n_1 + n_2 = 10 + 5 = 15$.

Step 6 — Divide: Combined mean $= \dfrac{75}{15} = 5$.

Why other options are wrong: 5.5 is the plain average of 4 and 7, ignoring the group sizes; 4.5 and 11 also ignore the weighting.

Final Answer: Combined mean $= 5 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q38

Q39.

Solution

Concept — Classical probability: For equally likely outcomes, $P = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$.

Step 1 — Count favourable outcomes: There are 3 red balls, so favourable $= 3$.

Step 2 — Count total outcomes: Total balls $= 3$ red $+ 5$ green $= 8$.

Step 3 — Form the probability: $P(\text{red}) = \dfrac{3}{8}$.

Why other options are wrong: $\frac{5}{8}$ is $P(\text{green})$; $\frac{1}{3}$ uses red over green ($3/5$ inverted in count) and $\frac{3}{5}$ uses red over green, both with the wrong total.

Final Answer: $P(\text{red}) = \dfrac{3}{8} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q39

Q40.

Solution

Concept — Without replacement: The probability of both events is the probability of the first draw times the conditional probability of the second (given the first has happened and is not replaced).

Step 1 — First draw is white: There are 4 white out of 6 total, so $P_1 = \dfrac{4}{6}$.

Step 2 — Simplify P_1: $\dfrac{4}{6} = \dfrac{2}{3}$.

Step 3 — Update the counts: After removing one white ball, 3 white remain out of 5 total.

Step 4 — Second draw is white: $P_2 = \dfrac{3}{5}$.

Step 5 — Multiply the two probabilities: $P(\text{both white}) = \dfrac{2}{3} \times \dfrac{3}{5}$.

Step 6 — Multiply numerators and denominators: $= \dfrac{2 \times 3}{3 \times 5} = \dfrac{6}{15}$.

Step 7 — Simplify: $\dfrac{6}{15} = \dfrac{2}{5}$.

Why other options are wrong: $\frac{4}{9}$ assumes drawing with replacement (using $\frac{4}{6} \times \frac{4}{6}$); $\frac{1}{3}$ and $\frac{1}{2}$ come from miscounting the remaining balls.

Final Answer: $P(\text{both white}) = \dfrac{2}{5} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	A
6	B	7	C	8	D	9	A	10	B
11	C	12	D	13	B	14	C	15	A
16	D	17	B	18	A	19	C	20	D
21	B	22	A	23	C	24	D	25	A
26	B	27	C	28	D	29	A	30	B
31	D	32	C	33	A	34	B	35	C
36	D	37	B	38	A	39	D	40	C