SRMJEEE 2026 June 10 Shift 1

Question Paper (Memory-Based) with Solutions

Conducted by SRMIST

General Instructions

(i) **Duration:** The total duration of the examination is 150 minutes.

(ii) **Total Marks:** The paper carries a maximum of 130 marks.

(iii) **Questions:** The paper has 130 questions.

(iv) **Marking Scheme:** Each question carries 1 mark and there is no negative marking for incorrect answers.

1. Monochromatic light of frequency 8×10^{14} Hz is incident on a metal surface whose threshold frequency is 5×10^{14} Hz. The stopping potential is approximately:
(Given $h = 6.6 \times 10^{-34}$ J s, $e = 1.6 \times 10^{-19}$ C)

(A) 0.62 V
(B) 1.24 V
(C) 2.48 V
(D) 3.72 V

Correct Answer: (B) 1.24 V

Solution:

Step 1: Understanding the Question:
The question asks for the stopping potential of photoelectrons emitted from a metal surface when monochromatic light of a given frequency is incident on it.

Step 2: Key Formula or Approach:

According to Einstein's photoelectric equation, the maximum kinetic energy of the emitted photoelectrons is given by:

$$K_{\max} = h\nu - h\nu_0$$

The stopping potential V_0 is related to the maximum kinetic energy by:

$$eV_0 = K_{\max} \implies V_0 = \frac{h(\nu - \nu_0)}{e}$$

where:

- h is Planck's constant
- ν is the frequency of incident light
- ν_0 is the threshold frequency
- e is the elementary charge

Step 3: Detailed Explanation:

Given:

- $\nu = 8 \times 10^{14}\,\text{Hz}$
- $\nu_0 = 5 \times 10^{14}\,\text{Hz}$
- $h = 6.6 \times 10^{-34}\,\text{J s}$
- $e = 1.6 \times 10^{-19}\,\text{C}$

Substitute the given values into the formula:

$$V_0 = \frac{6.6 \times 10^{-34} \times (8 \times 10^{14} - 5 \times 10^{14})}{1.6 \times 10^{-19}}$$

$$V_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^{14}}{1.6 \times 10^{-19}}$$

$$V_0 = \frac{19.8 \times 10^{-20}}{1.6 \times 10^{-19}}$$

$$V_0 = \frac{1.98}{1.6} \approx 1.2375\,\text{V}$$

Thus, the stopping potential is approximately $1.24\,\text{V}$.

Step 4: Final Answer:

(B) 1.24 V

Quick Tip: To perform quick calculations in photoelectric effect questions, convert energies into electron-volts (eV). Since $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$, dividing the energy in Joules by e directly gives the stopping potential in Volts.

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2. A planet has mass $81M_E$ and radius $9R_E$, where M_E and R_E are the mass and radius of Earth respectively. The escape velocity from the planet is:

(A) Equal to Earth's escape velocity
(B) 3 times Earth's escape velocity
(C) $\frac{1}{3}$ times Earth's escape velocity
(D) 9 times Earth's escape velocity

Correct Answer: (B) 3 times Earth's escape velocity

Solution:

Step 1: Understanding the Question:
We need to find the escape velocity of a planet given its mass and radius in terms of Earth's mass and radius.

Step 2: Key Formula or Approach:
The escape velocity v_e from the surface of a spherical body of mass M and radius R is given by:

$$v_e = \sqrt{\frac{2GM}{R}}$$

where G is the universal gravitational constant.

Step 3: Detailed Explanation:

Let the escape velocity of Earth be:

$$v_E = \sqrt{\frac{2GM_E}{R_E}}$$

For the given planet, mass $M_P = 81M_E$ and radius $R_P = 9R_E$.

The escape velocity from this planet is:

$$v_P = \sqrt{\frac{2GM_P}{R_P}}$$

Substitute the values of M_P and R_P:

$$v_P = \sqrt{\frac{2G(81M_E)}{9R_E}}$$

$$v_P = \sqrt{9 \times \frac{2GM_E}{R_E}}$$

$$v_P = 3\sqrt{\frac{2GM_E}{R_E}} = 3v_E$$

Thus, the escape velocity from the planet is 3 times the escape velocity of Earth.

Step 4: Final Answer:

(B) 3 times Earth's escape velocity

Quick Tip: Escape velocity scales as $v_e \propto \sqrt{\frac{M}{R}}$. If mass is multiplied by k_1 and radius by k_2, the escape velocity changes by a factor of $\sqrt{\frac{k_1}{k_2}}$. Here, $\sqrt{\frac{81}{9}} = \sqrt{9} = 3$.

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3. The depletion region width of a p-n junction is 4×10^{-6} m and the potential barrier is 0.8 V. The electric field intensity in the depletion region is:

(A) 1×10^5 V/m

(B) 2×10^5 V/m

(C) 4×10^5 V/m

(D) 8×10^5 V/m

Correct Answer: (B) 2×10^5 V/m

Solution:

Step 1: Understanding the Question:

We need to find the electric field intensity in the depletion region of a p-n junction when its width and the barrier potential are given.

Step 2: Key Formula or Approach:

The average electric field intensity E in the depletion region is related to the potential barrier V and the width d by the relation:

$$E = \frac{V}{d}$$

Step 3: Detailed Explanation:

Given:

- Potential barrier $V = 0.8\,\text{V}$
- Depletion region width $d = 4 \times 10^{-6}\,\text{m}$

Substitute these values into the formula:

$$E = \frac{0.8}{4 \times 10^{-6}}$$

$$E = 0.2 \times 10^6\,\text{V/m}$$

$$E = 2 \times 10^5\,\text{V/m}$$

Thus, the electric field intensity in the depletion region is $2 \times 10^5\,\text{V/m}$.

Step 4: Final Answer:

(B) $2 \times 10^5\,\text{V/m}$

Quick Tip: Always double-check the power of ten in units. Writing 0.8/4 as 0.2 and then shifting the decimal to get 2×10^5 prevents simple calculation errors.

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4. An AC source of frequency 50 Hz is connected to a pure capacitor. The phase difference between voltage and current is:

(A) 0°
(B) 45°
(C) 90° (current leads)
(D) 90° (voltage leads)

Correct Answer: (C) 90° (current leads)

Solution:

Step 1: Understanding the Question:
The question asks for the phase difference between current and voltage in an AC circuit consisting of an AC source and a pure capacitor.

Step 2: Detailed Explanation:
In a purely capacitive AC circuit, the relationship between charge q and voltage v is given by:

$$q = Cv$$

If the alternating voltage is:

$$v = V_0 \sin(\omega t)$$

Then the current i in the circuit is:

$$i = \frac{dq}{dt} = C\frac{dv}{dt} = C\frac{d}{dt}[V_0 \sin(\omega t)] = CV_0\omega \cos(\omega t)$$

Using trigonometric identities:

$$i = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Comparing the equations for v and i, we see that the phase of current is ahead of the phase of voltage by $\frac{\pi}{2}$ radians or $90°$.

Therefore, the current leads the voltage by $90°$.

Step 3: Final Answer:

(C) $90°$ (current leads)

Quick Tip: Remember the classic mnemonic "ELI the ICE man":

- In an Inductor (L), Voltage (E) leads Current (I).
- In a Capacitor (C), Current (I) leads Voltage (E).

For pure components, the lead/lag angle is always $90°$.

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5. An object of height 4 cm is placed 15 cm in front of a convex lens of focal length 10 cm. The height of the image formed is:

(A) 4 cm

(B) 6 cm

(C) 8 cm

(D) 12 cm

Correct Answer: (C) 8 cm

Solution:

Step 1: Understanding the Question:

An object of height 4 cm is kept in front of a convex lens. We need to find the height of the formed image.

Step 2: Key Formula or Approach:

Use the lens formula to find the image distance v:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Then, use the magnification formula to find the height of the image h_i:

$$m = \frac{v}{u} = \frac{h_i}{h_o}$$

Step 3: Detailed Explanation:

Given:

- Object height, $h_o = 4\,\text{cm}$
- Object distance, $u = -15\,\text{cm}$ (using coordinate sign convention)
- Focal length of convex lens, $f = +10\,\text{cm}$

Substitute f and u into the lens formula:

$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-15}$$
$$\frac{1}{10} = \frac{1}{v} + \frac{1}{15}$$
$$\frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{3-2}{30} = \frac{1}{30} \implies v = 30\,\text{cm}$$

Now, determine the magnification m:

$$m = \frac{v}{u} = \frac{30}{-15} = -2$$

Since $m = \frac{h_i}{h_o}$:

$$-2 = \frac{h_i}{4} \implies h_i = -8\,\text{cm}$$

The negative sign indicates that the image is inverted. The magnitude of the height of the image is $8\,\text{cm}$.

Step 4: Final Answer:

(C) $8\,\text{cm}$

Quick Tip: For a convex lens, if the object is placed between f and $2f$ (here, $10\,\text{cm} < 15\,\text{cm} < 20\,\text{cm}$), the image formed is real, inverted, magnified, and formed beyond $2f$. Since it is magnified, the height must be greater than 4 cm, ruling out option (A) immediately.

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6. The statement that is NOT correct is:

(A) Angular quantum number signifies the shape of the orbital
(B) Energies of stationary states in hydrogen like atoms is inversely proportional to the square of the principal quantum number
(C) Total number of nodes for 3s orbital is three.
(D) The radius of the first orbit of He^+ is half that of the first orbit of hydrogen atom.

Correct Answer: (C) Total number of nodes for 3s orbital is three.

Solution:

Step 1: Understanding the Question:
We need to examine the given statements about quantum mechanics and atomic structures to identify the incorrect statement.

Step 2: Detailed Explanation:
Let us analyze each option:
- **Option (A):** The angular quantum number l (orbital angular momentum quantum number) determines the shape of the orbital (e.g., $l = 0$ for spherical s, $l = 1$ for dumbbell p). This statement is correct.
- **Option (B):** The energy of stationary states in a hydrogen-like atom is given by $E_n = -13.6\frac{Z^2}{n^2}$ eV. The magnitude of energy is inversely proportional to n^2. This statement is correct.
- **Option (C):** The total number of nodes in any orbital is given by the formula $n-1$, where n

is the principal quantum number. For a 3s orbital, $n = 3$. Therefore, the total number of nodes is $3 - 1 = 2$. Thus, stating that the total number of nodes is three is incorrect.

- **Option (D):** The Bohr radius of the n-th orbit is given by $r_n = 0.529\frac{n^2}{Z}$ Å. For the first orbit ($n = 1$) of hydrogen ($Z = 1$), $r_H \propto 1$. For the first orbit ($n = 1$) of He^+ ($Z = 2$), $r_{He^+} \propto \frac{1}{2}$. Hence, the radius of He^+ is half that of hydrogen. This statement is correct.

Step 3: Final Answer:

(C) Total number of nodes for 3s orbital is three.

Quick Tip: Remember the formulas for nodes:

- Radial nodes $= n - l - 1$
- Angular nodes $= l$
- Total nodes = radial nodes + angular nodes $= n - 1$

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7. Which of following is correct?

(A) the lowering of vapour pressure is equal to the mole fraction of solute

(B) the relative lowering of vapour pressure is equal to the mole fraction of solute

(C) the relative lowering of vapour pressure is proportional to the amount of solute in solution

(D) the vapour pressure of the solution is equal to the mole fraction of solvent

Correct Answer: (B) the relative lowering of vapour pressure is equal to the mole fraction of solute

Solution:

Step 1: Understanding the Question:

The question asks for the correct statement regarding Raoult's law for solutions containing non-volatile solutes.

Step 2: Detailed Explanation:

According to Raoult's Law, when a non-volatile solute is added to a solvent, the vapour pressure of the solvent decreases.

The lowering of vapour pressure is given by:

$$\Delta P = P^0 - P_s$$

where P^0 is the vapour pressure of the pure solvent and P_s is the vapour pressure of the solution.

The relative lowering of vapour pressure is defined as:

$$\frac{P^0 - P_s}{P^0}$$

According to Raoult's law, this relative lowering of vapour pressure is equal to the mole fraction of the solute (χ_{solute}):

$$\frac{P^0 - P_s}{P^0} = \chi_{\text{solute}}$$

Hence, option (B) is the correct statement.

Step 3: Final Answer:

(B) the relative lowering of vapour pressure is equal to the mole fraction of solute

Quick Tip: Do not confuse "lowering of vapour pressure" ($P^0 - P_s$) with "relative lowering of vapour pressure" ($\frac{P^0 - P_s}{P^0}$). Only the relative lowering is equal to the mole fraction of the solute.

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8. K_2HgI_4 is 40% ionised in aqueous solution. The value of its van't Hoff factor (i) is:

(A) 1.6

(B) 1.8

(C) 2.2

(D) 2.0

Correct Answer: (B) 1.8

Solution:

Step 1: Understanding the Question:

We need to find the van't Hoff factor i for the complex salt K_2HgI_4 when it undergoes 40% ionization in water.

Step 2: Key Formula or Approach:

For association or dissociation, the van't Hoff factor i is related to the degree of ionization α by:

$$i = 1 + (n - 1)\alpha$$

where n is the number of ions produced per formula unit of solute.

Step 3: Detailed Explanation:

The ionization of the complex salt $K_2[HgI_4]$ in aqueous solution occurs as follows:

$$K_2[HgI_4] \rightleftharpoons 2K^+ + [HgI_4]^{2-}$$

From this equation, 1 molecule of the salt dissociates to give a total of 3 ions (2 potassium ions and 1 complex tetraiodomercurate ion).

Thus, the number of particles $n = 2 + 1 = 3$.

Given:

- Degree of ionization, $\alpha = 40\% = 0.4$

Using the relation:

$$i = 1 + (3 - 1)\alpha$$

$$i = 1 + 2(0.4)$$

$$i = 1 + 0.8 = 1.8$$

Thus, the van't Hoff factor is 1.8.

Step 4: Final Answer:

(B) 1.8

Quick Tip: Remember that the complex ion $[HgI_4]^{2-}$ remains intact as a single entity in aqueous solution; it does not dissociate further into mercury and iodine ions.

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9. The molal boiling point elevation constant for water is $0.510\,\text{K mol}^{-1}$ kg. The boiling point of a solution made by dissolving $6.0\,\text{g}$ urea in $200\,\text{g}$ water is:

(A) 100.255°C

(B) 100°C

(C) 0.255°C

(D) 99.1°C

Correct Answer: (A) 100.255°C

Solution:

Step 1: Understanding the Question:

We are asked to find the boiling point of an aqueous solution containing a known mass of urea dissolved in water, given the molal boiling point elevation constant (K_b) of water.

Step 2: Key Formula or Approach:

The elevation in boiling point (ΔT_b) is given by:

$$\Delta T_b = i \cdot K_b \cdot m$$

where:

- i is the van 't Hoff factor (for urea, which is a non-electrolyte, $i = 1$)

- K_b is the molal boiling point elevation constant
- m is the molality of the solution, calculated as:

$$m = \frac{\text{Moles of solute}}{\text{Mass of solvent in kg}} = \frac{w_2 \times 1000}{M_2 \times w_1}$$

where w_2 is the mass of solute, M_2 is the molar mass of solute, and w_1 is the mass of solvent in grams.

Step 3: Detailed Explanation:

Given:
- Mass of solute (urea), $w_2 = 6.0\,\text{g}$
- Molar mass of urea (NH_2CONH_2), $M_2 = 60\,\text{g/mol}$
- Mass of solvent (water), $w_1 = 200\,\text{g}$
- Molal boiling point elevation constant, $K_b = 0.510\,\text{K kg mol}^{-1}$

First, let's calculate the molality of the solution:

$$m = \frac{6.0 \times 1000}{60 \times 200} = \frac{6000}{12000} = 0.5\,\text{mol/kg}$$

Now, find the elevation in boiling point:

$$\Delta T_b = 1 \times 0.510 \times 0.5 = 0.255\,\text{K (or } 0.255^\circ\text{C)}$$

Since pure water boils at 100°C under normal atmospheric pressure, the boiling point of the solution (T_b) is:

$$T_b = T_b^0 + \Delta T_b = 100^\circ\text{C} + 0.255^\circ\text{C} = 100.255^\circ\text{C}$$

Step 4: Final Answer:

(A) 100.255°C

Quick Tip: Remember that urea is an organic non-electrolyte, so its van 't Hoff factor i is always 1. Be careful not to select the value of ΔT_b (which is 0.255°C, option C) as the final boiling point. Always add it to 100°C.

10. For an ideal binary liquid mixture:

(A) $\Delta S_{(\text{mix})} = 0; \Delta G_{(\text{mix})} = 0$

(B) $\Delta H_{(\text{mix})} = 0; \Delta S_{(\text{mix})} < 0$

(C) $\Delta V_{(\text{mix})} = 0; \Delta G_{(\text{mix})} > 0$

(D) $\Delta S_{(\text{mix})} > 0; \Delta G_{(\text{mix})} < 0$

Correct Answer: (D) $\Delta S_{(\text{mix})} > 0; \Delta G_{(\text{mix})} < 0$

Solution:

Step 1: Understanding the Question:

We need to identify the thermodynamic properties that characterize an ideal binary liquid mixture during mixing.

Step 2: Detailed Explanation:

For an ideal solution formed by mixing two liquids:

- There is no change in enthalpy during mixing, so $\Delta H_{\text{mix}} = 0$.
- There is no change in volume during mixing, so $\Delta V_{\text{mix}} = 0$.
- Since mixing of two components is a spontaneous physical process, the Gibbs free energy change of mixing must be negative, i.e., $\Delta G_{\text{mix}} < 0$.
- As molecules of two different liquids intermingle, randomness increases, leading to a positive entropy change of mixing, i.e., $\Delta S_{\text{mix}} > 0$.

Analyzing the options:

- Option (A) is incorrect because $\Delta S_{\text{mix}} \neq 0$ and $\Delta G_{\text{mix}} \neq 0$.
- Option (B) is incorrect because ΔS_{mix} must be positive (> 0).
- Option (C) is incorrect because ΔG_{mix} must be negative (< 0).
- Option (D) correctly lists $\Delta S_{\text{mix}} > 0$ and $\Delta G_{\text{mix}} < 0$.

Step 3: Final Answer:

(D) $\Delta S_{(\text{mix})} > 0; \Delta G_{(\text{mix})} < 0$

Quick Tip: For any spontaneous mixing process (ideal or non-ideal), ΔS_{mix} is always positive (> 0) and ΔG_{mix} is always negative (< 0). These two parameters do not depend on ideal behavior.

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11. The indefinite integral of $\sin(x)$ **w.r.t** $\cos(x)$ **is:**

(A) $\frac{\sin(2x)}{4} + \frac{x}{2} + c$

(B) $\frac{\sin(2x)}{4} - \frac{x}{2} + c$

(C) $2\sin(2x) + c$

(D) $\sin(x) + \cos(x) + c$

Correct Answer: (B) $\frac{\sin(2x)}{4} - \frac{x}{2} + c$

Solution:

Step 1: Understanding the Question:

The problem requires evaluating the indefinite integral of the function $\sin(x)$ with respect to $\cos(x)$.

Step 2: Key Formula or Approach:

The integral of a function $f(x)$ with respect to another function $g(x)$ is formulated as:

$$\int f(x)\, d(g(x)) = \int f(x) \cdot g'(x)\, dx$$

Step 3: Detailed Explanation:

Let $g(x) = \cos(x)$. Then its differential is:

$$d(\cos(x)) = -\sin(x)\, dx$$

Substitute this into the required integral:

$$I = \int \sin(x)\, d(\cos(x)) = \int \sin(x)(-\sin(x)\, dx)$$

$$I = -\int \sin^2(x)\,dx$$

Using the trigonometric identity $\sin^2(x) = \frac{1-\cos(2x)}{2}$:

$$I = -\int \frac{1-\cos(2x)}{2}\,dx$$

$$I = -\frac{1}{2}\int (1-\cos(2x))\,dx$$

$$I = -\frac{1}{2}\left(x - \frac{\sin(2x)}{2}\right) + c$$

$$I = -\frac{x}{2} + \frac{\sin(2x)}{4} + c$$

$$I = \frac{\sin(2x)}{4} - \frac{x}{2} + c$$

This matches option (B).

Step 4: Final Answer:

(B) $\frac{\sin(2x)}{4} - \frac{x}{2} + c$

Quick Tip: Using substitution $u = \cos(x)$ makes it very simple:

$$\int \sin(x)\,du = \int \sqrt{1-u^2}\,du$$

Alternatively, converting $d(\cos(x))$ directly to $-\sin(x)dx$ is often faster and less prone to algebraic errors.

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12. The equation of the lines through $(1,1)$ and making angles of 45° with the line $x+y=0$ are:

(A) $x-1=0, x-y=0$

(B) $x-y=0, y-1=0$

(C) $x+y-2=0, y-1=0$

(D) $x-1=0, y-1=0$

Correct Answer: (D) $x-1=0, y-1=0$

Solution:

Step 1: Understanding the Question:

We need to find the equations of two straight lines that pass through the point $(1,1)$ and make an angle of 45° with the line $x+y=0$.

Step 2: Key Formula or Approach:

The angle θ between two lines with slopes m_1 and m_2 is given by:

$$\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right|$$

Step 3: Detailed Explanation:

The given line is $x+y=0 \implies y=-x$.

The slope of this line is $m_1=-1$.

Let the slope of the required line be m. Given $\theta=45^\circ$:

$$\tan 45^\circ = \left|\frac{m-(-1)}{1+m(-1)}\right| \implies 1 = \left|\frac{m+1}{1-m}\right|$$

This gives two cases:

Case 1:

$$\frac{m+1}{1-m} = 1 \implies m+1=1-m \implies 2m=0 \implies m=0$$

The line with slope $m=0$ passing through $(1,1)$ is:

$$y-1=0(x-1) \implies y-1=0$$

Case 2:

$$\frac{m+1}{1-m} = -1 \implies m+1=m-1 \implies 1=-1 \text{ (No real solution)}$$

This indicates that the second line is vertical, having an undefined slope ($m\to\infty$).

The vertical line passing through $(1,1)$ is:

$$x - 1 = 0$$

Thus, the two required lines are $x-1=0$ and $y-1=0$.

Step 4: Final Answer:

(D) $x-1=0, y-1=0$

Quick Tip: The line $x+y=0$ has an inclination of 135° with the positive x-axis. Lines making an angle of 45° with it must have inclinations:

$135^\circ - 45^\circ = 90^\circ$ (vertical line)

$135^\circ + 45^\circ = 180^\circ$ (horizontal line)

The vertical and horizontal lines passing through $(1,1)$ are simply $x=1$ and $y=1$.

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13. If the standard deviation of n elements of the series $x_1, x_2, x_3, \ldots, x_n$ is σ, then find the variance of the series $ax_1, ax_2, ax_3, \ldots, ax_n$ is:

(A) $a^2\sigma$

(B) $a^2 n\sigma$

(C) $a\sigma$

(D) $a^2\sigma^2$

Correct Answer: (D) $a^2\sigma^2$

Solution:

Step 1: Understanding the Question:

The problem asks for the variance of a new series obtained by multiplying every term of a given series by a constant a.

Step 2: Key Formula or Approach:

If a series of observations is multiplied by a constant a, the new standard deviation σ' is:

$$\sigma' = |a|\sigma$$

The variance is the square of the standard deviation:

$$\text{Variance} = \sigma^2$$

Step 3: Detailed Explanation:

Given that the standard deviation of the series $x_1, x_2, \ldots, x_n$ is σ.

So, the variance of the original series is:

$$\text{Var}(X) = \sigma^2$$

When each element of the series is multiplied by a, the new elements are $ax_1, ax_2, \ldots, ax_n$.

Using the properties of variance:

$$\text{Var}(aX) = a^2\text{Var}(X) = a^2\sigma^2$$

Therefore, the variance of the scaled series is $a^2\sigma^2$.

Step 4: Final Answer:

(D) $a^2\sigma^2$

Quick Tip: Remember the scaling property of dispersion parameters:

- Standard Deviation scales linearly: $\text{S.D.}(aX) = |a|\text{S.D.}(X)$
- Variance scales quadratically: $\text{Var}(aX) = a^2\text{Var}(X)$

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14. If $1 + \sin\theta + \sin^2\theta + \ldots$ upto $\infty = 2\sqrt{3} + 4$, then $\theta =$

(A) $\frac{3\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

Correct Answer: (B) $\frac{\pi}{3}$

Solution:

Step 1: Understanding the Question:

We need to solve the given infinite series equation involving trigonometric terms to find the value of θ.

Step 2: Key Formula or Approach:

The sum of an infinite geometric progression (G.P.) with first term a and common ratio r (where $|r| < 1$) is given by:

$$S_\infty = \frac{a}{1-r}$$

Step 3: Detailed Explanation:

The given infinite series is:

$$1 + \sin\theta + \sin^2\theta + \cdots = 2\sqrt{3} + 4$$

This is an infinite G.P. where:

- First term, $a = 1$

- Common ratio, $r = \sin\theta$

Using the sum formula:

$$\frac{1}{1-\sin\theta} = 2\sqrt{3} + 4$$

Factorize the right-hand side:

$$\frac{1}{1-\sin\theta} = 2(2+\sqrt{3})$$

Taking reciprocal on both sides:

$$1 - \sin\theta = \frac{1}{2(2+\sqrt{3})}$$

Rationalize the denominator by multiplying the numerator and denominator by $(2-\sqrt{3})$:

$$1 - \sin\theta = \frac{2-\sqrt{3}}{2(2+\sqrt{3})(2-\sqrt{3})}$$

$$1 - \sin\theta = \frac{2-\sqrt{3}}{2(4-3)} = \frac{2-\sqrt{3}}{2} = 1 - \frac{\sqrt{3}}{2}$$

Comparing both sides:

$$\sin\theta = \frac{\sqrt{3}}{2}$$

Since θ lies in the standard domain of the options:

$$\theta = \frac{\pi}{3}$$

Step 4: Final Answer:

(B) $\frac{\pi}{3}$

Quick Tip: To save time during the exam, you can substitute the options directly:

- For $\theta = \frac{\pi}{3}$, $\sin\theta = \frac{\sqrt{3}}{2}$.

- Sum $S_\infty = \frac{1}{1-\sqrt{3}/2} = \frac{2}{2-\sqrt{3}} = 2(2+\sqrt{3}) = 4+2\sqrt{3}$, which matches perfectly!

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15. Find the value of $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2$

(A) $a^2b^2c^2$

(B) $4a^2b^2c^2$

(C) $2a^2b^2c^2$

(D) $(a+b+c)^2$

Correct Answer: (B) $4a^2b^2c^2$

Solution:

Step 1: Understanding the Question:

We need to evaluate the square of the determinant of a given 3×3 symmetric matrix.

Step 2: Key Formula or Approach:

First, calculate the determinant of the matrix D, then square it.

The determinant of a 3×3 matrix is evaluated by expanding along any row or column.

Step 3: Detailed Explanation:

Let the determinant be:

$$D = \begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}$$

Expanding along the first row:

$$D = 0 \cdot (0 \cdot 0 - a \cdot a) - c \cdot (c \cdot 0 - a \cdot b) + b \cdot (c \cdot a - 0 \cdot b)$$

$$D = 0 - c \cdot (-ab) + b \cdot (ca)$$

$$D = abc + abc = 2abc$$

Now, squaring the determinant:

$$D^2 = (2abc)^2 = 4a^2b^2c^2$$

This matches option (B).

Step 4: Final Answer:

(B) $4a^2b^2c^2$

Quick Tip: For any skew-symmetric-like structure with zeros on the diagonal and symmetric off-diagonal terms, the determinant often simplifies to $2abc$. Memorizing this standard form helps in solving such determinant questions instantly.
