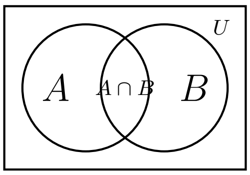
SRMJEEE Mathematics Sample Paper – 10

Duration: 47 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of **SRMJEEE** (SRM Joint Engineering Entrance Examination).
- Each correct answer carries **+1 mark**. There is **no negative marking**; an unattempted or wrong answer scores 0.
- Only **one** option is correct. Choose carefully.
- The actual SRMJEEE is a **computer-based test** conducted in remote-proctored online mode, with all sections sharing a common time window and no per-section limit.
- Personal calculators, mobile phones, log tables and other electronic gadgets are strictly prohibited.

Q1. In a survey of $n(U) = 50$ people, $n(A) = 18$, $n(B) = 22$ and $n(A \cap B) = 10$, as shown in the Venn diagram. The number of people who belong to *neither* set is:



(A) 20

(B) 30

(C) 10

(D) 40

Q2. On the set $X = \{1, 2, 3\}$, the relation $R = \{(1, 1), (2, 2), (3, 3)\}$ is best described as the:

(A) universal relation

(B) identity relation

(C) empty relation

(D) symmetric-only relation

Q3. If $f(x) = 2x + 1$ and $g(x) = x^2$, then the value of $f\big(g(3)\big)$ is:

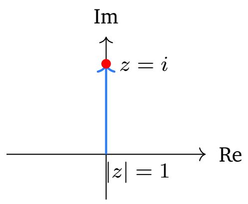
(A) 49

(B) 13

(C) 19

(D) 37

Q4. The modulus of the complex number $z = \dfrac{1+i}{1-i}$, plotted as a point on the Argand plane, is:



(A) 2

(B) $\sqrt{2}$

(C) 0

(D) 1

Q5. The value of i^{23} (where $i = \sqrt{-1}$) is:

(A) i

(B) $-i$

(C) 1

(D) -1

Q6. For which value of k does the quadratic equation $x^2 + (k-3)x + 9 = 0$ have roots that are equal in magnitude but opposite in sign?

(A) 3

(B) 0

(C) 9

(D) -3

Q7. If α and β are the roots of $x^2 - 7x + 12 = 0$, then the quadratic equation whose roots are $-\alpha$ and $-\beta$ is:

(A) $x^2 - 7x - 12 = 0$

(B) $x^2 + 7x - 12 = 0$

(C) $x^2 + 7x + 12 = 0$

(D) $x^2 - 7x + 12 = 0$

Q8. If $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix}$, then the product AB is:

(A) $\begin{pmatrix} 6 & 0 \\ 0 & 8 \end{pmatrix}$

(B) $\begin{pmatrix} 4 & 0 \\ 0 & 15 \end{pmatrix}$

(C) $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$

(D) $\begin{pmatrix} 8 & 0 \\ 0 & 15 \end{pmatrix}$

Q9. The solution $X = \begin{pmatrix} x \\ y \end{pmatrix}$ of the system $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} X = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$, obtained as $X = A^{-1}B$, is:

(A) $(5, 4)$

(B) $(5, 8)$

(C) $(5, 16)$

(D) $(10, 8)$

Q10. The value of the determinant $\begin{vmatrix} 3 & 5 & 7 \\ 0 & 2 & 6 \\ 0 & 0 & 4 \end{vmatrix}$ is:

(A) 9

(B) 24

(C) 14

(D) 0

Q11. The area of the triangle with vertices $(1, 1)$, $(5, 1)$ and $(1, 4)$ is:

(A) 12 sq. units

(B) 3 sq. units

(C) 4 sq. units

(D) 6 sq. units

Q12. The number of ways of filling 3 distinct places using 5 different books, i.e. ${}^5P_3$, is:

(A) 10

(B) 15

(C) 60

(D) 120

Q13. From 4 men and 3 women, the number of ways of forming a committee of 3 members that contains *at least one* woman is:

(A) 4

(B) 31

(C) 30

(D) 35

Q14. In how many ways can a President and a Vice-President (two *distinct* posts) be selected from a group of 6 members?

(A) 30

(B) 15

(C) 36

(D) 720

Q15. If the product of the roots of the cubic $2x^3 - 3x^2 + 4x + c = 0$ is 5, then the value of c is:

(A) 10

(B) -10

(C) 5

(D) -5

Q16. If α and β are the roots of $x^2 - 6x + 8 = 0$, then the quadratic equation whose roots are $(\alpha + \beta)$ and $\alpha\beta$ is:

(A) $x^2 - 6x + 8 = 0$

(B) $x^2 - 8x + 6 = 0$

(C) $x^2 + 14x + 48 = 0$

(D) $x^2 - 14x + 48 = 0$

Q17. The value of $\lim_{x \to 0} \frac{\sin 5x}{\sin 3x}$ is:

(A) $\frac{5}{3}$

(B) $\frac{3}{5}$

(C) 1

(D) 15

Q18. If $y = \frac{x}{\ln x}$, then $\frac{dy}{dx}$ equals:

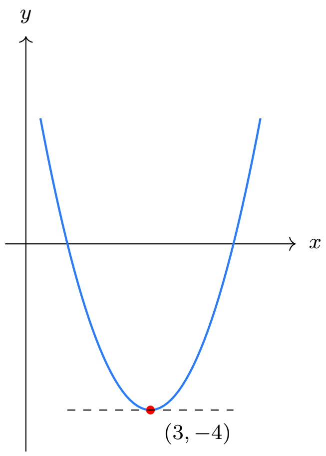
(A) $\frac{1}{\ln x}$

(B) $\frac{\ln x + 1}{(\ln x)^2}$

(C) $\dfrac{\ln x - 1}{(\ln x)^2}$

(D) $\dfrac{1 - \ln x}{(\ln x)^2}$

Q19. The point on the curve $y = x^2 - 6x + 5$, shown below, at which the tangent is parallel to the x-axis is:



(A) $(0, 5)$

(B) $(3, -4)$

(C) $(3, 5)$

(D) $(-3, -4)$

Q20. The angle between the curves $y = x^2$ and $x = y^2$ at their point of intersection $(1, 1)$ is given by $\tan^{-1}$ of:

(A) $\dfrac{4}{5}$

(B) $\dfrac{2}{3}$

(C) $\dfrac{3}{4}$

(D) 1

Q21. The order and degree of the differential equation $\left(\dfrac{d^3y}{dx^3}\right)^2 + \dfrac{dy}{dx} + y = 0$ are, respectively:

(A) order 2, degree 3

(B) order 3, degree 3

(C) order 2, degree 2

(D) order 3, degree 2

Q22. $\int 3^x \, dx$ equals:

(A) $\dfrac{3^x}{\ln 3} + C$

(B) $3^x \ln 3 + C$

(C) $\dfrac{3^{x+1}}{x+1} + C$

(D) $3^x + C$

Q23. The value of $\int_{-2}^{2} (x^5 + x) \, dx$ is:

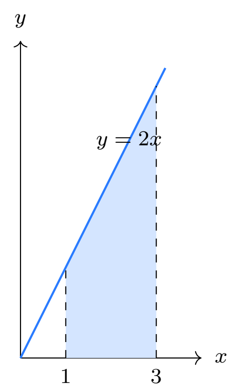
(A) 32

(B) 4

(C) 0

(D) 64

Q24. The area of the region bounded by the line $y = 2x$, the x-axis and the lines $x = 1$ and $x = 3$ (shaded) is:



(A) 4 sq. units

(B) 8 sq. units

(C) 16 sq. units

(D) 9 sq. units

Q25. The value of $\int_0^1 3x^2\,dx$ is:

(A) 3

(B) $\frac{1}{3}$

(C) $\frac{3}{2}$

(D) 1

Q26. The equation of the straight line with slope 2 and y-intercept -3 is:

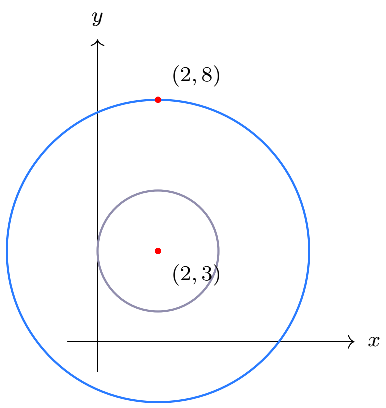
(A) $y = 2x - 3$

(B) $y = -3x + 2$

(C) $y = 2x + 3$

(D) $y = 3x - 2$

Q27. A circle is concentric with $x^2 + y^2 - 4x - 6y + 9 = 0$ and passes through the point $(2, 8)$. Its radius is:



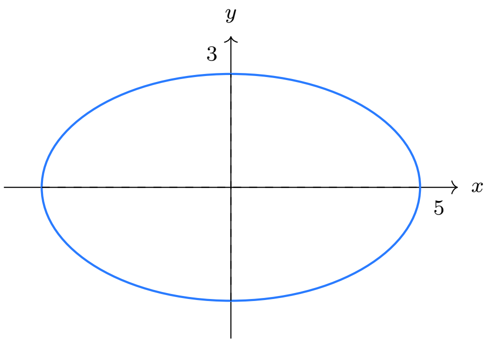
(A) 2

(B) 4

(C) 5

(D) 8

Q28. For the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, shown below, the lengths of the major and minor axes are, respectively:



(A) 5 and 3

(B) 10 and 6

(C) 25 and 9

(D) 6 and 10

Q29. The points $A(1, 2, 3)$, $B(2, 4, 6)$ and $C(4, 8, 12)$ are:

(A) vertices of a triangle

(B) never collinear

(C) collinear

(D) the corners of a square

Q30. The perpendicular distance of the point $(1, 2, 2)$ from the plane $x + 2y + 2z = 3$ is:

(A) 3

(B) $\frac{6}{\sqrt{5}}$

(C) 1

(D) 2

Q31. A constant force $\vec{F} = 3\hat{i}+2\hat{j}+\hat{k}$ displaces a particle through $\vec{d} = \hat{i}+2\hat{j}+3\hat{k}$. The work done $W = \vec{F} \cdot \vec{d}$ is:

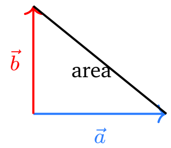
(A) 10 units

(B) 6 units

(C) 14 units

(D) 0 units

Q32. The area of the triangle whose two sides are represented by $\vec{a} = 4\hat{i}$ and $\vec{b} = 6\hat{j}$, equal to $\frac{1}{2}|\vec{a} \times \vec{b}|$, is:



(A) 24

(B) 12

(C) 10

(D) 6

Q33. The vectors $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{k}$ are coplanar if and only if the determinant $[\vec{a}\ \vec{b}\ \vec{c}]$ equals zero. For these vectors $[\vec{a}\ \vec{b}\ \vec{c}]$ is:

(A) 0 (coplanar)

(B) 1

(C) -2

(D) 2 (not coplanar)

Q34. The standard deviation of the data $2, 4, 6$ is:

(A) $\dfrac{2\sqrt{2}}{\sqrt{3}}$

(B) 2

(C) $\dfrac{8}{3}$

(D) 4

Q35. Two fair dice are rolled together. The probability that the sum of the numbers on the two dice is 7 is:

(A) $\frac{1}{12}$

(B) $\frac{5}{36}$

(C) $\frac{1}{6}$

(D) $\frac{7}{36}$

Q36. For a binomial distribution with $n = 10$ and $p = \frac{1}{2}$, the mean and variance are, respectively:

(A) 5 and 5

(B) 5 and 2.5

(C) 10 and 5

(D) 2.5 and 5

Q37. The general solution of $\cos\theta = 1$ is (where $n \in \mathbb{Z}$):

(A) $\theta = n\pi$

(B) $\theta = (2n+1)\pi$

(C) $\theta = 2n\pi + \frac{\pi}{2}$

(D) $\theta = 2n\pi$

Q38. If $\cos\theta = \frac{1}{\sqrt{2}}$, then the value of $\cos 2\theta$ is:

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) -1

Q39. The principal value of $\sec^{-1}(2)$ is:

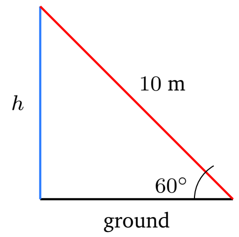
(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Q40. A ladder 10 m long leans against a vertical wall, making an angle of 60° with the ground, as shown. The height up the wall reached by the ladder is:



(A) 5 m

(B) 10 m

(C) $\frac{10}{\sqrt{3}}$ m

(D) $5\sqrt{3}$ m

Detailed Solutions

Q1.

Solution

Concept — Complement of a union: Every member of the universal set U either belongs to $A \cup B$ (at least one of the two sets) or to neither of them. Hence the count of people in neither set is the complement $n(U) - n(A \cup B)$. To find $n(A \cup B)$ we use the inclusion–exclusion principle $n(A \cup B) = n(A) + n(B) - n(A \cap B)$: simply adding $n(A)$ and $n(B)$ double-counts the overlap $A \cap B$, so we subtract it once to correct for that.

Step 1 — Read off the data: From the survey and the Venn diagram, $n(U) = 50$, $n(A) = 18$, $n(B) = 22$ and the overlap is $n(A \cap B) = 10$.

Step 2 — Apply inclusion–exclusion for the union:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) = 18 + 22 - 10 = 30.$$

Step 3 — Take the complement:

$$n(\text{neither}) = n(U) - n(A \cup B) = 50 - 30 = 20.$$

Step 4 — Cross-check via disjoint regions: Split the diagram into the three mutually exclusive pieces. Only-A is $n(A) - n(A \cap B) = 18 - 10 = 8$; only-$B$ is $22 - 10 = 12$; the intersection is 10. These sum to $8 + 12 + 10 = 30$, confirming $n(A \cup B) = 30$ and therefore neither $= 50 - 30 = 20$, exactly as before.

Why other options are wrong:

- (B) 30 is $n(A \cup B)$ itself; it counts those in at least one set, the opposite of "neither".
- (C) 10 wrongly reports only the intersection $n(A \cap B)$.
- (D) 40 comes from forgetting to subtract the overlap: $50 - (18 + 22 - 30)$ or $50 - 10$, both mishandling inclusion–exclusion.

Final Answer: neither $= 20 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q1

Q2.

Solution

Concept — Special relations on a set: A relation on X is any subset of $X \times X$. Certain subsets have standard names. The *identity* relation I_X consists of exactly the "diagonal" pairs (a, a) for every $a \in X$ and nothing else; it relates each element only to itself. The *universal* relation is the whole of $X \times X$ (every element related to every element), while the *empty* relation contains no pairs at all. Recognising a relation means matching its pair-list to one of these definitions.

Step 1 — List $X \times X$ for reference: With $X = \{1, 2, 3\}$, the full Cartesian product has $3 \times 3 = 9$ ordered pairs, and the diagonal pairs are $(1, 1), (2, 2), (3, 3)$.

Step 2 — Inspect the given relation: $R = \{(1, 1), (2, 2), (3, 3)\}$ contains precisely those three diagonal pairs and no off-diagonal pair such as $(1, 2)$.

Step 3 — Match to a definition: Containing every (a, a) and only those, R fits the identity-relation definition exactly. (It is incidentally reflexive, symmetric and transitive, hence an equivalence relation, but its sharpest single name is "identity".)

Why other options are wrong:

- (A) The universal relation would need all 9 pairs of $X \times X$, including pairs like $(1, 2)$ and $(2, 3)$, which R does not have.
- (C) The empty relation contains no pairs, whereas R has three.
- (D) "Symmetric-only" is wrong twice over: R is symmetric but also reflexive and transitive, and the precise name for this diagonal set is the identity relation.

Final Answer: identity relation $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept — Composition of functions: The composite $f(g(x))$, often written $(f \circ g)(x)$, means we apply g first and feed its output into f. The order is crucial: in $f(g(x))$ the inner function g acts on x, and only then does f act on the resulting number. Composition is generally not commutative, so $f(g(x)) \neq g(f(x))$ in most cases.

Step 1 — Evaluate the inner function: Substitute $x = 3$ into g:

$$g(3) = 3^2 = 9.$$

Step 2 — Feed the result into the outer function: Now apply f to the value 9:

$$f\big(g(3)\big) = f(9) = 2(9) + 1 = 18 + 1 = 19.$$

Step 3 — Cross-check with the general composite: Form $f(g(x)) = 2g(x) + 1 = 2x^2 + 1$, then put $x = 3$: $2(9) + 1 = 19$. This matches, confirming the answer and the order of operations.

Why other options are wrong:

- (A) $49 = \big(f(3)\big)^2 = 7^2$ applies g to the output of f, i.e. computes $g(f(3))$ in the squared sense, the wrong order.
- (B) $13 = g\big(f(3)\big)$? Actually $f(3) = 7$ and $g(7) = 49$; the value 13 comes from $2 \cdot 3^2 - 5$ or a similar slip and does not equal either composite.
- (D) $37 = 2(18) + 1$ doubles $g(3)$ before adding, mis-substituting $g(3) = 18$ instead of 9.

Final Answer: $f(g(3)) = 19 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q3

Q4.

Solution

Concept — Modulus of a quotient: The modulus $|z|$ is the distance of the point z from the origin in the Argand plane, $|a + bi| = \sqrt{a^2 + b^2}$. A key property is that the modulus is multiplicative, so it distributes over division: $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$. This lets us avoid messy division of complex numbers and instead take moduli of the simple numerator and denominator separately.

Step 1 — Simplify z to standard form (method 1): Multiply numerator and denominator by the conjugate $1 + i$:

$$z = \frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{(1+i)^2}{(1)^2 - (i)^2} = \frac{1 + 2i + i^2}{1 - (-1)} = \frac{2i}{2} = i.$$

So $z = i$, the point $(0, 1)$ on the imaginary axis.

Step 2 — Take the modulus of $z = i$:

$$|z| = |i| = \sqrt{0^2 + 1^2} = 1.$$

Step 3 — Verify with the quotient property (method 2): Without simplifying, $|1 + i| = \sqrt{1^2 + 1^2} = \sqrt{2}$ and $|1 - i| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$, hence

$$|z| = \frac{|1 + i|}{|1 - i|} = \frac{\sqrt{2}}{\sqrt{2}} = 1,$$

agreeing with Step 2.

Why other options are wrong:

- (A) 2 multiplies the two moduli $(\sqrt{2} \cdot \sqrt{2})$ instead of dividing them.
- (B) $\sqrt{2}$ takes only the numerator's modulus and forgets to divide by the denominator's.
- (C) 0 would require $z = 0$, but $z = i \neq 0$.

Final Answer: $|z| = 1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Powers of i: The imaginary unit satisfies $i^2 = -1$, and from this its powers repeat in a cycle of length 4: $i^1 = i$, $i^2 = -1$, $i^3 = i^2 \cdot i = -i$, $i^4 = (i^2)^2 = 1$, and then $i^5 = i$ again. Because $i^4 = 1$, raising i to any integer power depends only on the remainder of the exponent when divided by 4. So the strategy is to reduce the exponent modulo 4 and read off the answer from the short cycle.

Step 1 — Divide the exponent by 4:

$$23 = 4 \times 5 + 3,$$

so the remainder is 3.

Step 2 — Replace by the equivalent small power:

$$i^{23} = i^{4\times5+3} = (i^4)^5 \cdot i^3 = 1^5 \cdot i^3 = i^3.$$

Step 3 — Evaluate from the cycle:

$$i^3 = i^2 \cdot i = (-1)\,i = -i.$$

Why other options are wrong:

- (A) i corresponds to remainder 1 (e.g. i^{21}), not 3.
- (C) 1 corresponds to remainder 0, i.e. $i^4, i^8, \ldots$
- (D) -1 corresponds to remainder 2, i.e. $i^2, i^6, \ldots$; here the remainder is 3.

Final Answer: $i^{23} = -i \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q5

Q6.

Solution

Concept — Roots equal in magnitude, opposite in sign: If the two roots are α and $-\alpha$, then their sum is $\alpha + (-\alpha) = 0$. By Vieta's relations, for $ax^2 + bx + c = 0$ the sum of the roots is $-b/a$. So the condition "roots equal in magnitude but opposite in sign" is exactly the requirement that the sum of the roots vanish, i.e. the coefficient b of the linear term must be zero (while c stays such that real opposite roots exist).

Step 1 — Identify the coefficients: For $x^2 + (k-3)x + 9 = 0$ we have $a = 1$, $b = k - 3$, $c = 9$.

Step 2 — Write the sum of the roots:

$$\alpha + \beta = -\frac{b}{a} = -(k-3).$$

Step 3 — Impose the zero-sum condition:

$$-(k-3) = 0 \implies k - 3 = 0 \implies k = 3.$$

Step 4 — Verify: With $k = 3$ the equation becomes $x^2 + 9 = 0$, giving $x = \pm 3i$? These are $\pm$ a common value, equal in magnitude and opposite in sign, confirming the structure $\alpha, -\alpha$ (here purely imaginary but still opposite in sign, with product 9 as required by c/a).

Why other options are wrong:

- (B) $k = 0$ gives $b = k - 3 = -3$, sum $= 3 \neq 0$.
- (C) $k = 9$ gives $b = 6$, sum $= -6 \neq 0$.
- (D) $k = -3$ gives $b = -6$, sum $= 6 \neq 0$; none of these kills the linear term.

Final Answer: $k = 3 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q6

Q7.

Solution

Concept — Equation with negated roots: If α, β are roots of an equation $P(x) = 0$, then the equation whose roots are $-\alpha, -\beta$ is obtained by replacing x with $-x$, i.e. $P(-x) = 0$. The reason is that $x = -\alpha$ makes $P(-x) = P(\alpha) = 0$. Equivalently, one can build the new equation from the transformed sum and product of roots; both routes are shown below.

Step 1 — Substitution method, replace $x \to -x$:

$$(-x)^2 - 7(-x) + 12 = 0 \implies x^2 + 7x + 12 = 0.$$

Step 2 — Vieta method, transform sum and product: For $x^2 - 7x + 12 = 0$, $\alpha + \beta = 7$ and $\alpha\beta = 12$. The new roots $-\alpha, -\beta$ have

$$(-\alpha) + (-\beta) = -(\alpha + \beta) = -7, \qquad (-\alpha)(-\beta) = \alpha\beta = 12,$$

so the equation is $x^2 - (\text{sum})x + (\text{product}) = x^2 - (-7)x + 12 = x^2 + 7x + 12 = 0$, matching Step 1.

Step 3 — Concrete check: The original roots factor as $3, 4$. Their negatives $-3, -4$ have sum -7 and product 12, and indeed $(x + 3)(x + 4) = x^2 + 7x + 12$.

Why other options are wrong:

- (D) $x^2 - 7x + 12 = 0$ is the original equation, with roots $3, 4$, not their negatives.
- (A) $x^2 - 7x - 12 = 0$ flips only the constant's sign, giving the wrong product.
- (B) $x^2 + 7x - 12 = 0$ has the right linear term but the wrong product -12.

Final Answer: $x^2 + 7x + 12 = 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q7

Q8.

Solution

Concept — Product of diagonal matrices: A diagonal matrix has nonzero entries only on its main diagonal. When two diagonal matrices are multiplied, every off-diagonal entry of the product still comes out zero, and each diagonal entry is just the product of the corresponding diagonal entries of the factors. This follows directly from the row-times-column rule of matrix multiplication, where all cross terms involve a zero.

Step 1 — Apply the row-column rule explicitly: The $(1,1)$ entry is row 1 of A dotted with column 1 of B:

$$(2)(4) + (0)(0) = 8.$$

The $(2,2)$ entry is row 2 of A dotted with column 2 of B:

$$(0)(0) + (3)(5) = 15.$$

Step 2 — Off-diagonal entries: The $(1,2)$ entry is $(2)(0) + (0)(5) = 0$, and the $(2,1)$ entry is $(0)(4) + (3)(0) = 0$, so the product stays diagonal.

Step 3 — Assemble the product:

$$AB = \begin{pmatrix} 8 & 0 \\ 0 & 15 \end{pmatrix}.$$

Why other options are wrong:

- (A) $\begin{pmatrix} 6 & 0 \\ 0 & 8 \end{pmatrix}$ adds the diagonal entries $(2+4,\ 3+5)$ instead of multiplying.
- (B) $\begin{pmatrix} 4 & 0 \\ 0 & 15 \end{pmatrix}$ keeps the wrong top-left value (uses B's entry 4 rather than $2 \cdot 4$).
- (C) $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ repeats the first product in both slots, forgetting $3 \cdot 5 = 15$.

Final Answer: $\begin{pmatrix} 8 & 0 \\ 0 & 15 \end{pmatrix} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — Solving $AX = B$ with a diagonal coefficient matrix: The matrix equation $AX = B$ has solution $X = A^{-1}B$ whenever A is invertible. For a diagonal matrix the inverse is trivial: $\begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}^{-1} = \begin{pmatrix} 1/d_1 & 0 \\ 0 & 1/d_2 \end{pmatrix}$ (valid since $d_1, d_2 \neq 0$). Geometrically the system decouples into independent one-variable equations, so each unknown is found by a single division.

Step 1 — Write out the scalar equations: Multiplying $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$ gives

$$1 \cdot x = 5, \qquad 2 \cdot y = 8.$$

Step 2 — Solve each independently:

$$x = 5, \qquad y = \frac{8}{2} = 4.$$

Step 3 — Confirm via the inverse:

$$X = A^{-1}B = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 5 \\ 8 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix},$$

so $X = (5, 4)$, matching Step 2.

Why other options are wrong:

- (B) $(5, 8)$ leaves y unscaled, ignoring the factor 2 in the second equation.
- (C) $(5, 16)$ multiplies by 2 instead of dividing, giving $y = 8 \cdot 2$.
- (D) $(10, 8)$ wrongly doubles x and leaves y unsolved.

Final Answer: $X = (5, 4) \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q9**

Q10.

Solution

Concept — Determinant of a triangular matrix: A matrix is upper-triangular when every entry below the main diagonal is zero. For such a matrix (and likewise lower-triangular) the determinant is simply the product of the diagonal entries. The reason is that cofactor expansion down the first column meets a single nonzero entry at each stage, so the determinant collapses to the diagonal product.

Step 1 — Confirm the matrix is triangular: In $\begin{vmatrix} 3 & 5 & 7 \\ 0 & 2 & 6 \\ 0 & 0 & 4 \end{vmatrix}$ all entries below the diagonal are 0, so it is upper-triangular.

Step 2 — Multiply the diagonal entries:

$$\det = 3 \times 2 \times 4 = 24.$$

Step 3 — Cross-check by cofactor expansion along the first column: Only the top-left 3 survives:

$$3 \begin{vmatrix} 2 & 6 \\ 0 & 4 \end{vmatrix} = 3(2 \cdot 4 - 6 \cdot 0) = 3(8) = 24,$$

confirming the result.

Why other options are wrong:

- (A) $9 = 3 + 2 + 4$ adds the diagonal entries instead of multiplying.
- (C) 14 comes from mixing additions and products of the off-diagonal numbers.
- (D) 0 would arise only if some diagonal entry were zero; here none is.

Final Answer: determinant $= 24 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

Q11.

Solution

Concept — Area of a triangle by the determinant formula: For vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ the area is

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|.$$

The absolute value guarantees a non-negative area regardless of how the vertices are ordered, and the factor $\frac{1}{2}$ converts the parallelogram (cross-product) magnitude into the triangle's area.

Step 1 — Label and substitute: Take $(x_1, y_1) = (1, 1)$, $(x_2, y_2) = (5, 1)$, $(x_3, y_3) = (1, 4)$:

$$\text{Area} = \frac{1}{2}\left|1(1 - 4) + 5(4 - 1) + 1(1 - 1)\right|.$$

Step 2 — Simplify the bracket:

$$= \frac{1}{2}\left|1(-3) + 5(3) + 1(0)\right| = \frac{1}{2}\left|-3 + 15 + 0\right| = \frac{1}{2}|12|.$$

Step 3 — Evaluate:

$$\text{Area} = \frac{1}{2} \times 12 = 6 \text{ sq. units.}$$

Step 4 — Verify geometrically: The side from $(1, 1)$ to $(5, 1)$ is horizontal of length 4 (the base); the side from $(1, 1)$ to $(1, 4)$ is vertical of length 3 (the height of this right triangle). Then $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 3 = 6$, matching the determinant result.

Why other options are wrong:

- (A) 12 forgets the factor $\frac{1}{2}$ (it is the parallelogram/base$\times$height value).
- (B) 3 and (C) 4 use only one side length and mis-evaluate the determinant.

Final Answer: area $= 6$ sq. units $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q11

Q12.

Solution

Concept — Permutations: A permutation counts *ordered* arrangements: ${}^nP_r = \dfrac{n!}{(n-r)!}$ is the number of ways to fill r distinct, ordered places using n distinct objects. Equivalently, by the multiplication principle, the first place can be filled n ways, the next $n-1$ ways, and so on for r factors. Order matters here because the three places are distinct.

Step 1 — Write the formula with $n = 5,\ r = 3$:

$${}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!}.$$

Step 2 — Expand and cancel:

$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 5 \times 4 \times 3.$$

Step 3 — Evaluate:

$$5 \times 4 \times 3 = 60.$$

(Equivalently, by the slot method: 5 choices for the first place, 4 for the second, 3 for the third.)

Why other options are wrong:

- (A) $10 = \binom{5}{3}$ counts unordered selections, ignoring the order of the three places.
- (B) 15 is an arbitrary miscount.
- (D) $120 = 5!$ fills five ordered places, not three.

Final Answer: ${}^5P_3 = 60 \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q12

Q13.

Solution

Concept — "At least one" via the complement: Counting "at least one woman" directly would require adding the cases of exactly one, exactly two and exactly three women. The complement method is faster: the only outcome *excluded* by "at least one woman" is "no woman", i.e. an all-men committee. So

$$(\text{at least one woman}) = (\text{all committees}) - (\text{committees with no woman}).$$

Since members are chosen without regard to order, we use combinations $\binom{n}{r}$.

Step 1 — Total committees of 3 from all 7 people:

$$\binom{7}{3} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35.$$

Step 2 — Committees with no woman (all 3 from the 4 men):

$$\binom{4}{3} = 4.$$

Step 3 — Subtract:

$$35 - 4 = 31.$$

Step 4 — Cross-check by direct counting: Exactly one woman: $\binom{3}{1}\binom{4}{2} = 3 \times 6 = 18$. Exactly two women: $\binom{3}{2}\binom{4}{1} = 3 \times 4 = 12$. Exactly three women: $\binom{3}{3}\binom{4}{0} = 1$. Sum $= 18 + 12 + 1 = 31$, matching Step 3.

Why other options are wrong:

- (D) 35 is the unrestricted total, which still includes the all-men committees.
- (A) 4 is exactly the all-men (no-woman) count we needed to remove.
- (C) 30 is an arithmetic slip (e.g. $34 - 4$ or $35 - 5$).

Final Answer: 31 committees $\Rightarrow$ $\boxed{\text{B}}$

Answer: (B) Go Back to Q13

Q14.

Solution

Concept — Permutation versus combination: The deciding question is always: does order matter? Here the two posts, President and Vice-President, are *distinct* roles, so picking person X as President and Y as Vice-President differs from picking Y as President and X as Vice-President. Order matters, so we count ordered selections with ${}^nP_r$ rather than the unordered $\binom{n}{r}$.

Step 1 — Model the choice: We are filling $r = 2$ distinct ordered posts from $n = 6$ members, so the count is ${}^6P_2$.

Step 2 — Apply the multiplication principle: The President can be any of the 6 members; once chosen, the Vice-President can be any of the remaining 5:

$$ {}^6P_2 = 6 \times 5 = 30. $$

Step 3 — Relate to combinations as a check: Choosing an unordered pair is $\binom{6}{2} = 15$; each such pair can be arranged into the two ordered posts in $2! = 2$ ways, giving $15 \times 2 = 30$, confirming the answer.

Why other options are wrong:

- (B) $15 = \binom{6}{2}$ treats the two posts as identical, dropping the order.
- (C) $36 = 6^2$ wrongly allows the same person to hold both posts.
- (D) $720 = 6!$ arranges all six members, far more than the two posts require.

Final Answer: 30 ways $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q14

Q15.

Solution

Concept — Product of roots of a cubic (Vieta): For a cubic $ax^3 + bx^2 + cx + d = 0$ with roots r_1, r_2, r_3, Vieta's formulas give $r_1 + r_2 + r_3 = -b/a$, $r_1r_2 + r_2r_3 + r_3r_1 = c/a$, and the product $r_1r_2r_3 = -d/a$. The sign alternates with the number of roots, so for an odd-degree cubic the product picks up a minus sign. Here the constant term is itself called c, which we must not confuse with the generic coefficient names.

Step 1 — Match coefficients: Comparing $2x^3 - 3x^2 + 4x + c = 0$ with $ax^3 + bx^2 + \cdots + d = 0$ gives leading coefficient $a = 2$ and constant term $d = c$.

Step 2 — Write the product of roots:

$$r_1 r_2 r_3 = -\frac{d}{a} = -\frac{c}{2}.$$

Step 3 — Impose the given product $= 5$ and solve:

$$-\frac{c}{2} = 5 \implies c = -10.$$

Step 4 — Verify the sign: With $c = -10$ the constant term $d = -10$, so $-d/a = -(-10)/2 = 5$, exactly the required product. The negative constant is essential to make the product positive.

Why other options are wrong:

- (A) 10 drops the negative sign, using $+c/2 = 5$.
- (C) 5 ignores the leading coefficient $a = 2$ (treats product as $-c$).
- (D) -5 keeps the sign but forgets to multiply through by 2.

Final Answer: $c = -10 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q15

Q16.

Solution

Concept — Building a quadratic from its roots: Any monic quadratic with roots p and q can be written as $x^2 - (p+q)x + pq = 0$, i.e. $x^2 - (\text{sum})x + (\text{product}) = 0$. So once we know the two desired roots, we only need their sum and product. The roots here are themselves the sum and product of the roots of the original equation, which we get from Vieta.

Step 1 — Extract sum and product of the original roots: For $x^2 - 6x + 8 = 0$, Vieta gives $\alpha + \beta = 6$ and $\alpha\beta = 8$. Hence the two new roots are

$$p = \alpha + \beta = 6, \qquad q = \alpha\beta = 8.$$

Step 2 — Sum and product of the new roots:

$$p + q = 6 + 8 = 14, \qquad pq = 6 \times 8 = 48.$$

Step 3 — Assemble the new quadratic:

$$x^2 - (p+q)x + pq = 0 \implies x^2 - 14x + 48 = 0.$$

Step 4 — Verify by factoring: $x^2 - 14x + 48 = (x-6)(x-8)$, whose roots are exactly 6 and 8, as intended.

Why other options are wrong:

- (C) $x^2 + 14x + 48 = 0$ takes the sum with the wrong (positive) sign in the middle term.
- (A) $x^2 - 6x + 8 = 0$ just reuses the original equation.
- (B) $x^2 - 8x + 6 = 0$ swaps the roles of sum and product.

Final Answer: $x^2 - 14x + 48 = 0 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — The standard sine limit: The cornerstone result is $\lim_{t\to 0} \frac{\sin t}{t} = 1$. Direct substitution in $\frac{\sin 5x}{\sin 3x}$ gives the indeterminate form $\frac{0}{0}$, so we reshape the expression into copies of this standard limit. The general consequence is $\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$, because near 0 each $\sin(\cdot)$ behaves like its own argument.

Step 1 — Insert matching factors: Multiply and divide so each sine sits over its own argument:

$$\frac{\sin 5x}{\sin 3x} = \frac{\sin 5x}{5x} \cdot \frac{3x}{\sin 3x} \cdot \frac{5x}{3x} = \frac{\sin 5x}{5x} \cdot \frac{3x}{\sin 3x} \cdot \frac{5}{3}.$$

Step 2 — Take the limit of each factor: As $x \to 0$, $5x \to 0$ and $3x \to 0$, so $\frac{\sin 5x}{5x} \to 1$ and $\frac{3x}{\sin 3x} \to 1$. Thus

$$\lim_{x\to 0} \frac{\sin 5x}{\sin 3x} = 1 \cdot 1 \cdot \frac{5}{3} = \frac{5}{3}.$$

Step 3 — Cross-check by L'Hopital: Differentiating top and bottom, $\frac{5\cos 5x}{3\cos 3x} \to \frac{5\cos 0}{3\cos 0} = \frac{5}{3}$, the same value.

Why other options are wrong:

- (B) $\frac{3}{5}$ inverts the ratio (puts the 3 on top).
- (C) 1 would hold only if the coefficients were equal.
- (D) 15 multiplies 5×3 instead of dividing.

Final Answer: the limit is $\dfrac{5}{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q17

Q18.

Solution

Concept — The quotient rule: For a ratio of two differentiable functions, $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$. The order in the numerator matters: it is (derivative of top)×(bottom) minus (top)×(derivative of bottom), all over the bottom squared. Here $u = x$ and $v = \ln x$.

Step 1 — Differentiate the parts:

$$u = x \Rightarrow u' = 1, \qquad v = \ln x \Rightarrow v' = \frac{1}{x}.$$

Step 2 — Substitute into the rule:

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{(1)(\ln x) - (x)\left(\frac{1}{x}\right)}{(\ln x)^2}.$$

Step 3 — Simplify the numerator: Since $x \cdot \frac{1}{x} = 1$,

$$\frac{dy}{dx} = \frac{\ln x - 1}{(\ln x)^2}.$$

Step 4 — Sanity check at $x = e$: There $\ln x = 1$, so the derivative is $\dfrac{1-1}{1} = 0$; indeed $y = \dfrac{x}{\ln x}$ has its minimum at $x = e$, where the slope is zero, consistent with our formula.

Why other options are wrong:

- (D) $\dfrac{1 - \ln x}{(\ln x)^2}$ reverses the numerator order, flipping the sign.
- (A) $\dfrac{1}{\ln x}$ ignores the $-uv'$ term entirely.

- (B) $\frac{\ln x + 1}{(\ln x)^2}$ uses a plus sign where the rule demands a minus.

Final Answer: $\frac{\ln x - 1}{(\ln x)^2} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q18

Q19.

Solution

Concept — Horizontal tangent: The slope of the tangent at any point of $y = f(x)$ is $f'(x)$. A tangent parallel to the x-axis has slope 0, so we solve $f'(x) = 0$. For a parabola this happens exactly at the vertex, where the curve turns around, so the answer must be the vertex point.

Step 1 — Differentiate: For $y = x^2 - 6x + 5$,

$$f'(x) = 2x - 6.$$

Step 2 — Set the slope to zero:

$$2x - 6 = 0 \implies x = 3.$$

Step 3 — Find the y-coordinate:

$$f(3) = 3^2 - 6(3) + 5 = 9 - 18 + 5 = -4.$$

So the point is $(3, -4)$.

Step 4 — Cross-check by completing the square: $y = (x-3)^2 - 4$ has vertex $(3, -4)$, the lowest point, where the tangent is indeed horizontal, confirming the result.

Why other options are wrong:

- (A) $(0, 5)$ is the y-intercept, where the slope is $f'(0) = -6 \neq 0$.
- (C) $(3, 5)$ has the correct x but the wrong y (forgot to evaluate $f(3)$).
- (D) $(-3, -4)$ takes the wrong sign for x.

Final Answer: point $(3, -4) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q19

Q20.

Solution

Concept — Angle between two curves: The angle between two curves at a point of intersection is defined as the angle between their tangent lines there. If those tangents have slopes m_1 and m_2, the acute angle ϕ satisfies $\tan\phi = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$, the same formula as the angle between two straight lines. So the whole problem reduces to finding the two tangent slopes at the common point.

Step 1 — Slope of $y = x^2$ at $(1, 1)$:

$$\frac{dy}{dx} = 2x \implies m_1 = 2(1) = 2.$$

Step 2 — Slope of $x = y^2$ at $(1, 1)$: Writing $y = \sqrt{x}$,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \implies m_2 = \frac{1}{2\sqrt{1}} = \frac{1}{2}.$$

Step 3 — Apply the angle formula:

$$\tan\phi = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{2 - \frac{1}{2}}{1 + 2 \cdot \frac{1}{2}}\right| = \frac{3/2}{2} = \frac{3}{4}.$$

Why other options are wrong:

- (A) $\frac{4}{5}$ comes from a wrong denominator (e.g. using $1 + m_1 m_2$ incorrectly as $\frac{5}{2}$ over a wrong numerator).
- (B) $\frac{2}{3}$ inverts the fraction $\frac{3/2}{2}$.
- (D) 1 would require $\tan\phi = 1$ (a 45° angle), which these slopes do not give.

Final Answer: $\tan\phi = \frac{3}{4} \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q20

Q21.

Solution

Concept — Order and degree of a differential equation: The *order* is the order of the highest derivative that appears. The *degree* is the power to which that highest-order derivative is raised, but only after the equation has been made polynomial (rational and integral) in all its derivatives, free of radicals and fractions over derivatives. Here the equation is already polynomial in the derivatives, so we can read both off directly.

Step 1 — Locate the highest derivative: The derivatives present are $\frac{d^3y}{dx^3}$ and $\frac{dy}{dx}$. The highest is the third derivative, so the

$$\textbf{order} = 3.$$

Step 2 — Read the power of the highest derivative: The term $\left(\frac{d^3y}{dx^3}\right)^2$ shows the third derivative raised to the power 2, so the

$$\textbf{degree} = 2.$$

Step 3 — Confirm polynomial form: No square roots or derivative-in-denominator terms appear, so no algebraic clearing is needed; the degree is genuinely 2.

Why other options are wrong:

- (A) order 2, degree 3 misreads the highest derivative as second order and swaps the exponent.
- (C) order 2, degree 2 still misreads the order as 2.
- (B) order 3, degree 3 has the right order but wrongly takes the degree as 3.

Final Answer: order 3, degree 2 $\Rightarrow$ $\boxed{\text{D}}$

Answer: (D) Go Back to Q21

Q22.

Solution

Concept — Integral of an exponential a^x: The derivative of a^x is $a^x \ln a$, so integration (the reverse operation) divides by $\ln a$:

$$\int a^x \, dx = \frac{a^x}{\ln a} + C, \qquad a > 0, \ a \neq 1.$$

The base a^x is *not* a power of x, so the power rule $\int x^n \, dx = \frac{x^{n+1}}{n+1}$ does not apply; the variable sits in the exponent, not the base.

Step 1 — Apply the rule with $a = 3$:

$$\int 3^x \, dx = \frac{3^x}{\ln 3} + C.$$

Step 2 — Check by differentiating back:

$$\frac{d}{dx}\left(\frac{3^x}{\ln 3}\right) = \frac{3^x \ln 3}{\ln 3} = 3^x,$$

recovering the integrand, which confirms the antiderivative.

Why other options are wrong:

- (B) $3^x \ln 3 + C$ is the *derivative* of 3^x, multiplying by $\ln 3$ instead of dividing.
- (C) $\dfrac{3^{x+1}}{x+1} + C$ wrongly applies the power rule, treating the exponent as if the base were the variable.
- (D) $3^x + C$ omits the $\ln 3$ factor entirely.

Final Answer: $\dfrac{3^x}{\ln 3} + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q22

Q23.

Solution

Concept — Integral of an odd function over a symmetric interval: A function is odd if $f(-x) = -f(x)$; its graph has point symmetry about the origin. Over an interval $[-a, a]$ symmetric about 0, the area on the left of the y-axis exactly cancels the signed area on the right, so $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$. Recognising odd parity therefore lets us write the answer without any antidifferentiation.

Step 1 — Test the parity: Replace x by $-x$ in $f(x) = x^5 + x$:

$$f(-x) = (-x)^5 + (-x) = -x^5 - x = -(x^5 + x) = -f(x).$$

So f is odd (each term, an odd power of x, is odd).

Step 2 — Apply the symmetry property over $[-2, 2]$:

$$\int_{-2}^{2} (x^5 + x)\, dx = 0.$$

Step 3 — Verify by direct antidifferentiation:

$$\int_{-2}^{2} (x^5 + x)\, dx = \left[\frac{x^6}{6} + \frac{x^2}{2}\right]_{-2}^{2} = \left(\frac{64}{6} + 2\right) - \left(\frac{64}{6} + 2\right) = 0,$$

confirming the cancellation.

Why other options are wrong:

- (A) 32 and (D) 64 resemble one-sided values of $x^6/6$ but ignore the exact cancellation.
- (B) 4 keeps only the $\int x\, dx$ part over $[0, 2]$, doubling it, again ignoring symmetry.

Final Answer: the integral is $0 \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q23**

Q24.

Solution

Concept — Area under a curve: The area of the region bounded by $y = f(x)$, the x-axis and the vertical lines $x = a$ and $x = b$ (with $f \geq 0$ there) is the definite integral $\displaystyle\int_a^b y\, dx$. Here $y = 2x \geq 0$ on $[1, 3]$, so the shaded region's area is $\int_1^3 2x\, dx$.

Step 1 — Set up the definite integral:

$$\text{Area} = \int_1^3 2x\, dx.$$

Step 2 — Antiderivative: $\int 2x\,dx = x^2$, so

$$\text{Area} = \left[x^2\right]_1^3.$$

Step 3 — Apply the limits:

$$\left[x^2\right]_1^3 = 3^2 - 1^2 = 9 - 1 = 8 \text{ sq. units.}$$

Step 4 — Cross-check as a trapezium: The region is a trapezium with parallel vertical sides of heights $y(1) = 2$ and $y(3) = 6$ and width $3 - 1 = 2$. Its area is $\frac{1}{2}(2 + 6) \times 2 = \frac{1}{2} \times 8 \times 2 = 8$, matching the integral.

Why other options are wrong:

- (A) 4 uses the wrong width or only half the trapezium.
- (C) 16 squares the upper limit only ($\int_0^{\cdot}$.. to $x = ...$ confusion), ignoring the lower bound subtraction.
- (D) 9 keeps just 3^2 and forgets to subtract 1^2.

Final Answer: area $= 8$ sq. units $\Rightarrow$ $\boxed{\text{B}}$

Answer: (B) Go Back to Q24

Q25.

Solution

Concept — Power rule for definite integrals: The power rule $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ $(n \neq -1)$ gives the antiderivative; the Fundamental Theorem of Calculus then evaluates the definite integral as the antiderivative's change between the limits, $\left[F(x)\right]_a^b = F(b) - F(a)$. A constant multiplier (the 3) factors straight through the integral.

Step 1 — Find the antiderivative:

$$\int 3x^2\,dx = 3 \cdot \frac{x^3}{3} = x^3.$$

Step 2 — Apply the limits 0 to 1:

$$\int_0^1 3x^2\,dx = \left[x^3\right]_0^1 = 1^3 - 0^3.$$

Step 3 — Evaluate:

$$= 1 - 0 = 1.$$

Why other options are wrong:

- (A) 3 forgets to raise the power and divide (treats the integral as 3×1).
- (B) $\frac{1}{3}$ integrates x^2 to $\frac{x^3}{3}$ but drops the factor 3.
- (C) $\frac{3}{2}$ mis-evaluates the antiderivative (e.g. integrates $3x$ instead).

Final Answer: the integral is $1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q25

Q26.

Solution

Concept — Slope–intercept form: A straight line with slope m and y-intercept c has equation $y = mx + c$. Here m is the coefficient of x (the rate of rise) and c is the value of y where the line crosses the y-axis (at $x = 0$). Substituting the given numbers directly yields the equation.

Step 1 — Identify the data: slope $m = 2$ and y-intercept $c = -3$.

Step 2 — Substitute into $y = mx + c$:

$$y = (2)x + (-3) = 2x - 3.$$

Step 3 — Verify the intercept: Setting $x = 0$ gives $y = -3$, the stated y-intercept; and the coefficient of x is 2, the stated slope, so both conditions check out.

Why other options are wrong:

- (B) $y = -3x + 2$ swaps slope and intercept (uses -3 as slope, 2 as intercept).
- (D) $y = 3x - 2$ also swaps them, taking slope 3 and intercept -2.
- (C) $y = 2x + 3$ uses the wrong sign for the intercept ($+3$ instead of -3).

Final Answer: $y = 2x - 3 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q26

Q27.

Solution

Concept — Concentric circles: Concentric circles share the same centre but have different radii. For the general circle $x^2 + y^2 + 2gx + 2fy + c = 0$ the centre is $(-g, -f)$. A circle concentric with a given one keeps that centre; if it must also pass through a particular point, its radius is just the distance from the centre to that point.

Step 1 — Find the common centre: Compare $x^2 + y^2 - 4x - 6y + 9 = 0$ with $x^2 + y^2 + 2gx + 2fy + c = 0$:

$$2g = -4 \Rightarrow g = -2, \qquad 2f = -6 \Rightarrow f = -3,$$

so the centre is $(-g, -f) = (2, 3)$.

Step 2 — Radius = distance from $(2, 3)$ to $(2, 8)$:

$$r = \sqrt{(2-2)^2 + (8-3)^2} = \sqrt{0 + 25} = \sqrt{25} = 5.$$

Step 3 — Contrast with the original radius (check): The original circle has radius $\sqrt{g^2 + f^2 - c} = \sqrt{4 + 9 - 9} = \sqrt{4} = 2$, the smaller inner circle in the figure; our new circle, radius 5, is the larger one through $(2, 8)$. The two are concentric as required.

Why other options are wrong:

- (A) 2 is the radius of the *original* circle, not the new one through $(2, 8)$.
- (B) 4 and (D) 8 miscompute the vertical distance $8 - 3 = 5$ (e.g. using $8 - 4$ or just 8).

Final Answer: radius $= 5 \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q27

Q28.

Solution

Concept — Axes of an ellipse: For the standard ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$, the larger denominator sits under x^2, so the major axis lies along the x-axis with full length $2a$, while the minor axis lies along the y-axis with full length $2b$. The numbers a and b are the *semi*-axes, and the question asks for the *full* axis lengths, which are double those.

Step 1 — Read the semi-axes from the denominators:

$$a^2 = 25 \Rightarrow a = 5, \qquad b^2 = 9 \Rightarrow b = 3,$$

and since $25 > 9$ the major axis is along x.

Step 2 — Double to get full axis lengths:

$$\text{major} = 2a = 2(5) = 10, \qquad \text{minor} = 2b = 2(3) = 6.$$

Why other options are wrong:

- (A) 5 and 3 give the semi-axes a, b, not the full lengths.
- (C) 25 and 9 quote a^2, b^2 (the denominators), forgetting the square root.
- (D) 6 and 10 swaps major and minor.

Final Answer: 10 and $6 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q28

Q29.

Solution

Concept — Collinearity of points in space: Three points A, B, C lie on one straight line precisely when the vectors $\vec{AB}$ and $\vec{AC}$ point along the same line, i.e. one is a scalar multiple of the other. Equivalently, their direction ratios are proportional. If $\vec{AC} = \lambda\,\vec{AB}$ for some scalar λ, the three points are collinear.

Step 1 — Compute the two direction vectors from A:

$$\vec{AB} = B - A = (2-1,\ 4-2,\ 6-3) = (1, 2, 3),$$

$$\vec{AC} = C - A = (4-1,\ 8-2,\ 12-3) = (3, 6, 9).$$

Step 2 — Test proportionality: Compare component-by-component:

$$\frac{3}{1} = \frac{6}{2} = \frac{9}{3} = 3,$$

so $\vec{AC} = 3\,\vec{AB}$, a constant multiple.

Step 3 — Conclude: Since $\vec{AC}$ is parallel to $\vec{AB}$ and they share point A, the three points lie on a single line; they are collinear.

Why other options are wrong:

- (A) a triangle needs three non-collinear vertices, contradicting the exact proportionality.
- (B) "never collinear" is directly disproved by $\vec{AC} = 3\,\vec{AB}$.
- (D) a square needs four coplanar non-collinear corners; only three collinear points are given.

Final Answer: the points are collinear $\Rightarrow$ $\boxed{\text{C}}$

Answer: (C) Go Back to Q29

Q30.

Solution

Concept — Perpendicular distance from a point to a plane: For a plane written as $ax + by + cz = d$ and a point (x_0, y_0, z_0), the shortest (perpendicular) distance is

$$D = \frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}.$$

The numerator measures how far the point is from satisfying the plane equation, and dividing by the length of the normal vector (a, b, c) converts that into an actual geometric distance. The absolute value keeps the distance non-negative.

Step 1 — Identify coefficients: The plane $x + 2y + 2z = 3$ has $a = 1,\ b = 2,\ c = 2,\ d = 3$; the point is $(x_0, y_0, z_0) = (1, 2, 2)$.

Step 2 — Evaluate the numerator:

$$|ax_0 + by_0 + cz_0 - d| = |1(1) + 2(2) + 2(2) - 3| = |1 + 4 + 4 - 3| = |6| = 6.$$

Step 3 — Evaluate the denominator (normal length):

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3.$$

Step 4 — Divide:

$$D = \frac{6}{3} = 2.$$

Why other options are wrong:

- (A) 3 forgets to subtract d in the numerator or skips the division.
- (B) $\frac{6}{\sqrt{5}}$ miscounts the normal length as $\sqrt{5}$ (e.g. using $(1, 2, 0)$).
- (C) 1 divides 6 by the wrong number or mis-adds the numerator.

Final Answer: distance $= 2 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q30

Q31.

Solution

Concept — Work as a dot product: The work done by a constant force $\vec{F}$ over a displacement $\vec{d}$ is the scalar (dot) product $W = \vec{F}\cdot\vec{d}$. In components, $\vec{F}\cdot\vec{d} = F_1d_1 + F_2d_2 + F_3d_3$: multiply matching components and add. The result is a single number (scalar), and it equals $|\vec{F}||\vec{d}|\cos\theta$, so it is largest when force and displacement are aligned and zero when they are perpendicular.

Step 1 — Write the components: $\vec{F} = (3, 2, 1)$ and $\vec{d} = (1, 2, 3)$.

Step 2 — Multiply matching components:

$$F_1d_1 = (3)(1) = 3, \quad F_2d_2 = (2)(2) = 4, \quad F_3d_3 = (1)(3) = 3.$$

Step 3 — Add them:

$$W = 3 + 4 + 3 = 10 \text{ units}.$$

Why other options are wrong:

- (B) 6 drops a term (e.g. uses only $3 + 3$).
- (C) $14 = 3^2 + 2^2 + 1^2$ wrongly squares the force components instead of pairing with $\vec{d}$.
- (D) 0 would require $\vec{F} \perp \vec{d}$, but here their dot product is nonzero.

Final Answer: $W = 10$ units $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q31

Q32.

Solution

Concept — Triangle area via the cross product: If two sides of a triangle are the vectors $\vec{a}$ and $\vec{b}$ from a common vertex, the parallelogram they span has area $|\vec{a}\times\vec{b}|$, and the triangle is exactly half of that parallelogram, so area $= \frac{1}{2}|\vec{a}\times\vec{b}|$. The cross product of perpendicular unit vectors follows the right-hand rule: $\hat{i}\times\hat{j}=\hat{k}$.

Step 1 — Compute the cross product:

$$\vec{a}\times\vec{b} = (4\hat{i})\times(6\hat{j}) = 4\cdot 6\,(\hat{i}\times\hat{j}) = 24\,\hat{k}.$$

Step 2 — Take its magnitude:

$$|\vec{a}\times\vec{b}| = |24\hat{k}| = 24.$$

Step 3 — Halve for the triangle:

$$\text{Area} = \frac{1}{2}\times 24 = 12.$$

Step 4 — Cross-check geometrically: The sides $4\hat{i}$ and $6\hat{j}$ are perpendicular (along the axes), forming a right triangle with legs 4 and 6. Its area is $\frac{1}{2}\times 4\times 6 = 12$, matching the cross-product result.

Why other options are wrong:

- (A) 24 is the *parallelogram* area; it omits the factor $\frac{1}{2}$.
- (C) 10 and (D) 6 keep only one of the side lengths or mis-multiply 4×6.

Final Answer: area $= 12 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q32

Q33.

Solution

Concept — Scalar triple product and coplanarity: The scalar triple product $[\vec{a}\;\vec{b}\;\vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$ equals the determinant whose rows are the components of $\vec{a},\vec{b},\vec{c}$. Geometrically its absolute value is the volume of the parallelepiped built on the three vectors. Three vectors are coplanar exactly when that volume is zero, so coplanarity $\iff [\vec{a}\;\vec{b}\;\vec{c}] = 0$. A nonzero value means they are not coplanar.

Step 1 — Form the determinant of components: With $\vec{a} = (1, 1, 0)$, $\vec{b} = (0, 1, 1)$, $\vec{c} = (1, 0, 1)$,

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}.$$

Step 2 — Expand along the first row:

$$= 1\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}.$$

Step 3 — Evaluate the minors:

$$= 1(1 \cdot 1 - 1 \cdot 0) - 1(0 \cdot 1 - 1 \cdot 1) + 0 = 1(1) - 1(-1) + 0 = 1 + 1 = 2.$$

Step 4 — Interpret: Since $[\vec{a}\,\vec{b}\,\vec{c}] = 2 \neq 0$, the parallelepiped has nonzero volume, so the vectors are *not* coplanar.

Why other options are wrong:

- (A) 0 would mean coplanar, but the determinant is 2.
- (B) 1 and (C) -2 come from sign or minor errors in the cofactor expansion.

Final Answer: $[\vec{a}\,\vec{b}\,\vec{c}] = 2$ (not coplanar) $\Rightarrow$ $\boxed{\text{D}}$

Answer: (D) Go Back to Q33

Q34.

Solution

Concept — Standard deviation: The standard deviation σ measures the spread of data about its mean. We first find the mean $\bar{x}$, then average the squared deviations $(x_i - \bar{x})^2$ to get the variance $\sigma^2 = \dfrac{\sum(x_i - \bar{x})^2}{n}$, and finally take the square root, $\sigma = \sqrt{\sigma^2}$. The squaring prevents positive and negative deviations from cancelling, and the final square root restores the original units.

Step 1 — Compute the mean:

$$\bar{x} = \frac{2 + 4 + 6}{3} = \frac{12}{3} = 4.$$

Step 2 — Sum of squared deviations:

$$(2-4)^2+(4-4)^2+(6-4)^2=(-2)^2+0^2+2^2=4+0+4=8.$$

Step 3 — Variance:

$$\sigma^2=\frac{8}{3}.$$

Step 4 — Standard deviation:

$$\sigma=\sqrt{\frac{8}{3}}=\frac{\sqrt{8}}{\sqrt{3}}=\frac{2\sqrt{2}}{\sqrt{3}}.$$

Why other options are wrong:

- (C) $\frac{8}{3}$ is the variance σ^2, not the standard deviation; it skips the square root.
- (B) 2 would be $\sqrt{4}$, mis-summing the squared deviations or using $n-1$ wrongly.
- (D) 4 is the mean, not a measure of spread.

Final Answer: $\sigma=\dfrac{2\sqrt{2}}{\sqrt{3}}\Rightarrow\boxed{\text{A}}$

Answer: (A) **Go Back to Q34**

Q35.

Solution

Concept — Classical probability: When all outcomes are equally likely, $P(\text{event})=\dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$. Two distinguishable fair dice produce $6\times6=36$ equally likely ordered outcomes $(\text{die}_1,\text{die}_2)$, which forms the sample space.

Step 1 — List the favourable outcomes (sum $=7$): The ordered pairs whose entries add to 7 are

$$(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),$$

giving 6 favourable outcomes.

Step 2 — Form the probability:

$$P(\text{sum}=7)=\frac{6}{36}=\frac{1}{6}.$$

Step 3 — Sanity check: A sum of 7 is the most likely total with two dice (it has the most pairings), and $\frac{1}{6} \approx 0.167$ is the largest single-sum probability, which is consistent.

Why other options are wrong:

- (A) $\frac{1}{12}$ counts only 3 favourable pairs (forgets ordered duplicates like $(2, 5)$ vs $(5, 2)$).
- (B) $\frac{5}{36}$ is the count for a sum of 6 or 8, not 7.
- (D) $\frac{7}{36}$ over-counts by one favourable outcome.

Final Answer: $P = \dfrac{1}{6} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q35

Q36.

Solution

Concept — Mean and variance of a binomial distribution: For n independent trials each with success probability p (and failure probability $q = 1-p$), the number of successes follows a binomial distribution with mean $\mu = np$ and variance $\sigma^2 = npq$. The mean is the expected number of successes, while the variance is always smaller than (or equal to) the mean since it carries the extra factor $q \leq 1$.

Step 1 — Identify the parameters: $n = 10$, $p = \frac{1}{2}$, so $q = 1 - \frac{1}{2} = \frac{1}{2}$.

Step 2 — Mean:

$$\mu = np = 10 \times \frac{1}{2} = 5.$$

Step 3 — Variance:

$$\sigma^2 = npq = 10 \times \frac{1}{2} \times \frac{1}{2} = \frac{10}{4} = 2.5.$$

Step 4 — Consistency check: Variance $= \mu q = 5 \times \frac{1}{2} = 2.5$, smaller than the mean as expected when $q < 1$.

Why other options are wrong:

- (A) 5 and 5 wrongly uses variance $= np$ (forgetting the factor q).
- (C) 10 and 5 uses n as the mean and np as the variance.
- (D) 2.5 and 5 swaps the mean and variance.

Final Answer: mean 5, variance $2.5 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q36

Q37.

Solution

Concept — General solution of $\cos\theta = 1$: Cosine is periodic with period 2π and attains its maximum value 1 only at $\theta = 0$ within one period $[0, 2\pi)$. Because of the 2π periodicity, every solution is obtained by adding whole numbers of full revolutions to 0, giving $\theta = 2n\pi$ for integer n. (Only even multiples of π work; odd multiples give $\cos = -1$.)

Step 1 — Find the principal solution: On $[0, 2\pi)$, the only angle with $\cos\theta = 1$ is

$$\theta = 0.$$

Step 2 — Add full turns for the general solution: Since cosine repeats every 2π,

$$\theta = 0 + 2n\pi = 2n\pi, \qquad n \in \mathbb{Z}.$$

Step 3 — Verify a couple of values: $n = 1$ gives $\theta = 2\pi$ ($\cos 2\pi = 1$, good); $n = -1$ gives $\theta = -2\pi$ ($\cos(-2\pi) = 1$, good).

Why other options are wrong:

- (A) $\theta = n\pi$ includes $\theta = \pi$, where $\cos\pi = -1 \neq 1$.
- (B) $\theta = (2n+1)\pi$ gives odd multiples of π, where $\cos = -1$.
- (C) $\theta = 2n\pi + \frac{\pi}{2}$ gives $\cos = 0$, the solution set of $\cos\theta = 0$ instead.

Final Answer: $\theta = 2n\pi \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q37

Q38.

Solution

Concept — Double-angle identity for cosine: The identity $\cos 2\theta = 2\cos^2\theta - 1$ expresses $\cos 2\theta$ purely in terms of $\cos\theta$, which is exactly what we need since we are given $\cos\theta$. (It comes from $\cos 2\theta = \cos^2\theta - \sin^2\theta$ together with $\sin^2\theta = 1 - \cos^2\theta$.) So we only need to square the given cosine and substitute.

Step 1 — Square the given value: With $\cos\theta = \frac{1}{\sqrt{2}}$,

$$\cos^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}.$$

Step 2 — Substitute into the identity:

$$\cos 2\theta = 2\cos^2\theta - 1 = 2\cdot\frac{1}{2} - 1 = 1 - 1 = 0.$$

Step 3 — Cross-check via the angle: $\cos\theta = \frac{1}{\sqrt{2}}$ means $\theta = 45^\circ$, so $2\theta = 90^\circ$ and $\cos 90^\circ = 0$, confirming the result.

Why other options are wrong:

- (B) 1 would correspond to $\theta = 0$ ($\cos 0 = 1$), not 45°.
- (C) $\frac{1}{2}$ forgets the "-1" in the identity, stopping at $2\cos^2\theta = 1$ then mishandling it.
- (D) -1 over-subtracts (e.g. uses $2\cos^2\theta - 2$).

Final Answer: $\cos 2\theta = 0 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q38

Q39.

Solution

Concept — Principal value of $\sec^{-1}$: The inverse secant returns the unique angle in its principal range $[0,\pi]\setminus\{\frac{\pi}{2}\}$ whose secant equals the given value. Since $\sec\theta = \frac{1}{\cos\theta}$, it is easiest to convert to a cosine equation and solve within that range. The value $\frac{\pi}{2}$ is excluded because $\sec\frac{\pi}{2}$ is undefined.

Step 1 — Convert secant to cosine:

$$\sec\theta = 2 \implies \cos\theta = \frac{1}{2}.$$

Step 2 — Solve within the principal range: On $[0,\pi]$ the angle with $\cos\theta = \frac{1}{2}$ is

$$\theta = \frac{\pi}{3}.$$

This lies in the allowed range and is not the excluded $\frac{\pi}{2}$.

Step 3 — Verify: $\sec\frac{\pi}{3} = \dfrac{1}{\cos(\pi/3)} = \dfrac{1}{1/2} = 2$, exactly the given value.

Why other options are wrong:

- (A) $\frac{\pi}{6}$ has $\cos = \frac{\sqrt{3}}{2}$, so $\sec = \frac{2}{\sqrt{3}} \neq 2$.
- (B) $\frac{\pi}{4}$ has $\cos = \frac{1}{\sqrt{2}}$, so $\sec = \sqrt{2} \neq 2$.
- (D) $\frac{\pi}{2}$ is excluded from the range and $\sec\frac{\pi}{2}$ is undefined.

Final Answer: $\sec^{-1}(2) = \dfrac{\pi}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q39

Q40.

Solution

Concept — Ladder against a wall (right-triangle trigonometry): The ladder, the wall and the ground form a right triangle, with the ladder as the hypotenuse and the angle 60° measured at the foot (between ladder and ground). The wall height h is the side *opposite* that angle, so $\sin 60^\circ = \dfrac{h}{\text{ladder}}$, giving $h = (\text{ladder length})\sin 60^\circ$.

Step 1 — Set up the sine relation:

$$\sin 60^\circ = \frac{h}{10} \implies h = 10\sin 60^\circ.$$

Step 2 — Substitute the exact value $\sin 60^\circ = \dfrac{\sqrt{3}}{2}$:

$$h = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}\text{ m}.$$

Step 3 — Sanity check: $5\sqrt{3} \approx 8.66$ m, less than the 10 m ladder, as a height must be; and the base distance $10\cos 60^\circ = 5$ m, with $5^2 + (5\sqrt{3})^2 = 25 + 75 = 100 = 10^2$, satisfying Pythagoras.

Why other options are wrong:

- (A) 5 uses $\cos 60^\circ$, which gives the horizontal base distance, not the height.
- (B) 10 would be the height only if the ladder were vertical (90°).
- (C) $\frac{10}{\sqrt{3}}$ misuses $\tan$ instead of $\sin$.

Final Answer: height $= 5\sqrt{3}$ m $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	B
6	A	7	C	8	D	9	A	10	B
11	D	12	C	13	B	14	A	15	B
16	D	17	A	18	C	19	B	20	C
21	D	22	A	23	C	24	B	25	D
26	A	27	C	28	B	29	C	30	D
31	A	32	B	33	D	34	A	35	C
36	B	37	D	38	A	39	C	40	D