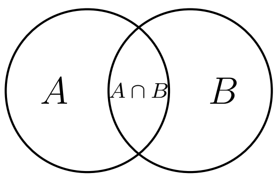
SRMJEEE Mathematics Sample Paper – 2

Duration: 47 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of **SRMJEEE** (SRM Joint Engineering Entrance Examination).
- Each correct answer carries **+1 mark**. There is **no negative marking**; an unattempted or wrong answer scores 0.
- Only **one** option is correct. Choose carefully.
- The actual SRMJEEE is a **computer-based test** conducted in remote-proctored online mode, with all sections sharing a common time window and no per-section limit.
- Personal calculators, mobile phones, log tables and other electronic gadgets are strictly prohibited.

Q1. Two sets A and B satisfy $n(A) = 18$, $n(B) = 14$ and $n(A \cup B) = 27$, as shown in the Venn diagram. The value of $n(A \cap B)$ is:



(A) 5

(B) 9

(C) 4

(D) 32

Q2. If a set A has 3 elements and a set B has 2 elements, then the total number of relations from A to B is:

(A) 32

(B) 64

(C) 6

(D) 8

Q3. The range of the real function $f(x) = x^2 + 1$ is:

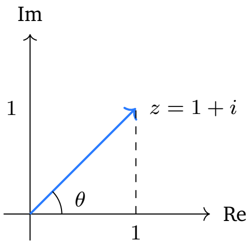
(A) $(1, \infty)$

(B) $\mathbb{R}$

(C) $[1, \infty)$

(D) $[0, \infty)$

Q4. The argument (amplitude) of the complex number $z = 1 + i$, shown as a point on the Argand plane, is:



(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$

Q5. If ω is a non-real cube root of unity, then the value of $\omega^4 + \omega^2$ is:

(A) -1

(B) 1

(C) 0

(D) 2

Q6. The nature of the roots of the quadratic equation $2x^2 - 7x + 3 = 0$ is:

(A) complex (non-real)

(B) real and distinct

(C) real and equal

(D) imaginary and equal

Q7. If α and β are the roots of $x^2 - 6x + 4 = 0$, then the value of $\alpha^2 + \beta^2$ is:

(A) 36

(B) 44

(C) 28

(D) 32

Q8. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$, then the $(1, 2)$ entry of the product AB is:

(A) 2

(B) 4

(C) 14

(D) 10

Q9. For any square matrix A of order n, the product $A \cdot (\text{adj } A)$ equals:

(A) $|A|\, I$

(B) I

(C) $|A|$

(D) A^2

Q10. The value of the determinant $\begin{vmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 2 & 1 & 0 \end{vmatrix}$ is:

(A) -11

(B) -13

(C) 13

(D) 7

Q11. The three points $(1, 2)$, $(2, 4)$ and $(3, 6)$ are collinear because the area of the triangle they form is:

(A) 1 sq. unit

(B) 3 sq. units

(C) 0

(D) 2 sq. units

Q12. The number of distinct arrangements of the letters of the word "LEVEL" is:

(A) 120

(B) 60

(C) 20

(D) 30

Q13. The number of ways of selecting a committee of 4 members from 7 persons is:

(A) 35

(B) 28

(C) 840

(D) 210

Q14. The number of distinct ways in which 6 different beads can be arranged on a garland (necklace) is:

(A) 120

(B) 60

(C) 720

(D) 360

Q15. If α, β, γ are the roots of $2x^3 - 3x^2 + 4x - 10 = 0$, then the product $\alpha\beta\gamma$ equals:

(A) -5

(B) 10

(C) 5

(D) -10

Q16. If α and β are the roots of $x^2 - 3x + 2 = 0$, then the quadratic equation whose roots are $\alpha + 2$ and $\beta + 2$ is:

(A) $x^2 - 3x + 2 = 0$

(B) $x^2 + 7x + 12 = 0$

(C) $x^2 - 5x + 6 = 0$

(D) $x^2 - 7x + 12 = 0$

Q17. The value of $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$ is:

(A) $\frac{1}{2}$

(B) 1

(C) 0

(D) 2

Q18. If $y = \sin(x^2)$, then $\frac{dy}{dx}$ equals:

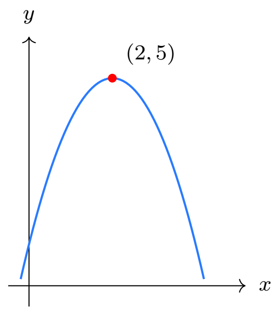
(A) $\cos(x^2)$

(B) $2x\cos(x^2)$

(C) $2x\sin(x^2)$

(D) $-2x\cos(x^2)$

Q19. The function $f(x) = -x^2 + 4x + 1$, whose graph is shown, attains its maximum value equal to:



(A) 4

(B) 1

(C) 5

(D) 2

Q20. The radius of a circle increases at the rate of 2 cm/s. The rate of increase of its area when the radius is 5 cm is:

(A) 10π cm^2/s

(B) 25π cm^2/s

(C) 5π cm^2/s

(D) 20π cm^2/s

Q21. The general solution of the differential equation $\dfrac{dy}{dx} = 3x^2$ is:

(A) $y = x^3 + C$

(B) $y = 6x + C$

(C) $y = 3x^3 + C$

(D) $y = x^2 + C$

Q22. $\displaystyle\int x^5\, dx$ equals:

(A) $5x^4 + C$

(B) $\dfrac{x^6}{6} + C$

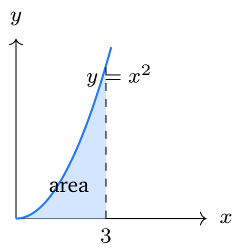
(C) $\dfrac{x^6}{5} + C$

(D) $6x^6 + C$

Q23. Using the property of definite integrals, the value of $\int_{-2}^{2} x^2\,dx$ is:

(A) 0
(B) $\frac{8}{3}$
(C) $\frac{16}{3}$
(D) 8

Q24. The area of the region bounded by the parabola $y = x^2$, the x-axis and the line $x = 3$ (shaded) is:



(A) 3 sq. units

(B) 27 sq. units

(C) 18 sq. units

(D) 9 sq. units

Q25. The value of $\int_{0}^{1} x^2\,dx$ is:

(A) $\frac{1}{3}$
(B) $\frac{1}{2}$
(C) 1
(D) $\frac{2}{3}$

Q26. The distance between the points $(2, 3)$ and $(5, 7)$ is:

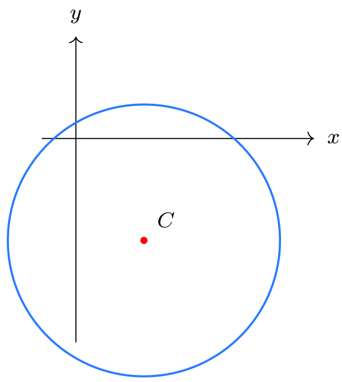
(A) 7

(B) 5

(C) $\sqrt{7}$

(D) 25

Q27. The centre of the circle $x^2 + y^2 - 4x + 6y - 3 = 0$, shown on the axes, is:



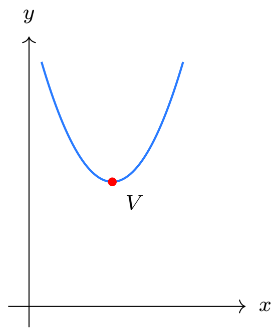
(A) $(4, -6)$

(B) $(-2, 3)$

(C) $(2, -3)$

(D) $(2, 3)$

Q28. The vertex of the parabola $y = (x - 2)^2 + 3$, shown below, is:



(A) $(0, 3)$

(B) $(-2, 3)$

(C) $(2, 0)$

(D) $(2, 3)$

Q29. The angle between two lines whose direction ratios are $(1, 0, 0)$ and $(0, 1, 0)$ is:

(A) 90°

(B) 0°

(C) 45°

(D) 60°

Q30. The angle between the planes $x + y + z = 1$ and $x + y + z = 5$ is:

(A) 90°

(B) 0°

(C) 45°

(D) 60°

Q31. If $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{i} - \hat{j}$, then the angle between $\vec{a}$ and $\vec{b}$ is:

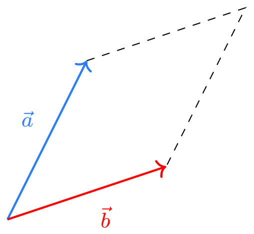
(A) 0°

(B) 45°

(C) 90°

(D) 180°

Q32. For the vectors $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = 3\hat{i} + \hat{j}$ shown as adjacent sides of a parallelogram, the cross product $\vec{a} \times \vec{b}$ is:



(A) $7\hat{k}$

(B) 0

(C) $\hat{k}$

(D) $-5\hat{k}$

Q33. The scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}]$ for $\vec{a} = \hat{i}$, $\vec{b} = \hat{j}$, $\vec{c} = \hat{k}$, evaluated as a 3×3 determinant, is:

(A) 1

(B) 0

(C) 3

(D) -1

Q34. The median of the data $7, 3, 9, 5, 11$ is:

(A) 9

(B) 7

(C) 5

(D) 11

Q35. For two events A and B with $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, the value of $P(A \cup B)$ is:

(A) 0.9

(B) 0.6

(C) 0.7

(D) 1.1

Q36. The mean of a binomial distribution with $n = 10$ and $p = 0.4$ is:

(A) 2.4

(B) 10

(C) 0.4

(D) 4

Q37. The value of $\tan 60°$ is:

(A) $\sqrt{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) 1

(D) $\frac{\sqrt{3}}{2}$

Q38. If $\cos\theta = \frac{1}{2}$, then the value of $\cos 2\theta$ is:

(A) 1

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 0

Q39. The principal value of $\cos^{-1}\left(\frac{1}{2}\right)$ is:

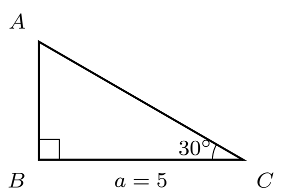
(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Q40. In triangle ABC, $\angle A = 30^\circ$, $\angle B = 90^\circ$ and side $a = BC = 5$, as shown. Using the sine rule, the side $b = CA$ is:



(A) 5

(B) $\frac{5}{2}$

(C) $5\sqrt{3}$

(D) 10

Detailed Solutions

Q1.

Solution

Concept — The inclusion–exclusion principle: When we add $n(A)$ and $n(B)$, every element lying in *both* sets is counted once in each total, hence twice altogether. To recover the true size of the union we must remove that double-counted overlap exactly once:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Rearranging this identity to isolate the overlap gives a direct formula for the intersection:

$$n(A \cap B) = n(A) + n(B) - n(A \cup B).$$

Step 1 — Read off the given data: The problem supplies $n(A) = 18$, $n(B) = 14$ and $n(A \cup B) = 27$. The unknown is $n(A \cap B)$, the shaded lens-shaped overlap in the Venn diagram.

Step 2 — Substitute into the rearranged identity:

$$n(A \cap B) = 18 + 14 - 27.$$

Step 3 — Simplify the arithmetic: First $18 + 14 = 32$, then $32 - 27 = 5$, so

$$n(A \cap B) = 5.$$

Step 4 — Cross-check by regions: If the overlap is 5, then A-only $= 18 - 5 = 13$ and B-only $= 14 - 5 = 9$. Adding the three disjoint regions gives $13 + 9 + 5 = 27$, which matches $n(A \cup B)$ exactly, confirming the answer.

Why other options are wrong:

- (B) 9 is the size of the B-only region, not the overlap.
- (C) 4 comes from a subtraction slip such as $14 + 18 - 28$.
- (D) $32 = n(A)+n(B)$ skips the subtraction entirely and so double-counts the overlap.

Final Answer: $n(A \cap B) = 5 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q1

Q2.

Solution

Concept — Counting all relations from A to B: A relation from A to B is, by definition, any subset of the Cartesian product $A \times B$. A set with k elements has exactly 2^k subsets, because each element is independently either in or out of a given subset. Since $|A \times B| = |A| \cdot |B| = mn$, the number of distinct relations is

$$2^{mn}.$$

Step 1 — Find the size of $A \times B$: Here $m = |A| = 3$ and $n = |B| = 2$, so the number of ordered pairs is

$$|A \times B| = mn = 3 \times 2 = 6.$$

Step 2 — Apply the subset count: Every one of these 6 ordered pairs may or may not belong to a relation, giving

$$2^{mn} = 2^6.$$

Step 3 — Evaluate the power: $2^6 = 64$.

Step 4 — Sanity check: The empty relation and the full relation $A \times B$ are both counted, and 64 comfortably exceeds the 6 "single-pair" relations, as it should since most relations contain several pairs.

Why other options are wrong:

- (A) $32 = 2^5$ miscounts the product as $mn = 5$ (perhaps $3 + 2$).
- (C) 6 is just mn, the number of pairs, not the number of subsets.
- (D) $8 = 2^3$ uses only $|A| = 3$ in the exponent and ignores B.

Final Answer: $2^6 = 64 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept — The range of a shifted square: The range of a function is the complete set of output values it can produce. For $f(x) = x^2 + 1$ the key fact is that a real square is never negative: $x^2 \geq 0$ for every real x. Adding 1 shifts every output up by one unit, so the graph is an upward parabola whose lowest point sits one unit above the x-axis. The range therefore starts at the minimum and extends upward without limit.

Step 1 — Locate the minimum: Since $x^2 \geq 0$ with equality only at $x = 0$, the smallest possible value of f occurs there:

$$f(0) = 0^2 + 1 = 1.$$

Step 2 — Check the upper behaviour: As $|x| \to \infty$, $x^2 \to \infty$, so $f(x) = x^2 + 1 \to \infty$. The function grows without any ceiling.

Step 3 — Assemble the range: Every output is at least 1, and the value 1 is genuinely reached (at $x = 0$), so it must be *included*. Hence

$$\text{range} = [1, \infty).$$

Step 4 — Verify with a sample: $f(2) = 5 \geq 1$ and $f(0.5) = 1.25 \geq 1$; no input can ever yield a number below 1, which agrees with the closed bracket at 1.

Why other options are wrong:

- (A) $(1, \infty)$ uses an open bracket and wrongly excludes the attained minimum 1.
- (B) $\mathbb{R}$ would allow negative outputs, which $x^2 + 1$ never produces.
- (D) $[0, \infty)$ includes values in $[0, 1)$ that the function cannot reach.

Final Answer: range $= [1, \infty) \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q3

Q4.

Solution

Concept — Argument of a complex number: The argument of $z = a + bi$ is the angle θ that the line from the origin to the point (a, b) makes with the positive real axis, measured anticlockwise. It satisfies $\tan\theta = \frac{b}{a}$, the ratio of the imaginary part (vertical leg) to the real part (horizontal leg). When the point lies in the first quadrant (both $a > 0$ and $b > 0$) the principal argument is simply

$$\arg z = \tan^{-1}\frac{b}{a}.$$

Step 1 — Identify the parts: For $z = 1 + i$ we have real part $a = 1$ and imaginary part $b = 1$. Both are positive, so z lies in the first quadrant, consistent with the Argand diagram.

Step 2 — Form the ratio:

$$\arg z = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{1}{1} = \tan^{-1} 1.$$

Step 3 — Evaluate the inverse tangent: The first-quadrant angle whose tangent is 1 is 45°, that is

$$\arg z = \frac{\pi}{4}.$$

Step 4 — Geometric cross-check: Because $a = b = 1$, the point sits exactly on the line $y = x$, which bisects the first quadrant; the bisector makes a $45^\circ = \pi/4$ angle with the real axis, matching the marked angle θ in the figure.

Why other options are wrong:

- (A) $\frac{\pi}{3}$ would require $\tan\theta = \sqrt{3}$, i.e. $b/a = \sqrt{3}$, not 1.
- (C) $\frac{\pi}{6}$ corresponds to $\tan\theta = \frac{1}{\sqrt{3}}$.
- (B) $\frac{\pi}{2}$ is the argument of a purely imaginary number ($a = 0$), which is not the case here.

Final Answer: $\arg z = \frac{\pi}{4} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Properties of the cube roots of unity: The non-real cube roots of unity satisfy two essential identities. First, $\omega^3 = 1$, because ω is a cube root of 1; this lets us reduce any high power of ω modulo 3. Second, the three cube roots $1, \omega, \omega^2$ sum to zero,

$$1 + \omega + \omega^2 = 0 \quad \Longrightarrow \quad \omega + \omega^2 = -1,$$

since they are the roots of $x^3 - 1 = (x-1)(x^2+x+1) = 0$ and the sum of roots of $x^2 + x + 1$ is -1.

Step 1 — Reduce the high power ω^4: Write the exponent as a multiple of 3 plus a remainder:

$$\omega^4 = \omega^3 \cdot \omega = 1 \cdot \omega = \omega.$$

Step 2 — Substitute back: The expression becomes

$$\omega^4 + \omega^2 = \omega + \omega^2.$$

Step 3 — Apply the sum identity: From $\omega + \omega^2 = -1$ we get

$$\omega^4 + \omega^2 = -1.$$

Step 4 — Verify with the explicit value: Taking $\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ gives $\omega^2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$, and their sum is $-1 + 0i = -1$, confirming the result.

Why other options are wrong:

- (B) 1 would follow only if $\omega^4 + \omega^2$ were treated as $\omega^6 = 1$, ignoring that the powers are added, not multiplied.
- (C) 0 wrongly assumes the two terms cancel.
- (D) 2 ignores the relation $1 + \omega + \omega^2 = 0$ altogether.

Final Answer: $\omega^4 + \omega^2 = -1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q5

Q6.

Solution

Concept — The discriminant test: For a quadratic $ax^2 + bx + c = 0$ the discriminant $D = b^2 - 4ac$ is the quantity under the square root in the quadratic formula $x = \dfrac{-b \pm \sqrt{D}}{2a}$. Its sign controls the nature of the roots: if $D > 0$ the roots are real and distinct; if $D = 0$ they are real and equal (a repeated root); and if $D < 0$ they are a pair of complex conjugates.

Step 1 — Identify the coefficients: Comparing $2x^2 - 7x + 3 = 0$ with $ax^2 + bx + c$ gives $a = 2$, $b = -7$, $c = 3$.

Step 2 — Compute the discriminant:

$$D = b^2 - 4ac = (-7)^2 - 4(2)(3) = 49 - 24 = 25.$$

Step 3 — Interpret the sign: Since $D = 25 > 0$, the roots are real and distinct. Moreover $25 = 5^2$ is a perfect square, so the roots are in fact rational.

Step 4 — Confirm by factoring: $2x^2 - 7x + 3 = (2x - 1)(x - 3)$, giving roots $x = \frac{1}{2}$ and $x = 3$, which are indeed two different real numbers, consistent with $D > 0$.

Why other options are wrong:

- (A) complex roots would require $D < 0$, but here $D = 25 > 0$.
- (C) real and equal roots need $D = 0$.
- (D) "imaginary and equal" is not possible for real coefficients and again needs $D = 0$.

Final Answer: real and distinct $\Rightarrow$ $\boxed{\text{B}}$

Answer: (B) Go Back to Q6

Q7.

Solution

Concept — Symmetric functions and Vieta's formulas: For $x^2 + px + q = 0$ with roots α, β, Vieta's formulas give the sum $\alpha + \beta = -p$ and the product $\alpha\beta = q$. Many expressions in the roots can be rewritten using only these two quantities, avoiding the need to solve for the roots themselves. In particular, the algebraic identity $(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$ rearranges to

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta.$$

Step 1 — Read off sum and product: Comparing $x^2 - 6x + 4 = 0$ with $x^2 + px + q$ gives $p = -6$ and $q = 4$, so

$$\alpha + \beta = 6, \qquad \alpha\beta = 4.$$

Step 2 — Substitute into the identity:

$$\alpha^2 + \beta^2 = (6)^2 - 2(4).$$

Step 3 — Simplify:

$$\alpha^2 + \beta^2 = 36 - 8 = 28.$$

Step 4 — Verify with the actual roots: Solving gives $\alpha, \beta = 3 \pm \sqrt{5}$. Then $\alpha^2 + \beta^2 = (3 + \sqrt{5})^2 + (3 - \sqrt{5})^2 = (14 + 6\sqrt{5}) + (14 - 6\sqrt{5}) = 28$, matching exactly.

Why other options are wrong:

- (A) 36 takes only $(\alpha + \beta)^2$ and forgets the $-2\alpha\beta$ term.
- (B) 44 adds $2\alpha\beta$ instead of subtracting it $(36 + 8)$.
- (D) 32 uses an incorrect product term such as $-2(2)$.

Final Answer: $\alpha^2 + \beta^2 = 28 \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q7**

Q8.

Solution

Concept — The row-by-column rule: In a matrix product AB, the entry sitting in row i and column j is obtained by pairing the i-th row of A with the j-th column of B, multiplying corresponding terms and summing. Symbolically,

$$(AB)_{ij} = \sum_k A_{ik}\, B_{kj}.$$

We only need the single $(1, 2)$ entry, so we never have to compute the full product.

Step 1 — Pick out the relevant row and column: Row 1 of A is $(1,\ 2)$, and column 2 of B is $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$.

Step 2 — Form the dot product term by term:

$$(AB)_{12} = (1)(0) + (2)(5).$$

Step 3 — Simplify:

$$(AB)_{12} = 0 + 10 = 10.$$

Step 4 — Cross-check with the full product: Computing all four entries gives $AB = \begin{pmatrix} 1\cdot2+2\cdot1 & 1\cdot0+2\cdot5 \\ 3\cdot2+4\cdot1 & 3\cdot0+4\cdot5 \end{pmatrix} = \begin{pmatrix} 4 & 10 \\ 10 & 20 \end{pmatrix}$, whose $(1,2)$ entry is indeed 10.

Why other options are wrong:

- (A) 2 takes only the first product $1\cdot0=0$ or A_{12} alone.
- (C) 14 pairs the wrong row, e.g. uses row 2 with column 1.
- (B) 4 is the $(1,1)$ entry, obtained by multiplying the wrong column.

Final Answer: $(1,2)$ entry $= 10 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — The fundamental adjoint identity: The adjoint (adjugate) of a square matrix A is the transpose of its cofactor matrix. A central theorem of matrix algebra states that multiplying a matrix by its adjoint, in either order, returns the determinant times the identity:

$$A\cdot(\text{adj }A) = (\text{adj }A)\cdot A = |A|\,I.$$

The reason is that the (i,i) entries of the product are cofactor expansions of $\det A$ (giving $|A|$ on the diagonal), while the off-diagonal entries are expansions along a "wrong" row and vanish.

Step 1 — Recall the structure of the result: The right-hand side is a *matrix*: the scalar $|A|$ multiplying the identity I, so every diagonal entry equals $|A|$ and every off-diagonal entry is 0.

Step 2 — Connect to the inverse: Dividing both sides by the scalar $|A|$ (when $|A|\neq0$) gives $A\cdot\dfrac{\text{adj }A}{|A|} = I$, which is exactly the formula for the inverse $A^{-1} =$

$\frac{1}{|A|}$ adj A. This dependence confirms the identity must read $|A|\, I$.

Step 3 — Quick 2×2 check: For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, adj $A = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ and A adj $A =$ $\begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix} = (ad-bc)I = |A|\, I$, confirming the rule.

Why other options are wrong:

- (B) I omits the scalar factor $|A|$, true only when $|A| = 1$.
- (C) $|A|$ alone is a scalar, but the product of two matrices must be a matrix.
- (D) A^2 has no connection to the adjoint identity.

Final Answer: $A \cdot (\text{adj } A) = |A|\, I \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Cofactor (Laplace) expansion: A 3×3 determinant can be evaluated by expanding along any row or column. Expanding along the first row, each entry is multiplied by its 2×2 minor and by the sign pattern $+, -, +$:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}.$$

Choosing a row with a zero entry reduces the work, since that term drops out.

Step 1 — Write the cofactor expansion: Expanding the given determinant along the first row $(1, 0, 2)$:

$$1\begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} + 2\begin{vmatrix} 0 & 3 \\ 2 & 1 \end{vmatrix}.$$

Step 2 — Evaluate each 2×2 minor: A 2×2 determinant is "main diagonal minus anti-diagonal":

$$\begin{vmatrix} 3 & 1 \\ 1 & 0 \end{vmatrix} = 3\cdot 0 - 1\cdot 1 = -1, \qquad \begin{vmatrix} 0 & 3 \\ 2 & 1 \end{vmatrix} = 0\cdot 1 - 3\cdot 2 = -6.$$

Step 3 — Combine: The middle term vanishes because its coefficient is 0, leaving

$$1(-1) - 0 + 2(-6) = -1 - 12 = -13.$$

Step 4 — Cross-check along the first column: Expanding along column 1 gives $1(3 \cdot 0 - 1 \cdot 1) - 0 + 2(0 \cdot 1 - 2 \cdot 3) = -1 + 2(-6) = -13$, the same value.

Why other options are wrong:

- (A) -11 mishandles a minor, e.g. taking the second minor as -5.
- (C) 13 drops the overall sign.
- (D) 7 comes from a sign error in the cofactor pattern.

Final Answer: determinant $= -13 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

Q11.

Solution

Concept — Area as a test for collinearity: Three points form a genuine triangle only when they are not on one line. The signed area of the triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is

$$\text{Area} = \tfrac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|.$$

If this area collapses to 0 the "triangle" is degenerate, meaning the three points lie on a single straight line, i.e. they are collinear.

Step 1 — Label the points: Take $(x_1, y_1) = (1, 2)$, $(x_2, y_2) = (2, 4)$, $(x_3, y_3) = (3, 6)$.

Step 2 — Substitute into the formula:

$$\text{Area} = \tfrac{1}{2}\left|1(4 - 6) + 2(6 - 2) + 3(2 - 4)\right|.$$

Step 3 — Simplify the bracket:

$$1(-2) + 2(4) + 3(-2) = -2 + 8 - 6 = 0, \qquad \text{so} \qquad \text{Area} = \tfrac{1}{2}\,|0| = 0.$$

Step 4 — Independent confirmation by slope: The slope from $(1, 2)$ to $(2, 4)$ is $\frac{4-2}{2-1} = 2$, and from $(2, 4)$ to $(3, 6)$ is $\frac{6-4}{3-2} = 2$. Equal slopes through a shared point means the points are collinear, so a zero area is exactly what we expect.

Why other options are wrong:

- (A) 1, (B) 3 and (D) 2 are all non-zero, which would contradict the collinearity that the slope check independently establishes.

Final Answer: area $= 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q11

Q12.

Solution

Concept — Permutations of a multiset: If a collection of n objects contains identical items, swapping those identical items produces no new arrangement, so the plain $n!$ overcounts. We divide by the factorial of each repeat count. For letters repeated $p, q, \ldots$ times the number of distinguishable arrangements is

$$\frac{n!}{p!\,q!\,\cdots}.$$

Step 1 — Audit the letters of "LEVEL": The word has 5 letters: L, E, V, E, L. The letter L appears twice and E appears twice; V appears once. So $n = 5$ with repeat counts 2 and 2.

Step 2 — Write the formula:

$$\frac{n!}{p!\,q!} = \frac{5!}{2!\,2!}.$$

Step 3 — Evaluate: With $5! = 120$ and $2!\,2! = 2 \times 2 = 4$,

$$\frac{120}{4} = 30.$$

Step 4 — Why the division is correct: The two L's can be interchanged ($2!$ ways) and the two E's interchanged ($2!$ ways) without changing the visible word, so each distinct word was counted $2! \times 2! = 4$ times in $5!$. Dividing restores the true count of 30.

Why other options are wrong:

- (A) $120 = 5!$ treats all letters as distinct, ignoring both repeats.
- (B) 60 divides by only one $2!$, accounting for a single repeated letter.
- (C) 20 divides by an extra factor, e.g. $\frac{5!}{3!}$.

Final Answer: $\frac{5!}{2!\,2!} = 30 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Combinations versus permutations: Selecting a committee is an unordered choice: only *who* is chosen matters, not the order of selection. The number of ways to choose r objects from n without regard to order is the binomial coefficient

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}.$$

A useful symmetry, $\binom{n}{r} = \binom{n}{n-r}$, often shortens the arithmetic.

Step 1 — Set up the count: We choose $r = 4$ members from $n = 7$ persons, so the answer is $\binom{7}{4}$.

Step 2 — Use the symmetry to ease computation: Since $\binom{7}{4} = \binom{7}{3}$, compute the smaller form:

$$\binom{7}{3} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1}.$$

Step 3 — Evaluate:

$$\frac{210}{6} = 35.$$

Step 4 — Cross-check from permutations: The number of *ordered* selections is ${}^7P_4 = 7 \cdot 6 \cdot 5 \cdot 4 = 840$; dividing by the $4! = 24$ orderings of any fixed group of 4 gives $840/24 = 35$, confirming the result.

Why other options are wrong:

- (C) $840 = {}^7P_4$ wrongly counts the order of selection.
- (D) $210 = {}^7P_3$ also counts order (and chooses 3).
- (B) $28 = \binom{8}{2}$ is an arithmetic slip with the wrong n or r.

Final Answer: $\binom{7}{4} = 35 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept — Circular and garland arrangements: Arranging n distinct objects in a *circle* gives $(n-1)!$ ways, because fixing one object removes the n equivalent rotations of a line arrangement. A *garland* (or necklace) can additionally be flipped over, so each circular arrangement and its mirror image are the same physical object. We therefore halve the circular count:

$$\frac{(n-1)!}{2}.$$

Step 1 — Apply the rotation reduction: For $n = 6$ beads, the number of circular arrangements is

$$(n-1)! = (6-1)! = 5! = 120.$$

Step 2 — Account for the flip symmetry: Since a garland read clockwise is identical to the same garland read anticlockwise, divide by 2:

$$\frac{5!}{2} = \frac{120}{2} = 60.$$

Step 3 — Interpret: Each distinct necklace has been counted exactly twice (once for each direction) among the 120 circular orders, so 60 genuinely different garlands exist.

Why other options are wrong:

- (A) $120 = (n-1)!$ is the circular count but forgets the division by 2 for the flip.
- (C) $720 = 6!$ treats the beads as a straight line, ignoring rotations entirely.
- (D) $360 = 6!/2$ halves the line count instead of the circular count.

Final Answer: $\frac{5!}{2} = 60 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q14

Q15.

Solution

Concept — Vieta's formulas for a cubic: For a cubic $ax^3 + bx^2 + cx + d = 0$ with roots α, β, γ, the symmetric functions of the roots relate to the coefficients with alternating signs:

$$\alpha + \beta + \gamma = -\frac{b}{a}, \qquad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}, \qquad \alpha\beta\gamma = -\frac{d}{a}.$$

The product picks up the sign $(-1)^3 = -1$, because the constant term equals $-a\,\alpha\beta\gamma$ when the polynomial is written as $a(x-\alpha)(x-\beta)(x-\gamma)$.

Step 1 — Identify the relevant coefficients: In $2x^3 - 3x^2 + 4x - 10 = 0$ the leading coefficient is $a = 2$ and the constant term is $d = -10$.

Step 2 — Apply the product formula:

$$\alpha\beta\gamma = -\frac{d}{a} = -\frac{-10}{2}.$$

Step 3 — Simplify:

$$\alpha\beta\gamma = \frac{10}{2} = 5.$$

Step 4 — Consistency check on signs: Expanding $a(x-\alpha)(x-\beta)(x-\gamma)$ the constant term is $-a\,\alpha\beta\gamma$; setting this equal to $d = -10$ with $a = 2$ gives $-2\,\alpha\beta\gamma = -10$, so $\alpha\beta\gamma = 5$, matching Step 3.

Why other options are wrong:

- (A) -5 drops the leading minus sign in $-d/a$.
- (D) -10 uses d directly without dividing by a or fixing the sign.
- (B) 10 forgets to divide by $a = 2$.

Final Answer: $\alpha\beta\gamma = 5 \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q15

Q16.

Solution

Concept — Transforming the roots of an equation: If a polynomial equation $f(x) = 0$ has root $x = r$, then the equation $f(x-k) = 0$ has root $x = r+k$, because substituting $x = r+k$ gives $f((r+k)-k) = f(r) = 0$. So to raise every root by 2 we replace x with $x-2$ throughout. (One can also rebuild the equation from the new sum and product of roots.)

Step 1 — Perform the substitution $x \to x-2$: Starting from $x^2 - 3x + 2 = 0$,

$$(x-2)^2 - 3(x-2) + 2 = 0.$$

Step 2 — Expand each piece:

$$(x-2)^2 = x^2 - 4x + 4, \qquad -3(x-2) = -3x + 6.$$

Step 3 — Combine and simplify:

$$x^2 - 4x + 4 - 3x + 6 + 2 = x^2 - 7x + 12 = 0.$$

Step 4 — Verify via Vieta: The original roots are 1 and 2, so the new roots should be 3 and 4. Their sum is 7 and product is 12, giving $x^2 - 7x + 12 = 0$, exactly the equation found above.

Why other options are wrong:

- (A) $x^2 - 3x + 2 = 0$ is the original equation, with roots $1, 2$ (unshifted).
- (B) $x^2 + 7x + 12 = 0$ has roots $-3, -4$, i.e. a shift by -2 instead of $+2$.
- (C) $x^2 - 5x + 6 = 0$ has roots $2, 3$, a shift of only $+1$.

Final Answer: $x^2 - 7x + 12 = 0 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — A $0/0$ trigonometric limit: Substituting $x = 0$ directly gives $\frac{1-1}{0} = \frac{0}{0}$, an indeterminate form, so the limit must be resolved algebraically. The key tool is the half-angle identity $1 - \cos x = 2\sin^2 \frac{x}{2}$ together with the fundamental limit $\lim_{t \to 0} \frac{\sin t}{t} = 1$ (with x in radians).

Step 1 — Rewrite the numerator: Using $1 - \cos x = 2\sin^2 \frac{x}{2}$,

$$\frac{1 - \cos x}{x^2} = \frac{2\sin^2(x/2)}{x^2}.$$

Step 2 — Engineer the standard limit: Write $x^2 = 4 \cdot (x/2)^2$ so that the denominator matches the inner angle:

$$\frac{2\sin^2(x/2)}{4\,(x/2)^2} = \frac{1}{2}\left(\frac{\sin(x/2)}{x/2}\right)^2.$$

Step 3 — Take the limit: As $x \to 0$, $x/2 \to 0$ and $\frac{\sin(x/2)}{x/2} \to 1$, hence

$$\frac{1}{2}\,(1)^2 = \frac{1}{2}.$$

Step 4 — Cross-check by Taylor series: Near 0, $\cos x \approx 1 - \frac{x^2}{2}$, so $1 - \cos x \approx \frac{x^2}{2}$ and $\frac{1 - \cos x}{x^2} \approx \frac{x^2/2}{x^2} = \frac{1}{2}$, confirming the limit.

Why other options are wrong:

- (B) 1 confuses this with $\lim \frac{\sin x}{x}$ and drops the factor $\frac{1}{2}$.
- (D) 2 inverts the constant.
- (C) 0 ignores that the numerator shrinks like x^2, matching the denominator's order.

Final Answer: the limit is $\frac{1}{2} \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q17

Q18.

Solution

Concept — The chain rule for composite functions: When y is a function of u and u is a function of x, the derivative multiplies the outer rate by the inner rate: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$. For the sine of an expression this reads

$$\frac{d}{dx}\sin(u) = \cos(u) \cdot \frac{du}{dx}.$$

The crucial point is not to forget the "inner derivative" $\frac{du}{dx}$.

Step 1 — Choose the inner function: Let $u = x^2$, so that $y = \sin u$.

Step 2 — Differentiate inner and outer separately:

$$\frac{du}{dx} = 2x, \qquad \frac{dy}{du} = \cos u = \cos(x^2).$$

Step 3 — Multiply the two rates:

$$\frac{dy}{dx} = \cos(x^2) \cdot 2x = 2x\cos(x^2).$$

Step 4 — Spot-check at a value: At $x = 0$, $\frac{dy}{dx} = 2(0)\cos 0 = 0$, which is correct since $\sin(x^2)$ has a horizontal tangent at the origin (its graph flattens there). This sanity check supports the formula.

Why other options are wrong:

- (A) $\cos(x^2)$ omits the inner derivative $2x$.
- (C) $2x\sin(x^2)$ wrongly differentiates $\sin$ as $\sin$ instead of $\cos$.
- (D) $-2x\cos(x^2)$ inserts a spurious minus sign (that belongs to $\cos$, not $\sin$).

Final Answer: $2x\cos(x^2) \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q18

Q19.

Solution

Concept — Maximising with calculus: A smooth function reaches an interior extreme value where its derivative vanishes. For $f(x) = -x^2 + 4x + 1$ the leading coefficient is negative, so the parabola opens downward and the single stationary point is a *maximum*. We find it by solving $f'(x) = 0$ and then substituting back to get the maximum value.

Step 1 — Differentiate:

$$f'(x) = -2x + 4.$$

Step 2 — Solve for the critical point:

$$-2x + 4 = 0 \implies x = 2.$$

Step 3 — Confirm it is a maximum: The second derivative is $f''(x) = -2 < 0$, so the curve is concave down and $x = 2$ gives a maximum, not a minimum.

Step 4 — Evaluate the maximum value:

$$f(2) = -(2)^2 + 4(2) + 1 = -4 + 8 + 1 = 5,$$

which matches the marked vertex $(2, 5)$ on the graph.

Why other options are wrong:

- (B) $1 = f(0)$ is merely the y-intercept, not the peak.
- (D) 2 is the x-coordinate of the vertex, not the function's value.
- (A) 4 is a partial evaluation (e.g. stopping at $4(2)/2$ or $-4 + 8$).

Final Answer: maximum value $= 5 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q19

Q20.

Solution

Concept — Related rates: When two quantities are linked by an equation and both change with time, differentiating the equation with respect to t links their rates of change. The area of a circle is $A = \pi r^2$; differentiating with respect to t and using the chain rule on r^2 gives

$$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt} = 2\pi r\,\frac{dr}{dt}.$$

Step 1 — List the known rates and values: The radius grows at $\frac{dr}{dt} = 2$ cm/s, and we evaluate at the instant $r = 5$ cm.

Step 2 — Substitute into the rate equation:

$$\frac{dA}{dt} = 2\pi(5)(2).$$

Step 3 — Simplify:

$$\frac{dA}{dt} = 2\pi \cdot 10 = 20\pi \text{ cm}^2/\text{s}.$$

Step 4 — Dimensional check: The units are (cm)×(cm/s)=cm^2/s, which is the correct unit for a rate of change of area, confirming the setup.

Why other options are wrong:

- (A) $10\pi = 2\pi r$ drops the factor $\frac{dr}{dt} = 2$.
- (B) $25\pi = \pi r^2$ is the area itself, not its rate of change.
- (C) 5π uses only πr and is far too small.

Final Answer: $\frac{dA}{dt} = 20\pi \text{ cm}^2/\text{s} \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q20**

Q21.

Solution

Concept — Solving $\frac{dy}{dx} = f(x)$ by direct integration: When the derivative of y depends only on x, the general solution is found by integrating the right-hand side once. Each indefinite integration introduces one arbitrary constant C, which is why the "general" solution of a first-order equation contains a single parameter. Here we integrate

$$\frac{dy}{dx} = 3x^2.$$

Step 1 — Integrate both sides with respect to x:

$$y = \int 3x^2\,dx.$$

Step 2 — Apply the power rule $\int x^n\,dx = \frac{x^{n+1}}{n+1}$:

$$y = 3 \cdot \frac{x^3}{3} + C.$$

Step 3 — Simplify: The factor 3 cancels the 3 in the denominator,

$$y = x^3 + C.$$

Step 4 — Verify by differentiating back: $\dfrac{d}{dx}(x^3 + C) = 3x^2$, which is exactly the original right-hand side, confirming the solution.

Why other options are wrong:

- (B) $6x + C$ differentiates $3x^2$ instead of integrating it.
- (C) $3x^3 + C$ forgets to divide by the new power 3.
- (D) $x^2 + C$ uses the wrong power (lowers instead of raises).

Final Answer: $y = x^3 + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept — The power rule for integration: To integrate a power of x, raise the exponent by one and divide by the new exponent:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad (n \neq -1).$$

This is the exact reverse of the differentiation power rule, and the $+C$ records the family of antiderivatives that all differ by a constant.

Step 1 — Raise the exponent: With $n = 5$ the new exponent is $n + 1 = 6$.

Step 2 — Divide by the new exponent and add the constant:

$$\int x^5\,dx = \frac{x^6}{6} + C.$$

Step 3 — Check by differentiating: $\frac{d}{dx}\left(\frac{x^6}{6} + C\right) = \frac{6x^5}{6} = x^5$, recovering the integrand and confirming the answer.

Why other options are wrong:

- (A) $5x^4 + C$ differentiates x^5 instead of integrating it.
- (C) $\frac{x^6}{5} + C$ divides by the old exponent $n = 5$ rather than $n + 1 = 6$.
- (D) $6x^6 + C$ multiplies by 6 instead of dividing by it.

Final Answer: $\frac{x^6}{6} + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q22

Q23.

Solution

Concept — Symmetry of definite integrals: A function is *even* if $f(-x) = f(x)$; its graph is symmetric about the y-axis, so the area to the left of 0 equals the area to the right. Over a symmetric interval $[-a, a]$ this lets us halve the work:

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx \quad (f \text{ even}), \qquad \int_{-a}^{a} f(x)\,dx = 0 \quad (f \text{ odd}).$$

Step 1 — Determine the parity of $f(x) = x^2$: $f(-x) = (-x)^2 = x^2 = f(x)$, so f

is even. Therefore

$$\int_{-2}^{2} x^2\,dx = 2\int_{0}^{2} x^2\,dx.$$

Step 2 — Find the antiderivative: $\int x^2\,dx = \frac{x^3}{3}$.

Step 3 — Apply the limits and the factor 2**:**

$$2\left[\frac{x^3}{3}\right]_0^2 = 2\left(\frac{8}{3} - 0\right) = \frac{16}{3}.$$

Step 4 — Direct cross-check: Without the symmetry trick, $\left[\frac{x^3}{3}\right]_{-2}^{2} = \frac{8}{3} - \frac{-8}{3} = \frac{16}{3}$, the same value.

Why other options are wrong:

- (A) 0 wrongly treats x^2 as an *odd* function.
- (B) $\frac{8}{3}$ computes only the half-integral and forgets to double it.
- (D) 8 mis-integrates (e.g. omits the division by 3).

Final Answer: the integral is $\frac{16}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q23**

Q24.

Solution

Concept — Area as a definite integral: The area trapped between a curve $y = f(x)$ that lies above the x-axis, the x-axis itself, and two vertical lines $x = 0$ and $x = a$ equals the definite integral $\int_0^a f(x)\,dx$. The integral sums infinitely many thin vertical strips of height y and width dx. Here $y = x^2 \geq 0$ on $[0, 3]$, so the integral gives the shaded area directly.

Step 1 — Set up the integral: The region runs from $x = 0$ to $x = 3$ under $y = x^2$, so

$$\text{Area} = \int_0^3 x^2\,dx.$$

Step 2 — Antidifferentiate: $\int x^2\,dx = \frac{x^3}{3}$.

Step 3 — Apply the limits:

$$\left[\frac{x^3}{3}\right]_0^3 = \frac{3^3}{3} - \frac{0^3}{3} = \frac{27}{3} = 9 \text{ sq. units.}$$

Step 4 — Reasonableness check: The bounding rectangle 3×9 (width 3, height $f(3) = 9$) has area 27; the area under a convex curve should be well below that, and $9 = \frac{1}{3} \cdot 27$ is the familiar "one-third of the box" result for $y = x^2$, confirming the answer.

Why other options are wrong:

- (B) 27 forgets to divide by 3 (uses x^3 at $x = 3$).
- (A) 3 integrates x instead of x^2.
- (C) 18 mis-integrates, e.g. doubling instead of dividing.

Final Answer: area $= 9$ sq. units $\Rightarrow$ D

Answer: (D) Go Back to Q24

Q25.

Solution

Concept — The Fundamental Theorem of Calculus: A definite integral is evaluated by finding any antiderivative F of the integrand and taking the difference of its values at the limits: $\int_a^b f(x)\,dx = F(b) - F(a)$. For $f(x) = x^2$ the antiderivative comes from the power rule, $F(x) = \frac{x^3}{3}$.

Step 1 — Write the antiderivative:

$$\int x^2\,dx = \frac{x^3}{3}.$$

Step 2 — Evaluate at the upper and lower limits:

$$\left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3} - 0.$$

Step 3 — Simplify:

$$\int_0^1 x^2\,dx = \frac{1}{3}.$$

Step 4 — Reasonableness check: On $[0, 1]$ the integrand x^2 stays between 0 and

1, so its integral (an average height times width 1) must lie in $(0, 1)$; $\frac{1}{3}$ fits, and it is smaller than $\int_0^1 x\,dx = \frac{1}{2}$ because $x^2 \le x$ there.

Why other options are wrong:

- (B) $\frac{1}{2}$ integrates x instead of x^2.
- (C) 1 skips the division by 3.
- (D) $\frac{2}{3}$ mis-divides (e.g. uses $\frac{2x^3}{3}$).

Final Answer: the integral is $\frac{1}{3} \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q25

Q26.

Solution

Concept — The distance formula from the Pythagorean theorem: The straight-line distance between two points is the length of the hypotenuse of the right triangle whose legs are the horizontal and vertical separations. With legs $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$, the Pythagorean theorem gives

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Step 1 — Compute the separations: From $(2, 3)$ to $(5, 7)$, $\Delta x = 5 - 2 = 3$ and $\Delta y = 7 - 3 = 4$.

Step 2 — Substitute into the formula:

$$d = \sqrt{3^2 + 4^2}.$$

Step 3 — Simplify:

$$d = \sqrt{9 + 16} = \sqrt{25} = 5.$$

Step 4 — Recognise the Pythagorean triple: The legs 3 and 4 form the classic 3–4–5 right triangle, so a hypotenuse of 5 is exactly as expected.

Why other options are wrong:

- (A) $7 = 3 + 4$ adds the legs instead of combining them under a square root.
- (D) 25 is d^2 and forgets the square root.
- (C) $\sqrt{7}$ subtracts the squares $(16 - 9)$ rather than adding them.

Final Answer: distance $= 5 \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q26

Q27.

Solution

Concept — The general equation of a circle: The equation $x^2+y^2+2gx+2fy+c=0$ is the expanded form of $(x+g)^2+(y+f)^2=g^2+f^2-c$. Reading off the completed squares, the centre is $(-g,-f)$ and the radius is $\sqrt{g^2+f^2-c}$. The essential step is to match the coefficients of x and y to $2g$ and $2f$, then halve and negate.

Step 1 — Match the linear coefficients: Compare $x^2+y^2-4x+6y-3=0$ with the standard form:

$$2g=-4 \Rightarrow g=-2, \qquad 2f=6 \Rightarrow f=3.$$

Step 2 — Apply the centre formula:

$$\text{centre}=(-g,-f)=(2,-3).$$

Step 3 — Verify by completing the square: $x^2-4x=(x-2)^2-4$ and $y^2+6y=(y+3)^2-9$, so the equation becomes $(x-2)^2+(y+3)^2=16$. The centre $(2,-3)$ and radius 4 read off here match the figure, where the marked centre lies at $(2,-3)$.

Why other options are wrong:

- (A) $(4,-6)$ uses $2g$ and $2f$ directly without halving.
- (D) $(2,3)$ keeps the sign of f instead of negating it.
- (B) $(-2,3)$ forgets to negate g.

Final Answer: centre $=(2,-3)\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q27

Q28.

Solution

Concept — Vertex form of a parabola: A parabola written as $y = a(x-h)^2 + k$ is in *vertex form*, and its turning point (vertex) sits at (h, k). The reason is that the squared term $(x-h)^2$ is smallest (zero) precisely when $x = h$, at which moment $y = k$; that lowest (or highest) point is the vertex. Note the minus sign inside the bracket: the x-coordinate is $+h$, not $-h$.

Step 1 — Match to vertex form: Compare $y = (x-2)^2 + 3$ with $y = (x-h)^2 + k$:

$$h = 2, \qquad k = 3.$$

Step 2 — Read off the vertex:

$$\text{vertex} = (h, k) = (2, 3).$$

Step 3 — Confirm it is the minimum: Since $(x-2)^2 \geq 0$, the smallest value of y is 3, attained at $x = 2$. So the curve dips to its lowest point at $(2, 3)$, matching the marked vertex V in the figure.

Why other options are wrong:

- (B) $(-2, 3)$ mishandles the sign inside the bracket (uses $-h$).
- (C) $(2, 0)$ ignores the vertical shift $k = 3$.
- (A) $(0, 3)$ reads the constant as the x-coordinate instead of the y-coordinate.

Final Answer: vertex $= (2, 3) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q28

Q29.

Solution

Concept — Angle between two lines in 3D: The angle between two lines is the angle between their direction vectors, obtained from the dot-product formula. For direction ratios (a_1, b_1, c_1) and (a_2, b_2, c_2),

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}}.$$

If the numerator (the dot product) is zero, the lines are perpendicular.

Step 1 — Compute the dot product of the direction ratios:

$$(1)(0) + (0)(1) + (0)(0) = 0.$$

Step 2 — Form the cosine: Both vectors are unit length, so $\cos\theta = \dfrac{0}{1 \cdot 1} = 0$.

Step 3 — Solve for the angle:

$$\cos\theta = 0 \implies \theta = 90^\circ.$$

Step 4 — Geometric check: The directions $(1, 0, 0)$ and $(0, 1, 0)$ are just the x- and y-axes, which meet at a right angle, confirming $\theta = 90^\circ$.

Why other options are wrong:

- (B) 0° would require parallel (proportional) direction ratios.
- (C) 45° and (D) 60° both need a non-zero dot product, but here it is exactly 0.

Final Answer: $\theta = 90^\circ \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q29

Q30.

Solution

Concept — Angle between two planes: The angle between two planes is defined as the angle between their normal vectors, which are read directly from the coefficients of x, y, z in each equation. If the normals point in the same (or proportional) direction the planes are parallel and the angle is 0°; if the normals are perpendicular the planes meet at 90°.

Step 1 — Extract the normals: For $x + y + z = 1$ the normal is $\vec{n}_1 = (1, 1, 1)$, and for $x + y + z = 5$ it is $\vec{n}_2 = (1, 1, 1)$. The two normals are identical.

Step 2 — Test for parallelism: Since $\vec{n}_2 = 1 \cdot \vec{n}_1$, the normals are scalar multiples, so they point in the same direction.

Step 3 — Conclude the angle: Parallel normals mean the planes are parallel, and the angle between parallel planes is

$$\theta = 0^\circ.$$

Step 4 — Cross-check with the cosine formula: $\cos\theta = \dfrac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \dfrac{1+1+1}{\sqrt{3}\cdot\sqrt{3}} = \dfrac{3}{3} = 1$, so $\theta = \cos^{-1} 1 = 0°$, confirming the result. (The planes differ only by the constant, so they never meet, exactly as parallel planes should.)

Why other options are wrong:

- (A) $90°$ would require perpendicular normals (zero dot product).
- (C) $45°$ and (D) $60°$ both need non-parallel normals.

Final Answer: angle $= 0° \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q30

Q31.

Solution

Concept — Angle between vectors from the dot product: The dot product encodes the angle between two vectors through $\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$, so

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}.$$

A zero dot product forces $\cos\theta = 0$ and hence a right angle, which is the geometric meaning of orthogonality.

Step 1 — Compute the dot product: With $\vec{a} = \hat{i}+\hat{j} = (1,1)$ and $\vec{b} = \hat{i}-\hat{j} = (1,-1)$,

$$\vec{a}\cdot\vec{b} = (1)(1) + (1)(-1) = 1 - 1 = 0.$$

Step 2 — Compute the magnitudes (for context): $|\vec{a}| = \sqrt{1^2+1^2} = \sqrt{2}$ and $|\vec{b}| = \sqrt{1^2+(-1)^2} = \sqrt{2}$, both non-zero, so the cosine is well defined.

Step 3 — Find the angle:

$$\cos\theta = \frac{0}{\sqrt{2}\cdot\sqrt{2}} = 0 \implies \theta = 90°.$$

Step 4 — Geometric picture: $\vec{a}$ points along the line $y = x$ and $\vec{b}$ along $y = -x$; these two diagonals of the unit square are perpendicular, confirming the right angle.

Why other options are wrong:

- (A) 0° and (D) 180° require parallel or antiparallel vectors (dot product $\pm|\vec{a}||\vec{b}|$).
- (B) 45° needs a non-zero dot product ($\cos 45^\circ = \frac{1}{\sqrt{2}} \neq 0$).

Final Answer: angle $= 90^\circ \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q31

Q32.

Solution

Concept — Cross product of planar vectors: The cross product of two vectors lying in the xy-plane points along $\hat{k}$ (perpendicular to that plane), with magnitude equal to the area of the parallelogram they span. From the determinant definition $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & 0 \\ b_1 & b_2 & 0 \end{vmatrix}$, only the $\hat{k}$ component survives:

$$\vec{a} \times \vec{b} = (a_1 b_2 - a_2 b_1)\,\hat{k}.$$

Step 1 — Read the components: For $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = 3\hat{i} + \hat{j}$, $a_1 = 1,\ a_2 = 2,\ b_1 = 3,\ b_2 = 1$.

Step 2 — Apply the cross-product formula:

$$a_1 b_2 - a_2 b_1 = (1)(1) - (2)(3).$$

Step 3 — Simplify:

$$(1) - (6) = -5, \qquad \text{so} \qquad \vec{a} \times \vec{b} = -5\,\hat{k}.$$

Step 4 — Interpret the sign and magnitude: The magnitude 5 is the area of the parallelogram with sides $\vec{a}$ and $\vec{b}$ (the dashed figure), and the negative sign means $\vec{a} \times \vec{b}$ points in the $-\hat{k}$ direction (into the page), consistent with $\vec{b}$ lying clockwise from $\vec{a}$.

Why other options are wrong:

- (A) $7\hat{k}$ adds the products $(1 + 6)$ instead of subtracting.
- (C) $\hat{k}$ uses the wrong terms, e.g. $a_1 b_1 - a_2 b_2$ done incorrectly.
- (B) 0 would require $\vec{a}$ and $\vec{b}$ to be parallel, which they are not.

Final Answer: $\vec{a} \times \vec{b} = -5\hat{k} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q32

Q33.

Solution

Concept — The scalar triple product: The scalar triple product $[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$ equals the signed volume of the parallelepiped built on the three vectors, and it is computed as the determinant whose rows are the components:

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}.$$

A non-zero value means the three vectors are non-coplanar.

Step 1 — Assemble the determinant: With $\vec{a} = \hat{\imath} = (1,0,0)$, $\vec{b} = \hat{\jmath} = (0,1,0)$, $\vec{c} = \hat{k} = (0,0,1)$,

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}.$$

Step 2 — Evaluate the determinant: This is the determinant of the identity matrix, which equals the product of its diagonal entries:

$$1 \cdot 1 \cdot 1 = 1.$$

Step 3 — Cross-check with the dot/cross definition: $\hat{\jmath} \times \hat{k} = \hat{\imath}$, so $\hat{\imath} \cdot (\hat{\jmath} \times \hat{k}) = \hat{\imath} \cdot \hat{\imath} = 1$, agreeing with the determinant.

Step 4 — Geometric meaning: The three unit vectors span a unit cube, whose volume is 1, exactly the value obtained, and the positive sign reflects the right-handed orientation $\hat{\imath}, \hat{\jmath}, \hat{k}$.

Why other options are wrong:

- (B) 0 would mean the vectors are coplanar, but unit axes are mutually perpendicular.
- (C) 3 wrongly sums the diagonal entries instead of multiplying.
- (D) -1 reverses the orientation (left-handed ordering).

Final Answer: $[\vec{a}\,\vec{b}\,\vec{c}] = 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q33

Q34.

Solution

Concept — The median of a data set: The median is the central value once the data are arranged in increasing order; it splits the data into a lower and an upper half. For an *odd* number n of observations the median is the single middle entry, located at position $\frac{n+1}{2}$. Crucially, the data must be sorted first, because the median depends on order, not on the position in which the values were listed.

Step 1 — Arrange the data in ascending order: The values $7, 3, 9, 5, 11$ become

$$3,\ 5,\ 7,\ 9,\ 11.$$

Step 2 — Locate the middle position: With $n = 5$, the median position is $\frac{n+1}{2} = \frac{6}{2} = 3$, the third entry.

Step 3 — Read off the median: The third value in the sorted list is

$$\text{median} = 7.$$

Step 4 — Check the split: Two values $(3, 5)$ lie below 7 and two $(9, 11)$ lie above, so 7 correctly divides the data in half.

Why other options are wrong:

- (A) 9 and (C) 5 are the values just above and below the centre, not the centre itself.
- (D) 11 is the maximum, not the median.

Final Answer: median $= 7 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q34

Q35.

Solution

Concept — The addition theorem of probability: This is the probability version of inclusion–exclusion. When we add $P(A)$ and $P(B)$, the outcomes common to both events are counted twice, so we subtract the overlap once to avoid double-counting:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Step 1 — List the given probabilities: $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.2$.

Step 2 — Substitute into the theorem:

$$P(A \cup B) = 0.5 + 0.4 - 0.2.$$

Step 3 — Simplify: $0.5 + 0.4 = 0.9$, then $0.9 - 0.2 = 0.7$, so

$$P(A \cup B) = 0.7.$$

Step 4 — Validity check: A probability must lie in $[0, 1]$; 0.7 does, and it is at least as large as each of $P(A) = 0.5$ and $P(B) = 0.4$, exactly as a union should be, which supports the answer.

Why other options are wrong:

- (A) 0.9 forgets to subtract the overlap $P(A \cap B)$.
- (D) 1.1 *adds* the overlap and even exceeds 1, which is impossible for a probability.
- (B) 0.6 subtracts the overlap twice.

Final Answer: $P(A \cup B) = 0.7 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q35

Q36.

Solution

Concept — Mean of a binomial distribution: A binomial distribution counts the number of successes in n independent trials, each with success probability p. Since each trial contributes an expected p successes and expectations add, the total expected (mean) number of successes is

$$\text{mean} = np.$$

By contrast, the variance is npq with $q = 1 - p$, a separate quantity not asked for here.

Step 1 — Identify the parameters: $n = 10$ trials and $p = 0.4$ success probability.

Step 2 — Apply the mean formula:

$$\text{mean} = np = 10 \times 0.4.$$

Step 3 — Compute:

$$\text{mean} = 4.$$

Step 4 — Sanity check against the variance: The variance would be $npq = 10(0.4)(0.6) = 2.4$, a different number; this contrast shows that 4 is the mean and not a misread variance.

Why other options are wrong:

- (A) $2.4 = npq$ is the variance, not the mean.
- (B) 10 is just n, the number of trials.
- (C) 0.4 is just p, the per-trial probability.

Final Answer: $\text{mean} = 4 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q36

Q37.

Solution

Concept — Standard trigonometric ratios: The tangent of an angle is the ratio of its sine to its cosine, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. The values at $60°$ come from the 30–60–90 right triangle, in which the sides are in the ratio $1 : \sqrt{3} : 2$, giving $\sin 60° = \dfrac{\sqrt{3}}{2}$ and $\cos 60° = \dfrac{1}{2}$.

Step 1 — Write tangent as a ratio:

$$\tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\sqrt{3}/2}{1/2}.$$

Step 2 — Simplify the compound fraction: Dividing by $\frac{1}{2}$ is the same as multiplying by 2, so the halves cancel:

$$\tan 60° = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}.$$

Step 3 — Cross-check geometrically: In the 30–60–90 triangle, $\tan 60° = \dfrac{\text{side opposite } 60°}{\text{side adjacent}} = \dfrac{\sqrt{3}}{1} = \sqrt{3}$, matching Step 2.

Why other options are wrong:

- (B) $\frac{1}{\sqrt{3}} = \tan 30°$, the complementary angle.
- (C) $1 = \tan 45°$.
- (D) $\frac{\sqrt{3}}{2} = \sin 60°$, confusing the sine with the tangent.

Final Answer: $\tan 60° = \sqrt{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q37

Q38.

Solution

Concept — The cosine double-angle identity: The identity $\cos 2\theta = 2\cos^2\theta - 1$ expresses the cosine of a doubled angle purely in terms of $\cos\theta$, so we can evaluate $\cos 2\theta$ without ever finding θ itself. It follows from $\cos 2\theta = \cos^2\theta - \sin^2\theta$ after replacing $\sin^2\theta$ with $1 - \cos^2\theta$.

Step 1 — Substitute the given value $\cos\theta = \frac{1}{2}$:

$$\cos 2\theta = 2\left(\tfrac{1}{2}\right)^2 - 1.$$

Step 2 — Evaluate the square: $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, so

$$\cos 2\theta = 2 \cdot \tfrac{1}{4} - 1 = \tfrac{1}{2} - 1.$$

Step 3 — Simplify:

$$\cos 2\theta = -\tfrac{1}{2}.$$

Step 4 — Cross-check via the explicit angle: $\cos\theta = \frac{1}{2}$ corresponds to $\theta = 60^\circ$, so $2\theta = 120^\circ$ and $\cos 120^\circ = -\frac{1}{2}$, confirming the identity-based result.

Why other options are wrong:

- (A) 1 drops the -1 term entirely.
- (C) $\frac{1}{2}$ keeps $2\cos^2\theta = \frac{1}{2}$ but forgets to subtract 1.
- (D) 0 miscomputes the square as $\frac{1}{2}$ instead of $\frac{1}{4}$.

Final Answer: $\cos 2\theta = -\dfrac{1}{2} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q38

Q39.

Solution

Concept — Principal value of the inverse cosine: The cosine function is many-to-one, so to make $\cos^{-1}$ a genuine function its output is restricted to the principal range $[0, \pi]$. The principal value of $\cos^{-1} x$ is therefore the *unique* angle in $[0, \pi]$ whose cosine is x. We must report the angle lying in that interval, not any coterminal alternative.

Step 1 — Set up the requirement: We need θ with $\cos\theta = \frac{1}{2}$ and $\theta \in [0, \pi]$.

Step 2 — Recall the standard angle: From the 30–60–90 triangle, $\cos 60^\circ = \frac{1}{2}$, and $60^\circ = \dfrac{\pi}{3}$ lies inside $[0, \pi]$:

$$\theta = \frac{\pi}{3}.$$

Step 3 — Confirm uniqueness: Within $[0, \pi]$ the cosine is strictly decreasing from 1 to -1, so it takes the value $\frac{1}{2}$ exactly once; $\dfrac{\pi}{3}$ is that single solution.

Why other options are wrong:

- (A) $\frac{\pi}{6}$ has $\cos = \frac{\sqrt{3}}{2}$, not $\frac{1}{2}$.
- (B) $\frac{\pi}{4}$ has $\cos = \frac{1}{\sqrt{2}}$.
- (D) $\frac{\pi}{2}$ has $\cos = 0$.

Final Answer: $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q39

Q40.

Solution

Concept — The sine rule: In any triangle the ratio of a side to the sine of its opposite angle is constant, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. Here side a is opposite angle A and side b is opposite angle B. Rearranging the first equality isolates the unknown side:

$$b = \frac{a \sin B}{\sin A}.$$

Step 1 — List the known quantities: $a = 5$, $A = 30°$, $B = 90°$, with $\sin 90° = 1$ and $\sin 30° = \frac{1}{2}$.

Step 2 — Substitute into the formula:

$$b = \frac{5 \sin 90°}{\sin 30°} = \frac{5 \cdot 1}{1/2}.$$

Step 3 — Simplify: Dividing by $\frac{1}{2}$ multiplies by 2:

$$b = 5 \times 2 = 10.$$

Step 4 — Cross-check with the right angle: Since $\angle B = 90°$, side $b = CA$ is the hypotenuse, and the side opposite the $30°$ angle is half the hypotenuse. Here $a = 5$ is opposite $30°$, so the hypotenuse is $2 \times 5 = 10$, confirming $b = 10$.

Why other options are wrong:

- (A) 5 keeps $b = a$, ignoring the differing opposite angles.
- (B) $\frac{5}{2}$ inverts the ratio (multiplies by $\sin 30°$ instead of dividing).
- (C) $5\sqrt{3}$ wrongly inserts $\sin 60°$ for the third side.

Final Answer: $b = 10 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	A
6	B	7	C	8	D	9	A	10	B
11	C	12	D	13	A	14	B	15	C
16	D	17	A	18	B	19	C	20	D
21	A	22	B	23	C	24	D	25	A
26	B	27	C	28	D	29	A	30	B
31	C	32	D	33	A	34	B	35	C
36	D	37	A	38	B	39	C	40	D