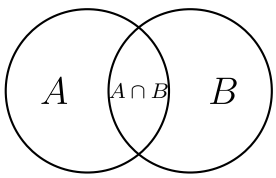
SRMJEEE Mathematics Sample Paper – 3

Duration: 47 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of **SRMJEEE** (SRM Joint Engineering Entrance Examination).
- Each correct answer carries **+1 mark**. There is **no negative marking**; an unattempted or wrong answer scores 0.
- Only **one** option is correct. Choose carefully.
- The actual SRMJEEE is a **computer-based test** conducted in remote-proctored online mode, with all sections sharing a common time window and no per-section limit.
- Personal calculators, mobile phones, log tables and other electronic gadgets are strictly prohibited.

Q1. For two finite sets, $n(A) = 18$, $n(B) = 14$ and $n(A \cap B) = 6$, as shown in the Venn diagram. The number of elements that belong to exactly one of the two sets is:



(A) 20

(B) 32

(C) 26

(D) 14

Q2. On the set of all real numbers $\mathbb{R}$, the relation R defined by "$a\,R\,b$ if and only if $a \leq b$" is:

(A) symmetric but not transitive

(B) reflexive and transitive but not symmetric

(C) symmetric and transitive only

(D) neither reflexive nor transitive

Q3. If $f(x) = 2x + 1$ and $g(x) = x^2$, then $(f \circ g)(x)$ equals:

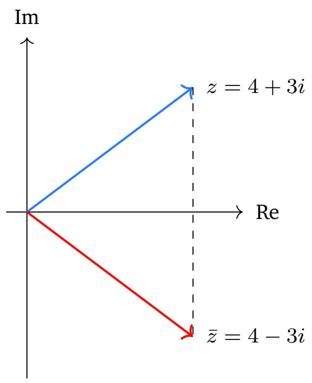
(A) $4x^2 + 4x + 1$

(B) $2x^2 + 1$ only at $x = 0$

(C) $2x^2 + 1$

(D) $(2x + 1)^2$

Q4. For the complex number $z = 4 + 3i$, shown with its conjugate $\bar{z}$ on the Argand plane, the value of $z\,\bar{z}$ is:



(A) 7

(B) 5

(C) $24i$

(D) 25

Q5. A quadratic equation with real coefficients has $2 + 3i$ as one of its roots. Its other root is:

(A) $2 - 3i$

(B) $-2 + 3i$

(C) $3 + 2i$

(D) $-2 - 3i$

Q6. The value of k for which the equation $x^2 - 6x + k = 0$ has equal roots is:

(A) 6

(B) 9

(C) 36

(D) 3

Q7. If α and β are the roots of $x^2 - 5x + 6 = 0$, then the equation whose roots are $\alpha + 1$ and $\beta + 1$ is:

(A) $x^2 - 5x + 6 = 0$

(B) $x^2 + 7x + 12 = 0$

(C) $x^2 - 7x + 12 = 0$

(D) $x^2 - 7x + 6 = 0$

Q8. Every diagonal entry of a skew-symmetric matrix (a matrix with $A^T = -A$) is necessarily:

(A) equal to 1

(B) positive

(C) undefined

(D) zero

Q9. The inverse of the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ is:

(A) $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$

(C) $\begin{pmatrix} -1 & 1 \\ 1 & -2 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

Q10. The value of the determinant $\begin{vmatrix} 5 & 7 & 2 \\ 3 & 1 & 4 \\ 5 & 7 & 2 \end{vmatrix}$ is:

(A) 0

(B) 14

(C) 70

(D) -14

Q11. Using Cramer's rule, the value of x in the system $2x + y = 7,\ x - y = 2$ is:

(A) 1

(B) 4

(C) 3

(D) 2

Q12. The value of ${}^7P_3$ is:

(A) 35

(B) 21

(C) 343

(D) 210

Q13. The number of diagonals of a regular octagon (8 sides) is:

(A) 20

(B) 28

(C) 16

(D) 8

Q14. The number of ways in which 5 persons can sit around a circular table such that two particular persons always sit together is:

(A) 24

(B) 12

(C) 48

(D) 6

Q15. If α, β, γ are the roots of $x^3 - 7x^2 + 14x - 8 = 0$, then $\alpha\beta + \beta\gamma + \gamma\alpha$ equals:

(A) 7

(B) 8

(C) 14

(D) -14

Q16. If the roots of $x^2 - 3x + 2 = 0$ are α and β, then the equation whose roots are $-\alpha$ and $-\beta$ is:

(A) $x^2 - 3x + 2 = 0$

(B) $x^2 - 3x - 2 = 0$

(C) $x^2 + 3x - 2 = 0$

(D) $x^2 + 3x + 2 = 0$

Q17. The value of $\lim_{x \to 0} \dfrac{e^x - 1}{x}$ is:

(A) 1

(B) 0

(C) e

(D) does not exist

Q18. If $y = \dfrac{x}{x+1}$, then $\dfrac{dy}{dx}$ is:

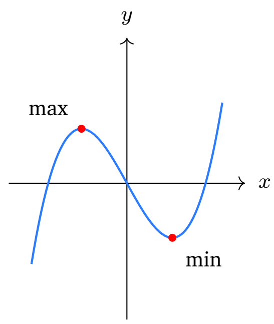
(A) $\dfrac{2x+1}{(x+1)^2}$

(B) $\dfrac{1}{(x+1)^2}$

(C) $\dfrac{-1}{(x+1)^2}$

(D) 1

Q19. For the function $f(x) = x^3 - 3x$, whose graph is shown, the x-coordinate of its local maximum is:



(A) 0

(B) 3

(C) 1

(D) -1

Q20. The equation of the tangent to the curve $y = x^2$ at the point $(2, 4)$ is:

(A) $y = 2x$

(B) $y = 4x + 4$

(C) $y = 4x - 4$

(D) $y = 4x$

Q21. The general solution of the differential equation $\dfrac{dy}{dx} = ky$ (with k a constant) is:

(A) $y = Ce^{kx}$

(B) $y = kx + C$

(C) $y = C + \frac{kx^2}{2}$

(D) $y = \frac{k}{x} + C$

Q22. $\int \frac{1}{x}\,dx$ equals:

(A) $-\frac{1}{x^2} + C$

(B) $\ln|x| + C$

(C) $\frac{x^0}{0} + C$

(D) $x \ln x + C$

Q23. Using the property of even functions, $\int_{-2}^{2} x^2\,dx$ equals:

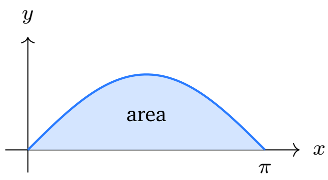
(A) 0

(B) $\frac{8}{3}$

(C) $\frac{16}{3}$

(D) $\frac{32}{3}$

Q24. The area of the region bounded by $y = \sin x$ and the x-axis from $x = 0$ to $x = \pi$ (shaded) is:



(A) 0

(B) π

(C) 1

(D) 2

Q25. The value of $\int_{1}^{e} \frac{1}{x}\,dx$ is:

(A) 1

(B) e

(C) $e-1$

(D) 0

Q26. The perpendicular distance of the point $(1,2)$ from the line $3x+4y-3=0$ is:

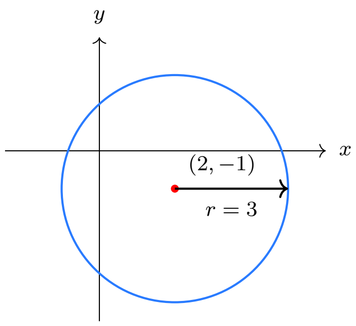
(A) $\frac{4}{5}$

(B) $\frac{8}{5}$

(C) $\frac{1}{5}$

(D) 2

Q27. The equation of the circle with centre $(2,-1)$ and radius 3, shown below, is:



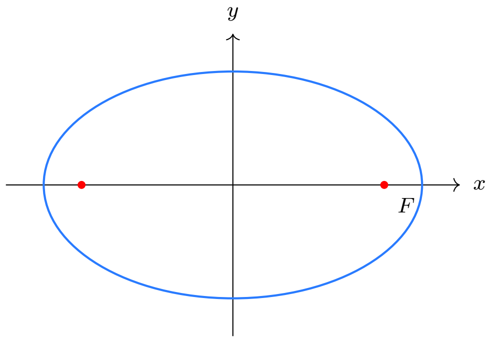
(A) $x^2+y^2-4x+2y+4=0$

(B) $x^2+y^2+4x-2y-4=0$

(C) $x^2+y^2-4x+2y-4=0$

(D) $x^2+y^2-4x-2y+5=0$

Q28. The eccentricity of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, shown below, is:



(A) $\frac{3}{5}$

(B) $\frac{5}{4}$

(C) $\frac{16}{25}$

(D) $\frac{4}{5}$

Q29. The direction ratios of the line joining the points $A(1, 2, 3)$ and $B(4, 6, 8)$ are:

(A) $3, 4, 5$

(B) $5, 8, 11$

(C) $4, 6, 8$

(D) $1, 2, 3$

Q30. The perpendicular distance of the point $(1, 1, 1)$ from the plane $2x + y + 2z - 9 = 0$ is:

(A) $\frac{5}{3}$

(B) $\frac{4}{3}$

(C) 4

(D) $\frac{2}{3}$

Q31. The projection of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ is:

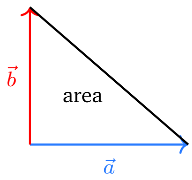
(A) $\frac{12}{17}$

(B) $\dfrac{12}{\sqrt{17}}$

(C) 4

(D) 12

Q32. The area of the triangle whose two sides are represented by $\vec{a} = 3\hat{i}$ and $\vec{b} = 4\hat{j}$ is $\frac{1}{2}|\vec{a} \times \vec{b}|$, equal to:



(A) 12

(B) 7

(C) 3

(D) 6

Q33. The volume of the parallelepiped whose coterminous edges are $\vec{a} = \hat{i}$, $\vec{b} = \hat{j}$ and $\vec{c} = \hat{k}$ is $[\vec{a}\ \vec{b}\ \vec{c}]$, equal to:

(A) 1

(B) 0

(C) 3

(D) $\sqrt{3}$

Q34. The mode of the data $4, 5, 5, 6, 6, 6, 7, 8$ is:

(A) 5

(B) 6

(C) 7

(D) 8

Q35. If $P(A \cap B) = 0.2$ and $P(B) = 0.5$, then the conditional probability $P(A \mid B)$ is:

(A) 0.1

(B) 0.7

(C) 0.25

(D) 0.4

Q36. Box I has 3 red and 2 black balls; Box II has 1 red and 4 black balls. A box is chosen at random and a ball drawn turns out red. The probability that it came from Box I is:

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{2}{5}$

Q37. The expression $\frac{1+\tan^2\theta}{\sec^2\theta}$ simplifies to:

(A) 1

(B) $\sec^2\theta$

(C) $\tan^2\theta$

(D) $\cos^2\theta$

Q38. Using $\sin(A+B) = \sin A\cos B + \cos A\sin B$, the value of $\sin 75^\circ$ is:

(A) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(B) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(C) $\frac{1}{2}$

(D) $\frac{\sqrt{3}}{2}$

Q39. The principal value of $\tan^{-1}(1)$ is:

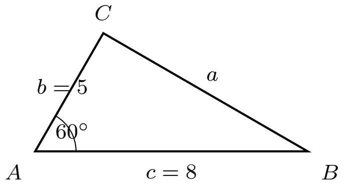
(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

Q40. In triangle ABC, $b = 5$, $c = 8$ and the included angle $A = 60°$, as shown. Using the cosine rule, the side a is:



(A) $\sqrt{129}$

(B) 13

(C) $\sqrt{89}$

(D) 7

Detailed Solutions

Q1.

Solution

Concept — Exactly one of two sets: A finite set splits cleanly into three disjoint regions: elements in A only, elements in B only, and elements in the overlap $A \cap B$. "Belonging to exactly one set" means the element lies in A or in B but not in both, which is precisely the symmetric difference $A \triangle B = (A \setminus B) \cup (B \setminus A)$. Since the overlap is counted once inside $n(A)$ and once again inside $n(B)$, it must be removed *twice* to leave only the two single-membership regions. This gives the working formula $n(A) + n(B) - 2\,n(A \cap B)$.

Step 1 — Read off the data: The Venn diagram and the statement give

$$n(A) = 18, \qquad n(B) = 14, \qquad n(A \cap B) = 6.$$

Step 2 — Substitute into the symmetric-difference formula:

$$n(A \triangle B) = n(A) + n(B) - 2\,n(A \cap B) = 18 + 14 - 2(6).$$

Step 3 — Simplify the arithmetic:

$$18 + 14 - 12 = 32 - 12 = 20.$$

Step 4 — Cross-check via the disjoint regions: Compute each region separately. A-only $= n(A) - n(A \cap B) = 18 - 6 = 12$; B-only $= n(B) - n(A \cap B) = 14 - 6 = 8$. Their sum

$$12 + 8 = 20$$

matches Step 3, confirming the count. (As a bonus, $n(A \cup B) = 12 + 8 + 6 = 26$.)

Why other options are wrong:

- (B) 32 comes from $n(A) + n(B) = 18 + 14$ without ever removing the shared 6 elements; it double-counts the overlap.
- (C) 26 subtracts the overlap only once, $18 + 14 - 6$; that is $n(A \cup B)$, the whole union, not the "exactly one" part.
- (D) 14 simply reports $n(B)$ and forgets the A-only elements entirely.

Final Answer: exactly one $= 20 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q1

Q2.

Solution

Concept — Classifying an order relation: A relation R on a set is *reflexive* if aRa for every a, *symmetric* if $aRb \Rightarrow bRa$, and *transitive* if aRb and $bRc \Rightarrow aRc$. The relation "$\leq$" is the prototype of a partial (here total) order, and orders are designed to be reflexive and transitive but deliberately *not* symmetric, because order would be meaningless if $a \leq b$ always forced $b \leq a$. We verify each of the three properties directly.

Step 1 — Reflexivity: For every real number a,

$$a \leq a$$

is true (any number equals itself). Hence aRa always holds and R is reflexive.

Step 2 — Symmetry: We need a single counterexample to refute it. Take $a = 2,\ b = 3$:

$$2 \leq 3 \text{ is true, but } 3 \leq 2 \text{ is false.}$$

So aRb does not imply bRa, and R is *not* symmetric. (In fact $a \leq b$ and $b \leq a$ together force $a = b$, the antisymmetry of an order.)

Step 3 — Transitivity: Suppose $a \leq b$ and $b \leq c$. Chaining the inequalities,

$$a \leq b \leq c \;\Rightarrow\; a \leq c,$$

so aRc follows. Therefore R is transitive.

Step 4 — Assemble the verdict: R is reflexive (Step 1) and transitive (Step 3) but not symmetric (Step 2). This exactly describes option (B).

Why other options are wrong:

- (A) claims "symmetric but not transitive": both halves are false: Step 2 kills symmetry and Step 3 confirms transitivity.
- (C) "symmetric and transitive only" again asserts symmetry, contradicted by $2 \leq 3,\ 3 \not\leq 2$.
- (D) "neither reflexive nor transitive" denies reflexivity, but $a \leq a$ in Step 1 shows reflexivity plainly holds.

Final Answer: reflexive and transitive but not symmetric $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept — Composition of functions: The composite $(f \circ g)(x)$ means "do g first, then feed the result into f"; symbolically $(f \circ g)(x) = f\big(g(x)\big)$. The inner function g supplies the input to the outer function f, so wherever f's rule shows the variable, we substitute the *entire* expression $g(x)$. Order matters: $f \circ g$ is generally different from $g \circ f$.

Step 1 — Write down the inner output: Here $g(x) = x^2$, so the quantity that enters f is x^2.

Step 2 — Substitute into the outer rule $f(t) = 2t + 1$: Replacing the input t by $g(x) = x^2$,

$$(f \circ g)(x) = f(x^2) = 2(x^2) + 1.$$

Step 3 — Simplify:

$$(f \circ g)(x) = 2x^2 + 1.$$

Step 4 — Cross-check with the reverse composite: For contrast, $(g \circ f)(x) = g(2x + 1) = (2x + 1)^2 = 4x^2 + 4x + 1$. This is option (A), confirming that swapping the order gives a genuinely different function and that our $f \circ g$ is the simpler $2x^2 + 1$. A numeric test at $x = 1$: $g(1) = 1$, then $f(1) = 2(1) + 1 = 3$, and indeed $2(1)^2 + 1 = 3$. ✓

Why other options are wrong:

- (A) $4x^2 + 4x + 1 = (2x + 1)^2$ is the *reverse* composite $g \circ f$, computed in Step 4, not $f \circ g$.
- (D) $(2x + 1)^2$ is the same reverse-order error left unexpanded.
- (B) writes the correct expression $2x^2 + 1$ but falsely restricts it to $x = 0$ only; the formula holds for all real x.

Final Answer: $(f \circ g)(x) = 2x^2 + 1 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q3

Q4.

Solution

Concept — Product of a complex number with its conjugate: For $z = a + bi$, the conjugate is $\bar{z} = a - bi$. Their product uses the difference-of-squares pattern $(a + bi)(a - bi) = a^2 - (bi)^2$. Since $i^2 = -1$, the term $-(bi)^2 = -b^2 i^2 = +b^2$, so the imaginary cross terms cancel and we are left with $z\,\bar{z} = a^2 + b^2$, which is exactly the squared modulus $|z|^2$ and is always a non-negative real number.

Step 1 — Identify the parts: Here $z = 4 + 3i$, so $a = 4$, $b = 3$, and $\bar{z} = 4 - 3i$.

Step 2 — Multiply out the brackets:

$$z\,\bar{z} = (4 + 3i)(4 - 3i) = 4^2 - (3i)^2 = 16 - 9i^2.$$

Step 3 — Use $i^2 = -1$ and simplify:

$$16 - 9(-1) = 16 + 9 = 25.$$

Step 4 — Verify via the modulus: The modulus is $|z| = \sqrt{a^2 + b^2} = \sqrt{16 + 9} = \sqrt{25} = 5$, so $|z|^2 = 5^2 = 25$, matching Step 3. The dashed segment in the Argand figure shows z and $\bar{z}$ as mirror images across the real axis, the geometric picture of conjugation.

Why other options are wrong:

- (A) 7 simply adds the parts $4 + 3$, ignoring squaring altogether.
- (B) 5 is the modulus $|z|$, not $|z|^2$; it forgot to square.
- (C) $24i$ keeps an imaginary cross term $(2 \cdot 4 \cdot 3i)$ that actually cancels in the conjugate product; that term would appear in z^2, not in $z\bar{z}$.

Final Answer: $z\,\bar{z} = 25 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Complex conjugate root theorem: If a polynomial has *real* coefficients and admits a non-real root $p + qi$ (with $q \neq 0$), then its complex conjugate $p - qi$ is automatically also a root. The reason is that complex roots of a real polynomial must occur in conjugate pairs, so that when the two linear factors are multiplied the imaginary parts cancel and leave a real quadratic factor. Only the *sign of the imaginary part* flips; the real part is untouched.

Step 1 — Apply the theorem: One root is $2+3i$, so the partner root is its conjugate

$$\overline{2 + 3i} = 2 - 3i.$$

Step 2 — Reconstruct the quadratic as a check: Sum of roots $= (2 + 3i) + (2 - 3i) = 4$ (real), product $= (2+3i)(2-3i) = 4-9i^2 = 4+9 = 13$ (real). The equation is therefore

$$x^2 - 4x + 13 = 0,$$

which has purely real coefficients, confirming the pair $2 \pm 3i$ is consistent.

Step 3 — Discriminant sanity check: $D = (-4)^2 - 4(1)(13) = 16 - 52 = -36 < 0$, so the roots are genuinely non-real and conjugate, exactly $2 \pm 3i$. ✓

Why other options are wrong:

- (B) $-2 + 3i$ wrongly negates the real part instead of the imaginary part.
- (D) $-2 - 3i$ negates both parts; that is $-z$, not the conjugate.
- (C) $3 + 2i$ swaps the real and imaginary components, which conjugation never does.

Final Answer: other root $= 2 - 3i \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q5**

Q6.

Solution

Concept — Condition for equal (repeated) roots: A quadratic $ax^2 + bx + c = 0$ has its two roots given by $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The two roots coincide exactly when the square-root term vanishes, i.e. when the discriminant $D = b^2 - 4ac = 0$. Geometrically this is when the parabola just touches the x-axis at a single point.

Step 1 — Read the coefficients: For $x^2 - 6x + k = 0$ we have $a = 1$, $b = -6$, $c = k$.

Step 2 — Impose $D = 0$:

$$b^2 - 4ac = (-6)^2 - 4(1)(k) = 36 - 4k = 0.$$

Step 3 — Solve for k:

$$4k = 36 \;\Rightarrow\; k = 9.$$

Step 4 — Verify: With $k = 9$ the equation becomes $x^2 - 6x + 9 = (x - 3)^2 = 0$, a perfect square with the single repeated root $x = 3$. The repeated root equals $-\dfrac{b}{2a} = \dfrac{6}{2} = 3$, consistent with equal roots. ✓

Why other options are wrong:

- (A) 6 is just $|b|$, the coefficient, not the value forcing $D = 0$.
- (C) 36 is b^2; setting $k = 36$ gives $D = 36 - 144 < 0$, complex roots.
- (D) 3 is the repeated *root* $-b/2a$, mistaken for the constant k.

Final Answer: $k = 9 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q6

Q7.

Solution

Concept — Building an equation from transformed roots: A monic quadratic with roots r_1, r_2 is $x^2 - (r_1 + r_2)x + r_1 r_2 = 0$. So to find the equation whose roots are $\alpha + 1, \beta + 1$ we only need their new sum and new product. Using Vieta's relations $S = \alpha + \beta$ and $P = \alpha\beta$, the shifted sum is $(\alpha + 1) + (\beta + 1) = S + 2$ and the shifted product is $(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1 = P + S + 1$.

Step 1 — Extract S and P from the given equation: For $x^2 - 5x + 6 = 0$, Vieta gives

$$S = \alpha + \beta = 5, \qquad P = \alpha\beta = 6.$$

Step 2 — New sum:

$$(\alpha + 1) + (\beta + 1) = S + 2 = 5 + 2 = 7.$$

Step 3 — New product:

$$(\alpha + 1)(\beta + 1) = P + S + 1 = 6 + 5 + 1 = 12.$$

Step 4 — Assemble the quadratic:

$$x^2 - (\text{sum})x + (\text{product}) = x^2 - 7x + 12 = 0.$$

Step 5 — Verify by direct substitution $x \to x - 1$: The roots of the original are $\alpha = 2, \beta = 3$, so the new roots are 3 and 4. Indeed $x^2 - 7x + 12 = (x - 3)(x - 4)$, which has roots $3, 4$. Equivalently, replacing x by $x - 1$ in $x^2 - 5x + 6$ gives $(x - 1)^2 - 5(x - 1) + 6 = x^2 - 7x + 12$. ✓

Why other options are wrong:

- (A) $x^2 - 5x + 6$ is the original, unshifted equation.
- (B) $x^2 + 7x + 12$ has the wrong sign on the linear term (its roots would be $-3, -4$).
- (D) $x^2 - 7x + 6$ uses the correct new sum but keeps the old constant 6 instead of 12.

Final Answer: $x^2 - 7x + 12 = 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q7

Q8.

Solution

Concept — Skew-symmetric matrices: A square matrix A is skew-symmetric (also called anti-symmetric) when its transpose equals its negative, $A^T = -A$. Reading this off entry-by-entry, the (i, j) entry of A^T is a_{ji} and the (i, j) entry of $-A$ is $-a_{ij}$, so the defining condition is $a_{ji} = -a_{ij}$ for all i, j. The diagonal is where $i = j$, and that special case forces a strong conclusion.

Step 1 — Set $i = j$ in the entry condition:

$$a_{ii} = -a_{ii}.$$

Step 2 — Solve for the diagonal entry: Adding a_{ii} to both sides,

$$a_{ii} + a_{ii} = 0 \Rightarrow 2a_{ii} = 0 \Rightarrow a_{ii} = 0.$$

Step 3 — Conclude: Since i was arbitrary, *every* diagonal entry is zero.

Step 4 — Concrete illustration: A typical 2×2 skew-symmetric matrix is $\begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix}$; its transpose $\begin{pmatrix} 0 & -5 \\ 5 & 0 \end{pmatrix}$ is indeed its negative, and both diagonal slots are 0. ✓

Why other options are wrong:

- (A) "equal to 1" and (B) "positive" both contradict $a_{ii} = -a_{ii}$, which leaves only 0.
- (C) "undefined" is wrong: the entries are perfectly well defined and equal 0.

Final Answer: each diagonal entry $= 0 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — Inverse of a 2×2 matrix via the adjoint: For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the inverse is $A^{-1} = \dfrac{1}{\det A}\,\mathrm{adj}(A)$, where the adjoint of a 2×2 matrix is obtained by *swapping the diagonal entries* and *negating the off-diagonal entries*: $\mathrm{adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. The inverse exists precisely when $\det A = ad - bc \neq 0$.

Step 1 — Compute the determinant: With $a = 2, b = 1, c = 1, d = 1$,

$$\det A = ad - bc = (2)(1) - (1)(1) = 2 - 1 = 1.$$

Non-zero, so A is invertible.

Step 2 — Form the adjoint: Swap $a \leftrightarrow d$ and negate b, c:

$$\text{adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}.$$

Step 3 — Divide by the determinant:

$$A^{-1} = \frac{1}{1}\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}.$$

Step 4 — Verify by multiplication $AA^{-1} = I$:

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 2-1 & -2+2 \\ 1-1 & -1+2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I.\checkmark$$

Why other options are wrong:

- (A) $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$ failed to swap the diagonal entries before negating.
- (C) $\begin{pmatrix} -1 & 1 \\ 1 & -2 \end{pmatrix}$ flips every sign (it is $-A^{-1}$).
- (D) $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ is just a relabelling, not satisfying $AA^{-1} = I$.

Final Answer: $A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q9

Q10.

Solution

Concept — Determinant with two identical rows: A fundamental property of determinants is that if any two rows (or any two columns) are identical, the determinant is zero. The reason: swapping two rows multiplies a determinant by -1; but swapping two *identical* rows leaves it unchanged, so $D = -D$, which forces $D = 0$. Spotting this saves a full cofactor expansion.

Step 1 — Inspect the rows: The matrix is

$$\begin{vmatrix} 5 & 7 & 2 \\ 3 & 1 & 4 \\ 5 & 7 & 2 \end{vmatrix}.$$

Row 1 is $(5, 7, 2)$ and row 3 is $(5, 7, 2)$: they are identical.

Step 2 — Apply the property: Two equal rows give

$$D = 0$$

immediately, with no arithmetic needed.

Step 3 — Confirm by direct expansion along row 2:

$$-3\begin{vmatrix} 7 & 2 \\ 7 & 2 \end{vmatrix} + 1\begin{vmatrix} 5 & 2 \\ 5 & 2 \end{vmatrix} - 4\begin{vmatrix} 5 & 7 \\ 5 & 7 \end{vmatrix} = -3(0) + 1(0) - 4(0) = 0.\checkmark$$

Each 2×2 minor itself has equal rows, so all vanish.

Why other options are wrong:

- (B) 14, (C) 70, (D) -14 are spurious values that arise only if one overlooks the repeated row and makes an expansion slip; the equal-row property guarantees exactly 0.

Final Answer: determinant $= 0 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q10

Q11.

Solution

Concept — Cramer's rule: For a 2×2 linear system, the solution is $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$, where D is the determinant of the coefficient matrix, D_x replaces the x-column with the column of constants, and D_y replaces the y-column. The rule is valid whenever $D \neq 0$, which guarantees a unique solution.

Step 1 — Write the system in standard form:

$$2x + y = 7, \qquad x - y = 2,$$

so the coefficient matrix is $\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$ and the constants are $7, 2$.

Step 2 — Coefficient determinant D:

$$D = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = (2)(-1) - (1)(1) = -2 - 1 = -3.$$

Step 3 — Determinant D_x (constants in the x-column):

$$D_x = \begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix} = (7)(-1) - (1)(2) = -7 - 2 = -9.$$

Step 4 — Divide:

$$x = \frac{D_x}{D} = \frac{-9}{-3} = 3.$$

Step 5 — Verify with the second equation: If $x = 3$, then from $x - y = 2$ we get $y = 1$, and the first equation checks: $2(3) + 1 = 7$. ✓

Why other options are wrong:

- (D) 2 is the right-hand constant of the second equation, not x.
- (A) 1 is the value of y, not x.
- (B) 4 arises from a sign slip in D or D_x (e.g. taking $D = +3,\ D_x = 12$).

Final Answer: $x = 3 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q11

Q12.

Solution

Concept — Permutations: The symbol ^nP_r counts the number of *ordered* arrangements of r objects chosen from n distinct objects. Its formula is $^nP_r = \frac{n!}{(n-r)!}$, which telescopes to the product of r consecutive descending integers $n(n-1)(n-2)\cdots(n-r+1)$. Because order matters, ^nP_r is always at least as large as the combination $\binom{n}{r}$, differing by the factor $r!$.

Step 1 — Substitute $n = 7$, $r = 3$: The product runs over $r = 3$ terms starting at 7:

$$^7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 7 \times 6 \times 5.$$

Step 2 — Multiply:

$$7 \times 6 = 42, \qquad 42 \times 5 = 210.$$

Step 3 — Cross-check via the combination: $\binom{7}{3} = \frac{7\times6\times5}{3!} = \frac{210}{6} = 35$, and multiplying back by $3! = 6$ returns $35 \times 6 = 210$. ✓

Why other options are wrong:

- (A) $35 = \binom{7}{3}$ is the *combination*, which ignores order; it is ${}^7P_3/3!$.
- (C) $343 = 7^3$ counts arrangements *with* repetition allowed, not the without-repetition permutation.
- (B) 21 is unrelated to this computation.

Final Answer: ${}^7P_3 = 210 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Number of diagonals of a polygon: Joining any two of the n vertices of a convex polygon gives a line segment, and there are $\binom{n}{2}$ such segments. Of these, n are the sides of the polygon (each pair of adjacent vertices). Everything left over is a diagonal, so the count is $\binom{n}{2} - n = \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$.

Step 1 — Total vertex pairs for $n = 8$:

$$\binom{8}{2} = \frac{8\times7}{2} = 28.$$

Step 2 — Remove the 8 sides:

$$28 - 8 = 20.$$

Step 3 — Confirm with the closed formula:

$$\frac{n(n-3)}{2} = \frac{8(8-3)}{2} = \frac{8\times5}{2} = \frac{40}{2} = 20. ✓$$

Why other options are wrong:

- (B) $28 = \binom{8}{2}$ counts all vertex pairs, i.e. it forgot to subtract the 8 sides.
- (C) $16 = 2 \times 8$ is an unfounded guess that under-counts.

- (D) 8 merely repeats the number of sides, not the diagonals.

Final Answer: number of diagonals $= 20 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept — Circular arrangements with a pair always together: Around a circular table, m distinct people can be seated in $(m-1)!$ ways, not $m!$, because rotations of the whole table are regarded as the same arrangement (we fix one person to kill the rotational symmetry). When two particular people must sit together, we "glue" them into a single block, reducing the count of units, then multiply by the internal orderings of the block.

Step 1 — Glue the pair into one block: Treating the two fixed people as a single unit leaves

$$5 - 2 + 1 = 4 \text{ units}$$

to be seated around the table.

Step 2 — Arrange the 4 units circularly:

$$(4-1)! = 3! = 6 \text{ arrangements.}$$

Step 3 — Order the two people inside the block: The pair can sit in either internal order, giving a factor

$$2! = 2.$$

Step 4 — Multiply for the total:

$$6 \times 2 = 12.$$

Why other options are wrong:

- (A) 24 uses $4! \times 2!/2$ or treats the table as a line, forgetting the circular fixing $(m-1)!$.
- (C) 48 takes $4! \times 2! = 48$, double-counting rotations of the block arrangement.
- (D) 6 stops at the $3!$ circular count and omits the $2!$ internal order of the pair.

Final Answer: 12 ways $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q14

Q15.

Solution

Concept — Vieta's relations for a cubic: For a monic cubic $x^3 + bx^2 + cx + d = 0$ with roots α, β, γ, the symmetric functions of the roots equal signed coefficients: $\alpha + \beta + \gamma = -b$, $\alpha\beta + \beta\gamma + \gamma\alpha = +c$, and $\alpha\beta\gamma = -d$. The sum of products taken two at a time is read directly as the coefficient c of x, with a $+$ sign for a monic cubic.

Step 1 — Match coefficients: Writing $x^3 - 7x^2 + 14x - 8 = 0$ in the form $x^3 + bx^2 + cx + d$,

$$b = -7, \qquad c = 14, \qquad d = -8.$$

Step 2 — Read off the required quantity:

$$\alpha\beta + \beta\gamma + \gamma\alpha = c = 14.$$

Step 3 — Verify with the actual roots: The cubic factors as $(x-1)(x-2)(x-4) = 0$, so the roots are $1, 2, 4$. Then

$$\alpha\beta + \beta\gamma + \gamma\alpha = (1)(2) + (2)(4) + (4)(1) = 2 + 8 + 4 = 14. \checkmark$$

(As further checks: sum $= 1 + 2 + 4 = 7 = -b$ and product $= 1 \cdot 2 \cdot 4 = 8 = -d$.)

Why other options are wrong:

- (A) 7 is the sum of the roots $-b$, not the pairwise-product sum.
- (B) 8 is the product of the roots $-d$.
- (D) -14 attaches the wrong sign; for a monic cubic the two-at-a-time sum equals $+c$.

Final Answer: $\alpha\beta + \beta\gamma + \gamma\alpha = 14 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q15

Q16.

Solution

Concept — Negating the roots of an equation: If α is a root of $P(x) = 0$, then $-\alpha$ is a root of $P(-x) = 0$, because substituting $x = -\alpha$ into $P(-x)$ gives $P(\alpha) = 0$. So to obtain the equation whose roots are $-\alpha, -\beta$, we simply replace every x by $-x$ in the original polynomial and tidy up. This negates the sign of every *odd*-degree term and leaves even-degree terms unchanged.

Step 1 — Substitute $x \to -x$ in $x^2 - 3x + 2 = 0$:

$$(-x)^2 - 3(-x) + 2 = 0.$$

Step 2 — Simplify each term: $(-x)^2 = x^2$ (even, unchanged) and $-3(-x) = +3x$ (odd, sign flipped), so

$$x^2 + 3x + 2 = 0.$$

Step 3 — Verify with Vieta: The original roots solve $x^2 - 3x + 2 = (x-1)(x-2) = 0$, giving $1, 2$. The new roots should be $-1, -2$, whose sum is -3 and product is 2. The equation with these is $x^2 - (-3)x + 2 = x^2 + 3x + 2 = 0$, matching Step 2. ✓

Why other options are wrong:

- (A) $x^2 - 3x + 2$ is the original equation, unchanged (roots still $1, 2$).
- (B) $x^2 - 3x - 2$ wrongly flips the constant's sign and leaves the linear term unchanged.
- (C) $x^2 + 3x - 2$ flips the linear term correctly but also corrupts the constant term, which should stay $+2$.

Final Answer: $x^2 + 3x + 2 = 0 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — A fundamental exponential limit: The value $\lim_{x \to 0} \dfrac{e^x - 1}{x} = 1$ is one of the standard limits of calculus. It is exactly the definition of the derivative of e^x at $x = 0$: $\dfrac{d}{dx} e^x \Big|_0 = \lim_{x \to 0} \dfrac{e^x - e^0}{x - 0} = e^0 = 1$. Substituting $x = 0$ directly gives the indeterminate form $\frac{0}{0}$, so we resolve it by a series or by L'Hopital's rule.

Step 1 — Maclaurin-series expansion: Using $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$,

$$e^x - 1 = x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots.$$

Step 2 — Divide by x:

$$\frac{e^x - 1}{x} = 1 + \frac{x}{2} + \frac{x^2}{6} + \cdots.$$

Step 3 — Take the limit: As $x \to 0$ every term after the first vanishes, leaving

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1.$$

Step 4 — Cross-check with L'Hopital: The form is $\frac{0}{0}$, so differentiate top and bottom: $\frac{\frac{d}{dx}(e^x - 1)}{\frac{d}{dx}x} = \frac{e^x}{1} \to e^0 = 1$. ✓

Why other options are wrong:

- (B) 0 comes from naively cancelling the -1 and ignoring the linear term in the expansion.
- (C) e evaluates e^x at $x = 1$ rather than taking the limit at 0.
- (D) "does not exist" is false: the two-sided limit equals 1 and the function is smooth.

Final Answer: the limit is $1 \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q17**

Q18.

Solution

Concept — Quotient rule: To differentiate a ratio $y = \frac{u}{v}$, use $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$. The numerator is "derivative of top times bottom, minus top times derivative of bottom"; the order of subtraction matters, and the denominator is always the square of the original bottom.

Step 1 — Identify u and v and their derivatives: With $u = x$ and $v = x + 1$,

$$u' = 1, \qquad v' = 1.$$

Step 2 — Substitute into the rule:

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{(1)(x+1) - (x)(1)}{(x+1)^2}.$$

Step 3 — Simplify the numerator:

$$\frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}.$$

Step 4 — Verify by rewriting: Write $y = \frac{x}{x+1} = 1 - \frac{1}{x+1} = 1 - (x+1)^{-1}$. Differentiating, $\frac{dy}{dx} = 0 - (-1)(x+1)^{-2} = \frac{1}{(x+1)^2}$, matching Step 3. ✓Note the derivative is positive everywhere, so the function is increasing on each branch.

Why other options are wrong:

- (C) $\frac{-1}{(x+1)^2}$ reverses the subtraction order in the numerator, flipping the sign.
- (A) $\frac{2x+1}{(x+1)^2}$ mishandles the numerator (adds instead of subtracts the x terms).
- (D) 1 ignores the quotient rule, as if the function were linear.

Final Answer: $\frac{1}{(x+1)^2} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q18

Q19.

Solution

Concept — Locating local extrema: Local maxima and minima of a differentiable function occur at critical points where $f'(x) = 0$. To classify each critical point, the second-derivative test inspects the sign of f'': if $f''(x_0) < 0$ the curve is concave down and x_0 is a local maximum; if $f''(x_0) > 0$ the curve is concave up and x_0 is a local minimum.

Step 1 — First derivative and critical points:

$$f(x) = x^3 - 3x \;\Rightarrow\; f'(x) = 3x^2 - 3.$$

Setting $f'(x) = 0$:

$$3x^2 - 3 = 0 \;\Rightarrow\; x^2 = 1 \;\Rightarrow\; x = \pm 1.$$

Step 2 — Second derivative:

$$f''(x) = 6x.$$

Step 3 — Classify each critical point: At $x = -1$, $f''(-1) = 6(-1) = -6 < 0$, so $x = -1$ is a *local maximum*. At $x = +1$, $f''(1) = 6 > 0$, so $x = +1$ is a local minimum.

Step 4 — Match the graph: The plotted curve shows its peak (marked "max") to the left of the origin and its trough ("min") to the right, exactly at $x = -1$ and $x = 1$ respectively, confirming the algebra. The maximum value is $f(-1) = -1 + 3 = 2$. ✓

Why other options are wrong:

- (C) $x = 1$ is the local *minimum*, not the maximum (there $f'' > 0$).
- (A) $x = 0$ is not even a critical point, since $f'(0) = -3 \neq 0$.
- (B) $x = 3$ is unrelated; it does not satisfy $f'(x) = 0$.

Final Answer: local maximum at $x = -1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q19

Q20.

Solution

Concept — Equation of a tangent line: The tangent to a curve $y = f(x)$ at a point (x_0, y_0) is the straight line through that point whose slope equals the instantaneous rate of change $m = \left.\frac{dy}{dx}\right|_{x_0} = f'(x_0)$. Using point-slope form, the tangent is $y - y_0 = m(x - x_0)$.

Step 1 — Find the slope function:

$$y = x^2 \;\Rightarrow\; \frac{dy}{dx} = 2x.$$

Step 2 — Evaluate the slope at the point of tangency $x_0 = 2$:

$$m = 2(2) = 4.$$

Step 3 — Write point-slope form through $(2, 4)$ and simplify:

$$y - 4 = 4(x - 2) \;\Rightarrow\; y - 4 = 4x - 8 \;\Rightarrow\; y = 4x - 4.$$

Step 4 — Verify the point lies on the line: Substituting $x = 2$ into $y = 4x - 4$ gives $y = 8 - 4 = 4$, so the line passes through $(2, 4)$, as a tangent must. ✓

Why other options are wrong:

- (D) $y = 4x$ has the right slope but omits the -4 intercept; it does not pass through $(2, 4)$ (it gives $y = 8$).
- (B) $y = 4x + 4$ flips the sign of the intercept; it gives $y = 12$ at $x = 2$.
- (A) $y = 2x$ uses the slope value 2 (the bare x-coordinate) instead of $f'(2) = 4$.

Final Answer: $y = 4x - 4 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q20

Q21.

Solution

Concept — The exponential growth/decay equation: The differential equation $\frac{dy}{dx} = ky$ says the rate of change is proportional to the current value, the hallmark of exponential behaviour. It is *separable*: collect all y terms on one side and all x terms on the other, then integrate both sides.

Step 1 — Separate the variables:

$$\frac{dy}{dx} = ky \;\Rightarrow\; \frac{dy}{y} = k\,dx.$$

Step 2 — Integrate both sides:

$$\int \frac{dy}{y} = \int k\,dx \;\Rightarrow\; \ln|y| = kx + c_1.$$

Step 3 — Exponentiate to solve for y:

$$|y| = e^{kx+c_1} = e^{c_1}e^{kx} \;\Rightarrow\; y = Ce^{kx}, \quad C = \pm e^{c_1}.$$

Step 4 — Verify by differentiating back: If $y = Ce^{kx}$ then $\frac{dy}{dx} = Cke^{kx} = k(Ce^{kx}) = ky$, recovering the original equation. ✓The constant $C = y(0)$ is the initial value.

Why other options are wrong:

- (B) $y = kx + C$ solves $y' = k$ (constant rate), not $y' = ky$.

- (C) $y = C + \frac{kx^2}{2}$ solves $y' = kx$ (rate proportional to x).
- (D) $y = \frac{k}{x} + C$ does not satisfy $y' = ky$ at all (substituting gives a mismatch).

Final Answer: $y = Ce^{kx} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept — The logarithmic antiderivative: The integral $\displaystyle\int \frac{1}{x}\,dx$ is the one case the power rule cannot handle. The power rule $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$ requires $n \neq -1$, since at $n = -1$ the denominator $n+1$ becomes 0. Instead we recall that $\dfrac{d}{dx}\ln|x| = \dfrac{1}{x}$ (the absolute value extends the result to negative x), so the antiderivative is the natural logarithm.

Step 1 — Recognise why the power rule fails: For $\dfrac{1}{x} = x^{-1}$ we have $n = -1$, and

$$\frac{x^{n+1}}{n+1} = \frac{x^0}{0}$$

is undefined (division by zero). So the polynomial rule does not apply.

Step 2 — Use the logarithmic derivative: Since $\dfrac{d}{dx}\ln|x| = \dfrac{1}{x}$,

$$\int \frac{1}{x}\,dx = \ln|x| + C.$$

Step 3 — Check by differentiating: $\dfrac{d}{dx}\big(\ln|x| + C\big) = \dfrac{1}{x} + 0 = \dfrac{1}{x}$, recovering the integrand. ✓The $|x|$ is essential so the formula is valid for both positive and negative x.

Why other options are wrong:

- (A) $-\dfrac{1}{x^2} + C$ is actually $\int x^{-3}\cdot(\cdots)$-type; differentiating it gives $\frac{2}{x^3}$, not $\frac{1}{x}$.
- (C) $\dfrac{x^0}{0} + C$ blindly applies the invalid power rule at $n = -1$, producing an undefined expression.
- (D) $x\ln x + C$ differentiates to $\ln x + 1$, not $\frac{1}{x}$.

Final Answer: $\ln|x| + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q22

Q23.

Solution

Concept — Even-function symmetry in definite integrals: A function is *even* if $f(-x) = f(x)$, meaning its graph is symmetric about the y-axis. For such a function the area from $-a$ to 0 mirrors the area from 0 to a, so $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$. (By contrast, an odd function would integrate to 0 over a symmetric interval.) Exploiting symmetry halves the work.

Step 1 — Confirm x^2 is even:

$$f(-x) = (-x)^2 = x^2 = f(x),$$

so the even-function property applies.

Step 2 — Reduce to half the interval:

$$\int_{-2}^{2} x^2\,dx = 2\int_{0}^{2} x^2\,dx.$$

Step 3 — Evaluate the antiderivative:

$$2\left[\frac{x^3}{3}\right]_0^2 = 2\left(\frac{2^3}{3} - 0\right) = 2 \cdot \frac{8}{3} = \frac{16}{3}.$$

Step 4 — Cross-check without symmetry: Directly, $\left[\frac{x^3}{3}\right]_{-2}^{2} = \frac{8}{3} - \frac{-8}{3} = \frac{16}{3}$, the same value. ✓

Why other options are wrong:

- (A) 0 is the answer for an *odd* integrand; x^2 is even, not odd.
- (B) $\frac{8}{3}$ is only the half-integral $\int_0^2 x^2\,dx$; it forgot the factor 2.
- (D) $\frac{32}{3}$ doubles once too often (it doubled the full integral).

Final Answer: the integral is $\frac{16}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q23

Q24.

Solution

Concept — Area under a curve: The area between a curve $y = f(x)$ and the x-axis from $x = a$ to $x = b$ is $\int_a^b y\,dx$, provided $y \geq 0$ throughout so the signed integral and the geometric area agree. On $[0, \pi]$ the sine curve lies entirely above the axis (it is non-negative there), so no splitting into positive and negative pieces is needed.

Step 1 — Confirm the sign and set up: For $0 \leq x \leq \pi$, $\sin x \geq 0$, so the shaded region's area is

$$A = \int_0^\pi \sin x\,dx.$$

Step 2 — Antiderivative: Since $\frac{d}{dx}(-\cos x) = \sin x$,

$$A = \big[-\cos x\big]_0^\pi.$$

Step 3 — Apply the limits:

$$A = -\cos\pi - (-\cos 0) = -(-1) - (-1) = 1 + 1 = 2.$$

Step 4 — Sanity check: The half-arch of sine has a height of 1 and a base of length $\pi \approx 3.14$; an area of 2 is comfortably less than the bounding rectangle of area π, which is reasonable for a curved hump. ✓

Why other options are wrong:

- (A) 0 integrates $\sin x$ over a full period $[0, 2\pi]$, where the positive and negative halves cancel; here the interval is only $[0, \pi]$.
- (B) π confuses the interval length with the area.
- (C) 1 takes only half the arch or stops at $-\cos\pi = 1$, forgetting the lower-limit contribution.

Final Answer: area $= 2 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q24

Q25.

Solution

Concept — A definite logarithmic integral: Because $\int \frac{1}{x}\,dx = \ln|x| + C$, the definite integral over $[1, e]$ is evaluated by the Fundamental Theorem of Calculus as $\big[\ln x\big]_1^e$. The upper limit e is chosen so that the natural log evaluates cleanly, since $\ln e = 1$ by definition of e.

Step 1 — Write the antiderivative with limits: On $[1, e]$, $x > 0$ so $|x| = x$, giving

$$\int_1^e \frac{1}{x}\,dx = \big[\ln x\big]_1^e.$$

Step 2 — Substitute the limits:

$$\ln e - \ln 1 = 1 - 0 = 1.$$

Step 3 — Interpretation: This says the area under $y = \frac{1}{x}$ from $x = 1$ to $x = e$ equals exactly 1; indeed $e \approx 2.718$ is the very number that makes this area one unit. ✓

Why other options are wrong:

- (B) e would be $\big[e^x\big]$ misapplied, treating the integrand as e^x.
- (C) $e - 1$ is $\int_0^1 e^x\,dx$ style: again integrating e^x instead of $\frac{1}{x}$.
- (D) 0 ignores that the limits differ, as if upper and lower coincided.

Final Answer: the integral is $1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q25

Q26.

Solution

Concept — Perpendicular distance of a point from a line: The shortest distance from a point (x_0, y_0) to the line $ax+by+c = 0$ is measured along the perpendicular and is given by $d = \dfrac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$. The numerator is the absolute value of substituting the point into the line's left-hand side, and the denominator normalises by the length of the normal vector (a, b).

Step 1 — Identify the coefficients and point: For $3x + 4y - 3 = 0$, $a = 3, b = 4, c = -3$; the point is $(x_0, y_0) = (1, 2)$.

Step 2 — Evaluate the numerator:

$$|ax_0 + by_0 + c| = |3(1) + 4(2) - 3| = |3 + 8 - 3| = |8| = 8.$$

Step 3 — Evaluate the denominator:

$$\sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

Step 4 — Divide:

$$d = \frac{8}{5}.$$

The convenient $3, 4, 5$ right-triangle makes the normalising length exactly 5. ✓

Why other options are wrong:

- (A) $\frac{4}{5}$ mis-sums the numerator (e.g. $|3 - 8 + \ldots|$), losing half the value.
- (C) $\frac{1}{5}$ drops the $4y$ term, using $|3 - 3| + \ldots$ incorrectly.
- (D) 2 reports a bare numerator-style figure while ignoring the denominator 5.

Final Answer: distance $= \frac{8}{5} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q26

Q27.

Solution

Concept — Equation of a circle from centre and radius: A circle with centre (h, k) and radius r consists of all points at distance r from the centre, giving the standard form $(x - h)^2 + (y - k)^2 = r^2$. Expanding this produces the general form $x^2 + y^2 + Dx + Ey + F = 0$, where the signs of D, E are opposite to those of the centre coordinates.

Step 1 — Substitute $(h, k) = (2, -1)$ and $r = 3$:

$$(x - 2)^2 + (y - (-1))^2 = 3^2 \Rightarrow (x - 2)^2 + (y + 1)^2 = 9.$$

Step 2 — Expand each square:

$$(x - 2)^2 = x^2 - 4x + 4, \qquad (y + 1)^2 = y^2 + 2y + 1.$$

Step 3 — Combine and move 9 across:

$$x^2 - 4x + 4 + y^2 + 2y + 1 = 9 \;\Rightarrow\; x^2 + y^2 - 4x + 2y + 5 - 9 = 0,$$

$$\Rightarrow\; x^2 + y^2 - 4x + 2y - 4 = 0.$$

Step 4 — Verify by recovering centre and radius: From the general form, centre $= (-\frac{D}{2}, -\frac{E}{2}) = (-\frac{-4}{2}, -\frac{2}{2}) = (2, -1)$, and $r = \sqrt{(\frac{D}{2})^2 + (\frac{E}{2})^2 - F} = \sqrt{4+1+4} = \sqrt{9} = 3$. ✓

Why other options are wrong:

- (A) $\cdots + 4 = 0$ keeps $+4$ instead of -4 (forgot to subtract 9 properly).
- (B) $\cdots + 4x - 2y - 4 = 0$ has the wrong linear signs, implying centre $(-2, 1)$.
- (D) $\cdots - 2y + 5 = 0$ uses $-2y$ (centre $y = +1$) and constant $+5$ (radius 0), describing a different circle.

Final Answer: $x^2 + y^2 - 4x + 2y - 4 = 0 \Rightarrow$ C

Answer: (C) Go Back to Q27

Q28.

Solution

Concept — Eccentricity of an ellipse: For $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$ (major axis along the x-axis), the eccentricity is $e = \sqrt{1 - \frac{b^2}{a^2}}$ and always satisfies $0 \le e < 1$. It measures how "stretched" the ellipse is: $e = 0$ is a circle and $e \to 1$ is a very elongated ellipse. The relation $b^2 = a^2(1 - e^2)$ links the two semi-axes to e.

Step 1 — Identify a^2 and b^2: Comparing $\frac{x^2}{25} + \frac{y^2}{9} = 1$ with the standard form, $a^2 = 25$ and $b^2 = 9$, so $a = 5$, $b = 3$ (and $a > b$, so the major axis is horizontal, matching the figure).

Step 2 — Apply the eccentricity formula:

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{25 - 9}{25}} = \sqrt{\frac{16}{25}}.$$

Step 3 — Simplify:

$$e = \frac{4}{5}.$$

Step 4 — Locate the foci as a check: $c = ae = 5 \cdot \frac{4}{5} = 4$, so the foci are at $(\pm 4, 0)$, exactly the red points F marked on the diagram. Also $b^2 = a^2 - c^2 = 25 - 16 = 9$. ✓

Why other options are wrong:

- (A) $\frac{3}{5} = \frac{b}{a}$ is the ratio of semi-axes, not the eccentricity.
- (C) $\frac{16}{25} = e^2$ forgot to take the square root.
- (B) $\frac{5}{4} > 1$ is impossible for an ellipse (that would be a hyperbola's regime).

Final Answer: $e = \dfrac{4}{5} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q28

Q29.

Solution

Concept — Direction ratios of a line in space: The direction of the line joining two points is captured by the displacement vector $\vec{AB} = B - A$. Its components, the differences of corresponding coordinates, are the direction ratios $(x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1)$. Direction ratios are defined only up to a non-zero scalar multiple, so any proportional triple describes the same direction.

Step 1 — Subtract corresponding coordinates of $A(1, 2, 3)$ and $B(4, 6, 8)$:

$$(x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1) = (4 - 1,\ 6 - 2,\ 8 - 3).$$

Step 2 — Simplify:

$$(3,\ 4,\ 5).$$

Step 3 — Verify direction is consistent: The distance $AB = \sqrt{3^2 + 4^2 + 5^2} = \sqrt{9 + 16 + 25} = \sqrt{50}$, and dividing the ratios by $\sqrt{50}$ gives the unit direction cosines, confirming $(3, 4, 5)$ is a valid direction vector. ✓

Why other options are wrong:

- (C) $4, 6, 8$ are the coordinates of B alone, not a difference.
- (D) $1, 2, 3$ are the coordinates of A alone.
- (B) $5, 8, 11$ *adds* the coordinates $(1 + 4, 2 + 6, 3 + 8)$ instead of subtracting.

Final Answer: direction ratios $3, 4, 5 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q29

Q30.

Solution

Concept — Distance of a point from a plane: The perpendicular distance from a point (x_0, y_0, z_0) to the plane $ax + by + cz + d = 0$ is $D = \dfrac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$. The numerator substitutes the point into the plane equation (its sign tells which side the point is on, the modulus discards that), and the denominator is the length of the plane's normal vector (a, b, c).

Step 1 — Identify coefficients and point: For $2x + y + 2z - 9 = 0$, $a = 2, b = 1, c = 2, d = -9$; the point is $(1, 1, 1)$.

Step 2 — Evaluate the numerator:

$$|2(1) + 1(1) + 2(1) - 9| = |2 + 1 + 2 - 9| = | - 4| = 4.$$

Step 3 — Evaluate the denominator:

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3.$$

Step 4 — Divide:

$$D = \frac{4}{3}.$$

The negative sign of -4 indicates the point lies on the opposite side of the plane from the normal's direction, but distance is the absolute value $\frac{4}{3}$. ✓

Why other options are wrong:

- (C) 4 keeps the numerator but forgets to divide by the normal's length 3.
- (D) $\frac{2}{3}$ and (A) $\frac{5}{3}$ both miscount the numerator (e.g. dropping or mis-adding a term so it reads 2 or 5 instead of 4).

Final Answer: distance $= \dfrac{4}{3} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q30

Q31.

Solution

Concept — Scalar projection of one vector on another: The (scalar) projection of $\vec{a}$ onto $\vec{b}$ is the signed length of the shadow of $\vec{a}$ along $\vec{b}$, given by $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = |\vec{a}|\cos\theta$, where θ is the angle between them. The dot product supplies $|\vec{a}||\vec{b}|\cos\theta$, and dividing by $|\vec{b}|$ strips off the length of $\vec{b}$, leaving the component of $\vec{a}$ in $\vec{b}$'s direction.

Step 1 — Compute the dot product $\vec{a}\cdot\vec{b}$: With $\vec{a} = 2\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+2\hat{k}$,

$$\vec{a}\cdot\vec{b} = (2)(1) + (3)(2) + (2)(2) = 2 + 6 + 4 = 12.$$

Step 2 — Compute $|\vec{b}|$:

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1+4+4} = \sqrt{9} = 3.$$

Step 3 — Divide to get the projection:

$$\text{proj} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{12}{3} = 4.$$

Step 4 — Plausibility check: Since $|\vec{a}| = \sqrt{4+9+4} = \sqrt{17} \approx 4.12$, the projection 4 is just shy of $|\vec{a}|$, meaning $\vec{a}$ points nearly along $\vec{b}$ ($\cos\theta = 4/\sqrt{17} \approx 0.97$), which is reasonable. ✓

Why other options are wrong:

- (D) 12 is just the dot product $\vec{a}\cdot\vec{b}$, before dividing by $|\vec{b}|$.
- (B) $\frac{12}{\sqrt{17}}$ divides by $|\vec{a}|$ instead of $|\vec{b}|$.
- (A) $\frac{12}{17}$ divides by $|\vec{a}|^2 = 17$, the wrong normaliser entirely.

Final Answer: projection $= 4 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q31

Q32.

Solution

Concept — Area of a triangle from two side vectors: If two sides of a triangle are the vectors $\vec{a}$ and $\vec{b}$ drawn from a common vertex, the parallelogram they span has area $|\vec{a} \times \vec{b}|$, and the triangle is exactly half of it: area $= \frac{1}{2}|\vec{a} \times \vec{b}|$. The cross product's magnitude is $|\vec{a}||\vec{b}|\sin\theta$, the natural "base times height" for the parallelogram.

Step 1 — Compute the cross product: Using $\hat{\imath} \times \hat{\jmath} = \hat{k}$,

$$\vec{a} \times \vec{b} = (3\hat{\imath}) \times (4\hat{\jmath}) = 12\,(\hat{\imath} \times \hat{\jmath}) = 12\hat{k}.$$

Step 2 — Take its magnitude:

$$|\vec{a} \times \vec{b}| = |12\hat{k}| = 12.$$

Step 3 — Halve for the triangle:

$$\text{area} = \frac{1}{2}(12) = 6.$$

Step 4 — Cross-check geometrically: The vectors $3\hat{\imath}$ and $4\hat{\jmath}$ are perpendicular, forming a right triangle with legs 3 and 4. Its area is $\frac{1}{2}\times\text{base}\times\text{height} = \frac{1}{2}(3)(4) = 6$, matching Step 3. ✓

Why other options are wrong:

- (A) 12 is the *parallelogram* area; it omits the factor $\frac{1}{2}$.
- (C) 3 keeps only one leg, not the product of both halved.
- (B) 7 wrongly adds the legs $(3 + 4)$ instead of using the cross product.

Final Answer: area $= 6 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q32

Q33.

Solution

Concept — Volume via the scalar triple product: The volume of a parallelepiped with coterminous edges $\vec{a}, \vec{b}, \vec{c}$ equals the absolute value of the scalar triple product $[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$, which is computed as the determinant whose rows are the three edge vectors' components. A zero value signals that the three edges are coplanar (degenerate, zero volume).

Step 1 — Write the edges as rows and form the determinant: With $\vec{a} = \hat{i} = (1,0,0)$, $\vec{b} = \hat{j} = (0,1,0)$, $\vec{c} = \hat{k} = (0,0,1)$,

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}.$$

Step 2 — Evaluate the determinant: This is the identity-matrix determinant, which equals the product of its diagonal entries,

$$1 \times 1 \times 1 = 1.$$

Step 3 — Interpret: The three mutually perpendicular unit vectors span a unit cube, whose volume is $1 \times 1 \times 1 = 1$, confirming the determinant. ✓

Why other options are wrong:

- (B) 0 would require the three edges to be coplanar; here they are linearly independent.
- (C) 3 wrongly sums the three unit lengths rather than multiplying via the determinant.
- (D) $\sqrt{3}$ is the cube's space diagonal length, not its volume.

Final Answer: volume $= 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q33

Q34.

Solution

Concept — The mode of a data set: The mode is the value that appears most frequently in a data set. It is a measure of central tendency that, unlike the mean, is unaffected by extreme values and can be read off directly from a frequency tally. (A data set may have one mode, several, or none if all values are equally frequent.)

Step 1 — Tally the frequency of each distinct value in $4, 5, 5, 6, 6, 6, 7, 8$:

$$4: 1, \quad 5: 2, \quad 6: 3, \quad 7: 1, \quad 8: 1.$$

Step 2 — Identify the maximum frequency: The largest count is 3, achieved uniquely by the value 6.

Step 3 — Confirm totals: The frequencies sum to $1 + 2 + 3 + 1 + 1 = 8$, matching the eight data points, so the tally is complete and correct. ✓

Why other options are wrong:

- (A) 5 appears only twice, fewer than 6.
- (C) 7 appears once.
- (D) 8 appears once; neither rivals the three occurrences of 6.

Final Answer: mode $= 6 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q34

Q35.

Solution

Concept — Conditional probability: The probability of A *given* that B has occurred is $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$, valid when $P(B) > 0$. Conditioning on B restricts the sample space to outcomes inside B, and we ask what fraction of that reduced space also lies in A, namely the overlap $A \cap B$ rescaled by $P(B)$.

Step 1 — Identify the given probabilities: $P(A \cap B) = 0.2$ and $P(B) = 0.5$.

Step 2 — Substitute into the formula:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.5}.$$

Step 3 — Simplify the ratio:

$$\frac{0.2}{0.5} = \frac{2}{5} = 0.4.$$

Step 4 — Sanity check: Since $A \cap B \subseteq B$, we must have $P(A \mid B) \leq 1$; indeed $0.4 \leq 1$. Also $0.4 > P(A \cap B) = 0.2$, as expected because dividing by $P(B) = 0.5 < 1$ inflates the value. ✓

Why other options are wrong:

- (A) 0.1 multiplies the two probabilities (0.2×0.5) instead of dividing.
- (C) 0.25 inverts the ratio, computing $P(A \cap B)/(2 \cdot ...)$ or 0.5×0.5.
- (B) 0.7 adds $0.2 + 0.5$, which is not how conditional probability works.

Final Answer: $P(A \mid B) = 0.4 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q35

Q36.

Solution

Concept — Bayes' theorem: When an effect (drawing a red ball) can arise from several mutually exclusive causes (Box I or Box II), Bayes' theorem reverses the conditioning to find the probability of a particular cause given the observed effect: $P(I \mid R) = \dfrac{P(I)P(R \mid I)}{P(I)P(R \mid I) + P(II)P(R \mid II)}$. The denominator is the total probability of drawing red, summed over all boxes.

Step 1 — Prior probabilities of the boxes: A box is chosen at random, so

$$P(I) = P(II) = \tfrac{1}{2}.$$

Step 2 — Likelihoods of drawing red from each box: Box I has 3 red of 5 and Box II has 1 red of 5:

$$P(R \mid I) = \frac{3}{5}, \qquad P(R \mid II) = \frac{1}{5}.$$

Step 3 — Total probability of red (denominator):

$$P(R) = \tfrac{1}{2} \cdot \tfrac{3}{5} + \tfrac{1}{2} \cdot \tfrac{1}{5} = \frac{3}{10} + \frac{1}{10} = \frac{4}{10}.$$

Step 4 — Apply Bayes' theorem:

$$P(I \mid R) = \frac{\frac{1}{2} \cdot \frac{3}{5}}{\frac{4}{10}} = \frac{\frac{3}{10}}{\frac{4}{10}} = \frac{3}{4}.$$

Step 5 — Consistency check: The complementary posterior $P(II \mid R) = \dfrac{1/10}{4/10} = \frac{1}{4}$, and $\frac{3}{4} + \frac{1}{4} = 1$, as the two boxes exhaust all possibilities. ✓

Why other options are wrong:

- (A) $\frac{1}{4}$ is actually $P(II \mid R)$, the wrong box.
- (B) $\frac{1}{2}$ is the prior $P(I)$, ignoring the red-ball evidence.
- (D) $\frac{2}{5}$ comes from using $P(R) = \frac{4}{10}$ as the answer rather than dividing the numerator by it.

Final Answer: $P(I \mid R) = \dfrac{3}{4} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q36

Q37.

Solution

Concept — Pythagorean identity: Dividing the fundamental identity $\sin^2\theta + \cos^2\theta = 1$ through by $\cos^2\theta$ yields $\tan^2\theta + 1 = \sec^2\theta$. So the numerator $1 + \tan^2\theta$ is just another name for $\sec^2\theta$, and the whole expression collapses to a ratio of equal quantities.

Step 1 — Replace the numerator using the identity:

$$1 + \tan^2\theta = \sec^2\theta.$$

Step 2 — Substitute and cancel:

$$\frac{1 + \tan^2\theta}{\sec^2\theta} = \frac{\sec^2\theta}{\sec^2\theta} = 1.$$

Step 3 — Numerical check at $\theta = 45°$: Here $\tan 45° = 1$ and $\sec 45° = \sqrt{2}$, so the expression is $\dfrac{1 + 1^2}{(\sqrt{2})^2} = \dfrac{2}{2} = 1$, confirming the simplification. ✓

Why other options are wrong:

- (B) $\sec^2\theta$ keeps only the numerator, forgetting to divide it away.

- (C) $\tan^2\theta$ drops the $+1$ and the denominator.
- (D) $\cos^2\theta$ inverts $\sec^2\theta$ but ignores the numerator entirely.

Final Answer: the expression equals $1 \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q37

Q38.

Solution

Concept — Sine of a compound angle: A non-standard angle can be split into a sum of two standard angles and evaluated with the addition formula $\sin(A+B) = \sin A\cos B + \cos A\sin B$. Writing $75^\circ = 45^\circ + 30^\circ$ lets us use the known exact values of 45° and 30°.

Step 1 — Apply the formula with $A = 45^\circ,\ B = 30^\circ$:

$$\sin 75^\circ = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ.$$

Step 2 — Insert the standard values $\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\sin 30^\circ = \frac{1}{2}$:

$$\sin 75^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}.$$

Step 3 — Combine over the common denominator:

$$\sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}.$$

Step 4 — Numerical check: $\dfrac{\sqrt{3}+1}{2\sqrt{2}} = \dfrac{1.732+1}{2.828} \approx \dfrac{2.732}{2.828} \approx 0.966$, which matches $\sin 75^\circ \approx 0.966$ from tables. ✓

Why other options are wrong:

- (A) $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$ is $\sin 15^\circ$ (it uses the minus sign from $45^\circ - 30^\circ$).
- (C) $\frac{1}{2} = \sin 30^\circ$, an unrelated standard value.
- (D) $\frac{\sqrt{3}}{2} = \sin 60^\circ$, also unrelated to 75°.

Final Answer: $\sin 75^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}} \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q38

Q39.

Solution

Concept — Principal value of the inverse tangent: The function $\tan^{-1}$ returns the unique angle whose tangent is the given number, chosen from the principal range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. This restriction is what makes $\tan^{-1}$ a genuine (single-valued) function, since $\tan$ itself repeats every π. So we seek the one angle in that open interval with tangent 1.

Step 1 — Set up the defining equation: We want θ with

$$\tan\theta = 1, \qquad \theta \in \left(-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right).$$

Step 2 — Identify the standard angle: Since $\tan\frac{\pi}{4} = \dfrac{\sin(\pi/4)}{\cos(\pi/4)} = \dfrac{1/\sqrt{2}}{1/\sqrt{2}} = 1$ and $\frac{\pi}{4}$ lies in the principal range,

$$\theta = \frac{\pi}{4}.$$

Step 3 — Rule out other solutions: Other angles with $\tan = 1$ (such as $\frac{5\pi}{4}$) lie outside $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so the principal value is uniquely $\frac{\pi}{4}$. ✓

Why other options are wrong:

- (A) $\frac{\pi}{3}$ has $\tan = \sqrt{3} \neq 1$.
- (B) $\frac{\pi}{6}$ has $\tan = \frac{1}{\sqrt{3}} \neq 1$.
- (D) $\frac{\pi}{2}$ is excluded: $\tan$ is undefined there (it diverges).

Final Answer: $\tan^{-1}(1) = \dfrac{\pi}{4} \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q39

Q40.

Solution

Concept — The cosine rule: In any triangle, the side opposite a known angle can be found from the other two sides and the included angle via $a^2 = b^2 + c^2 - 2bc\cos A$. It generalises the Pythagorean theorem: when $A = 90^\circ$, $\cos A = 0$ and it reduces to $a^2 = b^2 + c^2$. Here $A = 60^\circ$ is the angle between sides b and c.

Step 1 — Substitute $b = 5$, $c = 8$, $A = 60^\circ$ (with $\cos 60^\circ = \frac{1}{2}$):

$$a^2 = 5^2 + 8^2 - 2(5)(8)\cos 60^\circ = 25 + 64 - 80 \cdot \tfrac{1}{2}.$$

Step 2 — Simplify the arithmetic:

$$a^2 = 89 - 40 = 49.$$

Step 3 — Take the positive square root:

$$a = \sqrt{49} = 7.$$

Step 4 — Plausibility check (triangle inequality): The three sides $a = 7, b = 5, c = 8$ satisfy $5 + 7 > 8$, $5 + 8 > 7$, $7 + 8 > 5$, so a valid triangle exists; and $a = 7$ lies between $|c - b| = 3$ and $c + b = 13$, as it must. ✓

Why other options are wrong:

- (C) $\sqrt{89}$ drops the $-2bc\cos A$ term, treating the triangle as right-angled.
- (A) $\sqrt{129}$ uses $\cos 120^\circ = -\frac{1}{2}$, which would *add* 40 instead of subtracting.
- (B) $13 = b + c$ just sums the two sides, ignoring the cosine rule entirely.

Final Answer: $a = 7 \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q40**

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	A
6	B	7	C	8	D	9	B	10	A
11	C	12	D	13	A	14	B	15	C
16	D	17	A	18	B	19	D	20	C
21	A	22	B	23	C	24	D	25	A
26	B	27	C	28	D	29	A	30	B
31	C	32	D	33	A	34	B	35	D
36	C	37	A	38	B	39	C	40	D