

# SRMJEEE Mathematics Sample Paper – 4

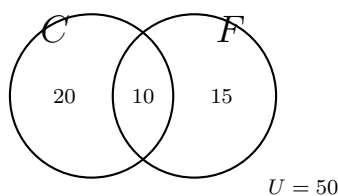
Duration: 47 Minutes

Maximum Marks: 40

## Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of **SRMJEEE** (SRM Joint Engineering Entrance Examination).
- Each correct answer carries **+1 mark**. There is **no negative marking**; an unattempted or wrong answer scores 0.
- Only **one** option is correct. Choose carefully.
- The actual SRMJEEE is a **computer-based test** conducted in remote-proctored online mode, with all sections sharing a common time window and no per-section limit.
- Personal calculators, mobile phones, log tables and other electronic gadgets are strictly prohibited.

**Q1.** In a class of 50 students, 30 play cricket and 25 play football; 10 play both, as shown in the Venn diagram. The number of students who play *neither* game is:



- (A) 15
- (B) 10
- (C) 5
- (D) 20

**Q2.** The relation  $R = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$  has domain and range, respectively:

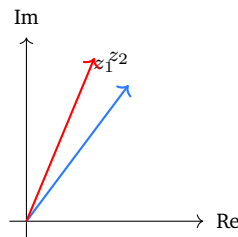


- (A)  $\{1, 2, 3, 4\}$  and  $\{2, 4, 6, 8\}$
- (B)  $\{2, 4, 6, 8\}$  and  $\{1, 2, 3, 4\}$
- (C)  $\{1, 2, 3, 4\}$  and  $\{1, 2, 3, 4\}$
- (D)  $\{1, 2, 4, 8\}$  and  $\{2, 4, 6, 8\}$

**Q3.** If  $f(x) = 3x - 5$ , then  $f^{-1}(x)$  is:

- (A)  $\frac{x - 5}{3}$
- (B)  $\frac{x + 5}{3}$
- (C)  $3x + 5$
- (D)  $\frac{x}{3} - 5$

**Q4.** If  $z_1 = 3 + 4i$  and  $z_2 = 5 + 12i$ , then  $|z_1 z_2|$  equals:



- (A) 17
- (B) 60
- (C) 169
- (D) 65

**Q5.** If  $\omega$  is a non-real cube root of unity, then the value of  $(1 + \omega)(1 + \omega^2)$  is:

- (A) 1
- (B) 0
- (C)  $-1$
- (D)  $\omega$

**Q6.** For the quadratic equation  $2x^2 - 7x + 3 = 0$ , the sum and the product of the roots are, respectively:



- (A)  $-\frac{7}{2}$  and  $\frac{3}{2}$
- (B)  $\frac{7}{2}$  and  $\frac{3}{2}$
- (C)  $\frac{7}{2}$  and  $-\frac{3}{2}$
- (D) 7 and 3

**Q7.** If  $\alpha$  and  $\beta$  are the roots of  $x^2 - 6x + 4 = 0$ , then  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$  equals:

- (A) 9
- (B)  $\frac{9}{2}$
- (C) 7
- (D)  $\frac{7}{2}$

**Q8.** For two matrices  $A$  and  $B$  for which the product  $AB$  is defined,  $(AB)^T$  equals:

- (A)  $A^T B^T$
- (B)  $AB$
- (C)  $BA$
- (D)  $B^T A^T$

**Q9.** If  $A$  is a square matrix of order 3 with  $|A| = 4$ , then  $|\text{adj } A|$  equals:

- (A) 16
- (B) 4
- (C) 64
- (D) 12

**Q10.** If  $A$  and  $B$  are square matrices of the same order with  $|A| = 2$  and  $|B| = 5$ , then  $|AB|$  equals:

- (A) 7
- (B) 10



- (C) 25
- (D)  $\frac{2}{5}$

- Q11.** A system of two linear equations in two unknowns possesses a unique solution if and only if the determinant of its coefficient matrix is:
- (A) zero
  - (B) equal to 1
  - (C) non-zero
  - (D) negative
- Q12.** The number of three-digit numbers having all distinct digits, formed using the digits 1, 2, 3, 4, 5 (no digit repeated), is:
- (A) 125
  - (B) 10
  - (C) 120
  - (D) 60
- Q13.** If every pair of people in a room of 10 persons shakes hands exactly once, the total number of handshakes is:
- (A) 90
  - (B) 45
  - (C) 100
  - (D) 20
- Q14.** The number of ways in which 3 men and 3 women can be seated in a row so that men and women occupy alternate seats is:
- (A) 72
  - (B) 36
  - (C) 144



(D) 720

**Q15.** If the roots of  $x^3 - 12x^2 + 39x - 28 = 0$  are in arithmetic progression, then the middle root is:

(A) 7

(B) 1

(C) 4

(D) 12

**Q16.** If  $\alpha$  and  $\beta$  are the roots of  $x^2 - 3x + 2 = 0$ , then the quadratic equation whose roots are  $2\alpha$  and  $2\beta$  is:

(A)  $x^2 - 3x + 2 = 0$

(B)  $x^2 - 6x + 2 = 0$

(C)  $x^2 - 3x + 8 = 0$

(D)  $x^2 - 6x + 8 = 0$

**Q17.** The value of  $\lim_{x \rightarrow 2} \frac{x^5 - 2^5}{x - 2}$  is:

(A) 80

(B) 32

(C) 16

(D) 5

**Q18.** If  $y = \tan^{-1} x$ , then  $\frac{dy}{dx}$  is:

(A)  $\frac{1}{\sqrt{1-x^2}}$

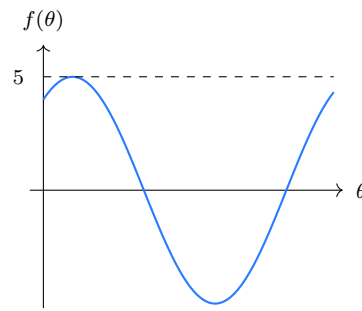
(B)  $\frac{1}{1+x^2}$

(C)  $\frac{-1}{1+x^2}$

(D)  $\frac{1}{1-x^2}$



**Q19.** The maximum value of  $f(\theta) = 3 \sin \theta + 4 \cos \theta$ , whose graph is shown, is:



- (A) 3
- (B) 7
- (C) 4
- (D) 5

**Q20.** The slope of the normal to the curve  $y = x^2$  at the point  $(1, 1)$  is:

- (A) 2
- (B)  $-2$
- (C)  $-\frac{1}{2}$
- (D)  $\frac{1}{2}$

**Q21.** The order and the degree of the differential equation  $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y = 0$  are, respectively:

- (A) order 3, degree 2
- (B) order 2, degree 3
- (C) order 3, degree 4
- (D) order 4, degree 1

**Q22.**  $\int x e^x dx$  equals:

- (A)  $x e^x + e^x + C$
- (B)  $x e^x - e^x + C$

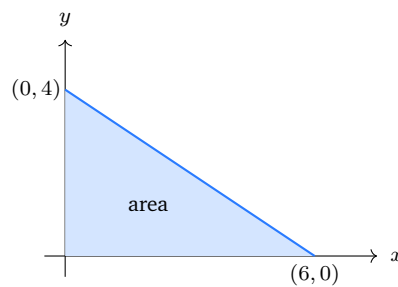


- (C)  $\frac{x^2}{2}e^x + C$   
(D)  $xe^x + C$

**Q23.** The value of  $\int_0^2 (3x^2 + 1) dx$  is:

- (A) 8  
(B) 9  
(C) 10  
(D) 12

**Q24.** The area of the triangle bounded by the line  $2x + 3y = 12$  and the coordinate axes (shaded) is:



- (A) 24 sq. units  
(B) 6 sq. units  
(C) 10 sq. units  
(D) 12 sq. units

**Q25.** The value of  $\int_0^\pi \sin x dx$  is:

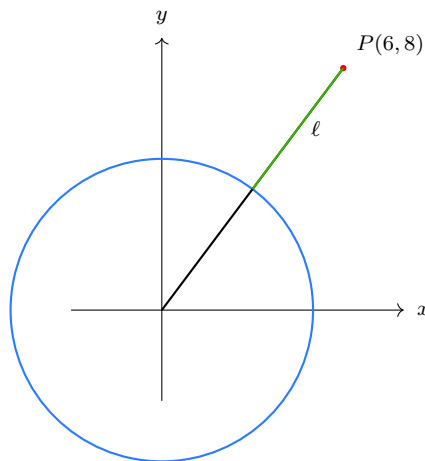
- (A) 2  
(B) 0  
(C) 1  
(D)  $\pi$

**Q26.** The acute angle between the two straight lines  $y = x$  and  $y = \sqrt{3}x$  is:



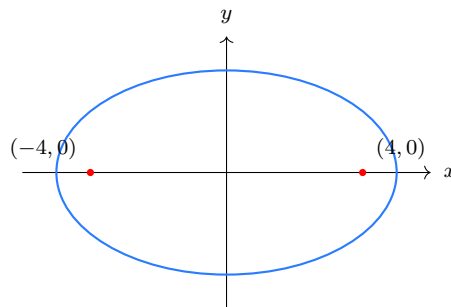
- (A)  $30^\circ$
- (B)  $15^\circ$
- (C)  $45^\circ$
- (D)  $60^\circ$

**Q27.** The length of the tangent drawn from the point  $P(6, 8)$  to the circle  $x^2 + y^2 = 25$  is:



- (A) 5
- (B)  $\sqrt{61}$
- (C) 10
- (D)  $5\sqrt{3}$

**Q28.** The foci of the ellipse  $\frac{x^2}{25} + \frac{y^2}{9} = 1$  are:



- (A)  $(\pm 5, 0)$
- (B)  $(0, \pm 4)$



(C)  $(\pm 4, 0)$

(D)  $(\pm 3, 0)$

**Q29.** If a line has direction cosines  $l = \frac{1}{2}$ ,  $m = \frac{1}{2}$  and  $n > 0$ , then the value of  $n$  is:

(A)  $\frac{1}{2}$

(B)  $\frac{1}{\sqrt{2}}$

(C)  $\frac{1}{4}$

(D)  $\frac{\sqrt{3}}{2}$

**Q30.** The equation of the plane passing through the point  $(1, 2, 3)$  with normal vector  $\vec{n} = 2\hat{i} + \hat{j} + 2\hat{k}$  is:

(A)  $2x + y + 2z = 10$

(B)  $2x + y + 2z = 6$

(C)  $x + 2y + 3z = 10$

(D)  $2x + y + 2z = 0$

**Q31.** The value of  $\lambda$  for which the vectors  $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$  are perpendicular is:

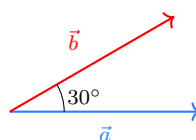
(A)  $-\frac{5}{2}$

(B) 2

(C)  $\frac{5}{2}$

(D) 5

**Q32.** If  $|\vec{a}| = 4$ ,  $|\vec{b}| = 5$  and the angle between  $\vec{a}$  and  $\vec{b}$  is  $30^\circ$ , then  $|\vec{a} \times \vec{b}|$  equals:



- (A) 20
- (B)  $\frac{20}{\sqrt{3}}$
- (C)  $10\sqrt{3}$
- (D) 10

**Q33.** The value of  $\lambda$  for which the vectors  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$  and  $\vec{c} = \hat{i} + \hat{j} + \lambda\hat{k}$  are coplanar is:

- (A) 0
- (B) 1
- (C) 2
- (D) 3

**Q34.** The variance of the observations 1, 3, 5, 7, 9 is:

- (A) 8
- (B) 5
- (C)  $\sqrt{8}$
- (D) 4

**Q35.** If  $A$  and  $B$  are independent events with  $P(A) = 0.3$  and  $P(B) = 0.4$ , then  $P(A \cap B)$  equals:

- (A) 0.7
- (B) 0.1
- (C) 0.58
- (D) 0.12

**Q36.** A fair coin is tossed 3 times. The probability of getting at least one head is:

- (A)  $\frac{1}{8}$
- (B)  $\frac{3}{8}$



- (C)  $\frac{7}{8}$   
(D)  $\frac{1}{2}$

**Q37.** The general solution of the equation  $\sin \theta = 0$  is:

- (A)  $\theta = n\pi, n \in \mathbb{Z}$   
(B)  $\theta = (2n + 1)\frac{\pi}{2}, n \in \mathbb{Z}$   
(C)  $\theta = 2n\pi, n \in \mathbb{Z}$   
(D)  $\theta = n\frac{\pi}{2}, n \in \mathbb{Z}$

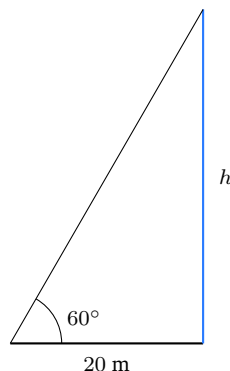
**Q38.** If  $\tan A = \frac{1}{2}$  and  $\tan B = \frac{1}{3}$ , then  $\tan(A + B)$  equals:

- (A)  $\frac{5}{6}$   
(B) 1  
(C)  $\frac{1}{6}$   
(D) 5

**Q39.** For every  $x \in [-1, 1]$ , the value of  $\sin^{-1} x + \cos^{-1} x$  is:

- (A) 0  
(B)  $\pi$   
(C)  $\frac{\pi}{4}$   
(D)  $\frac{\pi}{2}$

**Q40.** The angle of elevation of the top of a vertical tower from a point 20 m from its foot is  $60^\circ$ , as shown. The height of the tower is:



- (A) 20 m  
(B)  $\frac{20}{\sqrt{3}}$  m  
(C)  $20\sqrt{3}$  m  
(D) 40 m



## Detailed Solutions

Q1.

## Solution

**Concept — Inclusion–exclusion for “neither”:** When two sets overlap, simply adding their sizes double-counts the common members, so the correct size of the union is  $n(C \cup F) = n(C) + n(F) - n(C \cap F)$ . Everyone in the class is either inside this union (plays at least one game) or outside it (plays neither). Hence the “neither” count is the complement  $n(U) - n(C \cup F)$ , where  $U$  is the universal set of all 50 students. This is just a restatement of the fact that the four disjoint regions of the Venn diagram (cricket-only, football-only, both, neither) must add up to the total.

**Step 1 — Read the data from the diagram:** The class has  $n(U) = 50$  students. Cricket players total  $n(C) = 30$ , football players total  $n(F) = 25$ , and the overlap (both games) is  $n(C \cap F) = 10$ . The diagram already splits these as 20 cricket-only, 15 football-only and 10 in the middle.

**Step 2 — Size of the union:** Substitute into inclusion–exclusion:

$$n(C \cup F) = n(C) + n(F) - n(C \cap F) = 30 + 25 - 10 = 45.$$

**Step 3 — Subtract from the total:** The students playing neither game are

$$n(U) - n(C \cup F) = 50 - 45 = 5.$$

**Step 4 — Cross-check with disjoint regions:** Add the four labelled regions of the diagram directly: cricket-only 20, football-only 15, both 10, neither 5. Their sum is  $20 + 15 + 10 + 5 = 50 = n(U)$ , which confirms the count is internally consistent.

**Why other options are wrong:**

- (A) 15 comes from  $50 - (30 + 25 - 10)$  done as  $50 - 35 = 15$ , i.e. forgetting to subtract the overlap correctly (using 35 instead of 45).
- (B) 10 is simply the both-games overlap  $n(C \cap F)$  copied straight from the centre of the diagram, not the outside region.
- (D) 20 is the cricket-only region  $n(C) - n(C \cap F) = 30 - 10$ , again a region inside the union rather than the complement.

**Final Answer:** neither = 5  $\Rightarrow$   C

**Answer:** (C) [Go Back to Q1](#)



Q2.

**Solution**

**Concept — Domain and range of a relation:** A relation is just a set of ordered pairs  $(x, y)$ . By definition the *domain* is the set of all the first coordinates that actually appear (the inputs), and the *range* is the set of all the second coordinates that appear (the outputs). Each element is listed once in a set, so even if a value repeated across pairs we would write it only a single time. Order between the two sets matters: domain always refers to the  $x$ -entries and range to the  $y$ -entries.

**Step 1 — List the first coordinates:** Reading the first entry of each pair in  $R = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$  gives 1, 2, 3, 4. Collecting them as a set, the domain is

$$\text{Domain} = \{1, 2, 3, 4\}.$$

**Step 2 — List the second coordinates:** Reading the second entry of each pair gives 2, 4, 6, 8. As a set, the range is

$$\text{Range} = \{2, 4, 6, 8\}.$$

**Step 3 — Sanity check the rule:** Each output is double its input,  $y = 2x$ , so the inputs 1, 2, 3, 4 map to 2, 4, 6, 8. This pairing matches the listed ordered pairs exactly, confirming our two sets.

**Why other options are wrong:**

- (B)  $\{2, 4, 6, 8\}$  and  $\{1, 2, 3, 4\}$  swaps the two: it reports the range as the domain and vice versa.
- (C)  $\{1, 2, 3, 4\}$  and  $\{1, 2, 3, 4\}$  repeats the domain as the range, ignoring the actual second coordinates.
- (D)  $\{1, 2, 4, 8\}$  and  $\{2, 4, 6, 8\}$  misreads the first coordinates (it picks up an 8 that is a second entry and drops the 3).

**Final Answer:** domain  $\{1, 2, 3, 4\}$ , range  $\{2, 4, 6, 8\} \Rightarrow \boxed{A}$

**Answer:** (A) [Go Back to Q2](#)



Q3.

**Solution**

**Concept — Finding an inverse function:** The inverse  $f^{-1}$  undoes whatever  $f$  does, so if  $f$  maps  $x \mapsto y$  then  $f^{-1}$  maps  $y \mapsto x$ . The standard procedure is to write  $y = f(x)$ , solve algebraically for  $x$  in terms of  $y$ , and then swap the names  $x$  and  $y$  so the answer is expressed as a function of  $x$ . A linear function  $f(x) = 3x - 5$  is one-to-one (its graph is a straight line of non-zero slope), so its inverse exists and is also linear.

**Step 1 — Write  $y = f(x)$ :** Set

$$y = 3x - 5.$$

**Step 2 — Solve for  $x$ :** Add 5 to both sides, then divide by 3:

$$y + 5 = 3x \implies x = \frac{y + 5}{3}.$$

**Step 3 — Interchange  $x$  and  $y$ :** Replacing  $y$  by  $x$  gives the inverse function

$$f^{-1}(x) = \frac{x + 5}{3}.$$

**Step 4 — Verify by composition:** Check  $f(f^{-1}(x)) = 3\left(\frac{x + 5}{3}\right) - 5 = (x + 5) - 5 = x$ , and likewise  $f^{-1}(f(x)) = \frac{(3x - 5) + 5}{3} = \frac{3x}{3} = x$ . Both compositions return  $x$ , so the inverse is correct.

**Why other options are wrong:**

- (A)  $\frac{x - 5}{3}$  subtracts 5 instead of adding it; it would invert  $f(x) = 3x + 5$ , not  $3x - 5$ .
- (C)  $3x + 5$  just flips the sign of the constant in  $f$  itself; it is another function, not the inverse (its composition with  $f$  does not give  $x$ ).
- (D)  $\frac{x}{3} - 5$  divides only the  $x$  by 3 and forgets that the  $+5$  must also be divided by 3.

**Final Answer:**  $f^{-1}(x) = \frac{x + 5}{3} \Rightarrow \boxed{\text{B}}$

**Answer: (B)** [Go Back to Q3](#)



Q4.

**Solution**

**Concept — Modulus is multiplicative:** For any two complex numbers the modulus of a product equals the product of the moduli,  $|z_1 z_2| = |z_1| |z_2|$ . This follows because multiplying complex numbers in polar form multiplies their lengths (and adds their arguments). The modulus of  $z = a + bi$  is its distance from the origin in the Argand plane,  $|z| = \sqrt{a^2 + b^2}$ . So instead of multiplying the two complex numbers out, we can find each length separately and multiply.

**Step 1 — Modulus of  $z_1$ :** With  $z_1 = 3 + 4i$ ,

$$|z_1| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

**Step 2 — Modulus of  $z_2$ :** With  $z_2 = 5 + 12i$ ,

$$|z_2| = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13.$$

**Step 3 — Multiply the moduli:**

$$|z_1 z_2| = |z_1| |z_2| = 5 \times 13 = 65.$$

**Step 4 — Cross-check by direct multiplication:**  $z_1 z_2 = (3 + 4i)(5 + 12i) = 15 + 36i + 20i + 48i^2 = 15 + 56i - 48 = -33 + 56i$ . Then  $|z_1 z_2| = \sqrt{(-33)^2 + 56^2} = \sqrt{1089 + 3136} = \sqrt{4225} = 65$ , matching exactly.

**Why other options are wrong:**

- (A) 17 adds the moduli,  $5 + 13$  (wrongly using  $|z_1 + z_2|$ -style addition instead of multiplication).
- (B) 60 multiplies mismatched components such as  $5 \times 12$ , ignoring the Pythagorean moduli.
- (C)  $169 = 13^2$  uses only  $|z_2|^2$  and forgets the factor  $|z_1| = 5$ .

**Final Answer:**  $|z_1 z_2| = 65 \Rightarrow \boxed{D}$

**Answer:** (D) [Go Back to Q4](#)



Q5.

**Solution**

**Concept — Properties of cube roots of unity:** The three cube roots of unity are  $1, \omega, \omega^2$ . The two non-real roots satisfy two key identities: their sum with 1 vanishes,  $1 + \omega + \omega^2 = 0$ , and any of them cubed returns 1,  $\omega^3 = 1$ . From the first identity we can read off  $1 + \omega = -\omega^2$  and  $1 + \omega^2 = -\omega$ , which lets us replace awkward sums by single powers of  $\omega$ .

**Step 1 — Rewrite each factor:** Using  $1 + \omega + \omega^2 = 0$ ,

$$1 + \omega = -\omega^2, \quad 1 + \omega^2 = -\omega.$$

**Step 2 — Multiply the factors:**

$$(1 + \omega)(1 + \omega^2) = (-\omega^2)(-\omega) = \omega^2 \cdot \omega = \omega^3.$$

**Step 3 — Apply  $\omega^3 = 1$ :**

$$\omega^3 = 1.$$

**Step 4 — Verify by direct expansion:** Expand without the shortcut:  $(1 + \omega)(1 + \omega^2) = 1 + \omega^2 + \omega + \omega^3 = (1 + \omega + \omega^2) + \omega^3 = 0 + 1 = 1$ . Same answer, confirming the result.

**Why other options are wrong:**

- (B) 0 would result from stopping at  $1 + \omega + \omega^2$  and forgetting the extra  $\omega^3 = 1$  term in the expansion.
- (C)  $-1$  comes from a sign slip, e.g. taking  $(1 + \omega)(1 + \omega^2) = -\omega^3$ .
- (D)  $\omega$  leaves the answer unsimplified, ignoring that  $\omega^3 = 1$  is a real number.

**Final Answer:**  $(1 + \omega)(1 + \omega^2) = 1 \Rightarrow \boxed{A}$

**Answer: (A)** [Go Back to Q5](#)



Q6.

**Solution**

**Concept — Vieta's relations:** For a quadratic  $ax^2 + bx + c = 0$  with roots  $\alpha, \beta$ , comparing it with  $a(x - \alpha)(x - \beta) = 0$  gives the relations  $\alpha + \beta = -\frac{b}{a}$  and  $\alpha\beta = \frac{c}{a}$ . The crucial point is the minus sign in the sum and the division by the leading coefficient  $a$ , which is easy to forget when  $a \neq 1$ .

**Step 1 — Identify the coefficients:** Compare  $2x^2 - 7x + 3 = 0$  with  $ax^2 + bx + c = 0$ :

$$a = 2, \quad b = -7, \quad c = 3.$$

**Step 2 — Sum of the roots:**

$$\alpha + \beta = -\frac{b}{a} = -\frac{-7}{2} = \frac{7}{2}.$$

**Step 3 — Product of the roots:**

$$\alpha\beta = \frac{c}{a} = \frac{3}{2}.$$

**Step 4 — Verify with the actual roots:** Factor  $2x^2 - 7x + 3 = (2x - 1)(x - 3)$ , giving roots  $\frac{1}{2}$  and 3. Then sum  $= \frac{1}{2} + 3 = \frac{7}{2}$  and product  $= \frac{1}{2} \times 3 = \frac{3}{2}$ , matching exactly.

**Why other options are wrong:**

- (A)  $-\frac{7}{2}$  and  $\frac{3}{2}$  drops the minus sign in  $-b/a$ , giving a negative sum.
- (C)  $\frac{7}{2}$  and  $-\frac{3}{2}$  flips the sign of the product, treating  $c$  as  $-3$ .
- (D) 7 and 3 ignores the leading coefficient  $a = 2$  entirely, reading the relations as if  $a = 1$ .

**Final Answer:** sum  $\frac{7}{2}$ , product  $\frac{3}{2} \Rightarrow \boxed{\text{B}}$

**Answer: (B)** [Go Back to Q6](#)



Q7.

### Solution

**Concept — Symmetric functions of the roots:** The expression  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$  is symmetric in  $\alpha$  and  $\beta$ , so it can be written entirely in terms of the sum  $\alpha + \beta$  and product  $\alpha\beta$  without ever finding the roots themselves. Combine over a common denominator and use the identity  $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ :

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}.$$

**Step 1 — Read the sum and product:** For  $x^2 - 6x + 4 = 0$  (here  $a = 1$ ,  $b = -6$ ,  $c = 4$ ):

$$\alpha + \beta = -\frac{b}{a} = 6, \quad \alpha\beta = \frac{c}{a} = 4.$$

**Step 2 — Compute  $\alpha^2 + \beta^2$ :**

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 6^2 - 2(4) = 36 - 8 = 28.$$

**Step 3 — Divide by  $\alpha\beta$ :**

$$\frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{28}{4} = 7.$$

**Step 4 — Check with explicit roots:** The roots are  $3 \pm \sqrt{5}$ . Their product is  $(3)^2 - (\sqrt{5})^2 = 9 - 5 = 4$  and their squares sum to  $(3 + \sqrt{5})^2 + (3 - \sqrt{5})^2 = 2(9 + 5) = 28$ , so  $28/4 = 7$  again.

**Why other options are wrong:**

- (A) 9 uses  $(\alpha + \beta)^2/\alpha\beta = 36/4$ , forgetting to subtract  $2\alpha\beta$  in the numerator.
- (B)  $\frac{9}{2}$  divides 36 by  $2\alpha\beta = 8$  wrongly, mishandling the denominator.
- (D)  $\frac{7}{2}$  halves the correct value, e.g. by dividing 28 by 8 instead of 4.

**Final Answer:**  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 7 \Rightarrow \boxed{\text{C}}$

**Answer: (C)** [Go Back to Q7](#)



Q8.

**Solution**

**Concept — Reversal law for transposes:** Transposing swaps the rows and columns of a matrix. For a product, the transpose reverses the order of the factors:  $(AB)^T = B^T A^T$ . The reversal is forced by dimensions: if  $A$  is  $m \times n$  and  $B$  is  $n \times p$ , then  $AB$  is  $m \times p$  and  $(AB)^T$  is  $p \times m$ ; the only way to multiply  $A^T$  ( $n \times m$ ) and  $B^T$  ( $p \times n$ ) and get a  $p \times m$  result is to write them as  $B^T A^T$ .

**Step 1 — State the reversal law:** The factor that was on the right ( $B$ ) moves to the left after transposing, and each factor is individually transposed:

$$(AB)^T = B^T A^T.$$

**Step 2 — Justify via entries:** The  $(i, j)$  entry of  $(AB)^T$  equals the  $(j, i)$  entry of  $AB$ , namely  $\sum_k A_{jk} B_{ki} = \sum_k (B^T)_{ik} (A^T)_{kj}$ , which is exactly the  $(i, j)$  entry of  $B^T A^T$ .

**Step 3 — Quick numerical check:** Take  $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ . Then  $AB = \begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix}$  so  $(AB)^T = \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}$ , and  $B^T A^T = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}$ . They agree.

**Why other options are wrong:**

- (A)  $A^T B^T$  keeps the original order; in general the dimensions would not even match, so it is undefined or wrong.
- (B)  $AB$  confuses transposing with leaving the product unchanged.
- (C)  $BA$  commutes the matrices but does not transpose them, and matrix multiplication is not commutative anyway.

**Final Answer:**  $(AB)^T = B^T A^T \Rightarrow \boxed{D}$

**Answer: (D)** [Go Back to Q8](#)



Q9.

**Solution**

**Concept — Determinant of the adjugate:** For an  $n \times n$  matrix, the adjugate satisfies  $A(\text{adj } A) = |A| I_n$ . Taking determinants of both sides gives  $|A| |\text{adj } A| = |A|^n$  (since  $|cI_n| = c^n$ ). Dividing by  $|A|$  (assuming  $|A| \neq 0$ ) yields the standard result

$$|\text{adj } A| = |A|^{n-1}.$$

The exponent is  $n - 1$ , one less than the order, which is the single fact most often mis-remembered.

**Step 1 — Identify  $n$  and  $|A|$ :** The matrix has order  $n = 3$  and determinant  $|A| = 4$ .

**Step 2 — Apply the formula:**

$$|\text{adj } A| = |A|^{n-1} = 4^{3-1} = 4^2.$$

**Step 3 — Evaluate:**

$$4^2 = 16.$$

**Step 4 — Consistency check:** From  $A(\text{adj } A) = |A| I_3$ , the determinant of the right side is  $|A|^3 = 4^3 = 64$ , and the left side is  $|A| |\text{adj } A| = 4 \times 16 = 64$ . The two agree, confirming  $|\text{adj } A| = 16$ .

**Why other options are wrong:**

- (B) 4 uses exponent  $n - 2 = 1$  (i.e.  $|A|^1$ ), an off-by-one error.
- (C) 64 uses  $|A|^n = 4^3$ , forgetting to divide once by  $|A|$ .
- (D) 12 comes from multiplying  $|A| \times n = 4 \times 3$ , mixing the order with the determinant.

**Final Answer:**  $|\text{adj } A| = 16 \Rightarrow \boxed{\text{A}}$

**Answer: (A)** [Go Back to Q9](#)



Q10.

**Solution**

**Concept — Determinant is multiplicative:** For two square matrices of the same order, the determinant of the product equals the product of the determinants,  $|AB| = |A||B|$ . Geometrically the determinant is the scaling factor a matrix applies to volume; applying  $A$  then  $B$  scales volume by  $|A|$  and then by  $|B|$ , so the combined factor is  $|A||B|$ . This is a multiplication, never an addition.

**Step 1 — Note the determinants:** We are given  $|A| = 2$  and  $|B| = 5$ , and both are square of the same order so  $AB$  is defined.

**Step 2 — Apply the product rule:**

$$|AB| = |A||B| = 2 \times 5 = 10.$$

**Step 3 — Related cross-check:** The same rule gives  $|BA| = |B||A| = 5 \times 2 = 10$  as well, so even though  $AB \neq BA$  in general, their determinants are equal, which is a useful consistency observation.

**Why other options are wrong:**

- (A) 7 adds the determinants  $2 + 5$ , treating  $|AB|$  like a sum.
- (C) 25 squares  $|B|$  only ( $5^2$ ), dropping  $|A|$ .
- (D)  $\frac{2}{5}$  divides  $|A|$  by  $|B|$ , which is the rule for  $|AB^{-1}|$ , not  $|AB|$ .

**Final Answer:**  $|AB| = 10 \Rightarrow \boxed{\text{B}}$

**Answer: (B)** [Go Back to Q10](#)

Q11.

**Solution**

**Concept — Consistency and the coefficient determinant:** A system of two linear equations in two unknowns can be written as  $AX = B$ , where  $A$  is the  $2 \times 2$  matrix of coefficients. By Cramer's rule the solution is  $x = \frac{D_x}{D}$ ,  $y = \frac{D_y}{D}$  with  $D = |A|$ . A genuine unique answer exists precisely when we can divide, i.e. when  $D = |A| \neq 0$ . Geometrically the two equations represent two straight lines; a non-zero determinant means they are not parallel, so they meet in exactly one point.



**Step 1 — State the criterion:** A unique solution exists if and only if

$$|A| \neq 0.$$

**Step 2 — Interpret the failure case:** If  $|A| = 0$  the lines are parallel: either they never meet (no solution, inconsistent) or they coincide (infinitely many solutions). Neither is “unique”. So zero determinant is exactly the case that must be excluded.

**Step 3 — Why “non-zero” is the full answer:** The actual numerical value of the determinant does not matter; any non-zero value (positive, negative, 1,  $-7$ , etc.) gives a single intersection point. The only condition is non-vanishing.

**Why other options are wrong:**

- (A) zero is precisely the case that fails to give a unique solution (parallel or coincident lines).
- (B) equal to 1 is far too restrictive: a determinant of 3 or  $-2$  also gives a unique solution.
- (D) negative is also too restrictive: positive determinants such as  $+4$  work equally well.

**Final Answer:** determinant non-zero  $\Rightarrow$   C

Answer: (C) [Go Back to Q11](#)

Q12.

### Solution

**Concept — Permutations of distinct objects:** When order matters and repetition is not allowed, the number of ways to fill  $r$  ordered positions from  $n$  distinct items is the permutation  ${}^n P_r = \frac{n!}{(n-r)!} = n(n-1)(n-2)\cdots(n-r+1)$ . Here each three-digit number is an ordered arrangement (the hundreds, tens and units place are distinguishable), and “no digit repeated” is exactly the no-repetition condition. Since the digits are 1, 2, 3, 4, 5, none is 0, so there is no leading-zero restriction to worry about.

**Step 1 — Set up the slots:** We have  $r = 3$  places to fill from  $n = 5$  available digits with no repetition.

**Step 2 — Fill place by place:** The hundreds digit can be any of 5 choices; once used, the tens digit has 4 remaining choices; then the units digit has 3 choices:

$$5 \times 4 \times 3.$$



**Step 3 — Multiply:**

$$5 \times 4 \times 3 = 60.$$

**Step 4 — Confirm via the formula:**  ${}^5P_3 = \frac{5!}{(5-3)!} = \frac{120}{2} = 60$ , the same value.

**Why other options are wrong:**

- (A)  $125 = 5^3$  allows repetition (each place independently any of 5 digits), which the question forbids.
- (B)  $10 = \binom{5}{3}$  is a combination count, ignoring that the order of the three digits matters.
- (C)  $120 = 5!$  arranges all five digits, but we only place three of them.

**Final Answer:** 60 numbers  $\Rightarrow$  D

**Answer: (D)** [Go Back to Q12](#)

**Q13.**

### Solution

**Concept — Counting unordered pairs:** A handshake involves two distinct people, and the handshake between  $X$  and  $Y$  is the same event as the one between  $Y$  and  $X$ . So we are counting unordered pairs, which is the combination  $\binom{n}{2} = \frac{n(n-1)}{2}$ . The division by 2 is what removes the double-counting of ordered pairs.

**Step 1 — Count ordered pairs first:** Each of the  $n = 10$  people can shake hands with the other 9, giving  $10 \times 9 = 90$  ordered (person, partner) pairs.

**Step 2 — Remove double counting:** Each actual handshake was counted twice (once from each participant), so divide by 2:

$$\frac{10 \times 9}{2} = \frac{90}{2} = 45.$$

**Step 3 — Confirm with the combination formula:**

$$\binom{10}{2} = \frac{10!}{2!8!} = \frac{10 \times 9}{2} = 45.$$

**Step 4 — Small-case sanity check:** For  $n = 3$  people the formula gives  $\binom{3}{2} = 3$ , and indeed three people  $A, B, C$  make exactly the handshakes  $AB, AC, BC$  — three of them. The pattern holds.



**Why other options are wrong:**

- (A)  $90 = 10 \times 9$  counts each handshake twice (it is the ordered-pair count, not yet halved).
- (C)  $100 = 10^2$  even lets people shake their own hand, which is meaningless.
- (D)  $20 = 2 \times 10$  is an unrelated product with no combinatorial basis here.

**Final Answer:** 45 handshakes  $\Rightarrow$  B

Answer: (B) [Go Back to Q13](#)

**Q14.**

### Solution

**Concept — Alternate seating with a gender pattern:** “Men and women alternate” means no two of the same gender sit next to each other. In a row of 6 seats the genders must follow a strict alternating template, and with equal numbers (3 men, 3 women) there are exactly two possible templates depending on whether seat 1 holds a man or a woman. Within a fixed template, the three men can be permuted among the men’s seats ( $3!$  ways) and independently the three women among the women’s seats ( $3!$  ways).

**Step 1 — List the templates:** Because the count of men equals the count of women, both starting choices are valid:

Pattern I:  $M W M W M W$ ,      Pattern II:  $W M W M W M$ .

**Step 2 — Arrangements within one template:** Place the 3 men in their 3 fixed seats in  $3! = 6$  ways, and the 3 women in their 3 fixed seats in  $3! = 6$  ways:

$$3! \times 3! = 6 \times 6 = 36.$$

**Step 3 — Account for both templates:** Add the two mutually exclusive patterns:

$$2 \times 36 = 72.$$

**Step 4 — Cross-check by total minus nothing:** A free row of 6 distinct people has  $6! = 720$  arrangements; the alternating requirement keeps only 72 of them, i.e. 1 in 10, which is a reasonable fraction for such a strong constraint.

**Why other options are wrong:**



- (B) 36 counts only one starting pattern (e.g. man first) and forgets the woman-first arrangement.
- (C)  $144 = (3!)^2 \times 4$  over-counts by allowing too many templates.
- (D)  $720 = 6!$  treats all six seats as a free row with no alternation requirement at all.

**Final Answer:** 72 arrangements  $\Rightarrow$  **A**

**Answer: (A)** [Go Back to Q14](#)

**Q15.**

### Solution

**Concept — Roots in arithmetic progression:** The smart parametrisation for three numbers in AP is  $a - d, a, a + d$ , because their sum collapses to  $3a$  (the  $\pm d$  cancel). For a cubic  $x^3 + px^2 + qx + r = 0$  the sum of the roots equals  $-p$  (here the coefficient of  $x^2$  is  $-12$ , so the sum is  $+12$ ). This immediately pins down the common middle term  $a$  without solving the cubic.

**Step 1 — Use the sum of roots:** With roots  $a - d, a, a + d$ ,

$$(a - d) + a + (a + d) = 3a = 12 \quad \Rightarrow \quad a = 4.$$

So the middle root is  $a = 4$ .

**Step 2 — Find  $d$  from the product (optional):** The product of roots is  $-(-28) = 28$ , so  $(a - d)a(a + d) = a(a^2 - d^2) = 28$ . With  $a = 4$ :  $4(16 - d^2) = 28 \Rightarrow 16 - d^2 = 7 \Rightarrow d^2 = 9 \Rightarrow d = 3$ .

**Step 3 — Identify all three roots:** The roots are  $4 - 3, 4, 4 + 3 = 1, 4, 7$ . Check the sum  $1 + 4 + 7 = 12$  and the product  $1 \cdot 4 \cdot 7 = 28$ , both matching the cubic's coefficients.

**Step 4 — Verify  $x = 4$  is a root:** Substitute  $x = 4$ :  $4^3 - 12(4^2) + 39(4) - 28 = 64 - 192 + 156 - 28 = 0$ . Confirmed.

**Why other options are wrong:**

- (A) 7 is the largest root  $a + d$ , not the middle term.
- (B) 1 is the smallest root  $a - d$ , again an outer root.
- (D) 12 is the sum of all three roots, not any single root.

**Final Answer:** middle root = 4  $\Rightarrow$  **C**

**Answer: (C)** [Go Back to Q15](#)



Q16.

**Solution**

**Concept — Transforming the roots:** If a new equation is to have roots that are  $k$  times the old roots, every new root  $X$  relates to an old root  $x$  by  $X = kx$ , i.e.  $x = X/k$ . Substituting  $x \rightarrow X/k$  into the original equation forces  $X$  to range over the scaled roots. Equivalently, one can use Vieta: scaling each root by  $k$  multiplies the sum by  $k$  and the product by  $k^2$ . Both methods are shown below.

**Step 1 — Substitution method:** Replace  $x$  by  $\frac{x}{2}$  in  $x^2 - 3x + 2 = 0$ :

$$\left(\frac{x}{2}\right)^2 - 3\left(\frac{x}{2}\right) + 2 = 0 \implies \frac{x^2}{4} - \frac{3x}{2} + 2 = 0.$$

**Step 2 — Clear fractions:** Multiply every term by 4:

$$x^2 - 6x + 8 = 0.$$

**Step 3 — Vieta cross-check:** The original roots of  $x^2 - 3x + 2 = 0$  are 1 and 2 (sum 3, product 2). Doubling gives new roots 2 and 4, with sum 6 and product 8. The monic quadratic with these is  $x^2 - (\text{sum})x + (\text{product}) = x^2 - 6x + 8 = 0$ , matching Step 2.

**Why other options are wrong:**

- (A)  $x^2 - 3x + 2 = 0$  is the original equation, with roots 1, 2 rather than 2, 4.
- (B)  $x^2 - 6x + 2 = 0$  doubles the sum correctly but leaves the constant unchanged (product should be 8, not 2).
- (C)  $x^2 - 3x + 8 = 0$  scales the constant but not the linear coefficient (sum should be 6, not 3).

**Final Answer:**  $x^2 - 6x + 8 = 0 \Rightarrow$  D

Answer: (D) [Go Back to Q16](#)



Q17.

**Solution**

**Concept — The standard  $x^n - a^n$  limit:** Direct substitution gives the indeterminate form  $\frac{0}{0}$ , so we use the standard result  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n a^{n-1}$ . This is really the definition of the derivative of  $f(x) = x^n$  at  $x = a$ , since  $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$  and  $\frac{d}{dx} x^n = n x^{n-1}$ .

**Step 1 — Match the pattern:** Comparing  $\frac{x^5 - 2^5}{x - 2}$  with  $\frac{x^n - a^n}{x - a}$  gives  $n = 5$  and  $a = 2$ .

**Step 2 — Apply the formula:**

$$\lim_{x \rightarrow 2} \frac{x^5 - 2^5}{x - 2} = n a^{n-1} = 5 \cdot 2^4.$$

**Step 3 — Evaluate:**

$$5 \cdot 2^4 = 5 \cdot 16 = 80.$$

**Step 4 — Verify by factoring:** Since  $x^5 - 2^5 = (x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16)$ , the quotient simplifies to  $x^4 + 2x^3 + 4x^2 + 8x + 16$ , and at  $x = 2$  this is  $16 + 16 + 16 + 16 + 16 = 80$ . Same result, no indeterminate form remains.

**Why other options are wrong:**

- (B)  $32 = 2^5$  evaluates  $a^n$  and forgets the differentiation factor  $n$ .
- (C)  $16 = 2^4$  gives  $a^{n-1}$  but drops the leading factor  $n = 5$ .
- (D) 5 keeps only the factor  $n$  and forgets the power  $a^{n-1}$ .

**Final Answer:** the limit is 80  $\Rightarrow$

**Answer: (A)** [Go Back to Q17](#)

Q18.

**Solution**

**Concept — Derivative of inverse tangent:** The standard derivative is  $\frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$ . It is worth knowing where this comes from: if  $y = \tan^{-1} x$  then  $x = \tan y$ ; differentiating implicitly gives  $1 = \sec^2 y \frac{dy}{dx}$ , so  $\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$ , using the identity  $\sec^2 y = 1 + \tan^2 y$ .



**Step 1 — Start from  $y = \tan^{-1} x$ :** Equivalently  $\tan y = x$  with  $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ .

**Step 2 — Differentiate implicitly:**

$$\frac{d}{dx}(\tan y) = \frac{d}{dx}(x) \implies \sec^2 y \frac{dy}{dx} = 1.$$

**Step 3 — Solve and convert back to  $x$ :**

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}.$$

**Why other options are wrong:**

- (A)  $\frac{1}{\sqrt{1-x^2}}$  is the derivative of  $\sin^{-1} x$ , a different inverse function.
- (C)  $\frac{-1}{1+x^2}$  is the derivative of  $\cot^{-1} x$  (wrong sign for  $\tan^{-1}$ ).
- (D)  $\frac{1}{1-x^2}$  has the wrong denominator;  $1-x^2$  never appears for the inverse tangent.

**Final Answer:**  $\frac{1}{1+x^2} \Rightarrow \boxed{\text{B}}$

**Answer: (B)** [Go Back to Q18](#)

**Q19.**

### Solution

**Concept — Amplitude of  $a \sin \theta + b \cos \theta$ :** Any expression of the form  $a \sin \theta + b \cos \theta$  can be rewritten as a single sinusoid  $R \sin(\theta + \phi)$ , where  $R = \sqrt{a^2 + b^2}$  and  $\tan \phi = b/a$ . Because  $\sin(\theta + \phi)$  ranges between  $-1$  and  $1$ , the whole expression oscillates between  $-R$  and  $+R$ . Hence the maximum value is  $\sqrt{a^2 + b^2}$  and the minimum is  $-\sqrt{a^2 + b^2}$ .

**Step 1 — Identify  $a$  and  $b$ :** For  $f(\theta) = 3 \sin \theta + 4 \cos \theta$  we read off  $a = 3$ ,  $b = 4$ .

**Step 2 — Compute the amplitude:**

$$R = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

**Step 3 — Interpret the graph:** The dashed line at height 5 in the figure is exactly this amplitude; the curve  $f(\theta) = 5 \sin(\theta + \phi)$  just touches it, with  $\phi = \tan^{-1}(4/3) \approx 53.13^\circ$ , the phase shift shown.



**Step 4 — Verify at the peak:** The maximum occurs when  $\theta + \phi = 90^\circ$ . There  $\sin(\theta + \phi) = 1$ , so  $f = 5 \cdot 1 = 5$ , confirming the peak value.

**Why other options are wrong:**

- (A) 3 takes only the coefficient  $a$  of  $\sin \theta$ .
- (B) 7 adds  $a + b = 3 + 4$  without applying the square root of the sum of squares.
- (C) 4 takes only the coefficient  $b$  of  $\cos \theta$ .

**Final Answer:** maximum = 5  $\Rightarrow$  **D**

**Answer: (D)** [Go Back to Q19](#)

**Q20.**

### Solution

**Concept — Slope of the normal:** The tangent to a curve at a point has slope  $\frac{dy}{dx}$  evaluated there. The normal is the line perpendicular to the tangent at that same point, and perpendicular lines have slopes whose product is  $-1$ . Hence the normal's slope is the negative reciprocal of the tangent's slope,  $m_{\text{normal}} = -\frac{1}{(dy/dx)}$  (provided the tangent slope is non-zero).

**Step 1 — Differentiate the curve:** For  $y = x^2$ ,

$$\frac{dy}{dx} = 2x.$$

**Step 2 — Evaluate the tangent slope at (1, 1):**

$$\left. \frac{dy}{dx} \right|_{x=1} = 2(1) = 2.$$

**Step 3 — Take the negative reciprocal:**

$$m_{\text{normal}} = -\frac{1}{2}.$$

**Step 4 — Perpendicularity check:** The product of the tangent and normal slopes is  $2 \times \left(-\frac{1}{2}\right) = -1$ , confirming the two lines are perpendicular as required.

**Why other options are wrong:**

- (A) 2 is the tangent slope itself, not the normal's.



- (B)  $-2$  negates the tangent slope but forgets to take the reciprocal.
- (D)  $\frac{1}{2}$  takes the reciprocal but drops the minus sign (its product with 2 is  $+1$ , not  $-1$ ).

**Final Answer:** normal slope =  $-\frac{1}{2} \Rightarrow \boxed{\text{C}}$

**Answer: (C)** [Go Back to Q20](#)

**Q21.**

### Solution

**Concept — Order and degree of a differential equation:** The *order* is the order of the highest derivative appearing in the equation. The *degree* is the power (exponent) of that highest-order derivative, but only after the equation has been written as a polynomial in all its derivatives (free of radicals and fractional powers of derivatives). A common trap is to read the degree from whichever term carries the largest exponent; instead the degree is tied specifically to the *highest-order* derivative.

**Step 1 — Find the highest-order derivative:** The derivatives present are  $\frac{d^3y}{dx^3}$  and  $\frac{dy}{dx}$ . The highest order is the third derivative, so

$$\text{order} = 3.$$

**Step 2 — Confirm polynomial form:** The equation  $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y = 0$  is already polynomial in its derivatives (no roots, no fractions of derivatives), so the degree is well defined.

**Step 3 — Read the power of the highest derivative:** The third derivative appears as  $\left(\frac{d^3y}{dx^3}\right)^2$ , i.e. to the power 2. Therefore

$$\text{degree} = 2.$$

The fourth power sits on  $\frac{dy}{dx}$ , a lower-order term, and so does not affect the degree.

**Why other options are wrong:**

- (B) order 2, degree 3 misreads the highest order as 2 and invents a degree of 3.
- (C) order 3, degree 4 wrongly takes the degree from the 4th power on the first-derivative term.



- (D) order 4, degree 1 confuses the exponent 4 with the order of a derivative.

**Final Answer:** order 3, degree 2  $\Rightarrow$  **A**

**Answer: (A)** [Go Back to Q21](#)

**Q22.**

### Solution

**Concept — Integration by parts:** For a product of a polynomial and an exponential, integration by parts  $\int u dv = uv - \int v du$  is the right tool. The polynomial  $x$  should be chosen as  $u$  (so that  $du = dx$  simplifies it), and the exponential as  $dv$  (it integrates back to itself). The LIATE rule supports this choice: the algebraic factor is differentiated, the exponential is integrated.

**Step 1 — Choose the parts:**

$$u = x, \quad dv = e^x dx \implies du = dx, \quad v = \int e^x dx = e^x.$$

**Step 2 — Apply the formula:**

$$\int x e^x dx = uv - \int v du = x e^x - \int e^x dx.$$

**Step 3 — Finish the remaining integral:**

$$x e^x - \int e^x dx = x e^x - e^x + C = (x - 1)e^x + C.$$

**Step 4 — Verify by differentiation:**  $\frac{d}{dx} [(x - 1)e^x] = 1 \cdot e^x + (x - 1)e^x = e^x + x e^x - e^x = x e^x$ , recovering the integrand. The antiderivative is correct.

**Why other options are wrong:**

- (A)  $x e^x + e^x + C$  has the wrong sign on the second term; differentiating it gives  $x e^x + 2e^x \neq x e^x$ .
- (C)  $\frac{x^2}{2} e^x + C$  wrongly integrates  $x$  and  $e^x$  as if they were independent factors.
- (D)  $x e^x + C$  omits the  $-e^x$  term entirely; its derivative is  $e^x + x e^x \neq x e^x$ .

**Final Answer:**  $x e^x - e^x + C \Rightarrow$  **B**

**Answer: (B)** [Go Back to Q22](#)



Q23.

**Solution**

**Concept — Fundamental theorem of calculus:** A definite integral of a polynomial is found by integrating term by term to get an antiderivative  $F(x)$ , then evaluating  $F(b) - F(a)$  at the upper and lower limits. The power rule  $\int x^k dx = \frac{x^{k+1}}{k+1}$  handles each term, and the arbitrary constant cancels in the subtraction, so we may omit it.

**Step 1 — Antidifferentiate term by term:**

$$\int (3x^2 + 1) dx = 3 \cdot \frac{x^3}{3} + x = x^3 + x.$$

**Step 2 — Evaluate at the upper limit  $x = 2$ :**

$$F(2) = 2^3 + 2 = 8 + 2 = 10.$$

**Step 3 — Evaluate at the lower limit and subtract:**

$$F(0) = 0^3 + 0 = 0, \quad \int_0^2 (3x^2 + 1) dx = F(2) - F(0) = 10 - 0 = 10.$$

**Step 4 — Cross-check by splitting:**  $\int_0^2 3x^2 dx = [x^3]_0^2 = 8$  and  $\int_0^2 1 dx = [x]_0^2 = 2$ ; their sum  $8 + 2 = 10$  confirms the answer.

**Why other options are wrong:**

- (A) 8 keeps only the  $\int 3x^2 dx$  part and drops the constant term's contribution  $\int_0^2 1 dx = 2$ .
- (B) 9 comes from a slip such as evaluating  $x^3 + x$  at  $x$  wrongly (e.g.  $7 + 2$ ).
- (D) 12 over-counts, e.g. by using  $2 \cdot 2^3 / ?$  or adding an extra 2.

**Final Answer:** the integral is  $10 \Rightarrow$   C

Answer: (C) [Go Back to Q23](#)



Q24.

**Solution**

**Concept — Triangle bounded by a line and the axes:** A line cutting both coordinate axes forms a right-angled triangle with the origin, whose two legs are the  $x$ -intercept and the  $y$ -intercept. The area of a right triangle is half the product of its legs,  $\text{Area} = \frac{1}{2} \times (\text{x-intercept}) \times (\text{y-intercept})$ . The intercepts are found by setting one variable to zero at a time.

**Step 1 —  $x$ -intercept:** Put  $y = 0$  in  $2x + 3y = 12$ :

$$2x = 12 \implies x = 6, \quad \text{point } (6, 0).$$

**Step 2 —  $y$ -intercept:** Put  $x = 0$ :

$$3y = 12 \implies y = 4, \quad \text{point } (0, 4).$$

**Step 3 — Compute the area:** The legs are 6 and 4, so

$$\text{Area} = \frac{1}{2} \times 6 \times 4 = 12 \text{ sq. units.}$$

**Step 4 — Cross-check via intercept form:** Rewrite the line as  $\frac{x}{6} + \frac{y}{4} = 1$ , which directly reads off intercepts 6 and 4. The shaded region in the figure is exactly this triangle with vertices  $(0, 0)$ ,  $(6, 0)$ ,  $(0, 4)$ , area 12.

**Why other options are wrong:**

- (A) 24 multiplies the intercepts  $6 \times 4$  but forgets the factor  $\frac{1}{2}$ .
- (B) 6 uses only the  $x$ -intercept.
- (C) 10 adds the intercepts  $6 + 4$  instead of taking half their product.

**Final Answer:** area = 12 sq. units  $\Rightarrow$   D

**Answer: (D)** [Go Back to Q24](#)



Q25.

**Solution**

**Concept — Definite integral of  $\sin x$ :** The antiderivative of  $\sin x$  is  $-\cos x$  (since  $\frac{d}{dx}(-\cos x) = \sin x$ ). By the fundamental theorem of calculus the definite integral over  $[0, \pi]$  is  $[-\cos x]_0^\pi$ . Geometrically this is the area under one positive hump of the sine curve, which is a standard result equal to 2.

**Step 1 — Write the antiderivative:**

$$\int \sin x \, dx = -\cos x.$$

**Step 2 — Substitute the limits:**

$$[-\cos x]_0^\pi = (-\cos \pi) - (-\cos 0).$$

**Step 3 — Evaluate the cosines:** Using  $\cos \pi = -1$  and  $\cos 0 = 1$ ,

$$-(-1) - (-1) = 1 - (-1) = 1 + 1 = 2.$$

**Step 4 — Geometric check:** On  $[0, \pi]$  the sine curve is entirely above the axis, rising to 1 at  $x = \frac{\pi}{2}$  and returning to 0. The enclosed area is positive, and the exact value 2 matches the well-known “area of one sine arch”.

**Why other options are wrong:**

- (B) 0 would result from wrongly using  $\sin \pi - \sin 0 = 0$ , i.e. forgetting that  $\int \sin = -\cos$ .
- (C) 1 integrates over only half the interval  $[0, \frac{\pi}{2}]$ .
- (D)  $\pi$  confuses the width of the interval with the area under the curve.

**Final Answer:** the integral is 2  $\Rightarrow$  **A**

**Answer: (A)** [Go Back to Q25](#)



Q26.

**Solution**

**Concept — Angle between two straight lines:** If two lines have slopes  $m_1$  and  $m_2$ , the acute angle  $\phi$  between them satisfies  $\tan \phi = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$ . Equivalently, since the slope of a line is the tangent of its inclination, the angle between the lines is the difference of their inclinations. Either route gives the same acute angle.

**Step 1 — Read the slopes and inclinations:** For  $y = x$ ,  $m_1 = 1 = \tan 45^\circ$ , so its inclination is  $45^\circ$ . For  $y = \sqrt{3}x$ ,  $m_2 = \sqrt{3} = \tan 60^\circ$ , so its inclination is  $60^\circ$ .

**Step 2 — Subtract the inclinations:**

$$\phi = 60^\circ - 45^\circ = 15^\circ.$$

**Step 3 — Confirm with the slope formula:**

$$\tan \phi = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \right|.$$

Rationalising,  $\frac{1 - \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{(1 - \sqrt{3})^2}{1 - 3} = \frac{4 - 2\sqrt{3}}{-2} = \sqrt{3} - 2$ , whose magnitude is  $2 - \sqrt{3} = \tan 15^\circ$ . So  $\phi = 15^\circ$ , matching Step 2.

**Why other options are wrong:**

- (A)  $30^\circ$  doubles the answer or misreads  $m_2$ 's inclination.
- (C)  $45^\circ$  takes the inclination of  $y = x$  alone, ignoring the second line.
- (D)  $60^\circ$  takes the inclination of  $y = \sqrt{3}x$  alone.

**Final Answer:** angle =  $15^\circ \Rightarrow$  **B**

**Answer: (B)** [Go Back to Q26](#)

Q27.

**Solution**

**Concept — Length of the tangent from an external point:** For a circle  $x^2 + y^2 = r^2$  and an external point  $P(x_1, y_1)$ , the tangent line touches the circle at  $T$ , and  $OTP$  is right-angled at  $T$  (the radius is perpendicular to the tangent). By Pythagoras  $PT^2 = OP^2 - OT^2 = (x_1^2 + y_1^2) - r^2$ . So the tangent length is the square root of the “power of the point”,  $\sqrt{x_1^2 + y_1^2 - r^2}$ , which is real exactly when  $P$  lies outside the circle.



**Step 1 — Read the data:** The circle has  $r^2 = 25$  (so  $r = 5$ ), and the external point is  $P(6, 8)$ , giving  $x_1 = 6$ ,  $y_1 = 8$ .

**Step 2 — Substitute into the formula:**

$$PT = \sqrt{x_1^2 + y_1^2 - r^2} = \sqrt{6^2 + 8^2 - 25} = \sqrt{36 + 64 - 25} = \sqrt{75}.$$

**Step 3 — Simplify the surd:**

$$\sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}.$$

**Step 4 — Pythagorean cross-check:** The distance  $OP = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$ , and the radius is 5, so  $PT = \sqrt{OP^2 - r^2} = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}$ . Consistent.

**Why other options are wrong:**

- (A) 5 is the radius  $r$  of the circle, not the tangent length.
- (B)  $\sqrt{61}$  comes from subtracting the radius 5 instead of  $r^2 = 25$  ( $100 - ?$ ), mishandling the formula.
- (C) 10 is the full distance  $OP$  from  $P$  to the centre, the hypotenuse, not the tangent leg.

**Final Answer:** tangent length =  $\sqrt{75} = 5\sqrt{3} \Rightarrow$  D

Answer: (D) [Go Back to Q27](#)

Q28.

### Solution

**Concept — Foci of an ellipse:** For an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  with  $a^2 > b^2$ , the major axis lies along the  $x$ -axis and the foci are at  $(\pm c, 0)$ , where  $c$  is found from  $c^2 = a^2 - b^2$ . The larger denominator sits under  $x^2$ , so the foci must lie on the  $x$ -axis, never the  $y$ -axis. (Here  $a$  is the semi-major axis and  $b$  the semi-minor axis.)

**Step 1 — Identify  $a^2$  and  $b^2$ :** Comparing  $\frac{x^2}{25} + \frac{y^2}{9} = 1$  with the standard form gives  $a^2 = 25$  and  $b^2 = 9$ . Since  $25 > 9$ , the major axis is along the  $x$ -axis.

**Step 2 — Compute  $c$ :**

$$c^2 = a^2 - b^2 = 25 - 9 = 16 \implies c = 4.$$



**Step 3 — Write the foci:** On the  $x$ -axis,

$$\text{foci} = (\pm c, 0) = (\pm 4, 0).$$

**Step 4 — Eccentricity check:** The eccentricity is  $e = \frac{c}{a} = \frac{4}{5} = 0.8 < 1$ , which is valid for an ellipse, and the focal points  $(\pm 4, 0)$  match the red dots shown in the figure.

**Why other options are wrong:**

- (A)  $(\pm 5, 0)$  uses  $a = 5$  (a vertex), confusing the vertex with the focus.
- (B)  $(0, \pm 4)$  places the foci on the  $y$ -axis, the wrong (minor) axis.
- (D)  $(\pm 3, 0)$  uses  $b = 3$ , the semi-minor axis, instead of  $c$ .

**Final Answer:** foci  $= (\pm 4, 0) \Rightarrow$   C

Answer: (C) [Go Back to Q28](#)

**Q29.**

### Solution

**Concept — Direction cosines satisfy a normalisation:** The direction cosines  $l, m, n$  of a line are the cosines of the angles it makes with the  $x, y, z$  axes. They always obey  $l^2 + m^2 + n^2 = 1$ , because they are the components of a unit vector along the line. Given two of them, this identity fixes the third up to sign; the condition  $n > 0$  then selects the positive root.

**Step 1 — Substitute the known cosines:** With  $l = \frac{1}{2}$  and  $m = \frac{1}{2}$ ,

$$\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + n^2 = 1 \implies \frac{1}{4} + \frac{1}{4} + n^2 = 1.$$

**Step 2 — Solve for  $n^2$ :**

$$n^2 = 1 - \frac{1}{2} = \frac{1}{2}.$$

**Step 3 — Take the positive root:**

$$n = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \quad (\text{since } n > 0).$$

**Step 4 — Verify normalisation:** Check  $l^2 + m^2 + n^2 = \frac{1}{4} + \frac{1}{4} + \frac{1}{2} = 1$ , exactly as required for a genuine set of direction cosines.

**Why other options are wrong:**



- (A)  $\frac{1}{2}$  gives  $l^2 + m^2 + n^2 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} \neq 1$ .
- (C)  $\frac{1}{4}$  gives  $\frac{1}{4} + \frac{1}{4} + \frac{1}{16} = \frac{9}{16} \neq 1$ .
- (D)  $\frac{\sqrt{3}}{2}$  gives  $n^2 = \frac{3}{4}$ , so the sum is  $\frac{1}{4} + \frac{1}{4} + \frac{3}{4} = \frac{5}{4} > 1$ , too large.

**Final Answer:**  $n = \frac{1}{\sqrt{2}} \Rightarrow \boxed{\text{B}}$

**Answer: (B)** [Go Back to Q29](#)

**Q30.**

### Solution

**Concept — Equation of a plane from a normal and a point:** A plane is fixed by a point on it and a direction perpendicular to it (the normal). If  $\vec{n} = (a, b, c)$  is normal and the plane passes through  $(x_0, y_0, z_0)$ , then for any point  $(x, y, z)$  on the plane the vector  $(x - x_0, y - y_0, z - z_0)$  is perpendicular to  $\vec{n}$ , so their dot product is zero:  $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ . The coefficients of  $x, y, z$  are exactly the components of the normal.

**Step 1 — Read the normal and point:** The normal is  $\vec{n} = 2\hat{i} + \hat{j} + 2\hat{k}$ , so  $(a, b, c) = (2, 1, 2)$ , and the point is  $(x_0, y_0, z_0) = (1, 2, 3)$ .

**Step 2 — Write the point-normal form:**

$$2(x - 1) + 1(y - 2) + 2(z - 3) = 0.$$

**Step 3 — Expand and simplify:**

$$2x - 2 + y - 2 + 2z - 6 = 0 \implies 2x + y + 2z - 10 = 0 \implies 2x + y + 2z = 10.$$

**Step 4 — Verify the point lies on it:** Substitute  $(1, 2, 3)$  into the left side:  $2(1) + 2 + 2(3) = 2 + 2 + 6 = 10$ , which equals the right side. The plane indeed passes through the given point with the correct normal.

**Why other options are wrong:**

- (B)  $2x + y + 2z = 6$  miscalculates the constant (e.g. using  $2 + 1 + 3$  instead of  $2 + 2 + 6$ ).
- (C)  $x + 2y + 3z = 10$  wrongly uses the point's coordinates as the normal direction.
- (D)  $2x + y + 2z = 0$  drops the constant, giving a plane through the origin that misses  $(1, 2, 3)$ .



**Final Answer:**  $2x + y + 2z = 10 \Rightarrow \boxed{\text{A}}$

**Answer: (A)** [Go Back to Q30](#)

**Q31.**

### Solution

**Concept — Perpendicular vectors and the dot product:** The dot product of two vectors is  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ . When the vectors are perpendicular,  $\theta = 90^\circ$  and  $\cos 90^\circ = 0$ , so the dot product vanishes. Thus  $\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$ . In components the dot product is the sum of products of corresponding entries, giving a linear equation in  $\lambda$  to solve.

**Step 1 — Form the dot product in components:** With  $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ ,

$$\vec{a} \cdot \vec{b} = (2)(1) + (\lambda)(-2) + (1)(3) = 2 - 2\lambda + 3 = 5 - 2\lambda.$$

**Step 2 — Impose perpendicularity:** Set the dot product to zero:

$$5 - 2\lambda = 0.$$

**Step 3 — Solve for  $\lambda$ :**

$$2\lambda = 5 \implies \lambda = \frac{5}{2}.$$

**Step 4 — Check:** With  $\lambda = \frac{5}{2}$ ,  $\vec{a} \cdot \vec{b} = 2 - 2\left(\frac{5}{2}\right) + 3 = 2 - 5 + 3 = 0$ , confirming the vectors are perpendicular.

**Why other options are wrong:**

- (A)  $-\frac{5}{2}$  gets a sign wrong; it would give  $\vec{a} \cdot \vec{b} = 2 + 5 + 3 = 10 \neq 0$ .
- (B) 2 stops at an intermediate value and is not the solution of  $5 - 2\lambda = 0$ .
- (D) 5 forgets to divide by 2 when isolating  $\lambda$ .

**Final Answer:**  $\lambda = \frac{5}{2} \Rightarrow \boxed{\text{C}}$

**Answer: (C)** [Go Back to Q31](#)



Q32.

**Solution**

**Concept — Magnitude of a cross product:** The cross product magnitude is  $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$ , where  $\theta$  is the angle between the vectors. Geometrically this equals the area of the parallelogram spanned by  $\vec{a}$  and  $\vec{b}$ . Note the contrast with the dot product, which uses  $\cos\theta$ ; the cross product uses  $\sin\theta$ , so it is largest at  $90^\circ$  and zero for parallel vectors.

**Step 1 — Substitute the magnitudes and angle:** With  $|\vec{a}| = 4$ ,  $|\vec{b}| = 5$  and  $\theta = 30^\circ$ ,

$$|\vec{a} \times \vec{b}| = 4 \times 5 \times \sin 30^\circ.$$

**Step 2 — Use  $\sin 30^\circ = \frac{1}{2}$ :**

$$= 20 \times \frac{1}{2} = 10.$$

**Step 3 — Consistency note:** As a check, the parallelogram on sides 4 and 5 with a  $30^\circ$  angle has area (base  $\times$  height)  $= 4 \times (5 \sin 30^\circ) = 4 \times 2.5 = 10$ , matching the cross-product magnitude.

**Why other options are wrong:**

- (A) 20 omits the  $\sin 30^\circ$  factor (it would be the case  $\theta = 90^\circ$ ).
- (B)  $\frac{20}{\sqrt{3}}$  uses a reciprocal or  $\tan/\sin 60^\circ$ -type value rather than  $\sin 30^\circ$ .
- (C)  $10\sqrt{3}$  uses  $\sin 60^\circ = \frac{\sqrt{3}}{2}$  instead of  $\sin 30^\circ$ .

**Final Answer:**  $|\vec{a} \times \vec{b}| = 10 \Rightarrow \boxed{\text{D}}$

**Answer: (D)** [Go Back to Q32](#)

Q33.

**Solution**

**Concept — Coplanarity via the scalar triple product:** Three vectors are coplanar if and only if the volume of the parallelepiped they span is zero. That volume is the scalar triple product  $[\vec{a} \ \vec{b} \ \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$ , which equals the determinant whose rows are the vectors' components. Hence coplanarity is equivalent to setting this determinant to zero and solving for  $\lambda$ .

**Step 1 — Write the determinant:** Using the components of  $\vec{a} = (1, 1, 1)$ ,  $\vec{b} =$



$(1, 2, 3)$ ,  $\vec{c} = (1, 1, \lambda)$  as rows,

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & \lambda \end{vmatrix} = 0.$$

**Step 2 — Expand along the first row:**

$$1(2\lambda - 3) - 1(\lambda - 3) + 1(1 - 2).$$

**Step 3 — Simplify:**

$$2\lambda - 3 - \lambda + 3 - 1 = \lambda - 1.$$

**Step 4 — Solve the coplanarity condition:**

$$\lambda - 1 = 0 \implies \lambda = 1.$$

Check: at  $\lambda = 1$ ,  $\vec{c} = (1, 1, 1) = \vec{a}$ , so the three vectors trivially lie in one plane (in fact  $\vec{c}$  equals  $\vec{a}$ ), confirming coplanarity.

**Why other options are wrong:**

- (A)  $\lambda = 0$  gives determinant  $0 - 1 = -1 \neq 0$ , not coplanar.
- (C)  $\lambda = 2$  gives  $2 - 1 = 1 \neq 0$ , not coplanar.
- (D)  $\lambda = 3$  gives  $3 - 1 = 2 \neq 0$ , not coplanar.

**Final Answer:**  $\lambda = 1 \Rightarrow$

[Go Back to Q33](#)

**Q34.**

### Solution

**Concept — Variance of a data set:** Variance measures the average squared spread of the data about its mean:  $\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$ , where  $\bar{x}$  is the mean. It is the square of the standard deviation, so it carries squared units. The first step is always to compute the mean, then the deviations from it, square them, and average.

**Step 1 — Find the mean:**

$$\bar{x} = \frac{1 + 3 + 5 + 7 + 9}{5} = \frac{25}{5} = 5.$$



**Step 2 — Deviations from the mean:**  $1 - 5 = -4$ ,  $3 - 5 = -2$ ,  $5 - 5 = 0$ ,  $7 - 5 = 2$ ,  $9 - 5 = 4$ .

**Step 3 — Sum of squared deviations:**

$$(-4)^2 + (-2)^2 + 0^2 + 2^2 + 4^2 = 16 + 4 + 0 + 4 + 16 = 40.$$

**Step 4 — Divide by  $n$ :**

$$\sigma^2 = \frac{40}{5} = 8.$$

**Step 5 — Cross-check with the mean-of-squares formula:**  $\sigma^2 = \overline{x^2} - \bar{x}^2$ . Here  $\overline{x^2} = \frac{1 + 9 + 25 + 49 + 81}{5} = \frac{165}{5} = 33$ , so  $\sigma^2 = 33 - 5^2 = 33 - 25 = 8$ , matching.

**Why other options are wrong:**

- (B) 5 is the mean  $\bar{x}$ , not the variance.
- (C)  $\sqrt{8}$  is the standard deviation  $\sigma$ , i.e. the square root of the variance.
- (D) 4 halves the correct value (e.g. dividing 40 by 10 instead of 5).

**Final Answer:** variance = 8  $\Rightarrow$

[Go Back to Q34](#)

**Q35.**

### Solution

**Concept — Independent events:** Two events are independent when the occurrence of one does not affect the probability of the other. The defining multiplication rule is  $P(A \cap B) = P(A) \cdot P(B)$ . This is special to independence; for general events one would need  $P(A \cap B) = P(A)P(B | A)$ , but here  $P(B | A) = P(B)$ .

**Step 1 — Apply the multiplication rule:**

$$P(A \cap B) = P(A) \cdot P(B) = 0.3 \times 0.4.$$

**Step 2 — Multiply:**

$$0.3 \times 0.4 = 0.12.$$

**Step 3 — Consistency check with the union:** The union is  $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.4 - 0.12 = 0.58$ , a valid probability ( $\leq 1$ ), which confirms the intersection value is sensible.



**Why other options are wrong:**

- (A)  $0.7 = P(A) + P(B)$  adds the probabilities, the rule for mutually exclusive unions, not intersections.
- (B)  $0.1 = 0.4 - 0.3$  subtracts the probabilities, which has no basis here.
- (C)  $0.58$  is the union  $P(A \cup B)$ , not the intersection.

**Final Answer:**  $P(A \cap B) = 0.12 \Rightarrow$  D

Answer: (D) [Go Back to Q35](#)

**Q36.**

### Solution

**Concept — Complement for “at least one”:** Computing “at least one head” directly would require summing the cases of exactly one, exactly two and exactly three heads. It is far easier to use the complement: the only way to fail to get at least one head is to get no heads at all (all tails). So  $P(\text{at least one head}) = 1 - P(\text{no head})$ . Each toss of a fair coin is independent with  $P(\text{tail}) = \frac{1}{2}$ .

**Step 1 — Probability of no head:** All three tosses must be tails, and the tosses are independent:

$$P(\text{all tails}) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

**Step 2 — Take the complement:**

$$P(\text{at least one head}) = 1 - \frac{1}{8} = \frac{7}{8}.$$

**Step 3 — Verify by counting outcomes:** The 8 equally likely outcomes are HHH, HHT, HTH, THH, HTT, THT, TTH, TTT. Exactly one of them (TTT) has no head, so 7 of the 8 contain at least one head, giving  $\frac{7}{8}$  directly.

**Why other options are wrong:**

- (A)  $\frac{1}{8}$  is  $P(\text{no head})$ , the complement we subtracted, not the answer.
- (B)  $\frac{3}{8}$  is  $P(\text{exactly two heads}) = \binom{3}{2}\left(\frac{1}{2}\right)^3$ , only one of the favourable cases.
- (D)  $\frac{1}{2}$  is the probability of a head on a single toss, ignoring that there are three tosses.

**Final Answer:**  $P = \frac{7}{8} \Rightarrow$  C

Answer: (C) [Go Back to Q36](#)



Q37.

**Solution**

**Concept — General solution of  $\sin \theta = 0$ :** The sine function is zero exactly where its graph crosses the horizontal axis, which happens at  $\theta = 0, \pm\pi, \pm2\pi, \dots$ , i.e. at every integer multiple of  $\pi$ . Because sine is periodic with period  $2\pi$  but vanishes twice per period (at 0 and  $\pi$ ), the complete solution set is the union of these, compactly  $\theta = n\pi$  for all integers  $n$ . The phrase “general solution” means we must capture all of them, not just one.

**Step 1 — Locate the zeros in one period:** On  $[0, 2\pi)$ ,  $\sin \theta = 0$  at  $\theta = 0$  and  $\theta = \pi$  only.

**Step 2 — Extend periodically:** Adding any integer multiple of  $\pi$  to these (since 0 and  $\pi$  differ by  $\pi$  and the pattern repeats every  $\pi$ ) gives

$$\theta = n\pi, \quad n \in \mathbb{Z}.$$

**Step 3 — Spot-check:**  $\sin(0) = 0$ ,  $\sin(\pi) = 0$ ,  $\sin(2\pi) = 0$ ,  $\sin(-\pi) = 0$  — all of the form  $n\pi$ , confirming the family.

**Why other options are wrong:**

- (B)  $(2n + 1)\frac{\pi}{2}$  are the odd multiples of  $\frac{\pi}{2}$ , where  $\cos \theta = 0$ , not  $\sin \theta$ .
- (C)  $2n\pi$  captures only  $0, \pm2\pi, \dots$  and misses the odd multiples like  $\pi, 3\pi$  where sine is also zero.
- (D)  $n\frac{\pi}{2}$  includes  $\frac{\pi}{2}, \frac{3\pi}{2}, \dots$  where  $\sin \theta = \pm 1 \neq 0$ , so it adds non-solutions.

**Final Answer:**  $\theta = n\pi, n \in \mathbb{Z} \Rightarrow \boxed{\text{A}}$

**Answer: (A)** [Go Back to Q37](#)

Q38.

**Solution**

**Concept — Compound-angle formula for tangent:** The tangent of a sum is  $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ . The numerator adds the two tangents and the denominator subtracts their product from 1; forgetting the denominator (or its minus sign) is the usual error. Here both tangents are given, so it is a direct substitution.



**Step 1 — Substitute**  $\tan A = \frac{1}{2}$ ,  $\tan B = \frac{1}{3}$ :

$$\tan(A + B) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}$$

**Step 2 — Simplify numerator and denominator:**

$$\text{numerator} = \frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}, \quad \text{denominator} = 1 - \frac{1}{6} = \frac{5}{6}$$

**Step 3 — Divide:**

$$\tan(A + B) = \frac{\frac{5}{6}}{\frac{5}{6}} = 1.$$

**Step 4 — Interpret:**  $\tan(A + B) = 1$  means  $A + B = 45^\circ$  (the principal value). This is consistent:  $A = \tan^{-1} \frac{1}{2} \approx 26.57^\circ$  and  $B = \tan^{-1} \frac{1}{3} \approx 18.43^\circ$  sum to about  $45^\circ$ .

**Why other options are wrong:**

- (A)  $\frac{5}{6}$  keeps only the numerator and ignores the denominator entirely.
- (C)  $\frac{1}{6}$  is the product term  $\tan A \tan B$ , not the full expression.
- (D) 5 comes from dividing  $\frac{5}{6}$  by  $\frac{1}{6}$ , i.e. a sign error in the denominator ( $1 + \frac{1}{6}$  misread).

**Final Answer:**  $\tan(A + B) = 1 \Rightarrow$  B

**Answer: (B)** [Go Back to Q38](#)

**Q39.**

### Solution

**Concept — Complementary inverse-trig identity:** For every  $x \in [-1, 1]$ ,  $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ . The reason is that if  $\theta = \sin^{-1} x$  then  $\sin \theta = x$ , and using the co-function relation  $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta = x$  shows  $\frac{\pi}{2} - \theta = \cos^{-1} x$ , i.e.  $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$ . Rearranging gives the constant sum. Crucially the result does not depend on  $x$ .

**Step 1 — Let  $\theta = \sin^{-1} x$ :** Then  $\sin \theta = x$  with  $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .

**Step 2 — Use the co-function identity:**

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta = x \implies \cos^{-1} x = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \sin^{-1} x.$$



**Step 3 — Add the two inverse functions:**

$$\sin^{-1} x + \cos^{-1} x = \sin^{-1} x + \left( \frac{\pi}{2} - \sin^{-1} x \right) = \frac{\pi}{2}.$$

**Step 4 — Numerical spot-check:** At  $x = \frac{1}{2}$ :  $\sin^{-1} \frac{1}{2} = \frac{\pi}{6}$  and  $\cos^{-1} \frac{1}{2} = \frac{\pi}{3}$ ; their sum is  $\frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$ . At  $x = 0$ :  $0 + \frac{\pi}{2} = \frac{\pi}{2}$ . The sum is indeed constant.

**Why other options are wrong:**

- (A) 0 and (B)  $\pi$  ignore the identity; the sum is never 0 or  $\pi$ .
- (C)  $\frac{\pi}{4}$  halves the correct value  $\frac{\pi}{2}$ .

**Final Answer:**  $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \Rightarrow \boxed{\text{D}}$

**Answer: (D)** [Go Back to Q39](#)

**Q40.**

### Solution

**Concept — Heights and distances:** The line of sight to the top of the tower, the horizontal ground, and the vertical tower form a right-angled triangle with the right angle at the foot of the tower. The angle of elevation  $\theta$  is opposite the height  $h$  and adjacent to the horizontal distance  $d$ , so  $\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{h}{d}$ . Rearranging gives  $h = d \tan \theta$ .

**Step 1 — Set up the tangent relation:** With  $\theta = 60^\circ$  and horizontal distance  $d = 20$  m,

$$\tan 60^\circ = \frac{h}{20}.$$

**Step 2 — Use  $\tan 60^\circ = \sqrt{3}$ :**

$$\sqrt{3} = \frac{h}{20}.$$

**Step 3 — Solve for  $h$ :**

$$h = 20\sqrt{3} \text{ m} \approx 20 \times 1.732 = 34.64 \text{ m}.$$

**Step 4 — Sanity check:** Since  $60^\circ > 45^\circ$ , the height should exceed the base distance of 20 m; indeed  $20\sqrt{3} \approx 34.6 \text{ m} > 20 \text{ m}$ , which is consistent with a steeper line of sight.

**Why other options are wrong:**



- (A) 20 m uses  $\tan 45^\circ = 1$ , the wrong angle.
- (B)  $\frac{20}{\sqrt{3}}$  m uses  $\tan 30^\circ = \frac{1}{\sqrt{3}}$ , again the wrong angle (and gives a height less than the base).
- (D) 40 m comes from doubling the base, with no trigonometric basis.

**Final Answer:** height =  $20\sqrt{3}$  m  $\Rightarrow$

[Go Back to Q40](#)



## Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	B	4	D	5	A
6	B	7	C	8	D	9	A	10	B
11	C	12	D	13	B	14	A	15	C
16	D	17	A	18	B	19	D	20	C
21	A	22	B	23	C	24	D	25	A
26	B	27	D	28	C	29	B	30	A
31	C	32	D	33	B	34	A	35	D
36	C	37	A	38	B	39	D	40	C

