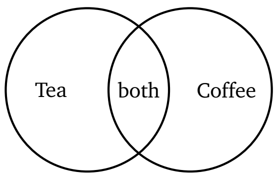
SRMJEEE Mathematics Sample Paper – 5

Duration: 47 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of **SRMJEEE** (SRM Joint Engineering Entrance Examination).
- Each correct answer carries **+1 mark**. There is **no negative marking**; an unattempted or wrong answer scores 0.
- Only **one** option is correct. Choose carefully.
- The actual SRMJEEE is a **computer-based test** conducted in remote-proctored online mode, with all sections sharing a common time window and no per-section limit.
- Personal calculators, mobile phones, log tables and other electronic gadgets are strictly prohibited.

Q1. In a class of 50 students, 30 like tea and 25 like coffee, while 10 like both, as shown in the Venn diagram. The number of students who like *only* tea is:



(A) 20

(B) 30

(C) 15

(D) 10

Q2. On the set $\{1, 2, 3\}$, the relation $R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)\}$ is:

(A) reflexive, symmetric and transitive

(B) reflexive and symmetric but not transitive

(C) symmetric and transitive but not reflexive

(D) neither reflexive nor symmetric

Q3. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3$ is:

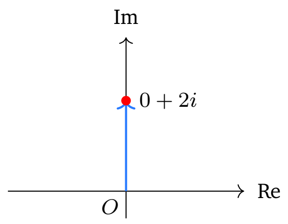
(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

Q4. Expressed in the form $a + ib$, the complex number $(1 + i)^2$ equals the point shown on the Argand plane, namely:



(A) 2

(B) $1 + 2i$

(C) -2

(D) $2i$

Q5. A quadratic equation with real coefficients has $2 + 3i$ as one of its roots. The equation is:

(A) $x^2 - 4x + 13 = 0$

(B) $x^2 + 4x + 13 = 0$

(C) $x^2 - 4x - 13 = 0$

(D) $x^2 - 4x + 5 = 0$

Q6. The value of k for which the equation $x^2 - kx + 9 = 0$ has equal roots (with $k > 0$) is:

(A) 3

(B) 9

(C) 6

(D) 12

Q7. If α, β are the roots of $x^2 - 5x + 6 = 0$, then the equation whose roots are α^2 and β^2 is:

(A) $x^2 - 25x + 36 = 0$

(B) $x^2 - 13x + 36 = 0$

(C) $x^2 + 13x + 36 = 0$

(D) $x^2 - 13x - 36 = 0$

Q8. A square matrix in which every diagonal entry equals the same scalar λ and every off-diagonal entry is 0 is called a scalar matrix. Such a matrix equals:

(A) the zero matrix

(B) a diagonal matrix with distinct entries

(C) a row matrix

(D) λI, where I is the identity matrix

Q9. If A and B are invertible matrices of the same order, then $(AB)^{-1}$ equals:

(A) $B^{-1}A^{-1}$

(B) $A^{-1}B^{-1}$

(C) $(BA)^{-1}$

(D) AB

Q10. If A and B are square matrices of the same order with $|A| = 3$ and $|B| = 4$, then $|AB|$ equals:

(A) 7

(B) 12

(C) 1

(D) $\frac{3}{4}$

Q11. The area of the triangle with vertices $(1, 1)$, $(4, 1)$ and $(1, 5)$, evaluated by the determinant form, is:

(A) 12 sq. units

(B) 3 sq. units

(C) 6 sq. units

(D) 4 sq. units

Q12. The number of arrangements of the letters of the word “ORANGE” (all distinct) in which the three vowels O, A, E are always together is:

(A) 720

(B) 36

(C) 120

(D) 144

Q13. The number of triangles that can be formed by joining 8 points, no three of which are collinear, is:

(A) 24

(B) 56

(C) 336

(D) 40

Q14. In how many ways can 5 persons be seated in a row so that two particular persons are never seated next to each other?

(A) 72

(B) 48

(C) 120

(D) 96

Q15. If the roots of $x^3 - 7x^2 + 14x - 8 = 0$ are in geometric progression, then the middle root is:

(A) 1

(B) 4

(C) 2

(D) 8

Q16. If α, β are the roots of $x^2 + 2x + 3 = 0$, then the equation whose roots are α^2 and β^2 is:

(A) $x^2 - 2x + 9 = 0$

(B) $x^2 + 2x - 9 = 0$

(C) $x^2 - 2x - 9 = 0$

(D) $x^2 + 2x + 9 = 0$

Q17. The value of $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ is:

(A) e

(B) 1

(C) 0

(D) ∞

Q18. If $y = x^x$ (with $x > 0$), then $\frac{dy}{dx}$ equals:

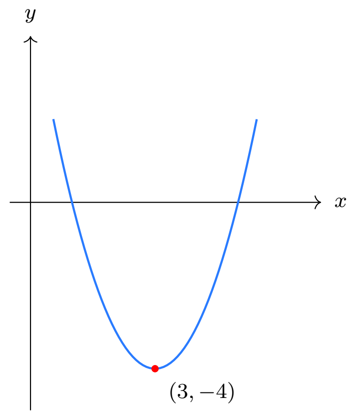
(A) $x\, x^{x-1}$

(B) $x^x(1 + \ln x)$

(C) $x^x \ln x$

(D) x^x

Q19. For the function $f(x) = x^2 - 6x + 5$, whose graph is shown, the interval in which f is *increasing* is:



(A) $(-\infty, 3)$

(B) $(-\infty, \infty)$

(C) $(0, 3)$

(D) $(3, \infty)$

Q20. For $f(x) = x^2 - 4x + 3$ on $[1, 3]$, the value of c guaranteed by Rolle's theorem is:

(A) 1

(B) 3

(C) 2

(D) $\frac{3}{2}$

Q21. The integrating factor of the linear differential equation $\frac{dy}{dx} + 2y = e^x$ is:

(A) e^{2x}

(B) e^{-2x}

(C) e^x

(D) $2x$

Q22. $\int 2x\, e^{x^2}\, dx$ equals:

(A) $\frac{1}{2}e^{x^2} + C$

(B) $e^{x^2} + C$

(C) $2e^{x^2} + C$

(D) $x^2e^{x^2} + C$

Q23. If $\int_2^5 f(x)\,dx = 7$, then $\int_5^2 f(x)\,dx$ equals:

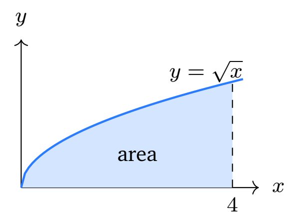
(A) 7

(B) 0

(C) 14

(D) -7

Q24. The area of the region bounded by $y = \sqrt{x}$, the x-axis and the lines $x = 0$ and $x = 4$ (shaded) is:



(A) 8 sq. units

(B) 4 sq. units

(C) $\frac{16}{3}$ sq. units

(D) $\frac{8}{3}$ sq. units

Q25. The value of $\int_0^2 2x\,dx$ is:

(A) 4

(B) 2

(C) 8

(D) 0

Q26. The midpoint of the line segment joining the points $(3, -2)$ and $(7, 6)$ is:

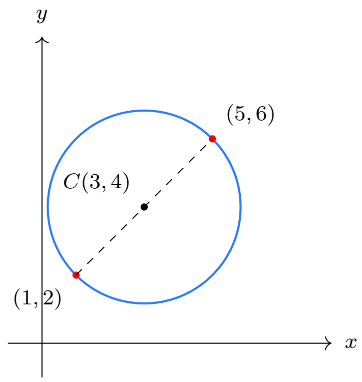
(A) $(10, 4)$

(B) $(5, 2)$

(C) $(2, 4)$

(D) $(5, 4)$

Q27. The equation of the circle having the segment joining $(1, 2)$ and $(5, 6)$ as a diameter (centre marked) is:



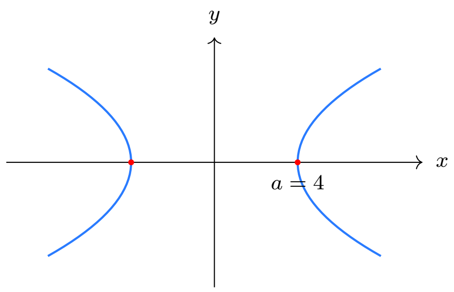
(A) $x^2 + y^2 - 6x - 8y + 25 = 0$

(B) $x^2 + y^2 + 6x + 8y + 17 = 0$

(C) $x^2 + y^2 - 6x - 8y + 17 = 0$

(D) $x^2 + y^2 - 6x - 8y = 0$

Q28. The eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$, sketched below, is:



(A) $\frac{4}{5}$

(B) $\frac{3}{4}$

(C) $\frac{4}{3}$

(D) $\frac{5}{4}$

Q29. Two lines with direction cosines (l_1, m_1, n_1) and (l_2, m_2, n_2) are perpendicular if and only if:

(A) $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

(B) $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

(C) $l_1 l_2 + m_1 m_2 + n_1 n_2 = 1$

(D) $l_1 + m_1 + n_1 = 0$

Q30. The shortest distance between the parallel-axis skew lines $\vec{r} = \hat{i} + \lambda(2\hat{i} + \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} + \mu(2\hat{i} + \hat{j} + \hat{k})$ is found by projecting the join $(2\hat{i} - \hat{i}) = \hat{i}$ on the common perpendicular. For two skew lines, this shortest distance is given by:

(A) $\frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 + \vec{b}_2|}$

(B) $\frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$

(C) $|\vec{a}_2 - \vec{a}_1|$

(D) $\frac{|\vec{b}_1 \times \vec{b}_2|}{|\vec{a}_2 - \vec{a}_1|}$

Q31. For any two vectors $\vec{a}$ and $\vec{b}$, the expression $|\vec{a} + \vec{b}|^2$ equals:

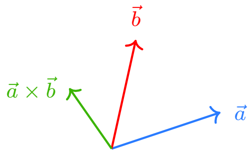
(A) $|\vec{a}|^2 + |\vec{b}|^2$

(B) $|\vec{a}|^2 + |\vec{b}|^2 - 2\,\vec{a} \cdot \vec{b}$

(C) $|\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a} \cdot \vec{b}$

(D) $|\vec{a}|^2 - |\vec{b}|^2$

Q32. A unit vector perpendicular to both $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$ is:



(A) $\dfrac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

(B) $\dfrac{1}{\sqrt{2}}(\hat{i} - \hat{k})$

(C) $\hat{i} - \hat{j} + \hat{k}$

(D) $\dfrac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

Q33. The scalar triple product $[\vec{a}\,\vec{b}\,\vec{c}]$ is necessarily zero when:

(A) two of the three vectors are equal

(B) the three vectors are mutually perpendicular

(C) all three are unit vectors

(D) $\vec{a} \cdot \vec{b} = 1$

Q34. The arithmetic mean of the first n natural numbers $1, 2, 3, \ldots, n$ is:

(A) $\dfrac{n}{2}$

(B) $\dfrac{n+1}{2}$

(C) $\dfrac{n(n+1)}{2}$

(D) $n + 1$

Q35. A card is drawn at random from a well-shuffled pack of 52 playing cards. The probability that it is a face card (Jack, Queen or King) is:

(A) $\dfrac{1}{13}$

(B) $\dfrac{1}{4}$

(C) $\dfrac{3}{13}$

(D) $\dfrac{4}{13}$

Q36. For a binomial distribution with $n = 10$, $p = 0.4$ (so $q = 0.6$), the variance npq equals:

(A) 4

(B) 6

(C) 1.6

(D) 2.4

Q37. The value of $\tan 45^\circ + \cot 45^\circ$ is:

(A) 2

(B) 1

(C) 0

(D) $\frac{1}{2}$

Q38. Using $\cos(A - B) = \cos A \cos B + \sin A \sin B$, the value of $\cos(60^\circ - 30^\circ)$ is:

(A) $\frac{1}{2}$

(B) 0

(C) $\frac{\sqrt{3}}{2}$

(D) 1

Q39. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is:

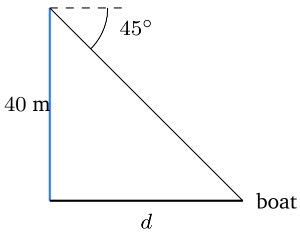
(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{5\pi}{6}$

Q40. From the top of a vertical cliff 40 m high, the angle of depression of a boat on the sea is 45°, as shown. The horizontal distance of the boat from the foot of the cliff is:



(A) 20 m

(B) $40\sqrt{3}$ m

(C) $\dfrac{40}{\sqrt{3}}$ m

(D) 40 m

Detailed Solutions

Q1.

Solution

Concept — Splitting a Venn diagram into disjoint regions: when two sets overlap, the people who like a particular drink split into two non-overlapping groups: those who like *only* that drink and those who like *both*. Since these groups share no member, the count for one set is the sum of its two pieces, so the "only" part is obtained by removing the intersection. Hence the number who like only tea is $n(\text{only Tea}) = n(\text{Tea}) - n(\text{Tea} \cap \text{Coffee})$. This is just the inclusion–exclusion idea applied to a single set.

Step 1 — Read the data: the diagram and the statement give the totals

$$n(\text{Tea}) = 30, \qquad n(\text{Coffee}) = 25, \qquad n(\text{Tea} \cap \text{Coffee}) = 10.$$

Step 2 — Remove the overlap from the tea-total: the 30 tea-drinkers include the 10 who also drink coffee, so subtract that overlap once:

$$n(\text{only Tea}) = n(\text{Tea}) - n(\text{Tea} \cap \text{Coffee}) = 30 - 10 = 20.$$

Step 3 — Cross-check with the three disjoint regions: only-tea $= 20$, only-coffee $= 25 - 10 = 15$, and both $= 10$. These add to

$$20 + 15 + 10 = 45,$$

which are exactly the students inside the two circles; the remaining $50 - 45 = 5$ like neither. The pieces are consistent, confirming only-tea $= 20$.

Why other options are wrong:

- (B) 30 is the *total* who like tea, i.e. $n(\text{Tea})$ before the overlap is removed, so it double-counts the 10 both-drinkers.
- (C) 15 is the only-*coffee* count $25 - 10$, the wrong circle's outer part.
- (D) 10 is $n(\text{Tea} \cap \text{Coffee})$, the both-count, i.e. the overlap we were meant to subtract, not the answer.

Final Answer: only tea $= 20 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q1

Q2.

Solution

Concept — The three relation properties: a relation R on a set S is *reflexive* if $(a,a) \in R$ for every $a \in S$; *symmetric* if $(a,b) \in R$ forces $(b,a) \in R$; and *transitive* if $(a,b) \in R$ and $(b,c) \in R$ force $(a,c) \in R$. A relation can satisfy any subset of these independently, so each must be tested separately against the actual list of ordered pairs. Here $S = \{1,2,3\}$ and we examine the seven given pairs.

Step 1 — Test reflexivity: we need all three "diagonal" pairs. The list contains

$$(1,1),\ (2,2),\ (3,3),$$

so every element relates to itself and R *is* reflexive.

Step 2 — Test symmetry: check each off-diagonal pair has its reverse. We have $(1,2)$ and $(2,1)$ both present, and $(2,3)$ and $(3,2)$ both present:

$$(1,2) \leftrightarrow (2,1), \qquad (2,3) \leftrightarrow (3,2).$$

No pair is left without its mirror image, so R *is* symmetric.

Step 3 — Test transitivity: look for a broken chain. Take $(1,2) \in R$ and $(2,3) \in R$; transitivity would demand $(1,3) \in R$, but

$$(1,3) \notin R.$$

The chain $1 \to 2 \to 3$ fails to close, so R is *not* transitive.

Step 4 — Conclude: R is reflexive and symmetric but not transitive, which is option (B).

Why other options are wrong:

- (A) claims all three hold, but Step 3 shows transitivity fails (missing $(1,3)$ and also $(3,1)$).
- (C) says not reflexive, contradicting Step 1 where all of $(1,1), (2,2), (3,3)$ appear.
- (D) says neither reflexive nor symmetric, contradicting both Step 1 and Step 2.

Final Answer: reflexive and symmetric but not transitive $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept — Injective and surjective maps: a function $f : \mathbb{R} \to \mathbb{R}$ is *one-one* (injective) when distinct inputs give distinct outputs, equivalently $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. It is *onto* (surjective) when every element of the codomain is hit, i.e. the range equals all of $\mathbb{R}$. A continuous, strictly monotonic function from $\mathbb{R}$ onto $\mathbb{R}$ is automatically a bijection, and $f(x) = x^3$ is exactly such a function.

Step 1 — Prove one-one algebraically: suppose $f(x_1) = f(x_2)$. Then

$$x_1^3 = x_2^3 \;\Rightarrow\; x_1^3 - x_2^3 = 0 \;\Rightarrow\; (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = 0.$$

The quadratic factor $x_1^2+x_1x_2+x_2^2 = \left(x_1+\frac{x_2}{2}\right)^2+\frac{3}{4}x_2^2$ is zero only when $x_1 = x_2 = 0$, so in every case $x_1 = x_2$. Hence f is one-one.

Step 2 — Confirm with the derivative: $f'(x) = 3x^2 \geq 0$, vanishing only at the single point $x = 0$, so f is strictly increasing on all of $\mathbb{R}$; a strictly increasing function cannot repeat a value, again giving injectivity.

Step 3 — Prove onto: take any target $y \in \mathbb{R}$. The real cube root $x = y^{1/3}$ satisfies

$$f(x) = \left(y^{1/3}\right)^3 = y,$$

so every real y is an image. The range is all of $\mathbb{R}$ and f is onto.

Step 4 — Conclude: being both injective and surjective, f is a bijection, which is option (C).

Why other options are wrong:

- (A) denies onto, but Step 3 produces a preimage for every y (e.g. $y = -8$ comes from $x = -2$).
- (B) denies one-one, but Step 1 shows $x_1^3 = x_2^3$ forces $x_1 = x_2$ (unlike x^2, x^3 never repeats a value).
- (D) denies both, contradicting Steps 1–3 simultaneously.

Final Answer: both one-one and onto $\Rightarrow$ $\boxed{\text{C}}$

Answer: (C) Go Back to Q3

Q4.

Solution

Concept — Squaring a complex number: a complex number $a + ib$ is squared like any binomial using $(p+q)^2 = p^2 + 2pq + q^2$, with the crucial rule $i^2 = -1$. The defining property $i^2 = -1$ is what collapses the real and imaginary parts together, so the answer is found purely by careful expansion, then writing the result in standard $a + ib$ form.

Step 1 — Expand the binomial: treat 1 and i as the two terms:

$$(1+i)^2 = 1^2 + 2(1)(i) + i^2 = 1 + 2i + i^2.$$

Step 2 — Apply $i^2 = -1$: replace i^2 by -1 and combine the real parts:

$$1 + 2i + (-1) = (1-1) + 2i = 0 + 2i = 2i.$$

Step 3 — Cross-check via modulus and polar form: $|1+i| = \sqrt{2}$ and $\arg(1+i) = 45^\circ$, so $(1+i)^2$ has modulus $(\sqrt{2})^2 = 2$ and argument 90°. A number of modulus 2 at 90° is $2(\cos 90^\circ + i \sin 90^\circ) = 2i$, matching Step 2.

Step 4 — Locate on the Argand plane: in $a + ib$ form the result is $0 + 2i$, a point on the positive imaginary axis at height 2, exactly the red dot in the figure.

Why other options are wrong:

- (A) 2 keeps the modulus but drops the factor i, ignoring that the argument is 90°.
- (B) $1 + 2i$ stops at $1 + 2i + i^2$ and forgets to substitute $i^2 = -1$.
- (C) -2 would arise from $(1+i) \cdot (\text{something})$ giving a real negative; here it wrongly treats the square as i^2·(real), squaring only the imaginary part.

Final Answer: $(1+i)^2 = 2i \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Conjugate-root theorem: if a polynomial has *real* coefficients and a complex number $\alpha = 2 + 3i$ as a root, then its conjugate $\bar{\alpha} = 2 - 3i$ must also be a root, because complex roots of real polynomials occur in conjugate pairs. Knowing both roots, a monic quadratic is rebuilt from $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$.

Step 1 — Write down both roots: the two roots are

$$\alpha = 2 + 3i, \qquad \bar{\alpha} = 2 - 3i.$$

Step 2 — Sum of the roots: the imaginary parts cancel,

$$\alpha + \bar{\alpha} = (2 + 3i) + (2 - 3i) = 4.$$

Step 3 — Product of the roots: use $(a + b)(a - b) = a^2 - b^2$ with $b = 3i$ and $i^2 = -1$:

$$\alpha\bar{\alpha} = (2 + 3i)(2 - 3i) = 2^2 - (3i)^2 = 4 - 9i^2 = 4 + 9 = 13.$$

Step 4 — Assemble the quadratic:

$$x^2 - (\alpha + \bar{\alpha})x + \alpha\bar{\alpha} = 0 \;\Rightarrow\; x^2 - 4x + 13 = 0.$$

Check the discriminant: $(-4)^2 - 4(13) = 16 - 52 = -36 < 0$, confirming complex roots, and $\sqrt{-36} = 6i$ gives $x = \frac{4 \pm 6i}{2} = 2 \pm 3i$ as required.

Why other options are wrong:

- (B) $x^2 + 4x + 13 = 0$ uses sum $= -4$; its roots are $-2 \pm 3i$, not $2 + 3i$.
- (C) $x^2 - 4x - 13 = 0$ takes product $= -13$, which would force real roots (positive discriminant).
- (D) $x^2 - 4x + 5 = 0$ has the right sum but product 5, giving roots $2 \pm i$, not $2 \pm 3i$.

Final Answer: $x^2 - 4x + 13 = 0 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q5

Q6.

Solution

Concept — The discriminant test: for a quadratic $ax^2 + bx + c = 0$ the nature of the roots is governed by the discriminant $D = b^2 - 4ac$. The two roots coincide (a repeated/equal root) exactly when $D = 0$, are real and distinct when $D > 0$, and are complex conjugates when $D < 0$. So "equal roots" translates directly into the single equation $D = 0$, which we solve for the unknown coefficient.

Step 1 — Identify the coefficients: comparing $x^2 - kx + 9 = 0$ with $ax^2 + bx + c$ gives

$$a = 1, \qquad b = -k, \qquad c = 9.$$

Step 2 — Write the discriminant:

$$D = b^2 - 4ac = (-k)^2 - 4(1)(9) = k^2 - 36.$$

Step 3 — Set $D = 0$ and solve:

$$k^2 - 36 = 0 \Rightarrow k^2 = 36 \Rightarrow k = \pm 6.$$

The condition $k > 0$ selects $k = 6$.

Step 4 — Verify the repeated root: with $k = 6$ the equation is $x^2 - 6x + 9 = 0 = (x-3)^2$, whose only root is $x = 3$ (a double root), confirming equal roots.

Why other options are wrong:

- (A) $k = 3$ gives $D = 9 - 36 = -27 < 0$, so the roots are complex, not equal.
- (B) $k = 9$ gives $D = 81 - 36 = 45 > 0$, two distinct real roots.
- (D) $k = 12$ gives $D = 144 - 36 = 108 > 0$, again distinct real roots.

Final Answer: $k = 6 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q6

Q7.

Solution

Concept — Building a new equation from transformed roots: by Vieta's formulas a monic quadratic $x^2 - Sx + P = 0$ has root-sum $S = \alpha + \beta$ and root-product $P = \alpha\beta$. To find the equation whose roots are α^2 and β^2 we only need their sum and product, computed via the identities $\alpha^2+\beta^2 = S^2-2P$ and $\alpha^2\beta^2 = (\alpha\beta)^2 = P^2$. We never need the individual roots, though here they are simple enough to verify with.

Step 1 — Read S and P: for $x^2 - 5x + 6 = 0$,

$$S = \alpha + \beta = 5, \qquad P = \alpha\beta = 6.$$

(Factoring, $x^2 - 5x + 6 = (x-2)(x-3)$, so the roots are 2 and 3.)

Step 2 — Sum of the squared roots:

$$\alpha^2 + \beta^2 = S^2 - 2P = 5^2 - 2(6) = 25 - 12 = 13.$$

Step 3 — Product of the squared roots:

$$\alpha^2\beta^2 = P^2 = 6^2 = 36.$$

Step 4 — Assemble and verify: the required equation is

$$x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0 \;\Rightarrow\; x^2 - 13x + 36 = 0.$$

The new roots should be $2^2 = 4$ and $3^2 = 9$; indeed $4 + 9 = 13$ and $4 \cdot 9 = 36$, and $x^2 - 13x + 36 = (x-4)(x-9)$, confirming the result.

Why other options are wrong:

- (A) $x^2-25x+36 = 0$ uses $S^2 = 25$ for the sum, forgetting the $-2P$ correction.
- (C) $x^2 + 13x + 36 = 0$ flips the linear sign; its roots are $-4, -9$, not $4, 9$.
- (D) $x^2 - 13x - 36 = 0$ flips the constant sign, giving product -36 instead of $+36$.

Final Answer: $x^2 - 13x + 36 = 0 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q7

Q8.

Solution

Concept — Diagonal, scalar and identity matrices: a *diagonal* matrix has zeros off the main diagonal; a *scalar* matrix is the special diagonal matrix in which every diagonal entry is the *same* number λ; and the *identity* matrix I is the scalar matrix with $\lambda = 1$. Because scalar multiplication of a matrix multiplies every entry, a matrix with all diagonal entries equal to λ and all other entries 0 is precisely λ times I.

Step 1 — Write the general scalar matrix: for order n it looks like

$$\begin{pmatrix} \lambda & 0 & \cdots & 0 \\ 0 & \lambda & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda \end{pmatrix}.$$

Step 2 — Factor out λ: pulling the common factor from every diagonal entry,

$$\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \lambda I,$$

and the same factoring works for any order. Hence a scalar matrix equals λI.

Step 3 — Sanity check via multiplication: for any matrix A of the same order, $(\lambda I)A = \lambda(IA) = \lambda A$, i.e. a scalar matrix acts by simply scaling, exactly the behaviour expected of λI.

Why other options are wrong:

- (A) the zero matrix is only the $\lambda = 0$ case; a general scalar matrix has $\lambda \neq 0$ too.
- (B) "distinct diagonal entries" violates the defining requirement that all diagonal entries be the same λ.
- (C) a row matrix has a single row and is generally not even square, so it cannot be a scalar matrix at all.

Final Answer: a scalar matrix equals $\lambda I \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — Reversal law for inverses: matrix multiplication is not commutative, so the inverse of a product does not simply distribute. The correct "socks-and-shoes" rule states that for invertible matrices A and B of the same order, the inverse of AB reverses the order: $(AB)^{-1} = B^{-1}A^{-1}$. Intuitively, to undo "first B then A" you must "first undo A then undo B". We verify this directly from the definition $XX^{-1} = I$.

Step 1 — Multiply on the right by the claimed inverse: using associativity,

$$(AB)\,(B^{-1}A^{-1}) = A\,(BB^{-1})\,A^{-1} = A\,I\,A^{-1} = AA^{-1} = I.$$

Step 2 — Multiply on the left as well: likewise

$$(B^{-1}A^{-1})\,(AB) = B^{-1}\,(A^{-1}A)\,B = B^{-1}I\,B = B^{-1}B = I.$$

Both products give I, so $B^{-1}A^{-1}$ is indeed the (two-sided) inverse of AB.

Step 3 — See why order matters: if we tried $A^{-1}B^{-1}$, then $(AB)(A^{-1}B^{-1}) = A(BA^{-1})B^{-1}$, and since $BA^{-1} \neq A^{-1}B$ in general this does not collapse to I. The reversal is essential.

Why other options are wrong:

- (B) $A^{-1}B^{-1}$ keeps the original order; Step 3 shows it fails to give I unless A, B commute.
- (C) $(BA)^{-1} = A^{-1}B^{-1}$, which is generally different from $B^{-1}A^{-1}$, so it is the inverse of BA, not of AB.
- (D) AB is the matrix itself, not its inverse; $(AB)(AB) = I$ only in special cases.

Final Answer: $(AB)^{-1} = B^{-1}A^{-1} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Multiplicative property of determinants: for two square matrices A and B of the same order, the determinant of their product equals the product of their determinants, $|AB| = |A|\,|B|$. This is a fundamental theorem of linear algebra; geometrically the determinant measures how a matrix scales volume, and applying two transformations multiplies their scaling factors. Note that determinants are scalars, so they commute even though the matrices may not.

Step 1 — State the given values:

$$|A| = 3, \qquad |B| = 4.$$

Step 2 — Apply the product rule:

$$|AB| = |A|\,|B| = 3 \times 4 = 12.$$

Step 3 — Consistency check: since determinants are scalars, $|BA| = |B|\,|A| = 4 \times 3 = 12 = |AB|$ as well, even though AB and BA may differ as matrices. The single value 12 is therefore unambiguous.

Why other options are wrong:

- (A) $7 = 3 + 4$ *adds* the determinants; determinants of a product multiply, they do not add.
- (C) 1 would be $|A|/|A|$ or similar; it ignores $|B|$ entirely.
- (D) $\frac{3}{4} = |A|/|B|$ divides the determinants, which is the rule for $|AB^{-1}|$, not $|AB|$.

Final Answer: $|AB| = 12 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

Q11.

Solution

Concept — Triangle area from coordinates: the signed area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by the determinant expansion

$$\text{Area} = \tfrac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|.$$

The absolute value discards orientation, and the factor $\frac{1}{2}$ converts the parallelogram (cross-product) area into the triangle area.

Step 1 — Label the vertices:

$$(x_1, y_1) = (1, 1), \quad (x_2, y_2) = (4, 1), \quad (x_3, y_3) = (1, 5).$$

Step 2 — Substitute into the formula:

$$\text{Area} = \tfrac{1}{2}\left|1(1 - 5) + 4(5 - 1) + 1(1 - 1)\right|.$$

Step 3 — Simplify each term:

$$\text{Area} = \tfrac{1}{2}\left|(-4) + 16 + 0\right| = \tfrac{1}{2}\,|12| = 6 \text{ sq. units.}$$

Step 4 — Cross-check with base and height: the side from $(1, 1)$ to $(4, 1)$ is horizontal of length 3, and from $(1, 1)$ to $(1, 5)$ is vertical of length 4, meeting at right angles at $(1, 1)$. So it is a right triangle with legs 3 and 4:

$$\text{Area} = \tfrac{1}{2} \cdot 3 \cdot 4 = 6 \text{ sq. units,}$$

matching the determinant result.

Why other options are wrong:

- (A) 12 is $|12|$ before applying the factor $\frac{1}{2}$ (the parallelogram area).
- (B) 3 is just the horizontal leg, not the area.
- (D) 4 is just the vertical leg, not the area.

Final Answer: area $= 6$ sq. units $\Rightarrow$ $\boxed{\text{C}}$

Answer: (C) **Go Back to Q11**

Q12.

Solution

Concept — The "group as one unit" technique: when certain letters must always stay together, glue them into a single super-letter (a block) and arrange the resulting units. Because the letters inside the block can also rearrange among themselves while staying together, we multiply by the number of internal arrangements. All six letters of ORANGE are distinct, so no repetition correction is needed.

Step 1 — Identify the block and the units: the vowels are O, A, E (three of them) and the consonants are R, N, G. Tying the three vowels together gives one block, so the items to arrange are

$$[\text{O A E}],\ \text{R},\ \text{N},\ \text{G} \quad \Longrightarrow \quad 4 \text{ units.}$$

Step 2 — Arrange the units: four distinct units in a row,

$$4! = 24 \text{ ways.}$$

Step 3 — Arrange inside the block: the three distinct vowels permute among themselves,

$$3! = 6 \text{ ways.}$$

Step 4 — Multiply (fundamental counting principle):

$$4! \times 3! = 24 \times 6 = 144.$$

Why other options are wrong:

- (A) $720 = 6!$ counts *all* arrangements of ORANGE, ignoring the together-condition entirely.
- (B) $36 = 6^2$ or $3! \cdot 3!$ is an unrelated product; it neither arranges the units nor the block correctly.
- (C) $120 = 5!$ treats only five items, as if the block held just two vowels.

Final Answer: 144 arrangements $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Counting triangles as combinations: a triangle is determined by choosing 3 of its vertices, and the *order* in which we pick the vertices does not matter (the same three points always give the same triangle). So this is a selection, counted by the combination $\binom{n}{3}$. The condition "no three collinear" guarantees every chosen triple actually forms a genuine triangle rather than a degenerate straight line.

Step 1 — Set up the combination: with $n = 8$ points,

$$\text{number of triangles} = \binom{8}{3} = \frac{8!}{3!\,(8-3)!} = \frac{8!}{3!\,5!}.$$

Step 2 — Cancel and compute:

$$\binom{8}{3} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = \frac{336}{6} = 56.$$

Step 3 — Why order must be removed: the permutation ${}^{8}P_{3} = 8 \cdot 7 \cdot 6 = 336$ would count each triangle $3! = 6$ times (once per ordering of its vertices); dividing by 6 returns 56, confirming the combination value.

Why other options are wrong:

- (A) 24 is far too small and matches no correct count here (an arithmetic slip).
- (C) $336 = {}^{8}P_{3}$ counts *ordered* triples, overcounting each triangle six-fold.
- (D) 40 is another miscalculation; the correct division $336/6$ gives 56, not 40.

Final Answer: $\binom{8}{3} = 56 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q13

Q14.

Solution

Concept — Complementary counting: "never adjacent" is awkward to count directly, so we use the complement. The arrangements in which the two particular persons are *never* together equal all arrangements minus those in which they *are* together:

$$N(\text{never together}) = N(\text{total}) - N(\text{together}).$$

The "together" case is handled by the block method.

Step 1 — Total arrangements: five distinct persons in a row give

$$5! = 120.$$

Step 2 — Arrangements with the two together: glue the two special persons into one block, leaving 4 units to arrange ($4!$ ways), and the two inside the block can swap ($2!$ ways):

$$4! \times 2! = 24 \times 2 = 48.$$

Step 3 — Subtract to get the complement:

$$120 - 48 = 72.$$

Step 4 — Cross-check by the gap method: seat the other 3 persons first in $3! = 6$ ways; they create 4 gaps (including the ends) into which the two special persons go in different gaps: ${}^4P_2 = 4 \cdot 3 = 12$ ways. Total $6 \times 12 = 72$, matching Step 3.

Why other options are wrong:

- (B) 48 is the count *with* the two together, i.e. the quantity we subtracted, not the answer.
- (C) 120 is the unrestricted total $5!$, ignoring the condition.
- (D) 96 comes from a wrong subtraction such as $120 - 24$ (forgetting the $2!$ inside the block).

Final Answer: 72 ways $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q14

Q15.

Solution

Concept — Symmetric parametrisation of a GP: three numbers in geometric progression can be written as $\frac{b}{r},\ b,\ br$, where b is the middle term and r the common ratio. The advantage is that their product telescopes to b^3, independent of r. For a cubic $x^3 + px^2 + qx + s = 0$, Vieta gives the product of roots as $-s$ (with leading coefficient 1), so the product is known directly.

Step 1 — Product of roots from Vieta: for $x^3 - 7x^2 + 14x - 8 = 0$, the product of the three roots is

$$-\frac{\text{constant}}{\text{leading}} = -\frac{-8}{1} = 8.$$

Step 2 — Use the GP form:

$$\frac{b}{r} \cdot b \cdot br = b^3 = 8.$$

Step 3 — Solve for the middle root:

$$b^3 = 8 \;\Rightarrow\; b = 2.$$

Step 4 — Verify by finding all roots: substituting $x = 2$ into the cubic, $8-28+28-8 = 0$, so 2 is genuinely a root. Factoring out $(x-2)$ leaves $x^2-5x+4 = (x-1)(x-4)$, so the roots are $1, 2, 4$ which are in GP with ratio 2. The middle one is indeed 2.

Why other options are wrong:

- (A) 1 is the smallest root $\frac{b}{r}$, not the middle term.
- (B) 4 is the largest root br, again an outer term.
- (D) 8 is the *product* of all three roots, not the middle root.

Final Answer: middle root $= 2 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q15

Q16.

Solution

Concept — Squared-root transformation (Vieta again): for a monic quadratic with root-sum $S = \alpha + \beta$ and root-product $P = \alpha\beta$, the equation whose roots are α^2, β^2 is built from the new sum $\alpha^2 + \beta^2 = S^2 - 2P$ and new product $\alpha^2\beta^2 = P^2$. Crucially, even when the original roots are complex, these symmetric combinations stay real because they are expressed through the real coefficients S and P.

Step 1 — Read S and P: for $x^2 + 2x + 3 = 0$ (so $b = +2, c = +3$),

$$S = \alpha + \beta = -b = -2, \qquad P = \alpha\beta = c = 3.$$

Step 2 — New sum of squares:

$$\alpha^2 + \beta^2 = S^2 - 2P = (-2)^2 - 2(3) = 4 - 6 = -2.$$

Step 3 — New product:

$$\alpha^2\beta^2 = P^2 = 3^2 = 9.$$

Step 4 — Assemble the equation:

$$x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0 \;\Rightarrow\; x^2 - (-2)x + 9 = 0 \;\Rightarrow\; x^2 + 2x + 9 = 0.$$

(Consistency: the original discriminant $4 - 12 = -8 < 0$ gives complex roots, and the new equation also has discriminant $4 - 36 = -32 < 0$, as expected for squares of complex conjugates.)

Why other options are wrong:

- (A) $x^2 - 2x + 9 = 0$ flips the linear sign, i.e. uses new sum $= +2$ instead of -2.
- (B) $x^2 + 2x - 9 = 0$ flips the constant sign, using product -9 instead of $+9$.
- (C) $x^2 - 2x - 9 = 0$ flips both signs at once.

Final Answer: $x^2 + 2x + 9 = 0 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — The 1^∞ indeterminate form: as $x \to \infty$ the base $1 + \frac{1}{x} \to 1$ while the exponent $x \to \infty$, giving the indeterminate form 1^∞. This is not automatically 1; the outcome depends on how fast the base approaches 1 relative to how fast the exponent grows. The standard limit resolves it to the special constant e, and this is in fact one of the definitions of e.

Step 1 — Recognise the standard form: the expression is exactly the defining limit

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e.$$

Step 2 — Confirm via logarithm: let $L = \lim(1 + \frac{1}{x})^x$. Then

$$\ln L = \lim_{x\to\infty} x \ln\left(1+\frac{1}{x}\right).$$

Put $t = \frac{1}{x} \to 0^+$, so $\ln L = \lim_{t\to 0} \dfrac{\ln(1+t)}{t} = 1$ (a known limit), hence $L = e^1 = e$.

Step 3 — Numerical sanity check: for $x = 1000$, $(1 + \frac{1}{1000})^{1000} \approx 2.7169$, close to $e \approx 2.71828$, confirming the limit is finite and equals e, not 1 or ∞.

Why other options are wrong:

- (B) 1 wrongly assumes $1^\infty = 1$, ignoring the growing exponent's effect.
- (C) 0 would need the base below 1; here the base exceeds 1.
- (D) ∞ overweights the exponent; the base shrinks toward 1 fast enough to keep the limit finite at e.

Final Answer: the limit is $e \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q17

Q18.

Solution

Concept — Logarithmic differentiation: when both the base and the exponent are variable, as in $y = x^x$, neither the power rule (exponent constant) nor the exponential rule (base constant) applies alone. Taking natural logs turns the exponent into a product, after which implicit differentiation and the product rule do the work. This technique is the standard route for any expression of the form $[f(x)]^{g(x)}$.

Step 1 — Take logarithms of both sides:

$$y = x^x \;\Rightarrow\; \ln y = \ln\big(x^x\big) = x \ln x.$$

Step 2 — Differentiate implicitly: the left side uses the chain rule, the right side the product rule:

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(x \ln x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1.$$

Step 3 — Solve for $\frac{dy}{dx}$: multiply through by $y = x^x$:

$$\frac{dy}{dx} = y(1 + \ln x) = x^x(1 + \ln x).$$

Step 4 — Spot-check at $x = 1$: there $y = 1^1 = 1$ and the formula gives $1^1(1 + \ln 1) = 1(1 + 0) = 1$, a finite positive slope, consistent with x^x rising for $x > 1$.

Why other options are wrong:

- (A) $x\,x^{x-1}$ wrongly applies the power rule, treating the exponent as the constant x.
- (C) $x^x \ln x$ drops the "+1", i.e. forgets the derivative of the base factor.
- (D) x^x is just y itself with no differentiation factor at all.

Final Answer: $x^x(1 + \ln x) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q18

Q19.

Solution

Concept — Monotonicity from the first derivative: a differentiable function is increasing on any interval where its derivative is positive and decreasing where the derivative is negative. For a parabola opening upward, the derivative changes sign exactly at the vertex: negative to the left (falling) and positive to the right (rising). So finding the increasing interval reduces to solving $f'(x) > 0$.

Step 1 — Differentiate: for $f(x) = x^2 - 6x + 5$,

$$f'(x) = 2x - 6.$$

Step 2 — Solve $f'(x) > 0$:

$$2x - 6 > 0 \;\Rightarrow\; 2x > 6 \;\Rightarrow\; x > 3,$$

so f is increasing on $(3, \infty)$.

Step 3 — Locate the turning point: $f'(x) = 0$ at $x = 3$, and $f(3) = 9 - 18 + 5 = -4$, giving the vertex $(3, -4)$ shown in the figure. To the left of $x = 3$, $f' < 0$ (decreasing); to the right, $f' > 0$ (increasing), matching the U-shape.

Why other options are wrong:

- (A) $(-\infty, 3)$ is precisely where $f'(x) < 0$, the *decreasing* branch.
- (B) $(-\infty, \infty)$ cannot be right because the function first falls then rises; it is not monotonic everywhere.
- (C) $(0, 3)$ lies entirely left of the vertex, still in the decreasing region.

Final Answer: increasing on $(3, \infty) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q19

Q20.

Solution

Concept — Rolle's theorem: if f is continuous on the closed interval $[a, b]$, differentiable on the open interval (a, b), and takes equal values at the endpoints, $f(a) = f(b)$, then there is at least one interior point $c \in (a, b)$ where the tangent is horizontal, i.e. $f'(c) = 0$. Geometrically, a smooth curve that returns to its starting height must have turned around somewhere, and at that turning point the slope is zero.

Step 1 — Verify the hypotheses: $f(x) = x^2 - 4x + 3$ is a polynomial, hence continuous on $[1, 3]$ and differentiable on $(1, 3)$. Check the endpoint values:

$$f(1) = 1 - 4 + 3 = 0, \qquad f(3) = 9 - 12 + 3 = 0.$$

Since $f(1) = f(3) = 0$, all three conditions of Rolle's theorem are met.

Step 2 — Differentiate:

$$f'(x) = 2x - 4.$$

Step 3 — Solve $f'(c) = 0$:

$$2c - 4 = 0 \;\Rightarrow\; c = 2.$$

This lies inside $(1,3)$, so it is the value guaranteed by the theorem.

Step 4 — Interpret: $c = 2$ is the vertex of the parabola, where $f(2) = 4 - 8 + 3 = -1$ is the minimum; the horizontal tangent there confirms $f'(2) = 0$.

Why other options are wrong:

- (A) 1 and (B) 3 are the *endpoints*; Rolle's theorem promises an *interior* point, and $f'(1) = -2 \neq 0$, $f'(3) = 2 \neq 0$ anyway.
- (D) $\frac{3}{2}$ gives $f'(\frac{3}{2}) = 2(\frac{3}{2}) - 4 = -1 \neq 0$, so the tangent is not horizontal there.

Final Answer: $c = 2 \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q20**

Q21.

Solution

Concept — Integrating factor of a linear ODE: a first-order linear equation in standard form $\frac{dy}{dx} + P(x)\,y = Q(x)$ is solved by multiplying through by the integrating factor $\mu = e^{\int P\,dx}$, chosen so the left side becomes the exact derivative $\frac{d}{dx}(\mu y)$. The factor depends only on the coefficient P of y, never on the right-hand side Q.

Step 1 — Put the equation in standard form and read P: the equation $\frac{dy}{dx} + 2y = e^x$ is already standard with

$$P(x) = 2, \qquad Q(x) = e^x.$$

Step 2 — Integrate P:

$$\int P\,dx = \int 2\,dx = 2x.$$

Step 3 — Exponentiate:

$$\mu = e^{\int P\,dx} = e^{2x}.$$

Step 4 — Verify the exactness: multiplying the ODE by e^{2x} gives $e^{2x}\frac{dy}{dx} + 2e^{2x}y = \frac{d}{dx}\left(e^{2x}y\right)$, confirming that e^{2x} is the correct integrating factor.

Why other options are wrong:

- (B) e^{-2x} uses $-P$; it would not make the left side a perfect derivative.
- (C) e^x wrongly uses the right-hand side $Q = e^x$ instead of P.

- (D) $2x$ is $\int P\,dx$ before exponentiating, forgetting the $e^{(\cdot)}$.

Final Answer: IF $= e^{2x} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept — Integration by substitution: when an integrand contains a composite function together with (a constant multiple of) the derivative of its inner part, substituting $u =$ inner function converts the integral into an elementary one. Here the inner function is x^2, whose derivative $2x$ is exactly the factor sitting in front of the exponential, so the substitution is perfectly set up.

Step 1 — Choose the substitution: let

$$u = x^2 \;\Rightarrow\; \frac{du}{dx} = 2x \;\Rightarrow\; du = 2x\,dx.$$

Step 2 — Rewrite the integral: the $2x\,dx$ in the integrand becomes du outright:

$$\int 2x\,e^{x^2}\,dx = \int e^u\,du.$$

Step 3 — Integrate and back-substitute:

$$\int e^u\,du = e^u + C = e^{x^2} + C.$$

Step 4 — Verify by differentiating: $\frac{d}{dx}\big(e^{x^2} + C\big) = e^{x^2} \cdot 2x = 2x\,e^{x^2}$, which is exactly the original integrand, confirming the answer.

Why other options are wrong:

- (A) $\frac{1}{2}e^{x^2} + C$ halves unnecessarily; the $2x$ already in the integrand supplies the full du, so no $\frac{1}{2}$ is needed (differentiating it gives only xe^{x^2}).
- (C) $2e^{x^2} + C$ keeps a spurious factor 2; its derivative is $4xe^{x^2}$, too big.
- (D) $x^2e^{x^2} + C$ comes from a wrong product-rule guess; its derivative is not $2xe^{x^2}$.

Final Answer: $e^{x^2} + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q22

Q23.

Solution

Concept — Orientation of definite integrals: a definite integral records direction. Swapping the upper and lower limits reverses the sign of the result, $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$. This follows from the Fundamental Theorem: if F is an antiderivative, then $\int_a^b f = F(b) - F(a)$, and interchanging the limits turns this into $F(a) - F(b)$, the negative of the original.

Step 1 — Note the given value: we are told

$$\int_2^5 f(x)\,dx = 7.$$

Step 2 — Apply the reversal property: the required integral runs from 5 down to 2, the reverse order, so

$$\int_5^2 f(x)\,dx = -\int_2^5 f(x)\,dx = -7.$$

Step 3 — Consistency check: adding the two should give zero over the closed loop: $\int_2^5 f + \int_5^2 f = 7 + (-7) = 0$, which is exactly $\int_2^2 f = 0$ as required by $\int_a^a f = 0$.

Why other options are wrong:

- (A) 7 ignores the sign change from reversing the limits.
- (B) 0 would require equal limits ($\int_a^a$), not reversed ones.
- (C) 14 doubles the value instead of negating it.

Final Answer: the integral is $-7 \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q23**

Q24.

Solution

Concept — Area as a definite integral: the region trapped between a curve $y = f(x) \geq 0$, the x-axis and two vertical lines $x = a,\ x = b$ has area $\int_a^b y\,dx$. Each thin vertical strip has height y and width dx, and summing (integrating) these strip-areas from a to b gives the total shaded area.

Step 1 — Set up the integral: with $y = \sqrt{x} = x^{1/2}$ from $x = 0$ to $x = 4$,

$$A = \int_0^4 \sqrt{x}\,dx = \int_0^4 x^{1/2}\,dx.$$

Step 2 — Apply the power rule: since $\int x^n\,dx = \dfrac{x^{n+1}}{n+1}$ with $n = \frac{1}{2}$,

$$\int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} = \frac{2}{3}x^{3/2}.$$

Step 3 — Evaluate at the limits: using $4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$ and $0^{3/2} = 0$,

$$A = \frac{2}{3}\left[x^{3/2}\right]_0^4 = \frac{2}{3}(8-0) = \frac{16}{3}\text{ sq. units.}$$

Step 4 — Reasonableness check: the region sits under the rectangle $4 \times 2 = 8$ (width 4, max height $\sqrt{4} = 2$); $\frac{16}{3} \approx 5.33$ is comfortably less than 8, as a curved region under that rectangle should be.

Why other options are wrong:

- (A) 8 is the bounding-rectangle area, not the area under the curve.
- (B) 4 ignores the integration altogether.
- (D) $\frac{8}{3}$ drops the factor 2 (using $\frac{1}{3}$ instead of $\frac{2}{3}$ from the power rule).

Final Answer: area $= \dfrac{16}{3}$ sq. units $\Rightarrow$ C

Answer: (C) Go Back to Q24

Q25.

Solution

Concept — Fundamental Theorem of Calculus: to evaluate a definite integral $\int_a^b f(x)\,dx$, find any antiderivative F with $F' = f$, then compute $F(b) - F(a)$. The constant of integration cancels in the subtraction, so it can be dropped for definite integrals.

Step 1 — Find the antiderivative: by the power rule,

$$\int 2x\,dx = 2 \cdot \frac{x^2}{2} = x^2.$$

Step 2 — Apply the limits 0 and 2:

$$\int_0^2 2x\,dx = \left[x^2\right]_0^2 = 2^2 - 0^2 = 4 - 0 = 4.$$

Step 3 — Geometric check: the integrand $y = 2x$ is a straight line through the origin; from $x = 0$ to $x = 2$ it bounds a right triangle with base 2 and height $2(2) = 4$, of area $\frac{1}{2} \cdot 2 \cdot 4 = 4$, matching the computed value.

Why other options are wrong:

- (B) 2 forgets to square the upper limit (using x instead of x^2).
- (C) 8 over-counts, e.g. by leaving the factor 2 in the antiderivative ($2x^2$ at $x = 2$ gives 8).
- (D) 0 ignores the upper limit and evaluates only at $x = 0$.

Final Answer: the integral is $4 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q25

Q26.

Solution

Concept — Midpoint as the average of coordinates: the midpoint of a segment is the point that divides it in ratio $1 : 1$, so each of its coordinates is the simple average of the corresponding endpoint coordinates:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

This is the section formula with internal ratio $1 : 1$.

Step 1 — Label the endpoints:

$$(x_1, y_1) = (3, -2), \qquad (x_2, y_2) = (7, 6).$$

Step 2 — Average each coordinate:

$$M = \left(\frac{3+7}{2}, \frac{-2+6}{2}\right) = \left(\frac{10}{2}, \frac{4}{2}\right).$$

Step 3 — Simplify:

$$M = (5, 2).$$

Step 4 — Check by symmetry: the midpoint should be equidistant from both ends. From $(5, 2)$ to $(3, -2)$ the displacement is $(-2, -4)$; to $(7, 6)$ it is $(+2, +4)$. These are exact opposites, so $(5, 2)$ sits squarely between the endpoints.

Why other options are wrong:

- (A) $(10, 4)$ is the sum of coordinates without dividing by 2.
- (C) $(2, 4)$ mishandles the x-coordinate (it looks like a difference, not an average).
- (D) $(5, 4)$ has the correct x but averages y wrongly ($\frac{-2+6}{2} = 2$, not 4).

Final Answer: midpoint $= (5, 2) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q26

Q27.

Solution

Concept — Diameter form of a circle: if a segment with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is a diameter, then for any point $P(x, y)$ on the circle the angle $\angle APB$ is a right angle (angle in a semicircle). Hence $\vec{PA} \perp \vec{PB}$, i.e. their dot product is zero, which gives the diameter form

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0.$$

Step 1 — Substitute the endpoints $(1, 2)$ and $(5, 6)$:

$$(x - 1)(x - 5) + (y - 2)(y - 6) = 0.$$

Step 2 — Expand each bracket:

$$(x^2 - 6x + 5) + (y^2 - 8y + 12) = 0.$$

Step 3 — Collect into general form:

$$x^2 + y^2 - 6x - 8y + (5 + 12) = 0 \ \Rightarrow\ x^2 + y^2 - 6x - 8y + 17 = 0.$$

Step 4 — Verify the centre and radius: the centre is $\left(\frac{1+5}{2}, \frac{2+6}{2}\right) = (3, 4)$, matching the marked $C(3, 4)$. The radius is half the diameter: $|AB| = \sqrt{(5-1)^2 + (6-2)^2} = \sqrt{16 + 16} = 4\sqrt{2}$, so $r = 2\sqrt{2}$ and $r^2 = 8$. The general-form constant should be $h^2 + k^2 - r^2 = 9 + 16 - 8 = 17$, exactly the constant obtained, confirming the equation.

Why other options are wrong:

- (A) constant 25 uses $5 + 12 = 17$ wrongly (or adds $9 + 16$); its circle would have zero radius.
- (B) $x^2 + y^2 + 6x + 8y + 17 = 0$ flips the signs of the linear terms, putting the centre at $(-3, -4)$.
- (D) $x^2 + y^2 - 6x - 8y = 0$ drops the $+17$, giving a circle through the origin, not through $(1, 2)$ and $(5, 6)$.

Final Answer: $x^2 + y^2 - 6x - 8y + 17 = 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q27

Q28.

Solution

Concept — Eccentricity of a hyperbola: for the standard hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the relation $c^2 = a^2 + b^2$ links the focal distance c to the semi-axes, and the eccentricity is $e = \frac{c}{a} = \sqrt{1 + \frac{b^2}{a^2}}$. Because $b^2 > 0$, the quantity under the root exceeds 1, so every hyperbola has $e > 1$, distinguishing it from an ellipse ($e < 1$).

Step 1 — Read the parameters: comparing $\frac{x^2}{16} - \frac{y^2}{9} = 1$ with the standard form,

$$a^2 = 16\ (a = 4), \qquad b^2 = 9\ (b = 3).$$

Step 2 — Apply the eccentricity formula:

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{16 + 9}{16}} = \sqrt{\frac{25}{16}}.$$

Step 3 — Simplify:

$$e = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4} = 1.25.$$

Step 4 — Sanity check: $e = \frac{5}{4} > 1$ as required for a hyperbola; equivalently $c = \sqrt{a^2 + b^2} = \sqrt{25} = 5$, so $e = c/a = 5/4$, consistent.

Why other options are wrong:

- (A) $\frac{4}{5} < 1$ is impossible for a hyperbola (that would be an ellipse's eccentricity).

- (B) $\frac{3}{4} < 1$ is likewise too small for a hyperbola.
- (C) $\frac{4}{3}$ comes from $\sqrt{1+\frac{b^2}{b^2}}$-type slips or mixing a and b; the correct ratio is $b^2/a^2 = 9/16$, giving $\frac{5}{4}$.

Final Answer: $e = \dfrac{5}{4} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q28

Q29.

Solution

Concept — Angle between lines via direction cosines: the direction cosines (l, m, n) of a line are the components of a unit vector along it. The cosine of the angle θ between two lines equals the dot product of these unit vectors:

$$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2.$$

Perpendicularity means $\theta = 90^\circ$, i.e. $\cos\theta = 0$, while parallelism means the direction vectors are proportional.

Step 1 — Impose $\theta = 90^\circ$: setting $\cos 90^\circ = 0$ in the dot-product formula gives the perpendicularity condition

$$l_1l_2 + m_1m_2 + n_1n_2 = 0.$$

Step 2 — Contrast with parallelism: if instead the lines were parallel, the direction cosines would be proportional, $\dfrac{l_1}{l_2} = \dfrac{m_1}{m_2} = \dfrac{n_1}{n_2}$, a different condition entirely.

Why other options are wrong:

- (B) $\dfrac{l_1}{l_2} = \dfrac{m_1}{m_2} = \dfrac{n_1}{n_2}$ is the *parallel* condition, not perpendicular.
- (C) $l_1l_2 + m_1m_2 + n_1n_2 = 1$ gives $\cos\theta = 1$, i.e. $\theta = 0^\circ$ (identical directions).
- (D) $l_1 + m_1 + n_1 = 0$ is a single-line relation with no bearing on the angle between two lines.

Final Answer: $l_1l_2 + m_1m_2 + n_1n_2 = 0 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q29

Q30.

Solution

Concept — Shortest distance between skew lines: two skew lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ do not intersect and are not parallel. Their common perpendicular points along $\vec{b}_1 \times \vec{b}_2$, which is simultaneously perpendicular to both direction vectors. The shortest distance is the length of the projection of the join $\vec{a}_2 - \vec{a}_1$ onto this common-perpendicular direction:

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}.$$

Step 1 — Find the common-perpendicular direction: a vector perpendicular to both lines is the cross product $\vec{b}_1 \times \vec{b}_2$; its unit version is $\dfrac{\vec{b}_1 \times \vec{b}_2}{|\vec{b}_1 \times \vec{b}_2|}$.

Step 2 — Project the join onto it: the scalar projection of $\vec{a}_2 - \vec{a}_1$ on this unit direction is

$$(\vec{a}_2 - \vec{a}_1) \cdot \frac{\vec{b}_1 \times \vec{b}_2}{|\vec{b}_1 \times \vec{b}_2|},$$

and taking the absolute value (distances are non-negative) gives the formula in the Concept.

Step 3 — Note the numerator is a box product: $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = [\vec{a}_2 - \vec{a}_1 \ \ \vec{b}_1 \ \ \vec{b}_2]$, the scalar triple product, which vanishes exactly when the lines are coplanar (intersecting or parallel), correctly giving $d = 0$ in that case.

Why other options are wrong:

- (A) omits the modulus (distance must be non-negative) and wrongly divides by $|\vec{b}_1 + \vec{b}_2|$ instead of $|\vec{b}_1 \times \vec{b}_2|$.
- (C) $|\vec{a}_2 - \vec{a}_1|$ is just the length of the join between the two reference points, not the perpendicular distance.
- (D) inverts numerator and denominator, giving wrong dimensions (not a length).

Final Answer: $\dfrac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q30

Q31.

Solution

Concept — Square of a vector sum: the squared magnitude of any vector equals its dot product with itself, $|\vec{u}|^2 = \vec{u}\cdot\vec{u}$. Because the dot product is distributive and commutative, expanding $|\vec{a}+\vec{b}|^2$ proceeds just like the algebraic identity $(p+q)^2 = p^2+2pq+q^2$, with multiplication replaced by the dot product.

Step 1 — Write as a self dot product:

$$|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}).$$

Step 2 — Distribute the dot product:

$$= \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}.$$

Step 3 — Use commutativity $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ and $\vec{u}\cdot\vec{u}=|\vec{u}|^2$:

$$= |\vec{a}|^2+2\,\vec{a}\cdot\vec{b}+|\vec{b}|^2.$$

Step 4 — Cross-check with the companion identity: the analogous expansion of $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2+|\vec{b}|^2-2\,\vec{a}\cdot\vec{b}$ differs only in the sign of the middle term; adding the two recovers the parallelogram law $|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2 = 2|\vec{a}|^2+2|\vec{b}|^2$, a useful consistency check.

Why other options are wrong:

- (A) $|\vec{a}|^2+|\vec{b}|^2$ drops the cross term $2\,\vec{a}\cdot\vec{b}$; it holds only when $\vec{a}\perp\vec{b}$.
- (B) $|\vec{a}|^2+|\vec{b}|^2-2\,\vec{a}\cdot\vec{b}$ has the wrong sign; that is $|\vec{a}-\vec{b}|^2$.
- (D) $|\vec{a}|^2-|\vec{b}|^2$ is not a valid expansion of a squared magnitude at all.

Final Answer: $|\vec{a}|^2+|\vec{b}|^2+2\,\vec{a}\cdot\vec{b} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q31

Q32.

Solution

Concept — Cross product gives a perpendicular direction: the cross product $\vec{a} \times \vec{b}$ is, by definition, perpendicular to both $\vec{a}$ and $\vec{b}$. To obtain a *unit* vector in that direction we divide by its magnitude: $\hat{n} = \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$. (The opposite vector $-\hat{n}$ is also valid, but only one of these appears among the options.)

Step 1 — Compute the cross product by the determinant: with $\vec{a} = \hat{i} + \hat{j}$ (components $1, 1, 0$) and $\vec{b} = \hat{j} + \hat{k}$ (components $0, 1, 1$),

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \hat{i}\,(1 \cdot 1 - 0 \cdot 1) - \hat{j}\,(1 \cdot 1 - 0 \cdot 0) + \hat{k}\,(1 \cdot 1 - 1 \cdot 0).$$

Step 2 — Simplify each component:

$$\vec{a} \times \vec{b} = \hat{i}(1) - \hat{j}(1) + \hat{k}(1) = \hat{i} - \hat{j} + \hat{k}.$$

Step 3 — Find the magnitude and normalise:

$$|\hat{i} - \hat{j} + \hat{k}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3},$$

so the required unit vector is $\dfrac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$.

Step 4 — Verify perpendicularity by dot products: $(\hat{i} - \hat{j} + \hat{k}) \cdot \vec{a} = 1 - 1 + 0 = 0$ and $(\hat{i} - \hat{j} + \hat{k}) \cdot \vec{b} = 0 - 1 + 1 = 0$. Both vanish, confirming the vector is perpendicular to $\vec{a}$ and $\vec{b}$.

Why other options are wrong:

- (A) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ has $+\hat{j}$; its dot with $\vec{a}$ is $1 + 1 = 2 \neq 0$, so it is not perpendicular.
- (B) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{k})$ dotted with $\vec{a}$ gives $1 \neq 0$, so it fails perpendicularity.
- (C) $\hat{i} - \hat{j} + \hat{k}$ points the right way but has magnitude $\sqrt{3} \neq 1$, so it is not a unit vector.

Final Answer: $\dfrac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k}) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q32

Q33.

Solution

Concept — Scalar triple product and coplanarity: the scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$ equals the 3×3 determinant formed from the three vectors' components, and geometrically it is the (signed) volume of the parallelepiped they span. It is zero precisely when the three vectors are coplanar, since a flat box has zero volume. A determinant with two identical rows is always zero, so a repeated vector forces the triple product to vanish.

Step 1 — Suppose two vectors are equal: say $\vec{a} = \vec{b}$. Then the determinant

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

has its first two rows identical.

Step 2 — Apply the determinant property: a determinant with two equal rows is 0, hence

$$[\vec{a}\ \vec{b}\ \vec{c}] = 0.$$

Step 3 — Geometric reading: two equal (or even parallel) vectors cannot help span a 3D box; the three vectors lie in a common plane, so the enclosed volume, and therefore the triple product, is zero.

Why other options are wrong:

- (B) three *mutually perpendicular* vectors span the largest box for their lengths, giving a *non-zero* triple product (e.g. $[\hat{i}\ \hat{j}\ \hat{k}] = 1$).
- (C) being unit vectors fixes only their lengths, not coplanarity; $[\hat{i}\ \hat{j}\ \hat{k}] = 1 \neq 0$.
- (D) $\vec{a}\cdot\vec{b} = 1$ constrains one dot product but says nothing about the volume spanned by all three.

Final Answer: zero when two vectors are equal $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q33

Q34.

Solution

Concept — Arithmetic mean of a list: the mean is the sum of all values divided by how many there are. For the first n natural numbers we use the well-known sum $1+2+\cdots+n=\dfrac{n(n+1)}{2}$ (an arithmetic series), then divide by the count n.

Step 1 — Sum the first n naturals:

$$1+2+3+\cdots+n=\frac{n(n+1)}{2}.$$

Step 2 — Divide by the count n:

$$\bar{x}=\frac{1}{n}\cdot\frac{n(n+1)}{2}=\frac{n(n+1)}{2n}.$$

Step 3 — Cancel n:

$$\bar{x}=\frac{n+1}{2}.$$

Step 4 — Sanity check for small n: for $n=4$ the numbers are $1,2,3,4$ with sum 10 and mean $10/4=2.5$; the formula gives $\frac{4+1}{2}=2.5$. It also equals the average of the first and last terms, $\frac{1+n}{2}$, as expected for an arithmetic progression.

Why other options are wrong:

- (A) $\frac{n}{2}$ drops the $+1$ (it would be the mean of $0,1,\ldots,n-1$-type lists, not $1,\ldots,n$).
- (C) $\frac{n(n+1)}{2}$ is the *sum*, not the mean; it was never divided by n.
- (D) $n+1$ forgets the division by 2 (it is twice the mean).

Final Answer: mean $=\dfrac{n+1}{2}\Rightarrow\boxed{\text{B}}$

Answer: (B) Go Back to Q34

Q35.

Solution

Concept — Classical (equally-likely) probability: when every outcome is equally likely, the probability of an event is the ratio of favourable outcomes to the total number of outcomes, $P = \dfrac{\text{favourable}}{\text{total}}$. A standard pack has 52 equally likely cards, so we just count how many are face cards.

Step 1 — Total outcomes: a well-shuffled standard pack has

$$\text{total} = 52 \text{ cards.}$$

Step 2 — Count favourable (face) cards: the face cards are Jack, Queen and King; each of the 4 suits (spades, hearts, diamonds, clubs) contains one of each, so

$$\text{favourable} = 4 \times 3 = 12.$$

Step 3 — Form and reduce the probability:

$$P = \frac{12}{52} = \frac{12 \div 4}{52 \div 4} = \frac{3}{13}.$$

Step 4 — Reasonableness check: $\frac{3}{13} \approx 0.23$, a bit under a quarter, which fits intuition since 12 of 52 cards is roughly 23%.

Why other options are wrong:

- (A) $\frac{1}{13} = \frac{4}{52}$ counts only one rank (say only Kings), missing Jacks and Queens.
- (B) $\frac{1}{4} = \frac{13}{52}$ counts an entire suit, not the face cards.
- (D) $\frac{4}{13} = \frac{16}{52}$ over-counts, treating 16 cards as face cards instead of 12.

Final Answer: $P = \dfrac{3}{13} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q35

Q36.

Solution

Concept — Mean and variance of a binomial distribution: for n independent trials each with success probability p (and failure probability $q = 1-p$), the number of successes X has mean $\mu = np$ and variance $\sigma^2 = npq$. The variance is always smaller than the mean here because the extra factor $q < 1$ scales it down.

Step 1 — List the parameters:

$$n = 10, \qquad p = 0.4, \qquad q = 1 - p = 0.6.$$

Step 2 — Apply the variance formula:

$$\text{Var}(X) = npq = 10 \times 0.4 \times 0.6.$$

Step 3 — Multiply step by step:

$$10 \times 0.4 = 4, \qquad 4 \times 0.6 = 2.4.$$

Step 4 — Cross-check against the mean: the mean is $np = 4$, and the variance $npq = 2.4$ is indeed the mean multiplied by $q = 0.6$ ($4 \times 0.6 = 2.4$), and it is smaller than the mean as expected.

Why other options are wrong:

- (A) $4 = np$ is the *mean*, not the variance (it omits the factor q).
- (B) $6 = nq = 10 \times 0.6$ omits the factor p.
- (C) 1.6 is a miscomputation (e.g. 4×0.4); the correct product uses $q = 0.6$, giving 2.4.

Final Answer: variance $= 2.4 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q36

Q37.

Solution

Concept — Standard angle values and reciprocal identities: at 45° the right triangle is isosceles, so the opposite and adjacent sides are equal and $\tan 45^\circ = 1$. Since $\cot\theta = \dfrac{1}{\tan\theta}$ is the reciprocal of tangent, $\cot 45^\circ = \dfrac{1}{1} = 1$ as well. Both are exact, so the sum is exact.

Step 1 — Evaluate each term:

$$\tan 45^\circ = 1, \qquad \cot 45^\circ = \frac{1}{\tan 45^\circ} = \frac{1}{1} = 1.$$

Step 2 — Add them:

$$\tan 45^\circ + \cot 45^\circ = 1 + 1 = 2.$$

Step 3 — Confirm via sine/cosine: $\tan 45^\circ = \dfrac{\sin 45^\circ}{\cos 45^\circ} = \dfrac{1/\sqrt{2}}{1/\sqrt{2}} = 1$ and $\cot 45^\circ = \dfrac{\cos 45^\circ}{\sin 45^\circ} = 1$, again summing to 2.

Why other options are wrong:

- (B) 1 uses only one of the two terms.
- (C) 0 would need the terms to cancel, but both are $+1$.
- (D) $\frac{1}{2}$ misreads the standard values; neither $\tan 45^\circ$ nor $\cot 45^\circ$ equals $\frac{1}{4}$.

Final Answer: $\tan 45^\circ + \cot 45^\circ = 2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q37

Q38.

Solution

Concept — Cosine difference identity: the compound-angle formula $\cos(A - B) = \cos A\cos B + \sin A\sin B$ expresses the cosine of a difference in terms of the sines and cosines of the separate angles. Here $A = 60^\circ$ and $B = 30^\circ$ are both standard angles, so every term is a known exact value, and the result must agree with the direct evaluation $\cos(60^\circ - 30^\circ) = \cos 30^\circ$.

Step 1 — Substitute the standard values: using $\cos 60^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\sin 30^\circ = \frac{1}{2}$,

$$\cos(60^\circ - 30^\circ) = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ = \frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{1}{2}.$$

Step 2 — Multiply each product:

$$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}.$$

Step 3 — Add:

$$= \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}.$$

Step 4 — Direct check: $60^\circ - 30^\circ = 30^\circ$, and $\cos 30^\circ = \frac{\sqrt{3}}{2}$, exactly matching the identity-based answer.

Why other options are wrong:

- (A) $\frac{1}{2} = \cos 60^\circ$ uses the wrong angle (the original A, not the difference).
- (B) $0 = \cos 90^\circ$ would correspond to $A + B$, not $A - B$.
- (D) $1 = \cos 0^\circ$ would need the angles to be equal, but $60^\circ \neq 30^\circ$.

Final Answer: $\cos(60^\circ - 30^\circ) = \frac{\sqrt{3}}{2} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q38

Q39.

Solution

Concept — Principal value of $\cos^{-1}$: the inverse cosine is made single-valued by restricting its output to the principal range $[0, \pi]$. So $\cos^{-1}(x)$ is the *unique* angle in $[0, \pi]$ whose cosine is x. On this interval cosine decreases from 1 (at 0) to -1 (at π), so a negative input corresponds to an angle in the second quadrant $\left(\frac{\pi}{2}, \pi\right)$.

Step 1 — State what to solve: we need $\theta \in [0, \pi]$ with

$$\cos\theta = -\frac{1}{2}.$$

Step 2 — Use the reference angle: $\cos\frac{\pi}{3} = \frac{1}{2}$, so the related angle with cosine $-\frac{1}{2}$ in the second quadrant is

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}.$$

Step 3 — Verify: $\cos\frac{2\pi}{3} = \cos 120^\circ = -\frac{1}{2}$, and $\frac{2\pi}{3} \in [0, \pi]$, so it is the valid principal value.

Why other options are wrong:

- (A) $\frac{\pi}{3}$ has $\cos = +\frac{1}{2}$, the wrong sign.
- (C) $\frac{\pi}{6}$ has $\cos = \frac{\sqrt{3}}{2}$, not $-\frac{1}{2}$.
- (D) $\frac{5\pi}{6}$ has $\cos = -\frac{\sqrt{3}}{2}$, the wrong magnitude.

Final Answer: $\cos^{-1}\left(-\frac{1}{2}\right) = \dfrac{2\pi}{3} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q39

Q40.

Solution

Concept — Angle of depression and the right triangle: the angle of depression is measured from the horizontal line at the observer's eye down to the object. Because that horizontal line is parallel to the sea, the angle of depression equals the angle of elevation from the boat (alternate interior angles). In the resulting right triangle the cliff is the vertical side (opposite the angle at the boat) and the horizontal distance is the adjacent side, so $\tan(\text{angle}) = \dfrac{\text{height}}{\text{horizontal distance}}$.

Step 1 — Set up the tangent relation: with height 40 m and horizontal distance d,

$$\tan 45^\circ = \frac{40}{d}.$$

Step 2 — Insert the standard value: since $\tan 45^\circ = 1$,

$$1 = \frac{40}{d}.$$

Step 3 — Solve for d:

$$d = 40 \text{ m}.$$

Step 4 — Interpret: a 45° line of sight means the triangle is isosceles, so the horizontal distance equals the vertical height, 40 m, confirming the result.

Why other options are wrong:

- (A) 20 m wrongly halves the height, as if using $\tan\theta = 2$.
- (B) $40\sqrt{3}$ m corresponds to $\tan 30^\circ = \frac{40}{d}$ solved as $d = 40\sqrt{3}$, i.e. the wrong angle.
- (C) $\frac{40}{\sqrt{3}}$ m corresponds to $\tan 60^\circ = \frac{40}{d}$, again the wrong angle.

Final Answer: distance $= 40$ m $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	A
6	C	7	B	8	D	9	A	10	B
11	C	12	D	13	B	14	A	15	C
16	D	17	A	18	B	19	D	20	C
21	A	22	B	23	D	24	C	25	A
26	B	27	C	28	D	29	A	30	B
31	C	32	D	33	A	34	B	35	C
36	D	37	A	38	C	39	B	40	D