

SRMJEEE Mathematics Sample Paper – 6

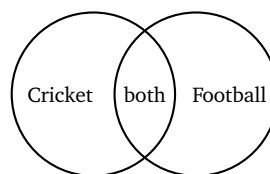
Duration: 47 Minutes

Maximum Marks: 40

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of **SRMJEEE** (SRM Joint Engineering Entrance Examination).
- Each correct answer carries **+1 mark**. There is **no negative marking**; an unattempted or wrong answer scores 0.
- Only **one** option is correct. Choose carefully.
- The actual SRMJEEE is a **computer-based test** conducted in remote-proctored online mode, with all sections sharing a common time window and no per-section limit.
- Personal calculators, mobile phones, log tables and other electronic gadgets are strictly prohibited.

Q1. In a class of 50 students, 30 like cricket and 25 like football, while 15 like both, as shown in the Venn diagram. How many students like at least one of the two games?



- (A) 40
- (B) 55
- (C) 45
- (D) 35

Q2. The number of distinct equivalence relations that can be defined on the set $\{1, 2, 3\}$ is:

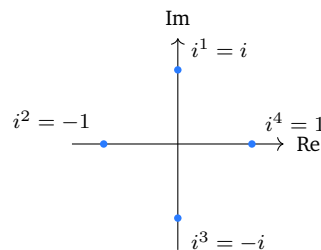


- (A) 3
- (B) 5
- (C) 6
- (D) 8

Q3. The domain of the real function $f(x) = \log(x - 3)$ is:

- (A) $[3, \infty)$
- (B) $(-\infty, 3)$
- (C) $(3, \infty)$
- (D) \mathbb{R}

Q4. Using the cycle of powers of i shown on the Argand plane, the value of i^{10} is:



- (A) 1
- (B) i
- (C) $-i$
- (D) -1

Q5. If ω is a non-real root of $x^2 + x + 1 = 0$, then the value of $(1 + \omega)(1 + \omega^2)$ is:

- (A) 1
- (B) 0
- (C) -1
- (D) ω



- Q6.** The minimum value of the expression $x^2 - 6x + 11$ for real x is:
- (A) 11
 - (B) 2
 - (C) 3
 - (D) -2
- Q7.** If α and β are the roots of $x^2 - 7x + 10 = 0$, then $(\alpha - \beta)^2$ equals:
- (A) 49
 - (B) 40
 - (C) 9
 - (D) 29
- Q8.** For any square matrix A , the matrix $A + A^T$ is always:
- (A) skew-symmetric
 - (B) singular
 - (C) the zero matrix
 - (D) symmetric
- Q9.** The matrix $\begin{pmatrix} x & 2 \\ 8 & 4 \end{pmatrix}$ is singular when x equals:
- (A) 4
 - (B) 2
 - (C) 16
 - (D) 8
- Q10.** The value of the determinant $\begin{vmatrix} 3 & 6 \\ 1 & 5 \end{vmatrix}$, after taking out the common factor 3 from the first row, is:
- (A) 3



- (B) 9
- (C) 27
- (D) 5

Q11. The homogeneous system $\begin{cases} 2x + ky = 0 \\ 3x + 6y = 0 \end{cases}$ has a non-trivial solution when k equals:

- (A) 2
- (B) 9
- (C) 4
- (D) 6

Q12. The number of 3-digit numbers (no digit repeated) that can be formed using the digits 1, 2, 3, 4, 5 is:

- (A) 125
- (B) 10
- (C) 120
- (D) 60

Q13. If ${}^{12}C_r = {}^{12}C_{r+2}$, then the value of r is:

- (A) 5
- (B) 6
- (C) 7
- (D) 2

Q14. The number of distinct 2-digit numbers, formed using the digits 1, 2, 5 without repetition, that are divisible by 5 is:

- (A) 1
- (B) 2



(C) 3

(D) 6

Q15. If $x = 1$ is a root of $x^3 - 2x^2 - 5x + 6 = 0$, then the sum of the other two roots is:

(A) 2

(B) -6

(C) 1

(D) 5

Q16. If the roots of $x^2 - 6x + 5 = 0$ are diminished by 2, the transformed equation (in $y = x - 2$) is:

(A) $y^2 - 6y + 5 = 0$

(B) $y^2 + 2y - 3 = 0$

(C) $y^2 - 6y - 3 = 0$

(D) $y^2 - 2y - 3 = 0$

Q17. The value of $\lim_{x \rightarrow 0} \frac{\tan x}{x}$ is:

(A) 1

(B) 0

(C) ∞

(D) does not exist

Q18. If $y = x^5$, then the value of $\frac{dy}{dx}$ at $x = 2$ is:

(A) 32

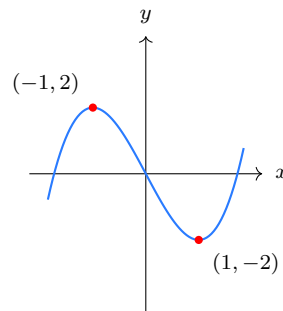
(B) 80

(C) 10

(D) 40



Q19. The function $f(x) = x^3 - 3x$, whose graph is shown, attains a local maximum value equal to:



- (A) -2
- (B) 0
- (C) 2
- (D) 1

Q20. For $f(x) = x^2$ on the interval $[1, 3]$, the value of c given by Lagrange's mean value theorem is:

- (A) 1
- (B) 3
- (C) $\sqrt{2}$
- (D) 2

Q21. The differential equation of the family of straight lines $y = mx$ (with m an arbitrary constant) is:

- (A) $x \frac{dy}{dx} = y$
- (B) $\frac{dy}{dx} = x$
- (C) $y \frac{dy}{dx} = x$
- (D) $\frac{dy}{dx} = 0$

Q22. $\int \sin x \, dx$ equals:

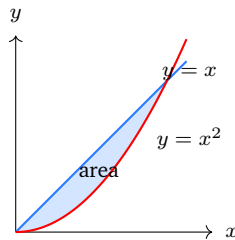


- (A) $\cos x + C$
- (B) $-\cos x + C$
- (C) $\sin x + C$
- (D) $-\sin x + C$

Q23. The value of $\int_0^5 4 dx$ is:

- (A) 4
- (B) 5
- (C) 20
- (D) 9

Q24. The area of the region enclosed between the curves $y = x^2$ and $y = x$ (shaded) is:



- (A) $\frac{1}{2}$ sq. unit
- (B) 1 sq. unit
- (C) $\frac{1}{3}$ sq. unit
- (D) $\frac{1}{6}$ sq. unit

Q25. The value of $\int_0^{\pi/4} \sec^2 x dx$ is:

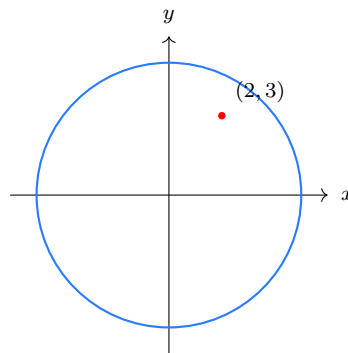
- (A) 1
- (B) 0
- (C) $\frac{\pi}{4}$
- (D) $\sqrt{2}$



Q26. The line through the origin perpendicular to the line $y = 2x + 3$ has slope:

- (A) 2
- (B) $-\frac{1}{2}$
- (C) $\frac{1}{2}$
- (D) -2

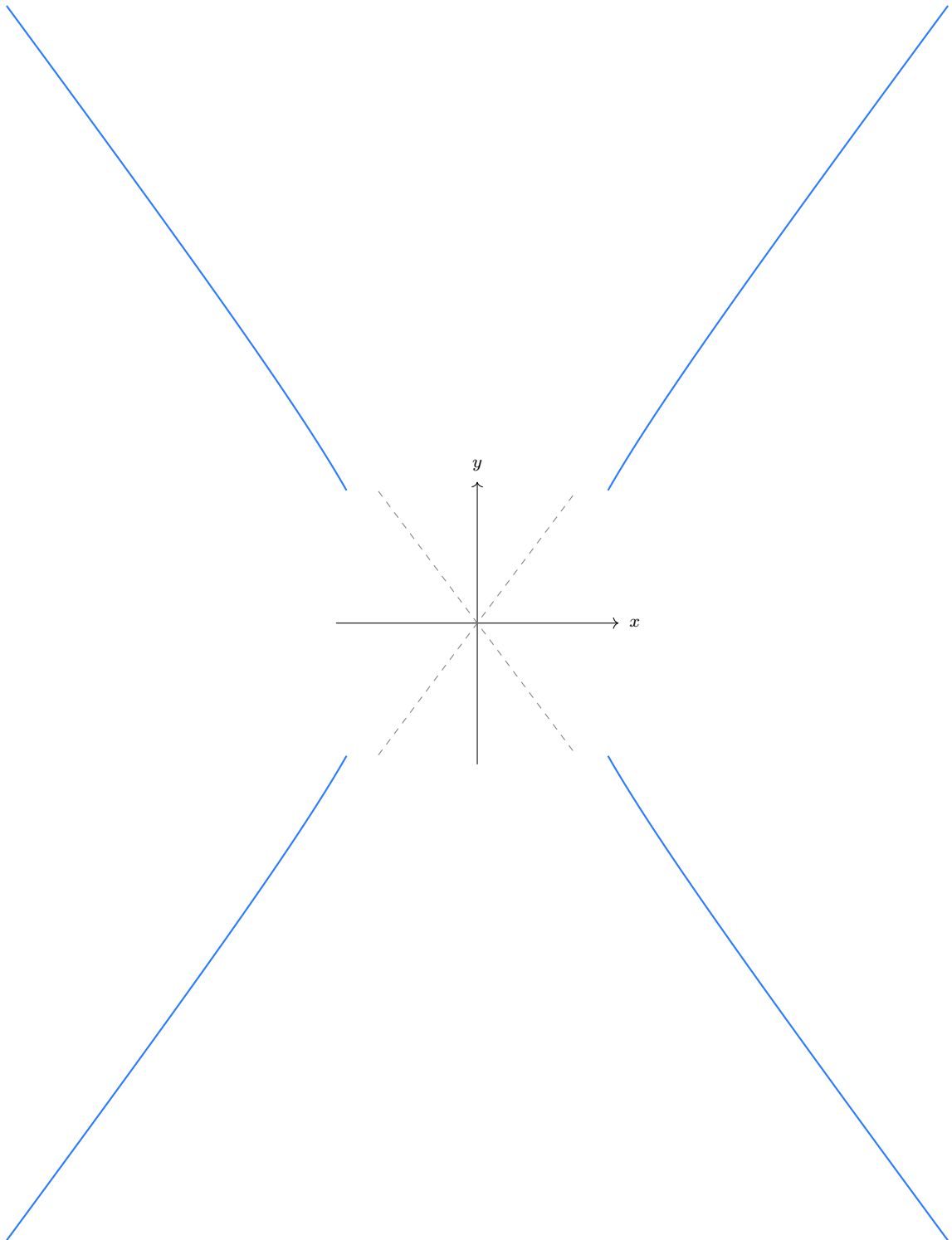
Q27. For the circle $x^2 + y^2 = 25$ shown, the point $(2, 3)$ lies:



- (A) on the circle
- (B) outside the circle
- (C) inside the circle
- (D) at the centre

Q28. The equations of the asymptotes of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$, drawn as dashed lines, are:





- (A) $y = \pm \frac{3}{4}x$
(B) $y = \pm x$
(C) $y = \pm \frac{9}{16}x$
(D) $y = \pm \frac{4}{3}x$

Q29. The line with direction ratios $(2, -1, 3)$ is parallel to the line with direc-



tion ratios:

- (A) $(4, -2, 6)$
- (B) $(2, 1, 3)$
- (C) $(1, 1, 1)$
- (D) $(3, -1, 2)$

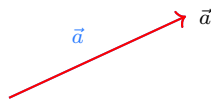
Q30. A line is perpendicular to a plane. The angle between the line and the plane is:

- (A) 0°
- (B) 90°
- (C) 45°
- (D) 60°

Q31. A unit vector in the direction of $\vec{a} = 3\hat{i} + 4\hat{k}$ is:

- (A) $3\hat{i} + 4\hat{k}$
- (B) $\frac{1}{7}(3\hat{i} + 4\hat{k})$
- (C) $\frac{1}{5}(3\hat{i} + 4\hat{k})$
- (D) $\frac{1}{25}(3\hat{i} + 4\hat{k})$

Q32. For any vector \vec{a} (drawn below), the cross product $\vec{a} \times \vec{a}$ equals:



- (A) $|\vec{a}|^2$
- (B) 1
- (C) $2\vec{a}$
- (D) $\vec{0}$



- Q33.** The volume of the tetrahedron with coterminous edge vectors $\vec{a} = 2\hat{i}$, $\vec{b} = 3\hat{j}$ and $\vec{c} = 4\hat{k}$ is:
- (A) 4 cubic units
(B) 24 cubic units
(C) 8 cubic units
(D) 12 cubic units
- Q34.** A group of 4 numbers has mean 5 and another group of 6 numbers has mean 10. The mean of all 10 numbers is:
- (A) 7.5
(B) 8
(C) 15
(D) 7
- Q35.** If $P(A) = \frac{2}{7}$, then $P(\text{not } A)$ equals:
- (A) $\frac{2}{7}$
(B) $\frac{7}{5}$
(C) $\frac{5}{7}$
(D) 1
- Q36.** A fair coin is tossed three times. The probability of getting heads on all three tosses is:
- (A) $\frac{3}{8}$
(B) $\frac{1}{2}$
(C) $\frac{1}{4}$
(D) $\frac{1}{8}$



Q37. The value of $\sec^2 40^\circ - \tan^2 40^\circ$ is:

- (A) 1
- (B) 0
- (C) $\sec 80^\circ$
- (D) 40

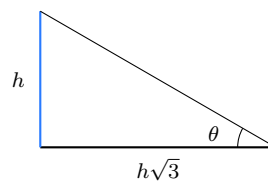
Q38. Using $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$, the value of $\sin 90^\circ$ obtained with $\theta = 30^\circ$ is:

- (A) 0
- (B) 1
- (C) $\frac{1}{2}$
- (D) $\frac{\sqrt{3}}{2}$

Q39. The principal value of $\tan^{-1}(\sqrt{3})$ is:

- (A) $\frac{\pi}{6}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{3}$
- (D) $\frac{\pi}{2}$

Q40. A pole of height h casts a shadow of length $h\sqrt{3}$ on the ground, as shown. The angle of elevation of the sun is:



- (A) 60°
- (B) 45°
- (C) 90°
- (D) 30°



Detailed Solutions

Q1.

Solution

Concept — The inclusion–exclusion principle. When we simply add the number of cricket-lovers to the number of football-lovers, every student who likes *both* games is counted twice—once inside each circle of the Venn diagram. To repair this double counting we subtract the overlap exactly once, which gives the size of the union $n(C \cup F) = n(C) + n(F) - n(C \cap F)$. This single correction is what the principle of inclusion–exclusion guarantees for the union of two sets, and the phrase “at least one” is precisely the union of the two groups.

Step 1 — Read the data from the problem. The class has 50 students. We are told $n(C) = 30$ like cricket, $n(F) = 25$ like football, and $n(C \cap F) = 15$ like both games, the central region of the diagram.

Step 2 — Substitute into the formula.

$$n(C \cup F) = n(C) + n(F) - n(C \cap F) = 30 + 25 - 15.$$

Step 3 — Simplify.

$$n(C \cup F) = 55 - 15 = 40.$$

Step 4 — Cross-check using disjoint regions. Split the union into three non-overlapping parts: only cricket = $30 - 15 = 15$, only football = $25 - 15 = 10$, and both = 15. Their sum is

$$15 + 10 + 15 = 40,$$

which matches Step 3 and confirms the count. (As a bonus, $50 - 40 = 10$ students like neither game.)

Why other options are wrong:

- (B) 55 comes from $30 + 25$ without ever subtracting the common 15, so the overlap is counted twice.
- (C) 45 would arise from subtracting only 10, and (D) 35 from subtracting 20; neither matches the stated overlap of 15.

Final Answer: 40 students like at least one game \Rightarrow A

Answer: (A) [Go Back to Q1](#)



Q2.

Solution

Concept — Equivalence relations correspond exactly to partitions. A relation is an equivalence relation when it is reflexive, symmetric and transitive. The classical theorem is that every equivalence relation on a set splits it into disjoint equivalence classes, and conversely every way of splitting the set into non-empty disjoint blocks (a partition) yields exactly one equivalence relation, in which two elements are related iff they sit in the same block. Hence counting equivalence relations is the same as counting partitions, and the total number of partitions of an n -element set is the Bell number B_n ; here $B_3 = 5$.

Step 1 — List the partitions of $\{1, 2, 3\}$ by block sizes. The all-in-one partition has one block: $\{\{1, 2, 3\}\}$. The partitions with a block of size 2 and a singleton are

$$\{\{1, 2\}, \{3\}\}, \quad \{\{1, 3\}, \{2\}\}, \quad \{\{2, 3\}, \{1\}\},$$

three of them. The all-singletons partition is $\{\{1\}, \{2\}, \{3\}\}$.

Step 2 — Add them up.

$$1 + 3 + 1 = 5 \text{ partitions} \implies 5 \text{ equivalence relations.}$$

Step 3 — Sanity check via the matrix picture. Each partition gives a symmetric reflexive “related” table. For example the partition $\{\{1, 2\}, \{3\}\}$ relates $1 \sim 2$ (and themselves) but isolates 3; transitivity holds automatically because blocks are disjoint. Listing all five tables reproduces exactly the same five relations, with no duplicates.

Why other options are wrong:

- (A) 3 counts only the three two-block partitions and forgets the coarsest and finest ones.
- (C) $6 = 3!$ counts orderings/permutations, not partitions; (D) $8 = 2^3$ would count arbitrary reflexive symmetric relations and ignores transitivity.

Final Answer: 5 equivalence relations \implies B

Answer: (B) [Go Back to Q2](#)



Q3.

Solution

Concept — The domain of a logarithm. The real logarithm $\log u$ is defined only for $u > 0$, because no real power of the base produces zero or a negative number; as $u \rightarrow 0^+$ the logarithm tends to $-\infty$, and for $u \leq 0$ it does not exist at all. So for a composite function $\log(g(x))$ the domain is precisely the set of x for which the inside expression $g(x)$ is strictly positive. Here $g(x) = x - 3$.

Step 1 — Impose the positivity condition. We need the argument to be strictly greater than zero:

$$x - 3 > 0.$$

Step 2 — Solve the inequality. Adding 3 to both sides gives

$$x > 3.$$

Step 3 — Write the domain in interval form. The solution set $\{x : x > 3\}$ is the open interval

$$(3, \infty).$$

Step 4 — Check the boundary. At $x = 3$ the argument is $x - 3 = 0$ and $\log 0$ is undefined, so 3 must be excluded; for any x slightly above 3, say $x = 3.01$, the argument $0.01 > 0$ is valid. This confirms the open (not closed) left end.

Why other options are wrong:

- (A) $[3, \infty)$ wrongly includes $x = 3$, where $\log 0$ is undefined.
- (B) $(-\infty, 3)$ is where the argument is negative, so the log fails everywhere there; (D) \mathbb{R} ignores the requirement $x - 3 > 0$ entirely.

Final Answer: domain = $(3, \infty) \Rightarrow$ C

Answer: (C) [Go Back to Q3](#)



Q4.

Solution

Concept — Powers of i are periodic with period 4. By definition $i = \sqrt{-1}$, so $i^2 = -1$, $i^3 = i^2 \cdot i = -i$, and $i^4 = (i^2)^2 = (-1)^2 = 1$. After $i^4 = 1$ the pattern restarts, because multiplying by $i^4 = 1$ changes nothing. The Argand diagram makes this visible: the four powers sit at the four “compass points” $1, i, -1, -i$ and any higher power lands back on one of them. Therefore i^n depends only on the remainder of n when divided by 4.

Step 1 — Divide the exponent by 4. For $n = 10$,

$$10 = 4 \cdot 2 + 2,$$

so the remainder is 2.

Step 2 — Use periodicity to reduce.

$$i^{10} = i^{4 \cdot 2 + 2} = (i^4)^2 \cdot i^2 = 1^2 \cdot i^2 = i^2.$$

Step 3 — Read off the value.

$$i^2 = -1.$$

Step 4 — Verify by direct stepping. Squaring is quick: $i^{10} = (i^5)^2$, and $i^5 = i^4 \cdot i = i$, so $(i^5)^2 = i^2 = -1$, the same result.

Why other options are wrong:

- (A) 1 requires the exponent to be a multiple of 4 (remainder 0); 10 has remainder 2.
- (B) i corresponds to remainder 1 and (C) $-i$ to remainder 3, neither of which is the remainder of 10.

Final Answer: $i^{10} = -1 \Rightarrow \boxed{\text{D}}$

Answer: (D) [Go Back to Q4](#)



Q5.

Solution

Concept — The non-real cube roots of unity. The complex roots of $x^2 + x + 1 = 0$ are the two non-real cube roots of unity, usually written ω and ω^2 . They satisfy two key identities that come straight from the equation and from $x^3 - 1 = (x - 1)(x^2 + x + 1)$: first, since ω is a root, $\omega^2 + \omega + 1 = 0$, i.e. $1 + \omega + \omega^2 = 0$; second, $\omega^3 = 1$. We use both below.

Step 1 — Rewrite each factor using $1 + \omega + \omega^2 = 0$. From the sum identity,

$$1 + \omega = -\omega^2, \quad 1 + \omega^2 = -\omega.$$

Step 2 — Multiply the two factors.

$$(1 + \omega)(1 + \omega^2) = (-\omega^2)(-\omega) = \omega^2 \cdot \omega = \omega^3.$$

Step 3 — Apply $\omega^3 = 1$.

$$\omega^3 = 1.$$

Step 4 — Cross-check by direct expansion. Expanding first,

$$(1 + \omega)(1 + \omega^2) = 1 + \omega + \omega^2 + \omega^3.$$

The middle three terms $\omega + \omega^2$ combine with the leading 1 to give $1 + \omega + \omega^2 = 0$, leaving $\omega^3 = 1$. Same answer, confirming the result.

Why other options are wrong:

- (B) 0 is the value of the *sum* $1 + \omega + \omega^2$, not of this product.
- (C) -1 would need an odd leftover sign that does not occur; (D) ω fails to use $\omega^3 = 1$ to collapse the product to a number.

Final Answer: $(1 + \omega)(1 + \omega^2) = 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) [Go Back to Q5](#)



Q6.

Solution

Concept — Minimising an upward parabola. The graph of $x^2 + bx + c$ (with positive leading coefficient) is a parabola opening upward, so it has a single lowest point, the vertex. Completing the square rewrites it as $(x + \frac{b}{2})^2 + (c - \frac{b^2}{4})$; because the squared bracket is never negative, the smallest the expression can be is the constant $c - \frac{b^2}{4}$, attained when the bracket is zero. This is faster and surer than guessing values.

Step 1 — Complete the square. Take half of the x -coefficient ($-6/2 = -3$) and square it (9):

$$x^2 - 6x + 11 = (x^2 - 6x + 9) + (11 - 9) = (x - 3)^2 + 2.$$

Step 2 — Read off the minimum. Since $(x - 3)^2 \geq 0$ for all real x , with equality only at $x = 3$,

$$x^2 - 6x + 11 = (x - 3)^2 + 2 \geq 0 + 2 = 2.$$

The least value is 2, occurring at $x = 3$.

Step 3 — Verify with calculus. Differentiate: $f'(x) = 2x - 6 = 0 \Rightarrow x = 3$, and $f''(x) = 2 > 0$ confirms a minimum. Then

$$f(3) = 9 - 18 + 11 = 2,$$

matching Step 2.

Why other options are wrong:

- (A) 11 is the value $f(0)$ at the y -intercept, not the lowest point.
- (C) 3 is the x -coordinate of the vertex, not the minimum value; (D) -2 has the wrong sign—the vertex value is $+2$.

Final Answer: minimum value = 2 \Rightarrow **B**

Answer: (B) [Go Back to Q6](#)



Q7.

Solution

Concept — Vieta's formulas and a symmetric-function identity. For a monic quadratic $x^2 - (\text{sum})x + (\text{product}) = 0$, the sum and product of the roots are read directly off the coefficients: $\alpha + \beta = 7$ and $\alpha\beta = 10$ here. The quantity $(\alpha - \beta)^2$ is symmetric, so it can be expressed through the sum and product without finding the roots individually, using the algebraic identity $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$. This avoids square roots and sign casework.

Step 1 — Identify sum and product. Comparing $x^2 - 7x + 10 = 0$ with $x^2 - (\alpha + \beta)x + \alpha\beta$,

$$\alpha + \beta = 7, \quad \alpha\beta = 10.$$

Step 2 — Apply the identity.

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 7^2 - 4(10).$$

Step 3 — Simplify.

$$(\alpha - \beta)^2 = 49 - 40 = 9.$$

Step 4 — Cross-check by factoring. The equation factors as $(x - 2)(x - 5) = 0$, so $\alpha = 5$, $\beta = 2$. Then $\alpha - \beta = 3$ and $(\alpha - \beta)^2 = 9$, confirming the answer.

Why other options are wrong:

- (A) 49 takes only $(\alpha + \beta)^2$ and forgets to subtract $4\alpha\beta$.
- (B) 40 is just $4\alpha\beta$; (D) 29 comes from using $2\alpha\beta$ instead of $4\alpha\beta$ ($49 - 20$).

Final Answer: $(\alpha - \beta)^2 = 9 \Rightarrow \boxed{\text{C}}$

Answer: (C) [Go Back to Q7](#)

Q8.

Solution

Concept — Testing symmetry through the transpose. A square matrix M is called symmetric when it is unchanged by transposition, i.e. $M^T = M$, and skew-symmetric when $M^T = -M$. Two transpose rules drive the proof: $(P + Q)^T = P^T + Q^T$ (transpose distributes over addition) and $(P^T)^T = P$ (transposing twice returns the original). We apply these to $M = A + A^T$.



Step 1 — Transpose the matrix. Using distributivity,

$$(A + A^T)^T = A^T + (A^T)^T.$$

Step 2 — Simplify the double transpose. Since $(A^T)^T = A$,

$$(A + A^T)^T = A^T + A = A + A^T.$$

Step 3 — Conclude. The matrix equals its own transpose, so by definition $A + A^T$ is symmetric, for every square A .

Step 4 — Numerical check. Take $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$. Then $A^T = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$ and

$$A + A^T = \begin{pmatrix} 2 & 2 \\ 2 & 6 \end{pmatrix},$$

whose off-diagonal entries match ($2 = 2$): symmetric, as claimed.

Why other options are wrong:

- (A) skew-symmetric describes the *difference* $A - A^T$, since $(A - A^T)^T = -(A - A^T)$.
- (B) singular and (C) the zero matrix need not hold; in the example above $A + A^T$ has determinant $12 - 4 = 8 \neq 0$ and is clearly non-zero.

Final Answer: $A + A^T$ is always symmetric \Rightarrow D

Answer: (D) [Go Back to Q8](#)

Q9.

Solution

Concept — Singular matrices have zero determinant. A square matrix is singular (non-invertible) exactly when its determinant vanishes, because the inverse formula $A^{-1} = \frac{1}{\det A} \text{adj}(A)$ breaks down only when $\det A = 0$. For a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant is $ad - bc$. So to find the value of the unknown that makes the matrix singular, set this determinant equal to zero and solve.



Step 1 — Write the determinant. For $\begin{pmatrix} x & 2 \\ 8 & 4 \end{pmatrix}$,

$$\det = (x)(4) - (2)(8) = 4x - 16.$$

Step 2 — Set it to zero.

$$4x - 16 = 0.$$

Step 3 — Solve for x .

$$4x = 16 \implies x = 4.$$

Step 4 — Verify. With $x = 4$ the matrix is $\begin{pmatrix} 4 & 2 \\ 8 & 4 \end{pmatrix}$, whose rows are proportional ($(8, 4) = 2 \cdot (4, 2)$). Proportional rows force the determinant to zero, confirming singularity.

Why other options are wrong:

- (B) 2 gives $\det = 8 - 16 = -8 \neq 0$; (C) 16 gives $\det = 64 - 16 = 48 \neq 0$; (D) 8 gives $\det = 32 - 16 = 16 \neq 0$.

Final Answer: $x = 4 \Rightarrow \boxed{A}$

Answer: (A) [Go Back to Q9](#)

Q10.

Solution

Concept — Pulling a common factor out of a row. A basic determinant property states that if every entry of a single row is multiplied by a scalar k , the determinant is multiplied by k . Read in reverse, a factor common to one row can be taken *outside* the determinant once. The first row here is $(3, 6) = 3 \cdot (1, 2)$, so a factor of 3 comes out, leaving a simpler determinant to evaluate.

Step 1 — Factor 3 out of the first row.

$$\begin{vmatrix} 3 & 6 \\ 1 & 5 \end{vmatrix} = 3 \begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix}.$$



Step 2 — Evaluate the inner 2×2 determinant.

$$\begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix} = (1)(5) - (2)(1) = 5 - 2 = 3.$$

Step 3 — Multiply by the extracted factor.

$$3 \times 3 = 9.$$

Step 4 — Cross-check without factoring. Directly,

$$\begin{vmatrix} 3 & 6 \\ 1 & 5 \end{vmatrix} = (3)(5) - (6)(1) = 15 - 6 = 9,$$

which agrees, confirming the factor was taken out exactly once.

Why other options are wrong:

- (A) 3 keeps only the extracted factor and forgets the inner value 3.
- (C) $27 = 3 \cdot 3 \cdot 3$ wrongly takes the factor out twice; (D) 5 ignores the factor and miscalculates the 2×2 value.

Final Answer: determinant = 9 \Rightarrow **B**

Answer: (B) [Go Back to Q10](#)

Q11.

Solution

Concept — When a homogeneous system has non-trivial solutions. A homogeneous linear system always has the trivial solution $x = y = 0$. It admits a non-trivial (non-zero) solution if and only if the coefficient matrix is singular, that is, its determinant is zero; geometrically the two lines through the origin must coincide rather than meet only at the origin. So we form the coefficient determinant and set it to zero to find the special value of k .

Step 1 — Form the coefficient determinant. The coefficients give

$$\begin{vmatrix} 2 & k \\ 3 & 6 \end{vmatrix} = (2)(6) - (k)(3) = 12 - 3k.$$



Step 2 — Set the determinant to zero.

$$12 - 3k = 0.$$

Step 3 — Solve for k .

$$3k = 12 \implies k = 4.$$

Step 4 — Verify. With $k = 4$ the equations are $2x + 4y = 0$ and $3x + 6y = 0$, both equivalent to $x = -2y$. They describe the same line, so infinitely many non-trivial solutions exist (e.g. $x = -2$, $y = 1$), confirming $k = 4$.

Why other options are wrong:

- (A) 2 gives $\det = 12 - 6 = 6 \neq 0$; (B) 9 gives $\det = 12 - 27 = -15 \neq 0$; (D) 6 gives $\det = 12 - 18 = -6 \neq 0$. A non-zero determinant forces only the trivial solution.

Final Answer: $k = 4 \Rightarrow$ C

Answer: (C) [Go Back to Q11](#)

Q12.

Solution

Concept — Ordered selection without repetition. Forming a number is an *ordered* arrangement, because the position of each digit matters ($123 \neq 321$). When digits may not repeat, filling r places from n distinct digits is a permutation, counted by ${}^n P_r = \frac{n!}{(n-r)!}$. Equivalently, fill the places one at a time: the first place has n choices, the next $n-1$, and so on, with each choice removing one digit from the pool.

Step 1 — Identify n and r . We have $n = 5$ available digits $\{1, 2, 3, 4, 5\}$ and $r = 3$ places (hundreds, tens, units).

Step 2 — Count place by place.

$$\underbrace{5}_{\text{hundreds}} \times \underbrace{4}_{\text{tens}} \times \underbrace{3}_{\text{units}}.$$

Step 3 — Multiply.

$$5 \times 4 \times 3 = 60.$$



Step 4 — Confirm with the permutation formula.

$${}^5P_3 = \frac{5!}{(5-3)!} = \frac{120}{2} = 60,$$

the same value. (No leading-zero issue arises since 0 is not among the digits.)

Why other options are wrong:

- (A) $125 = 5^3$ counts arrangements *with* repetition allowed.
- (C) $120 = 5!$ fills all five places, not three; (B) $10 = {}^5C_3$ counts unordered selections, ignoring digit order.

Final Answer: 60 three-digit numbers \Rightarrow D

Answer: (D) [Go Back to Q12](#)

Q13.

Solution

Concept — The symmetry of binomial coefficients. Choosing r items from n is the same as choosing which $n - r$ to leave out, so ${}^nC_r = {}^nC_{n-r}$. Consequently, if ${}^nC_a = {}^nC_b$, there are only two ways the equality can hold: either the lower indices are equal, $a = b$, or they are “complementary,” $a + b = n$. Here the two lower indices r and $r + 2$ obviously differ, so only the complementary case can apply.

Step 1 — Rule out $a = b$. Setting $a = r$ and $b = r + 2$, the case $a = b$ would require $r = r + 2$, which is impossible. So we must use $a + b = n$.

Step 2 — Apply the complementary condition with $n = 12$.

$$r + (r + 2) = 12.$$

Step 3 — Solve the linear equation.

$$2r + 2 = 12 \implies 2r = 10 \implies r = 5.$$

Step 4 — Verify. With $r = 5$, $r + 2 = 7$ and $5 + 7 = 12$, so ${}^{12}C_5 = {}^{12}C_7$ by symmetry. Indeed both equal 792, confirming the result.

Why other options are wrong:

- (B) 6 gives $6 + 8 = 14 \neq 12$; (C) 7 gives $7 + 9 = 16 \neq 12$; (D) 2 gives $2 + 4 = 6 \neq 12$. None satisfies $r + (r + 2) = 12$.



Final Answer: $r = 5 \Rightarrow$ A

Answer: (A) [Go Back to Q13](#)

Q14.

Solution

Concept — The divisibility-by-5 rule constrains the units digit. An integer is divisible by 5 exactly when its last (units) digit is 0 or 5. In a counting problem this fixes the units place first, and only after that do we count the ways to fill the remaining places from the digits still available. Since the digit 0 is not in our set $\{1, 2, 5\}$, the units digit is forced to be 5, which sharply limits the possibilities.

Step 1 — Fix the units place. For divisibility by 5 the units digit must be 0 or 5; among $\{1, 2, 5\}$ only 5 qualifies, so

$$\text{units digit} = 5.$$

Step 2 — Fill the tens place without repetition. Having used 5, the tens digit may be either remaining digit, 1 or 2:

$$\text{tens digit} \in \{1, 2\} \Rightarrow 2 \text{ choices.}$$

Step 3 — List and count. The numbers are 15 and 25, so there are

$$1 \times 2 = 2 \text{ numbers.}$$

Step 4 — Check divisibility. $15 = 5 \times 3$ and $25 = 5 \times 5$; both are indeed multiples of 5, confirming the count.

Why other options are wrong:

- (A) 1 counts only one of 15, 25 and misses the other tens choice.
- (C) 3 would count an extra number that does not end in 5; (D) 6 is the total of *all* two-digit numbers from the digits, ignoring the divisibility rule.

Final Answer: 2 such numbers \Rightarrow B

Answer: (B) [Go Back to Q14](#)



Q15.

Solution

Concept — Vieta's formula for the sum of roots of a cubic. For a cubic $x^3 + px^2 + qx + r = 0$ with roots α, β, γ , the sum of the roots equals the negative of the coefficient of x^2 : $\alpha + \beta + \gamma = -p$. We do not need to find the individual roots; knowing the total sum and one root immediately gives the sum of the remaining two by subtraction.

Step 1 — Compute the total sum of roots. Here $p = -2$, so

$$\alpha + \beta + \gamma = -(-2) = 2.$$

Step 2 — Use the given root. We are told $x = 1$ is a root; call it $\gamma = 1$. (Check: $1 - 2 - 5 + 6 = 0$, valid.)

Step 3 — Subtract to isolate the other two.

$$\alpha + \beta = (\alpha + \beta + \gamma) - \gamma = 2 - 1 = 1.$$

Step 4 — Cross-check by factoring. Dividing out $(x - 1)$ gives $x^2 - x - 6 = (x - 3)(x + 2)$, so the other roots are 3 and -2 . Their sum is $3 + (-2) = 1$, matching Step 3.

Why other options are wrong:

- (A) 2 is the total of all three roots, not the sum of the other two.
- (B) -6 is the product $\alpha\beta$ of the remaining roots (constant term of $x^2 - x - 6$), not their sum; (D) 5 ignores the given root entirely.

Final Answer: sum of the other two roots = 1 \Rightarrow C

Answer: (C) [Go Back to Q15](#)

Q16.

Solution

Concept — Diminishing the roots of an equation. “Diminishing the roots by h ” means producing a new equation whose roots are each h less than the original roots. If x is an old root, then $y = x - h$ is a new root, so $x = y + h$. Substituting $x = y + h$ into the original polynomial and simplifying produces the transformed equation in y . The problem states $y = x - 2$, i.e. $h = 2$ and $x = y + 2$.



Step 1 — Substitute $x = y + 2$.

$$(y + 2)^2 - 6(y + 2) + 5 = 0.$$

Step 2 — Expand each term.

$$(y^2 + 4y + 4) - (6y + 12) + 5 = 0.$$

Step 3 — Collect like terms.

$$y^2 + (4y - 6y) + (4 - 12 + 5) = y^2 - 2y - 3 = 0.$$

Step 4 — Verify through the roots. The original $x^2 - 6x + 5 = (x - 1)(x - 5)$ has roots 1, 5. Diminished by 2 they become $-1, 3$, and indeed $y^2 - 2y - 3 = (y + 1)(y - 3)$ has roots $-1, 3$. The transformation is correct.

Why other options are wrong:

- (A) $y^2 - 6y + 5 = 0$ leaves the equation unchanged, as if $h = 0$.
- (B) $y^2 + 2y - 3 = 0$ flips the sign of the linear term; (C) $y^2 - 6y - 3 = 0$ keeps the wrong linear coefficient from a botched expansion.

Final Answer: the transformed equation is $y^2 - 2y - 3 = 0 \Rightarrow$ D

Answer: (D) [Go Back to Q16](#)

Q17.

Solution

Concept — A standard trigonometric limit. As $x \rightarrow 0$ both $\tan x$ and x tend to 0, giving the indeterminate form $\frac{0}{0}$, so the value must be teased out rather than read off. The cornerstone is the standard limit $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ (valid with x in radians), which says that for small angles the sine is essentially equal to the angle. Writing $\tan x = \frac{\sin x}{\cos x}$ lets us reuse this fact.

Step 1 — Split the tangent.

$$\frac{\tan x}{x} = \frac{\sin x}{x} \cdot \frac{1}{\cos x}.$$

Step 2 — Take the limit factor by factor. As $x \rightarrow 0$, $\frac{\sin x}{x} \rightarrow 1$ and $\cos x \rightarrow$



$\cos 0 = 1$, so

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \cdot \frac{1}{1} = 1.$$

Step 3 — Confirm with L'Hopital's rule. Differentiating top and bottom of $\frac{\tan x}{x}$ gives $\frac{\sec^2 x}{1}$, and at $x = 0$ this is $\sec^2 0 = 1$, the same value.

Why other options are wrong:

- (B) 0 wrongly assumes the numerator shrinks faster, but $\tan x$ and x vanish at the same rate.
- (C) ∞ and (D) “does not exist” both contradict the finite two-sided value 1 that the function approaches from either side.

Final Answer: the limit equals 1 \Rightarrow A

Answer: (A) [Go Back to Q17](#)

Q18.

Solution

Concept — The power rule of differentiation. For any constant power n , $\frac{d}{dx}(x^n) = nx^{n-1}$: bring the exponent down as a multiplier and reduce the exponent by one. To find the slope at a particular point we first differentiate to get the derivative function, then substitute the given x -value. Differentiating before substituting is essential—plugging in the number first would lose the variable we need to differentiate.

Step 1 — Differentiate. With $n = 5$,

$$y = x^5 \implies \frac{dy}{dx} = 5x^{5-1} = 5x^4.$$

Step 2 — Substitute $x = 2$.

$$\left. \frac{dy}{dx} \right|_{x=2} = 5 \cdot 2^4.$$

Step 3 — Simplify.

$$5 \cdot 16 = 80.$$

Step 4 — Sanity check by magnitude. Near $x = 2$ a small step Δx changes y by about $5 \cdot 2^4 \Delta x = 80 \Delta x$; for instance $2.01^5 \approx 32.80$ versus $2^5 = 32$, a rise of about 0.80 for $\Delta x = 0.01$, consistent with slope 80.

Why other options are wrong:



- (A) $32 = 2^5$ is the value of y itself, not its derivative.
- (C) 10 comes from the wrong product $5 \cdot 2$; (D) $40 = 5 \cdot 2^3$ uses x^3 instead of the correct x^4 .

Final Answer: $\left. \frac{dy}{dx} \right|_{x=2} = 80 \Rightarrow \boxed{\text{B}}$

Answer: (B) [Go Back to Q18](#)

Q19.

Solution

Concept — Locating a local maximum with derivatives. At a smooth local extremum the tangent is horizontal, so the first derivative is zero there: solve $f'(x) = 0$ to get the critical points. The second-derivative test then classifies each: where $f'' < 0$ the curve is concave down, giving a local maximum; where $f'' > 0$ it is concave up, a local minimum. The question asks for the maximum *value*, which is f evaluated at the maximising x .

Step 1 — Find the critical points.

$$f'(x) = 3x^2 - 3 = 0 \implies x^2 = 1 \implies x = \pm 1.$$

Step 2 — Classify with the second derivative.

$$f''(x) = 6x, \quad f''(-1) = -6 < 0 \text{ (max)}, \quad f''(1) = 6 > 0 \text{ (min)}.$$

So the local maximum is at $x = -1$.

Step 3 — Compute the maximum value.

$$f(-1) = (-1)^3 - 3(-1) = -1 + 3 = 2.$$

Step 4 — Match against the graph. The figure marks the turning point $(-1, 2)$ as the crest of the curve and $(1, -2)$ as the trough, exactly the local maximum and minimum we found.

Why other options are wrong:

- (A) -2 is the local *minimum* value $f(1)$, the opposite extremum.
- (B) 0 and (D) 1 are not values of f at either critical point, so they are not extreme values at all.

Final Answer: local maximum value = 2 $\Rightarrow \boxed{\text{C}}$



Answer: (C) [Go Back to Q19](#)

Q20.

Solution

Concept — Lagrange’s mean value theorem. If f is continuous on $[a, b]$ and differentiable on (a, b) , the theorem guarantees at least one interior point c where the instantaneous rate of change equals the average rate over the whole interval: $f'(c) = \frac{f(b) - f(a)}{b - a}$. Geometrically the tangent at $x = c$ is parallel to the chord joining the endpoints. Here $f(x) = x^2$ is a polynomial, hence continuous and differentiable everywhere, so the hypotheses hold on $[1, 3]$.

Step 1 — Compute the average rate (slope of the chord).

$$\frac{f(3) - f(1)}{3 - 1} = \frac{9 - 1}{2} = \frac{8}{2} = 4.$$

Step 2 — Set the derivative equal to this slope. Since $f'(x) = 2x$,

$$f'(c) = 2c = 4.$$

Step 3 — Solve for c .

$$2c = 4 \implies c = 2.$$

Step 4 — Check the conclusion. The value $c = 2$ lies inside the open interval $(1, 3)$, as required, and the midpoint of $[1, 3]$ coincides with it—expected, since for a parabola the mean-value point is always the interval’s midpoint.

Why other options are wrong:

- (A) 1 and (B) 3 are the endpoints, but the theorem demands c strictly inside $(1, 3)$.
- (C) $\sqrt{2} \approx 1.41$ gives $f'(\sqrt{2}) = 2\sqrt{2} \approx 2.83 \neq 4$, so it does not satisfy the equation.

Final Answer: $c = 2 \Rightarrow$ **D**

Answer: (D) [Go Back to Q20](#)



Q21.

Solution

Concept — Forming a differential equation from a family of curves. A family described by an equation with one arbitrary constant satisfies a first-order differential equation obtained by eliminating that constant. The recipe: differentiate the given relation once (introducing $\frac{dy}{dx}$), then use the original relation to remove the parameter. The resulting equation holds for every member of the family regardless of the parameter's value. Here the parameter is the slope m .

Step 1 — Differentiate the given relation. From $y = mx$,

$$\frac{dy}{dx} = m.$$

Step 2 — Express the parameter from the original equation. Provided $x \neq 0$,

$$m = \frac{y}{x}.$$

Step 3 — Eliminate m . Equating the two expressions for m ,

$$\frac{dy}{dx} = \frac{y}{x} \implies x \frac{dy}{dx} = y.$$

Step 4 — Verify a member. Take $m = 3$, so $y = 3x$. Then $\frac{dy}{dx} = 3$ and $x \frac{dy}{dx} = 3x = y$. The relation $x \frac{dy}{dx} = y$ is satisfied, with no m remaining.

Why other options are wrong:

- (B) $\frac{dy}{dx} = x$ has the wrong right-hand side—the slope is constant m , not x .
- (C) $y \frac{dy}{dx} = x$ swaps the roles of x and y ; (D) $\frac{dy}{dx} = 0$ forces $m = 0$, describing only horizontal lines, not the whole family.

Final Answer: $x \frac{dy}{dx} = y \Rightarrow \boxed{\text{A}}$

Answer: (A) [Go Back to Q21](#)



Q22.

Solution

Concept — Integration reverses differentiation. An antiderivative of f is any function whose derivative is f ; the indefinite integral collects all of them by adding an arbitrary constant C . To integrate $\sin x$ we ask which function differentiates to $\sin x$. Recalling $\frac{d}{dx}(\cos x) = -\sin x$, the minus sign tells us we need $-\cos x$, since $\frac{d}{dx}(-\cos x) = +\sin x$.

Step 1 — Identify the antiderivative.

$$\frac{d}{dx}(-\cos x) = \sin x,$$

so $-\cos x$ is an antiderivative of $\sin x$.

Step 2 — Add the constant of integration.

$$\int \sin x \, dx = -\cos x + C.$$

Step 3 — Check by differentiating the answer. Differentiating $-\cos x + C$ gives $-(-\sin x) + 0 = \sin x$, which recovers the integrand exactly, confirming correctness.

Why other options are wrong:

- (A) $\cos x + C$ differentiates to $-\sin x$, the wrong sign.
- (C) $\sin x + C$ and (D) $-\sin x + C$ differentiate to $\cos x$ and $-\cos x$ respectively; both are antiderivatives of (plus or minus) $\cos x$, not of $\sin x$.

Final Answer: $\int \sin x \, dx = -\cos x + C \Rightarrow \boxed{\text{B}}$

Answer: (B) [Go Back to Q22](#)

Q23.

Solution

Concept — Definite integral of a constant as area. The definite integral $\int_a^b f(x) \, dx$ measures the signed area under the graph of f between $x = a$ and $x = b$. When f is a constant k , the region is simply a rectangle of height k and width $(b - a)$, so its area is $k(b - a)$. Formally, an antiderivative of k is kx , and $[kx]_a^b = kb - ka = k(b - a)$.



Step 1 — Identify the constant and limits. Here $k = 4$, lower limit $a = 0$, upper limit $b = 5$.

Step 2 — Apply the formula.

$$\int_0^5 4 \, dx = 4(5 - 0).$$

Step 3 — Simplify.

$$4 \times 5 = 20.$$

Step 4 — Cross-check via the antiderivative.

$$\int_0^5 4 \, dx = [4x]_0^5 = 4(5) - 4(0) = 20 - 0 = 20,$$

matching the rectangle-area reasoning.

Why other options are wrong:

- (A) 4 is just the height and forgets to multiply by the width 5.
- (B) 5 uses only the upper limit and drops the constant 4; (D) 9 comes from adding $4 + 5$ instead of multiplying.

Final Answer: the integral equals 20 \Rightarrow C

Answer: (C) [Go Back to Q23](#)

Q24.

Solution

Concept — Area enclosed between two curves. The region trapped between two curves over $[a, b]$ has area $\int_a^b (y_{\text{upper}} - y_{\text{lower}}) \, dx$, integrating the vertical gap between the higher and lower curve. The limits are the x -coordinates where the curves cross, and one must first decide which curve is on top in that interval, since reversing the subtraction would give a negative (wrong) value.

Step 1 — Find the intersection points. Set the curves equal:

$$x^2 = x \implies x^2 - x = 0 \implies x(x - 1) = 0 \implies x = 0, 1.$$

Step 2 — Decide the upper curve. Test $x = \frac{1}{2}$: the line gives $y = 0.5$ while the parabola gives $y = 0.25$, so on $(0, 1)$ the line $y = x$ is above $y = x^2$. Thus



$$y_{\text{upper}} - y_{\text{lower}} = x - x^2.$$

Step 3 — Integrate the gap.

$$\int_0^1 (x - x^2) dx = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1.$$

Step 4 — Evaluate.

$$\left(\frac{1}{2} - \frac{1}{3} \right) - 0 = \frac{3-2}{6} = \frac{1}{6} \text{ sq. unit.}$$

Why other options are wrong:

- (A) $\frac{1}{2}$ integrates only x (the line) and forgets to subtract the parabola.
- (C) $\frac{1}{3}$ integrates only x^2 ; (B) 1 ignores the subtraction of areas altogether.

Final Answer: area = $\frac{1}{6}$ sq. unit \Rightarrow D

Answer: (D) [Go Back to Q24](#)

Q25.

Solution

Concept — A standard definite integral via the fundamental theorem. The fundamental theorem of calculus evaluates a definite integral by finding an antiderivative F and computing $F(b) - F(a)$. Since $\frac{d}{dx}(\tan x) = \sec^2 x$, the antiderivative of $\sec^2 x$ is $\tan x$. We then substitute the limits 0 and $\frac{\pi}{4}$, using the exact values $\tan \frac{\pi}{4} = 1$ and $\tan 0 = 0$.

Step 1 — Antidifferentiate.

$$\int \sec^2 x dx = \tan x.$$

Step 2 — Apply the limits.

$$\int_0^{\pi/4} \sec^2 x dx = [\tan x]_0^{\pi/4} = \tan \frac{\pi}{4} - \tan 0.$$

Step 3 — Evaluate.

$$1 - 0 = 1.$$



Step 4 — Reasonableness check. On $[0, \frac{\pi}{4}]$ the integrand $\sec^2 x$ runs from 1 up to $\sec^2 \frac{\pi}{4} = 2$ over a width $\frac{\pi}{4} \approx 0.785$, so the area lies between 0.785 and 1.57; the exact answer 1 sits sensibly in that band.

Why other options are wrong:

- (B) 0 would require equal antiderivative values at both ends, but $\tan \frac{\pi}{4} \neq \tan 0$.
- (C) $\frac{\pi}{4}$ confuses the integral with $\int_0^{\pi/4} 1 dx$; (D) $\sqrt{2}$ is the value $\sec \frac{\pi}{4}$, not the integral.

Final Answer: the integral equals 1 \Rightarrow

[Go Back to Q25](#)

Q26.

Solution

Concept — The slope condition for perpendicular lines. Two non-vertical lines are perpendicular exactly when the product of their slopes is -1 , i.e. $m_1 m_2 = -1$. Equivalently, the perpendicular slope is the *negative reciprocal*: flip the fraction and change the sign. The intercept of the given line is irrelevant to its slope, and a line “through the origin” simply means the new line passes through $(0, 0)$, which fixes its intercept but not its slope.

Step 1 — Read off the given slope. Comparing $y = 2x + 3$ with $y = mx + c$,

$$m_1 = 2.$$

Step 2 — Use the perpendicularity condition.

$$m_1 m_2 = -1 \implies m_2 = -\frac{1}{m_1} = -\frac{1}{2}.$$

Step 3 — Verify. Multiplying the slopes back:

$$2 \times \left(-\frac{1}{2}\right) = -1,$$

which confirms the two lines meet at a right angle.

Why other options are wrong:

- (A) 2 is the slope of a line *parallel* to the given one, not perpendicular.



- (C) $\frac{1}{2}$ takes the reciprocal but drops the negative sign; (D) -2 negates the slope without taking the reciprocal.

Final Answer: perpendicular slope = $-\frac{1}{2} \Rightarrow$ **B**

Answer: (B) [Go Back to Q26](#)

Q27.

Solution

Concept — Locating a point relative to a circle. Write the circle as $S \equiv x^2 + y^2 - 25 = 0$. The sign of S evaluated at a point tells us its position: substituting the point's coordinates compares its distance from the centre with the radius. If $S < 0$ the point is closer than the radius (inside); $S = 0$ means it is exactly on the circle; $S > 0$ means farther than the radius (outside). For $x^2 + y^2 = 25$ the centre is $(0, 0)$ and the radius is 5.

Step 1 — Substitute $(2, 3)$ into S .

$$S = 2^2 + 3^2 - 25 = 4 + 9 - 25.$$

Step 2 — Simplify.

$$S = 13 - 25 = -12.$$

Step 3 — Interpret the sign. Since $S = -12 < 0$, the point lies *inside* the circle.

Step 4 — Distance cross-check. The distance from the origin is $\sqrt{2^2 + 3^2} = \sqrt{13} \approx 3.61$, which is less than the radius 5. A point nearer the centre than the radius is interior, confirming the conclusion.

Why other options are wrong:

- (A) “on the circle” would need $S = 0$, i.e. distance exactly 5.
- (B) “outside” would need $S > 0$ (distance > 5); (D) “at the centre” is false because the centre is $(0, 0)$, not $(2, 3)$.

Final Answer: the point lies inside the circle \Rightarrow **C**

Answer: (C) [Go Back to Q27](#)



Q28.

Solution

Concept — Asymptotes of a horizontal hyperbola. For the standard hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the two straight lines the branches approach but never touch are the asymptotes $y = \pm \frac{b}{a}x$. They come from setting the right-hand side to 0 instead of 1: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ factors as $(\frac{x}{a} - \frac{y}{b})(\frac{x}{a} + \frac{y}{b}) = 0$, giving $y = \pm \frac{b}{a}x$. Crucially one uses a and b themselves, the square roots of the denominators, not a^2 and b^2 .

Step 1 — Read off a and b .

$$a^2 = 9 \Rightarrow a = 3, \quad b^2 = 16 \Rightarrow b = 4.$$

Step 2 — Form the slope $\frac{b}{a}$.

$$\frac{b}{a} = \frac{4}{3}.$$

Step 3 — Write the asymptotes.

$$y = \pm \frac{b}{a}x = \pm \frac{4}{3}x,$$

the dashed lines in the figure.

Step 4 — Consistency check. A point far out on the upper-right branch, say $x = 6$, gives $y = 4\sqrt{\frac{36}{9} - 1} = 4\sqrt{3} \approx 6.93$, while the asymptote predicts $\frac{4}{3}(6) = 8$. The branch stays just below the line and the gap narrows as x grows, exactly as an asymptote should.

Why other options are wrong:

- (A) $\pm \frac{3}{4}x$ inverts the ratio to $\frac{a}{b}$ instead of $\frac{b}{a}$.
- (B) $\pm x$ ignores a, b (it would need $a = b$); (C) $\pm \frac{9}{16}x$ wrongly uses $\frac{a^2}{b^2}$ without taking square roots.

Final Answer: $y = \pm \frac{4}{3}x \Rightarrow$ D

Answer: (D) [Go Back to Q28](#)



Q29.

Solution

Concept — Parallelism via proportional direction ratios. A line in space is fixed in direction by its direction ratios (a, b, c) . Two lines are parallel precisely when one set of direction ratios is a non-zero scalar multiple of the other, i.e. the corresponding components are proportional: $\frac{a_2}{a_1} = \frac{b_2}{b_1} = \frac{c_2}{c_1}$. A constant common ratio across all three components signals the same direction.

Step 1 — Test option (A) component-wise. Compare $(4, -2, 6)$ with $(2, -1, 3)$:

$$\frac{4}{2} = 2, \quad \frac{-2}{-1} = 2, \quad \frac{6}{3} = 2.$$

All three ratios equal 2, so $(4, -2, 6) = 2(2, -1, 3)$.

Step 2 — Conclude. A common factor 2 throughout means the directions coincide, so the line with ratios $(4, -2, 6)$ is parallel to the given line.

Why other options are wrong:

- (B) $(2, 1, 3)$: ratios are 1, -1 , 1—not constant (the middle sign differs).
- (C) $(1, 1, 1)$: ratios $\frac{1}{2}$, -1 , $\frac{1}{3}$ disagree; (D) $(3, -1, 2)$: ratios $\frac{3}{2}$, 1 , $\frac{2}{3}$ disagree. None is a scalar multiple of $(2, -1, 3)$.

Final Answer: $(4, -2, 6)$ is parallel to the given line \Rightarrow **A**

Answer: (A) [Go Back to Q29](#)

Q30.

Solution

Concept — The angle between a line and a plane. This angle is defined as the angle between the line and its projection onto the plane, and it equals the complement of the angle ϕ that the line makes with the plane's normal: angle-with-plane = $90^\circ - \phi$. A line perpendicular to a plane points exactly along the normal direction, a fact we use directly here.

Step 1 — Relate the line to the normal. “Perpendicular to the plane” means the line coincides with the normal, so the angle between the line and the normal is

$$\phi = 0^\circ.$$



Step 2 — Take the complement.

$$\text{angle with the plane} = 90^\circ - \phi = 90^\circ - 0^\circ = 90^\circ.$$

Step 3 — Geometric check. A pole standing straight up out of flat ground is perpendicular to the ground; it clearly rises at a right angle (90°) to the surface, matching the result.

Why other options are wrong:

- (A) 0° is the angle the line makes with the *normal*, not with the plane.
- (C) 45° and (D) 60° would require a line that is slanted, not perpendicular, to the plane.

Final Answer: the angle between the line and the plane is $90^\circ \Rightarrow$ **B**

Answer: (B) [Go Back to Q30](#)

Q31.

Solution

Concept — Constructing a unit vector. A unit vector has length 1 but points in a chosen direction. To obtain the unit vector along \vec{a} we divide \vec{a} by its own magnitude: $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$. The magnitude of $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is $\sqrt{a_1^2 + a_2^2 + a_3^2}$. Note that $\vec{a} = 3\hat{i} + 4\hat{k}$ has no \hat{j} component, so its j -term is 0.

Step 1 — Compute the magnitude.

$$|\vec{a}| = \sqrt{3^2 + 0^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

Step 2 — Divide the vector by its magnitude.

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{5} (3\hat{i} + 4\hat{k}).$$

Step 3 — Verify it is a unit vector. The components are $\frac{3}{5}$ and $\frac{4}{5}$, and

$$\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = 1,$$

so $|\hat{a}| = 1$ as required.

Why other options are wrong:



- (A) $3\hat{i} + 4\hat{k}$ is the original vector (length 5), not normalised.
- (B) $\frac{1}{7}$ uses $3 + 4 = 7$ instead of $\sqrt{3^2 + 4^2}$; (D) $\frac{1}{25}$ divides by $|\vec{a}|^2 = 25$ rather than by $|\vec{a}| = 5$.

Final Answer: $\frac{1}{5}(3\hat{i} + 4\hat{k}) \Rightarrow \boxed{\text{C}}$

Answer: (C) [Go Back to Q31](#)

Q32.

Solution

Concept — The cross product of a vector with itself. The magnitude of a cross product is $|\vec{u} \times \vec{v}| = |\vec{u}| |\vec{v}| \sin \theta$, where θ is the angle between the vectors, and its direction is perpendicular to both. The factor $\sin \theta$ measures how non-parallel the vectors are: when they point the same way ($\theta = 0$) the sine vanishes and the product is the zero vector. Geometrically $|\vec{u} \times \vec{v}|$ is the area of the parallelogram they span, which collapses to a line (zero area) when the vectors coincide.

Step 1 — Find the angle for $\vec{a} \times \vec{a}$. A vector is parallel to itself, so the angle between \vec{a} and \vec{a} is

$$\theta = 0^\circ, \quad \sin 0^\circ = 0.$$

Step 2 — Evaluate the magnitude.

$$|\vec{a} \times \vec{a}| = |\vec{a}| |\vec{a}| \sin 0^\circ = |\vec{a}|^2 \cdot 0 = 0.$$

A vector of zero magnitude is the zero vector, so $\vec{a} \times \vec{a} = \vec{0}$.

Step 3 — Confirm by the antisymmetry rule. The cross product satisfies $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$. Putting $\vec{v} = \vec{a}$ gives $\vec{a} \times \vec{a} = -(\vec{a} \times \vec{a})$, so $2(\vec{a} \times \vec{a}) = \vec{0}$, hence $\vec{a} \times \vec{a} = \vec{0}$ —the same conclusion by pure algebra.

Why other options are wrong:

- (A) $|\vec{a}|^2$ is the value of the *dot* product $\vec{a} \cdot \vec{a}$, a scalar, not the cross product.
- (B) 1 and (C) $2\vec{a}$ are non-zero, but the area spanned is zero; only the zero vector fits.

Final Answer: $\vec{a} \times \vec{a} = \vec{0} \Rightarrow \boxed{\text{D}}$

Answer: (D) [Go Back to Q32](#)



Q33.

Solution

Concept — Volume of a tetrahedron from its edge vectors. Three edge vectors $\vec{a}, \vec{b}, \vec{c}$ meeting at a common vertex span a parallelepiped whose volume is the absolute scalar triple product $|[\vec{a} \ \vec{b} \ \vec{c}]| = |\vec{a} \cdot (\vec{b} \times \vec{c})|$. The tetrahedron occupies exactly one-sixth of that box, so $V = \frac{1}{6}|[\vec{a} \ \vec{b} \ \vec{c}]|$. When the three edges are along the coordinate axes (as here), the triple product is just the product of the three lengths.

Step 1 — Compute the scalar triple product. With $\vec{a} = 2\hat{i}$, $\vec{b} = 3\hat{j}$, $\vec{c} = 4\hat{k}$, the triple product is the determinant

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{vmatrix} = 2 \cdot 3 \cdot 4 = 24.$$

Step 2 — Apply the one-sixth factor.

$$V = \frac{1}{6}|24| = \frac{24}{6} = 4 \text{ cubic units.}$$

Step 3 — Check via the right-tetrahedron formula. Because the edges are mutually perpendicular, the tetrahedron is a corner with legs 2, 3, 4, and $V = \frac{1}{6}(\text{leg}_1)(\text{leg}_2)(\text{leg}_3) = \frac{1}{6}(2)(3)(4) = 4$, matching Step 2.

Why other options are wrong:

- (B) 24 is the parallelepiped (box) volume, before dividing by 6.
- (C) 8 uses a factor $\frac{1}{3}$ (the cone/pyramid base-times-height factor misapplied); (D) 12 uses $\frac{1}{2}$.

Final Answer: $V = 4$ cubic units \Rightarrow

Answer: (A) [Go Back to Q33](#)



Q34.

Solution

Concept — The combined (weighted) mean. The mean of a merged data set is its total sum divided by its total count, not the plain average of the two group means. Since each group sum equals (its count) \times (its mean), the combined mean is $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$. The larger group pulls the combined mean toward its own mean, which is why a simple average of 5 and 10 is generally wrong.

Step 1 — Recover each group's total.

$$\text{Group 1 sum} = n_1\bar{x}_1 = 4 \times 5 = 20, \quad \text{Group 2 sum} = n_2\bar{x}_2 = 6 \times 10 = 60.$$

Step 2 — Add sums and counts.

$$\text{Total sum} = 20 + 60 = 80, \quad \text{Total count} = 4 + 6 = 10.$$

Step 3 — Divide.

$$\bar{x} = \frac{80}{10} = 8.$$

Step 4 — Plausibility check. The answer 8 lies between the two means 5 and 10, and leans toward 10 because the 6-element group is larger than the 4-element group—exactly the expected behaviour of a weighted average.

Why other options are wrong:

- (A) $7.5 = \frac{5+10}{2}$ averages the means directly, ignoring the unequal group sizes.
- (C) $15 = 5 + 10$ simply adds the two means; (D) 7 does not result from any correct weighting.

Final Answer: combined mean = 8 \Rightarrow **B**

Answer: (B) [Go Back to Q34](#)



Q35.

Solution

Concept — The complement rule of probability. An event A and its complement “not A ” (written A' or \bar{A}) are mutually exclusive and together cover the whole sample space, so their probabilities add to 1. Rearranging $P(A) + P(\text{not } A) = 1$ gives the complement rule $P(\text{not } A) = 1 - P(A)$. This is often the quickest route to “at least one” or “does not happen” probabilities.

Step 1 — Apply the rule.

$$P(\text{not } A) = 1 - P(A) = 1 - \frac{2}{7}.$$

Step 2 — Use a common denominator.

$$1 - \frac{2}{7} = \frac{7}{7} - \frac{2}{7} = \frac{5}{7}.$$

Step 3 — Check the total. The two probabilities should sum to 1:

$$\frac{2}{7} + \frac{5}{7} = \frac{7}{7} = 1,$$

confirming the computation. Also $0 \leq \frac{5}{7} \leq 1$, a valid probability.

Why other options are wrong:

- (A) $\frac{2}{7}$ is $P(A)$ itself, not its complement.
- (B) $\frac{7}{5}$ exceeds 1, which no probability can do; (D) 1 forgets to subtract $P(A)$.

Final Answer: $P(\text{not } A) = \frac{5}{7} \Rightarrow \boxed{\text{C}}$

Answer: (C) [Go Back to Q35](#)

Q36.

Solution

Concept — Probability of a sequence of independent events. Separate coin tosses are independent: the outcome of one does not affect another. For independent events the probability of all of them happening is the product of their individual probabilities. A fair coin gives heads with probability $\frac{1}{2}$ each toss, so the probability of heads on every one of n tosses is $(\frac{1}{2})^n$.



Step 1 — Multiply the per-toss probabilities. For three tosses,

$$P(\text{HHH}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^3.$$

Step 2 — Simplify.

$$\left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

Step 3 — Verify by counting outcomes. Three tosses have $2^3 = 8$ equally likely outcomes (HHH, HHT, ..., TTT). Exactly one of them, HHH, is all heads, so the probability is $\frac{1}{8}$, matching Step 2.

Why other options are wrong:

- (A) $\frac{3}{8}$ counts the three outcomes with *exactly two* heads, not all three.
- (B) $\frac{1}{2}$ is the chance of heads on a single toss; (C) $\frac{1}{4} = \left(\frac{1}{2}\right)^2$ uses only two tosses.

Final Answer: $P(\text{HHH}) = \frac{1}{8} \Rightarrow \boxed{\text{D}}$

Answer: (D) [Go Back to Q36](#)

Q37.

Solution

Concept — A Pythagorean trigonometric identity. Dividing the fundamental identity $\sin^2 \theta + \cos^2 \theta = 1$ through by $\cos^2 \theta$ produces $\tan^2 \theta + 1 = \sec^2 \theta$, which rearranges to $\sec^2 \theta - \tan^2 \theta = 1$. This holds for *every* angle θ where the functions are defined ($\cos \theta \neq 0$); the particular value 40° is irrelevant, and the result is always the constant 1.

Step 1 — Apply the identity at $\theta = 40^\circ$.

$$\sec^2 40^\circ - \tan^2 40^\circ = 1.$$

Step 2 — Numerical sanity check. Using $\cos 40^\circ \approx 0.766$ and $\tan 40^\circ \approx 0.839$,

$$\sec^2 40^\circ \approx \frac{1}{0.766^2} \approx 1.704, \quad \tan^2 40^\circ \approx 0.704,$$

and $1.704 - 0.704 = 1.000$, confirming the identity numerically.

Why other options are wrong:



- (B) 0 confuses this with $\sin^2 \theta - \cos^2 \theta$ type expressions, which are not identically zero.
- (C) $\sec 80^\circ$ misreads $\sec^2 40^\circ$ as a double-angle; (D) 40 wrongly treats the angle's number as the value.

Final Answer: the value is 1 \Rightarrow

Answer: (A) [Go Back to Q37](#)

Q38.

Solution

Concept — The sine triple-angle formula. The identity $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$ expresses the sine of a tripled angle purely in terms of $\sin \theta$. Choosing $\theta = 30^\circ$ makes $3\theta = 90^\circ$, so the formula should reproduce the well-known value $\sin 90^\circ = 1$; using the exact value $\sin 30^\circ = \frac{1}{2}$ keeps the arithmetic clean.

Step 1 — Substitute $\theta = 30^\circ$ and $\sin 30^\circ = \frac{1}{2}$.

$$\sin 90^\circ = 3 \left(\frac{1}{2} \right) - 4 \left(\frac{1}{2} \right)^3 = \frac{3}{2} - 4 \cdot \frac{1}{8}.$$

Step 2 — Evaluate the cube term.

$$4 \cdot \frac{1}{8} = \frac{1}{2}.$$

Step 3 — Combine.

$$\sin 90^\circ = \frac{3}{2} - \frac{1}{2} = \frac{2}{2} = 1.$$

Step 4 — Cross-check against the unit circle. The known value $\sin 90^\circ = 1$ (the highest point of the unit circle) agrees exactly, confirming both the formula and the substitution.

Why other options are wrong:

- (A) 0 is $\sin 0^\circ$ or $\sin 180^\circ$; (C) $\frac{1}{2}$ is $\sin 30^\circ$; (D) $\frac{\sqrt{3}}{2}$ is $\sin 60^\circ$. None is the sine of 90° .

Final Answer: $\sin 90^\circ = 1 \Rightarrow$

Answer: (B) [Go Back to Q38](#)



Q39.

Solution

Concept — Principal value of the inverse tangent. The function \tan^{-1} is made single-valued by restricting its output to the principal range $(-\frac{\pi}{2}, \frac{\pi}{2})$. So $\tan^{-1}(\sqrt{3})$ is the *unique* angle in that open interval whose tangent equals $\sqrt{3}$. Although infinitely many angles share the same tangent (they differ by multiples of π), only one lies in the principal range.

Step 1 — Recall a standard tangent value. From the special triangles,

$$\tan \frac{\pi}{3} = \sqrt{3}.$$

Step 2 — Confirm it lies in the principal range. The angle $\frac{\pi}{3} = 60^\circ$ satisfies $-\frac{\pi}{2} < \frac{\pi}{3} < \frac{\pi}{2}$, so it is the principal value:

$$\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}.$$

Step 3 — Reject the other coterminal candidate. The angle $\frac{\pi}{3} + \pi = \frac{4\pi}{3}$ also has tangent $\sqrt{3}$, but it lies outside $(-\frac{\pi}{2}, \frac{\pi}{2})$, so it is not the principal value.

Why other options are wrong:

- (A) $\frac{\pi}{6}$ has $\tan = \frac{1}{\sqrt{3}}$, not $\sqrt{3}$.
- (B) $\frac{\pi}{4}$ has $\tan = 1$; (D) $\frac{\pi}{2}$ is excluded because \tan is undefined there and it lies outside the open range.

Final Answer: $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) [Go Back to Q39](#)

Q40.

Solution

Concept — Angle of elevation in a right triangle. The pole, its shadow and the line from the shadow's tip to the pole's top form a right triangle, with the right angle at the foot of the pole. The angle of elevation θ of the sun is measured at the shadow's tip; the pole is the side *opposite* this angle and the shadow is the side *adjacent*, so $\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{height}}{\text{shadow length}}$. The unknown lengths share the factor h , which cancels.



Step 1 — Write the tangent ratio.

$$\tan \theta = \frac{\text{height}}{\text{shadow}} = \frac{h}{h\sqrt{3}}.$$

Step 2 — Cancel h .

$$\tan \theta = \frac{1}{\sqrt{3}}.$$

Step 3 — Solve for θ . The acute angle whose tangent is $\frac{1}{\sqrt{3}}$ is

$$\theta = 30^\circ.$$

Step 4 — Reasonableness check. A long shadow ($h\sqrt{3} \approx 1.73h$, longer than the pole) corresponds to a low sun, so a small elevation like 30° is exactly what we expect; a high sun would cast a short shadow.

Why other options are wrong:

- (A) 60° has $\tan = \sqrt{3}$, which would mean the shadow is shorter than the pole, the reverse of the given figure.
- (B) 45° needs equal height and shadow ($\tan = 1$); (C) 90° would mean the sun directly overhead with zero shadow, impossible here.

Final Answer: angle of elevation = $30^\circ \Rightarrow$ D

Answer: (D) [Go Back to Q40](#)



Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	A
6	B	7	C	8	D	9	A	10	B
11	C	12	D	13	A	14	B	15	C
16	D	17	A	18	B	19	C	20	D
21	A	22	B	23	C	24	D	25	A
26	B	27	C	28	D	29	A	30	B
31	C	32	D	33	A	34	B	35	C
36	D	37	A	38	B	39	C	40	D

