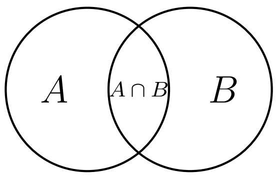
SRMJEEE Mathematics Sample Paper – 7

Duration: 47 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of **SRMJEEE** (SRM Joint Engineering Entrance Examination).
- Each correct answer carries **+1 mark**. There is **no negative marking**; an unattempted or wrong answer scores 0.
- Only **one** option is correct. Choose carefully.
- The actual SRMJEEE is a **computer-based test** conducted in remote-proctored online mode, with all sections sharing a common time window and no per-section limit.
- Personal calculators, mobile phones, log tables and other electronic gadgets are strictly prohibited.

Q1. For two sets, $n(B) = 18$ and $n(A \cap B) = 7$, as shown in the Venn diagram. The number of elements that belong to B only is:



(A) 25

(B) 11

(C) 7

(D) 18

Q2. If $R = \{(1,2),(3,4),(5,6)\}$ is a relation, then its inverse R^{-1} is:

(A) $\{(2,1),(4,3),(6,5)\}$

(B) $\{(1,2),(3,4),(5,6)\}$

(C) $\{(1,1),(3,3),(5,5)\}$

(D) $\{(2,2),(4,4),(6,6)\}$

Q3. The function $f(x) = x^3 + x$ is:

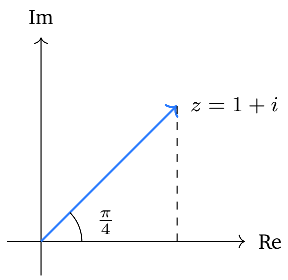
(A) an even function

(B) neither even nor odd

(C) an odd function

(D) both even and odd

Q4. The modulus–argument (polar) form of the complex number $z = 1 + i$, shown on the Argand plane, is:



(A) $2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(B) $\sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(C) $\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

(D) $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Q5. If ω is a non-real cube root of unity, then the value of $\omega + \omega^2$ is:

(A) 0

(B) 1

(C) -1

(D) 2

Q6. The quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ has real and distinct roots if and only if:

(A) $b^2 - 4ac > 0$

(B) $b^2 - 4ac = 0$

(C) $b^2 - 4ac < 0$

(D) $b^2 - 4ac \leq 0$

Q7. If α and β are the roots of $x^2 - 3x + 2 = 0$, then the value of $\alpha^3 + \beta^3$ is:

(A) 27

(B) 9

(C) 7

(D) 19

Q8. The trace of the matrix $A = \begin{pmatrix} 2 & 5 & 1 \\ 0 & 4 & 7 \\ 3 & 6 & 9 \end{pmatrix}$ is:

(A) 6

(B) 9

(C) 37

(D) 15

Q9. A square matrix A is said to be orthogonal if:

(A) $AA^T = I$

(B) $A = A^T$

(C) $A = -A^T$

(D) $A^2 = A$

Q10. The determinant of any skew-symmetric matrix of odd order is:

(A) always 1

(B) always positive

(C) always 0

(D) always -1

Q11. The value of x satisfying the system $2x + y = 7$ and $x - y = 2$ is:

(A) 2

(B) 3

(C) 1

(D) 4

Q12. The number of three-digit numbers that can be formed using the digits $1, 2, 3, 4, 5$ without repetition is:

(A) 125

(B) 15

(C) 10

(D) 60

Q13. If ${}^{12}C_r = {}^{12}C_5$ and $r \neq 5$, then the value of r is:

(A) 7

(B) 5

(C) 12

(D) 17

Q14. In how many ways can 6 persons be seated around a circular table if one particular person always occupies a fixed seat?

(A) 720

(B) 120

(C) 24

(D) 6

Q15. If α, β, γ are the roots of $x^3 - 6x^2 + 11x - 6 = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ equals:

(A) 36

(B) 11

(C) 14

(D) 22

Q16. If the equation $2x^2+5x+2=0$ is a reciprocal equation, then the product of its two roots equals:

(A) -1

(B) $\frac{5}{2}$

(C) 2

(D) 1

Q17. The value of $\lim_{x\to 0}\frac{\log(1+x)}{x}$ is:

(A) 1

(B) 0

(C) e

(D) does not exist

Q18. If $y=\sin x$, then $\frac{d^2y}{dx^2}$ is:

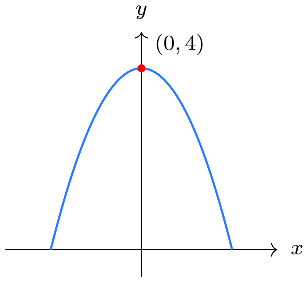
(A) $\cos x$

(B) $\sin x$

(C) $-\sin x$

(D) $-\cos x$

Q19. The greatest value of $f(x)=4-x^2$ on the interval $[-2,2]$, whose graph is shown, is:



(A) 0

(B) 4

(C) 2

(D) 8

Q20. For $f(x) = x^2$ on $[1, 3]$, the value of c in $(1, 3)$ guaranteed by the Lagrange Mean Value Theorem is:

(A) 1

(B) 3

(C) $\sqrt{3}$

(D) 2

Q21. The differential equation $\dfrac{dy}{dx} = \dfrac{x+y}{x}$ is homogeneous and is solved by the substitution:

(A) $y = vx$

(B) $x = vy^2$

(C) $y = v + x$

(D) $x + y = v$

Q22. $\displaystyle\int \frac{1}{1+x^2}\,dx$ equals:

(A) $\log(1 + x^2) + C$

(B) $\dfrac{1}{2}\log(1 + x^2) + C$

(C) $\tan^{-1} x + C$

(D) $\sin^{-1} x + C$

Q23. The value of $\int_0^{\pi/2} \sin^2 x \, dx$ is:

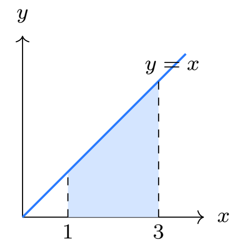
(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) 1

(D) $\frac{\pi}{3}$

Q24. The area of the region bounded by the line $y = x$, the x-axis and the lines $x = 1$ and $x = 3$ (shaded) is:



(A) 2 sq. units

(B) 8 sq. units

(C) 4 sq. units

(D) 9 sq. units

Q25. The value of $\int_0^{1} e^x \, dx$ is:

(A) e

(B) 1

(C) $e + 1$

(D) $e - 1$

Q26. The x- and y-intercepts of the line $\frac{x}{4} + \frac{y}{3} = 1$ are, respectively:

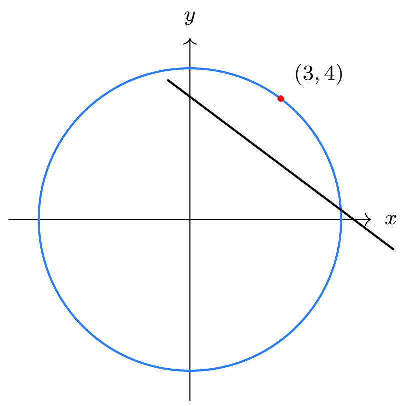
(A) 4 and 3

(B) 3 and 4

(C) $\frac{1}{4}$ and $\frac{1}{3}$

(D) -4 and -3

Q27. The equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$ on it, as shown, is:



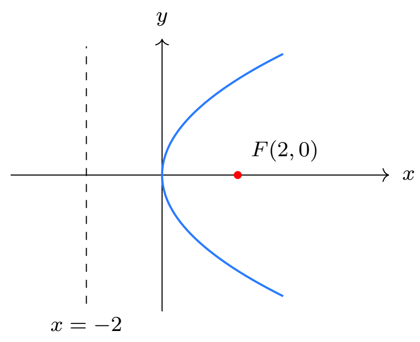
(A) $4x + 3y = 25$

(B) $3x + 4y = 25$

(C) $3x - 4y = 25$

(D) $3x + 4y = 5$

Q28. The equation of the parabola with focus $(2, 0)$ and directrix $x = -2$, as shown, is:



(A) $y^2 = 4x$

(B) $y^2 = 2x$

(C) $x^2 = 8y$

(D) $y^2 = 8x$

Q29. The direction cosines of a line whose direction ratios are $1, 2, 2$ are:

(A) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

(B) $1, 2, 2$

(C) $\frac{1}{9}, \frac{2}{9}, \frac{2}{9}$

(D) $\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}$

Q30. A line with direction ratios a_1, b_1, c_1 is perpendicular to a plane with normal direction ratios a_2, b_2, c_2 if:

(A) $a_1a_2 + b_1b_2 + c_1c_2 = 0$

(B) $a_1a_2 = b_1b_2 = c_1c_2$

(C) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(D) $a_1 + b_1 + c_1 = a_2 + b_2 + c_2$

Q31. The component (scalar projection) of $\vec{a} = 3\hat{\imath} + 4\hat{\jmath}$ along the direction of $\hat{\imath}$ is:

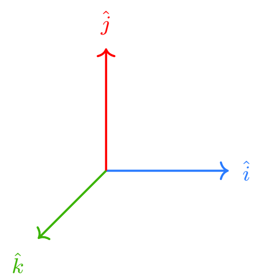
(A) 4

(B) 3

(C) 5

(D) 7

Q32. For the standard right-handed unit vectors shown, the value of $\hat{\imath} \times \hat{\jmath} + \hat{\jmath} \times \hat{k} + \hat{k} \times \hat{\imath}$ is:



(A) $\hat{i} + \hat{j}$

(B) $\vec{0}$

(C) $3\hat{k}$

(D) $\hat{i} + \hat{j} + \hat{k}$

Q33. For any three vectors, the scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}]$ is equal to:

(A) $[\vec{b}\ \vec{c}\ \vec{a}]$

(B) $[\vec{b}\ \vec{a}\ \vec{c}]$

(C) $-[\vec{c}\ \vec{a}\ \vec{b}]$

(D) 0

Q34. The range of the data $7, 12, 3, 20, 8, 15$ is:

(A) 20

(B) 3

(C) 17

(D) 23

Q35. A fair coin is tossed twice. The probability of getting at least one head is:

(A) $\frac{1}{4}$

(B) $\frac{3}{4}$

(C) $\frac{1}{2}$

(D) 1

Q36. For a binomial distribution, the mean is 4 and the variance is 2. The number of trials n is:

(A) 4

(B) 2

(C) 16

(D) 8

Q37. The value of $\dfrac{\sin 70^\circ}{\cos 20^\circ}$ is:

(A) 1

(B) 0

(C) $\dfrac{1}{2}$

(D) $\tan 20^\circ$

Q38. The value of $\cos 15^\circ$ is:

(A) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(B) $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$

(C) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(D) $\dfrac{1}{2}$

Q39. The value of $\sin^{-1}\left(\sin \dfrac{2\pi}{3}\right)$ is:

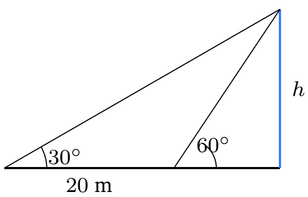
(A) $\dfrac{2\pi}{3}$

(B) $\dfrac{\pi}{3}$

(C) $\dfrac{\pi}{6}$

(D) $-\dfrac{\pi}{3}$

Q40. The angle of elevation of the top of a tower from a point on the ground is 30°; on walking 20 m towards the tower the angle becomes 60°, as shown. The height of the tower is:



(A) $20\sqrt{3}$ m

(B) 20 m

(C) $\frac{20}{\sqrt{3}}$ m

(D) $10\sqrt{3}$ m

Detailed Solutions

Q1.

Solution

Concept — Elements lying in one set only: Any element of B either also lies in A (the overlap region $A \cap B$) or lies in B but outside A (the region "B only"). These two regions are disjoint and together make up all of B, so $n(B) = n(B \text{ only}) + n(A \cap B)$. Rearranging this gives the working formula $n(B \text{ only}) = n(B) - n(A \cap B)$. Geometrically, on the Venn diagram we are simply removing the shaded lens $A \cap B$ from the circle B and counting what remains.

Step 1 — Read the given data: The problem states the total size of set B and the size of the overlap:

$$n(B) = 18, \qquad n(A \cap B) = 7.$$

Step 2 — Apply the subtraction formula: The part of B outside A is everything in B minus what it shares with A:

$$n(B \text{ only}) = n(B) - n(A \cap B) = 18 - 7.$$

Step 3 — Simplify:

$$n(B \text{ only}) = 11.$$

Step 4 — Cross-check: Add the two disjoint pieces of B back together: the "B only" region (11) plus the overlap (7) gives $11 + 7 = 18 = n(B)$, which matches the given total. The decomposition is consistent.

Why other options are wrong:

- (A) 25 comes from adding, $18 + 7 = 25$, instead of subtracting; addition would double-count the overlap and exceed $n(B)$ itself, which is impossible.
- (C) 7 is the size of the overlap $A \cap B$, not the part of B lying outside A.
- (D) 18 is the size of all of B; it forgets to remove the 7 elements shared with A.

Final Answer: $n(B \text{ only}) = 11 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q1

Q2.

Solution

Concept — Inverse of a relation: A relation R from set X to set Y is just a set of ordered pairs (a, b). Its inverse R^{-1} is the relation from Y to X obtained by reversing the direction of every pair, so that $(a, b) \in R$ if and only if $(b, a) \in R^{-1}$. Intuitively, if R records "a is related to b," then R^{-1} records "b is related back to a." The number of pairs never changes; only the order of the two coordinates in each pair is swapped.

Step 1 — List the original pairs: The given relation is

$$R = \{(1, 2),\ (3, 4),\ (5, 6)\}.$$

Step 2 — Reverse every ordered pair: Swap the first and second coordinate of each pair:

$$(1, 2) \to (2, 1), \qquad (3, 4) \to (4, 3), \qquad (5, 6) \to (6, 5).$$

Step 3 — Collect the inverse:

$$R^{-1} = \{(2, 1),\ (4, 3),\ (6, 5)\}.$$

Step 4 — Verify by inverting twice: Reversing each pair of R^{-1} should return the original R. Indeed $(2, 1) \to (1, 2)$, $(4, 3) \to (3, 4)$, $(6, 5) \to (5, 6)$ recovers R, confirming $(R^{-1})^{-1} = R$ as expected.

Why other options are wrong:

- (B) $\{(1, 2), (3, 4), (5, 6)\}$ is simply R unchanged; inversion was never carried out.
- (C) $\{(1, 1), (3, 3), (5, 5)\}$ keeps only the first coordinates and pairs each with itself, which is not the reversal rule.
- (D) $\{(2, 2), (4, 4), (6, 6)\}$ does the same mistake with the second coordinates; diagonal pairs are not how inversion works.

Final Answer: $R^{-1} = \{(2, 1), (4, 3), (6, 5)\} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q2

Q3.

Solution

Concept — Even and odd functions: A function is classified by how it behaves when x is replaced by $-x$. It is *even* if $f(-x) = f(x)$ for all x (its graph is symmetric about the y-axis) and *odd* if $f(-x) = -f(x)$ for all x (its graph has point symmetry about the origin). A useful shortcut is that a sum of odd powers of x is always odd, while a sum of even powers is even. Here $f(x) = x^3 + x$ contains only odd powers (x^3 and x^1), which already suggests the function is odd; we verify this directly.

Step 1 — Replace x by $-x$:

$$f(-x) = (-x)^3 + (-x) = -x^3 - x.$$

Step 2 — Factor out the negative sign:

$$f(-x) = -(x^3 + x) = -f(x).$$

Step 3 — Conclude: Since $f(-x) = -f(x)$ holds for every x, the function satisfies the defining condition of an odd function.

Step 4 — Numerical check: Take $x = 2$: $f(2) = 2^3 + 2 = 10$ and $f(-2) = (-2)^3 + (-2) = -10 = -f(2)$. The values are negatives of each other, exactly as an odd function requires.

Why other options are wrong:

- (A) even would require $f(-x) = f(x)$, but we found $f(-x) = -10 \neq 10 = f(x)$ at $x = 2$.
- (B) "neither" is false because the odd condition $f(-x) = -f(x)$ is satisfied for all x.
- (D) "both even and odd" forces $f(x) = -f(x)$, i.e. $f \equiv 0$; our f is not the zero function.

Final Answer: f is an odd function $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q3

Q4.

Solution

Concept — Polar (modulus–argument) form: Any complex number $z = a + ib$ can be written as $z = r(\cos\theta + i\sin\theta)$, where $r = |z| = \sqrt{a^2 + b^2}$ is the distance from the origin and $\theta = \arg z$ is the angle the position vector makes with the positive real axis. The conversion works because, by definition of the polar coordinates, $a = r\cos\theta$ and $b = r\sin\theta$. The quadrant of z fixes which value of θ to take; for $1 + i$ both parts are positive, placing z in the first quadrant.

Step 1 — Identify the real and imaginary parts: For $z = 1 + i$ we have $a = 1$ and $b = 1$.

Step 2 — Compute the modulus:

$$r = \sqrt{a^2 + b^2} = \sqrt{1^2 + 1^2} = \sqrt{2}.$$

Step 3 — Compute the argument: Since z lies in the first quadrant,

$$\theta = \tan^{-1}\frac{b}{a} = \tan^{-1}\frac{1}{1} = \frac{\pi}{4},$$

matching the 45° angle marked on the Argand diagram.

Step 4 — Assemble the polar form:

$$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right).$$

Step 5 — Convert back to verify: $\sqrt{2}\cos\frac{\pi}{4} = \sqrt{2}\cdot\frac{1}{\sqrt{2}} = 1$ and $\sqrt{2}\sin\frac{\pi}{4} = \sqrt{2}\cdot\frac{1}{\sqrt{2}} = 1$, giving $1 + i$, the original number.

Why other options are wrong:

- (A) uses modulus $r = 2$ instead of $\sqrt{2}$; that would describe a point twice as far from the origin.
- (B) uses the angle $\frac{\pi}{3}$ (60°); but $\tan\frac{\pi}{3} = \sqrt{3} \neq 1$, so it is the wrong argument.
- (C) drops the modulus $\sqrt{2}$ entirely, describing a point on the unit circle rather than $1 + i$.

Final Answer: $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q4**

Q5.

Solution

Concept — Cube roots of unity: The equation $x^3 = 1$ has three roots: $1,\ \omega,\ \omega^2$, where ω is non-real. Because $x^3 - 1 = (x-1)(x^2+x+1)$, the two non-real roots satisfy $x^2+x+1=0$. Summing all three roots of $x^3-1=0$ (whose x^2-coefficient is 0) gives $1+\omega+\omega^2=0$ by Vieta's formula. This single identity is the key relation among the cube roots of unity, and we also have $\omega^3 = 1$.

Step 1 — Start from the fundamental identity:

$$1+\omega+\omega^2=0.$$

Step 2 — Isolate the required sum: Subtract 1 from both sides:

$$\omega+\omega^2=-1.$$

Step 3 — Alternative check via the quadratic: Since ω is a root of $x^2+x+1=0$, the sum of *its* two roots ω and ω^2 equals $-\dfrac{(\text{coeff. of } x)}{(\text{coeff. of } x^2)} = -\dfrac{1}{1} = -1$, the same answer.

Why other options are wrong:

- (A) 0 is the sum of *all three* roots $1+\omega+\omega^2$, not just $\omega+\omega^2$.
- (B) 1 would force $1+\omega+\omega^2=2$, contradicting the identity.
- (D) 2 similarly contradicts $1+\omega+\omega^2=0$, which requires the pair to sum to -1.

Final Answer: $\omega+\omega^2=-1 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q5

Q6.

Solution

Concept — Discriminant and nature of roots: The roots of $ax^2+bx+c=0$ $(a \neq 0)$ are given by the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$. Everything about the nature of the roots is controlled by the quantity under the square root, the discriminant $D = b^2-4ac$. If $D > 0$ the $\pm\sqrt{D}$ contributes two different real numbers, giving real and distinct roots. If $D = 0$ the square root vanishes and the two roots coincide. If $D < 0$ the square root is imaginary, giving a conjugate pair of complex roots.

Step 1 — Look at the formula: The two roots differ only through the term $\pm\sqrt{b^2 - 4ac}$.

Step 2 — Require two different real values: For the roots to be real the radicand must be non-negative, and for them to be *distinct* the radicand must be strictly positive, so that $+\sqrt{D}$ and $-\sqrt{D}$ are genuinely different:

$$b^2 - 4ac > 0.$$

Step 3 — Confirm with an example: For $x^2 - 3x + 2 = 0$, $D = (-3)^2 - 4(1)(2) = 9 - 8 = 1 > 0$, and indeed the roots 1 and 2 are real and distinct, illustrating the rule.

Why other options are wrong:

- (B) $D = 0$ makes $\pm\sqrt{D} = 0$, so the two roots are equal (a repeated root), not distinct.
- (C) $D < 0$ makes $\sqrt{D}$ imaginary, giving non-real complex-conjugate roots.
- (D) $D \leq 0$ combines the equal-roots and complex-roots cases, so it never yields two distinct real roots.

Final Answer: $b^2 - 4ac > 0 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q6

Q7.

Solution

Concept — Sum of cubes via symmetric functions: For a quadratic $x^2 - (\text{sum})x + (\text{product}) = 0$, Vieta's formulas give $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ directly from the coefficients, without solving for the roots. The sum of cubes is then handled by the algebraic identity $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$, which expresses a symmetric expression of the roots purely in terms of their sum and product. This avoids ever finding α and β individually.

Step 1 — Read off the elementary symmetric sums: For $x^2 - 3x + 2 = 0$ (so $a = 1,\ b = -3,\ c = 2$),

$$\alpha + \beta = -\frac{b}{a} = 3, \qquad \alpha\beta = \frac{c}{a} = 2.$$

Step 2 — Apply the identity:

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta).$$

Step 3 — Substitute and simplify:

$$\alpha^3 + \beta^3 = 3^3 - 3(2)(3) = 27 - 18 = 9.$$

Step 4 — Verify directly: The equation factors as $(x-1)(x-2) = 0$, so $\alpha = 1,\ \beta = 2$. Then $\alpha^3 + \beta^3 = 1 + 8 = 9$, matching the identity-based result exactly.

Why other options are wrong:

- (A) 27 keeps only $(\alpha + \beta)^3 = 27$ and forgets to subtract the $3\alpha\beta(\alpha + \beta) = 18$ term.
- (C) 7 would come from $\alpha^2 + \beta^2 = 9 - 2 \cdot 2 = 5$-type slips or mis-cubing; the direct value $1 + 8 = 9$ rules it out.
- (D) 19 corresponds to no correct combination of $\alpha + \beta$ and $\alpha\beta$; it is an arithmetic error.

Final Answer: $\alpha^3 + \beta^3 = 9 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q7

Q8.

Solution

Concept — Trace of a square matrix: The trace of an $n \times n$ matrix $A = [a_{ij}]$ is the sum of its main-diagonal entries, $\text{tr}(A) = a_{11} + a_{22} + \cdots + a_{nn}$. The main diagonal consists of the entries whose row index equals their column index (top-left to bottom-right). Only these entries are added; every off-diagonal entry is ignored. The trace is an important invariant equal to the sum of the eigenvalues of the matrix.

Step 1 — Locate the main-diagonal entries: In

$$A = \begin{pmatrix} 2 & 5 & 1 \\ 0 & 4 & 7 \\ 3 & 6 & 9 \end{pmatrix}$$

the diagonal positions a_{11}, a_{22}, a_{33} hold the values $2,\ 4,\ 9$.

Step 2 — Add the diagonal entries:

$$\text{tr}(A) = 2 + 4 + 9.$$

Step 3 — Simplify:

$$\text{tr}(A) = 15.$$

Step 4 — Note what is excluded: The off-diagonal entries $5, 1, 0, 7, 3, 6$ play no role; including them would (wrongly) give the sum of all nine entries.

Why other options are wrong:

- (A) 6 adds only $2 + 4$ and omits the corner entry 9.
- (B) 9 keeps only the single entry $a_{33} = 9$.
- (C) 37 sums all nine entries $2 + 5 + 1 + 0 + 4 + 7 + 3 + 6 + 9 = 37$, not just the diagonal.

Final Answer: trace $= 15 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — Orthogonal matrix: A real square matrix A is called orthogonal when its transpose acts as its inverse, that is $AA^T = A^T A = I$. Equivalently, the rows of A form a set of mutually perpendicular unit vectors, and so do the columns. Such matrices represent length-preserving transformations (rotations and reflections), and their determinant is always ± 1. The single matrix equation $AA^T = I$ captures all of this.

Step 1 — Recall the definition of the inverse: A^{-1} is the unique matrix with $AA^{-1} = I$. Orthogonality asserts that this inverse is precisely the transpose, $A^{-1} = A^T$.

Step 2 — Translate into a testable condition: Substituting $A^{-1} = A^T$ into $AA^{-1} = I$ gives the defining relation

$$AA^T = I.$$

Step 3 — Interpret geometrically: The (i, j) entry of AA^T is the dot product of row i with row j. Setting $AA^T = I$ forces each row to have unit length (diagonal

$= 1$) and distinct rows to be perpendicular (off-diagonal $= 0$), the hallmark of an orthonormal system.

Why other options are wrong:

- (B) $A = A^T$ is the definition of a *symmetric* matrix, unrelated to orthogonality.
- (C) $A = -A^T$ is the definition of a *skew-symmetric* matrix.
- (D) $A^2 = A$ defines an *idempotent* matrix (e.g. a projection), again a different class.

Final Answer: $AA^T = I \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Determinant of a skew-symmetric matrix: A matrix is skew-symmetric if $A^T = -A$. Two determinant facts drive the result: first, transposing leaves the determinant unchanged, $\det(A^T) = \det(A)$; second, scaling an $n \times n$ matrix by a constant k multiplies the determinant by k^n, so $\det(-A) = (-1)^n \det(A)$. Combining these for an odd-order skew-symmetric matrix forces the determinant to be zero.

Step 1 — Take the determinant of both sides of $A^T = -A$:

$$\det(A^T) = \det(-A).$$

Step 2 — Apply the two rules: Using $\det(A^T) = \det(A)$ on the left and $\det(-A) = (-1)^n \det(A)$ on the right:

$$\det(A) = (-1)^n \det(A).$$

Step 3 — Use that n is odd: For odd order, $(-1)^n = -1$, so

$$\det(A) = -\det(A).$$

Step 4 — Solve: Adding $\det(A)$ to both sides gives $2\det(A) = 0$, hence

$$\det(A) = 0.$$

Step 5 — Concrete check: The 3×3 skew-symmetric matrix $\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$ has determinant 0 for all a, b, c, confirming the general result.

Why other options are wrong:

- (A) "always 1" and (D) "always -1" are non-zero, contradicting $\det A = -\det A$, which only 0 satisfies.
- (B) "always positive" fails for the same reason; a positive determinant cannot equal its own negative.

Final Answer: the determinant is $0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q10

Q11.

Solution

Concept — Solving a 2×2 linear system by elimination: A pair of simultaneous linear equations can be solved by combining them so that one variable cancels. When the coefficients of a variable are equal and opposite in the two equations, adding the equations eliminates that variable, leaving a single equation in the other. Here the y-coefficients are $+1$ and -1, which are perfectly set up to cancel on addition.

Step 1 — Write the system:

$$2x + y = 7 \qquad \text{and} \qquad x - y = 2.$$

Step 2 — Add the two equations to eliminate y:

$$(2x + y) + (x - y) = 7 + 2 \implies 3x = 9.$$

Step 3 — Solve for x:

$$x = \frac{9}{3} = 3.$$

Step 4 — Back-substitute and verify: From $x - y = 2$ with $x = 3$ we get $y = 1$. Checking the first equation, $2(3) + 1 = 7$ ✓, and the second, $3 - 1 = 2$ ✓; both hold, so $x = 3$ is correct.

Why other options are wrong:

- (C) 1 is the value of y, not x; it answers the wrong variable.
- (A) 2 gives $2(2)+y=7 \Rightarrow y=3$, but then $x-y=2-3=-1 \neq 2$, so it fails the second equation.
- (D) 4 gives $2(4)+y=7 \Rightarrow y=-1$, but then $x-y=4-(-1)=5 \neq 2$, again inconsistent.

Final Answer: $x=3 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q11

Q12.

Solution

Concept — Fundamental counting principle without repetition: To build a three-digit number we must fill three ordered positions (hundreds, tens, units). By the multiplication principle, the total count is the product of the number of independent choices for each position. "Without repetition" means once a digit is used it cannot be reused, so each filled position reduces the pool available to the next. This is exactly the permutation ${}^5P_3 = \frac{5!}{(5-3)!}$.

Step 1 — Hundreds place: Any of the five digits $1,2,3,4,5$ may go here, giving 5 choices.

Step 2 — Tens place: One digit is now used, leaving 4 choices.

Step 3 — Units place: Two digits are used, leaving 3 choices.

Step 4 — Multiply:

$$5 \times 4 \times 3 = 60.$$

Step 5 — Cross-check with the permutation formula:

$${}^5P_3 = \frac{5!}{2!} = \frac{120}{2} = 60,$$

the same total.

Why other options are wrong:

- (A) $125 = 5^3$ allows repetition (each place independently 5 ways), which the problem forbids.
- (B) 15 comes from adding $5+4+3+3$-type slips or 5×3; it ignores the product rule.
- (C) 10 equals ${}^5C_3$, the number of unordered selections, but here order matters (different arrangements are different numbers).

Final Answer: 60 numbers $\Rightarrow \boxed{D}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Symmetry of combinations: The identity ${}^nC_r = {}^nC_{n-r}$ holds because choosing r objects to include is the same as choosing the $n-r$ objects to leave out; both describe the same selection. Consequently the equation ${}^nC_x = {}^nC_y$ forces either $x = y$ or $x + y = n$. Applying this to ${}^{12}C_r = {}^{12}C_5$ gives two possible values of r, one of which is ruled out by the condition $r \neq 5$.

Step 1 — Write both possibilities: From ${}^{12}C_r = {}^{12}C_5$ we get

$$r = 5 \qquad \text{or} \qquad r = 12 - 5.$$

Step 2 — Compute the second root:

$$r = 12 - 5 = 7.$$

Step 3 — Apply the restriction: Since the problem requires $r \neq 5$, the admissible value is $r = 7$.

Step 4 — Verify: By symmetry ${}^{12}C_7 = {}^{12}C_{12-7} = {}^{12}C_5$, so $r = 7$ genuinely satisfies the original equation.

Why other options are wrong:

- (B) 5 does satisfy the equation but is explicitly excluded by $r \neq 5$.
- (C) 12 gives ${}^{12}C_{12} = 1$, while ${}^{12}C_5 = 792$, so it fails.
- (D) 17 is not even a valid index since r cannot exceed $n = 12$.

Final Answer: $r = 7 \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept — Circular arrangements with a fixed seat: In a circular arrangement, rotations of the same configuration are usually counted as identical, which is why n people around a round table give $(n-1)!$ arrangements rather than $n!$. Fixing one particular person in a definite seat is the standard way to remove this rotational duplication: it acts as a reference point. Once that person's place is locked, the remaining $n-1$ people simply fill the remaining $n-1$ distinct seats in $(n-1)!$ linear-style ways, and no further division is needed.

Step 1 — Fix the reference person: Seat the one particular person in the chosen fixed seat; this contributes a factor of 1 (there is only one way to place a fixed person).

Step 2 — Arrange the rest: The remaining $6-1=5$ people occupy the other 5 seats, now all distinguishable relative to the fixed person:

$$5! = 5 \times 4 \times 3 \times 2 \times 1.$$

Step 3 — Evaluate:

$$5! = 120.$$

Step 4 — Consistency check: This matches the general circular-permutation count $(n-1)! = (6-1)! = 5! = 120$, confirming the fixed-seat reasoning.

Why other options are wrong:

- (A) $720 = 6!$ treats all six seats as distinct positions in a line, ignoring the fixed reference (over-counts by a factor of 6).
- (C) $24 = 4!$ arranges only 4 people, removing one person too many.
- (D) 6 wrongly uses $3!$ or simply the number of people, with no permutation logic.

Final Answer: 120 ways $\Rightarrow$ $\boxed{\text{B}}$

Answer: (B) Go Back to Q14

Q15.

Solution

Concept — Sum of squares of cubic roots: For a cubic $x^3 - px^2 + qx - r = 0$ with roots α, β, γ, Vieta's formulas give the elementary symmetric sums $\alpha + \beta + \gamma = p$, $\alpha\beta + \beta\gamma + \gamma\alpha = q$, $\alpha\beta\gamma = r$. The sum of squares is obtained from the algebraic identity $(\alpha+\beta+\gamma)^2 = \alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$, rearranged as $\alpha^2+\beta^2+\gamma^2 = (\sum\alpha)^2 - 2\sum\alpha\beta$. This lets us answer without finding the roots themselves.

Step 1 — Read the symmetric sums: For $x^3 - 6x^2 + 11x - 6 = 0$,

$$\alpha + \beta + \gamma = 6, \qquad \alpha\beta + \beta\gamma + \gamma\alpha = 11.$$

Step 2 — Apply the identity:

$$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha).$$

Step 3 — Substitute and simplify:

$$\alpha^2 + \beta^2 + \gamma^2 = 6^2 - 2(11) = 36 - 22 = 14.$$

Step 4 — Verify with the actual roots: The cubic factors as $(x-1)(x-2)(x-3) = 0$, so the roots are $1, 2, 3$. Then $1^2 + 2^2 + 3^2 = 1 + 4 + 9 = 14$, matching the identity result exactly.

Why other options are wrong:

- (A) 36 keeps only $(\sum\alpha)^2$ and forgets to subtract $2\sum\alpha\beta = 22$.
- (B) 11 is the value of $\sum\alpha\beta$ itself, not the sum of squares.
- (D) 22 is $2\sum\alpha\beta$, the term that should be subtracted, not the answer.

Final Answer: $\alpha^2 + \beta^2 + \gamma^2 = 14 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q15

Q16.

Solution

Concept — Reciprocal equations: A polynomial equation is called reciprocal when its coefficient sequence is palindromic (reads the same forwards and backwards). The defining feature is that if α is a root then so is its reciprocal $\frac{1}{\alpha}$, because the equation is unchanged on replacing x by $\frac{1}{x}$ and clearing denominators. For a reciprocal quadratic the two roots are a reciprocal pair α and $\frac{1}{\alpha}$, so their product is automatically 1. This also agrees with Vieta's product-of-roots formula $\frac{c}{a}$.

Step 1 — Confirm the reciprocal structure: The equation $2x^2 + 5x + 2 = 0$ has coefficients $2, 5, 2$, which read identically left-to-right and right-to-left, so it is a reciprocal equation.

Step 2 — Use the reciprocal-pair property: The roots come in the pair α, $\frac{1}{\alpha}$, hence

$$\text{product} = \alpha \cdot \frac{1}{\alpha} = 1.$$

Step 3 — Cross-check with Vieta: For $ax^2 + bx + c = 0$ the product of roots is $\frac{c}{a} = \frac{2}{2} = 1$, the same value.

Step 4 — Optional direct check: Solving, $x = \dfrac{-5 \pm \sqrt{25 - 16}}{4} = \dfrac{-5 \pm 3}{4}$ gives roots $-\frac{1}{2}$ and -2, whose product is $(-\frac{1}{2})(-2) = 1$, and indeed $-2 = \frac{1}{-1/2}$, a reciprocal pair.

Why other options are wrong:

- (A) -1 has the wrong sign; the product $\frac{c}{a} = +1$ is positive.
- (B) $\frac{5}{2}$ is $-\frac{b}{a}$, the *sum* of the roots, not their product.
- (C) 2 would require $\frac{c}{a} = 2$, but here $\frac{c}{a} = 1$.

Final Answer: product of roots $= 1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — A standard logarithmic limit: The limit $\lim_{x\to 0}\dfrac{\log(1+x)}{x} = 1$ is one of the basic limits of calculus (here $\log$ denotes the natural logarithm $\ln$). It is a $\frac{0}{0}$ indeterminate form because both numerator and denominator tend to 0. The cleanest justification uses the Maclaurin series $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$, whose leading term is x, so dividing by x leaves 1 in the limit. This limit underlies the derivative of $\log(1+x)$ at $x = 0$.

Step 1 — Recognise the $\frac{0}{0}$ form: As $x \to 0$, $\log(1+x) \to \log 1 = 0$ and the denominator $x \to 0$, so direct substitution is indeterminate.

Step 2 — Expand the numerator: Using the series,

$$\frac{\log(1+x)}{x} = \frac{x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \cdots}{x} = 1 - \frac{x}{2} + \frac{x^2}{3} - \cdots.$$

Step 3 — Take the limit: As $x \to 0$ every term after the first vanishes:

$$\lim_{x\to 0}\frac{\log(1+x)}{x} = 1.$$

Step 4 — Confirm by L'Hôpital's rule: Differentiating top and bottom, $\dfrac{\frac{1}{1+x}}{1} = \dfrac{1}{1+x} \to 1$ as $x \to 0$, the same value.

Why other options are wrong:

- (B) 0 ignores the leading term x in the numerator's expansion.
- (C) e confuses this with $\lim_{x\to 0}(1+x)^{1/x} = e$, a different standard limit.
- (D) "does not exist" is false; both the series and L'Hôpital give a definite value 1.

Final Answer: the limit is $1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q17

Q18.

Solution

Concept — Repeated differentiation of $\sin x$: The second derivative is found by differentiating the function twice in succession. The trigonometric derivatives follow the cycle $\sin x \xrightarrow{d/dx} \cos x \xrightarrow{d/dx} -\sin x \xrightarrow{d/dx} -\cos x \xrightarrow{d/dx} \sin x$, which repeats every four differentiations. So differentiating $\sin x$ twice lands two steps into this cycle, at $-\sin x$. Equivalently, $\sin x$ satisfies the differential equation $y'' = -y$.

Step 1 — First derivative:

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x) = \cos x.$$

Step 2 — Second derivative: Differentiate $\cos x$:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\cos x) = -\sin x.$$

Step 3 — Consistency with $y'' = -y$: Since $y = \sin x$, the result $-\sin x = -y$ confirms the well-known relation $y'' + y = 0$ satisfied by the sine function.

Why other options are wrong:

- (A) $\cos x$ is only the *first* derivative; differentiation must be applied once more.
- (B) $\sin x$ has the correct magnitude but the wrong sign; the cycle introduces a minus.
- (D) $-\cos x$ is the *third* derivative (one differentiation too many).

Final Answer: $\frac{d^2y}{dx^2} = -\sin x \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q18

Q19.

Solution

Concept — Extreme values on a closed interval: A continuous function on a closed interval $[a, b]$ attains its greatest (and least) value either at an interior critical point, where $f'(x) = 0$, or at one of the endpoints. The procedure (the "closed-interval method") is to find all critical points inside the interval, evaluate f there and at both endpoints, and pick the largest. Here $f(x) = 4 - x^2$ is a downward-opening parabola, so its peak is at the vertex.

Step 1 — Differentiate and find critical points:

$$f'(x) = -2x = 0 \implies x = 0,$$

which lies inside $[-2, 2]$.

Step 2 — Evaluate at the critical point:

$$f(0) = 4 - 0^2 = 4.$$

Step 3 — Evaluate at the endpoints:

$$f(-2) = 4 - (-2)^2 = 0, \qquad f(2) = 4 - 2^2 = 0.$$

Step 4 — Compare: Among $\{4, 0, 0\}$ the greatest value is 4, attained at $x = 0$, the vertex $(0, 4)$ marked on the graph.

Why other options are wrong:

- (A) 0 is the *least* value (attained at the endpoints), not the greatest.
- (C) 2 is never an output of f at the relevant points; it is not attained as a maximum.
- (D) 8 exceeds $f(0) = 4$, the actual peak; the parabola never reaches 8 on $[-2, 2]$.

Final Answer: greatest value $= 4 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q19

Q20.

Solution

Concept — Lagrange Mean Value Theorem (LMVT): If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists at least one $c \in (a, b)$ where the instantaneous rate of change equals the average rate of change over the whole interval: $f'(c) = \dfrac{f(b) - f(a)}{b - a}$. Geometrically, the tangent at $x = c$ is parallel to the chord joining the endpoints. Here $f(x) = x^2$ is a polynomial, so it is continuous and differentiable everywhere and the theorem applies on $[1, 3]$.

Step 1 — Compute the average rate of change (slope of the chord):

$$\frac{f(3) - f(1)}{3 - 1} = \frac{3^2 - 1^2}{2} = \frac{9 - 1}{2} = 4.$$

Step 2 — Write the derivative:

$$f'(x) = 2x.$$

Step 3 — Set $f'(c)$ equal to the average rate and solve:

$$2c = 4 \implies c = 2.$$

Step 4 — Check the interval: $c = 2$ lies strictly inside $(1, 3)$, so it is the valid value guaranteed by LMVT.

Why other options are wrong:

- (A) 1 and (B) 3 are the endpoints a and b; the theorem requires c to be *interior*, i.e. in the open interval.
- (C) $\sqrt{3} \approx 1.73$ does not satisfy $2c = 4$ (it gives slope $2\sqrt{3} \neq 4$).

Final Answer: $c = 2 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

Q21.

Solution

Concept — Solving homogeneous differential equations: A first-order equation is homogeneous when it can be written as $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$, i.e. the right side depends only on the ratio $\frac{y}{x}$. The standard technique is the substitution $y = vx$, where v is a new function of x. Differentiating by the product rule gives $\frac{dy}{dx} = v + x\frac{dv}{dx}$, and substituting turns the equation into one whose variables v and x separate, allowing integration.

Step 1 — Confirm homogeneity: Rewrite the right-hand side:

$$\frac{dy}{dx} = \frac{x+y}{x} = 1 + \frac{y}{x},$$

which is indeed a function of $\frac{y}{x}$ alone, so the equation is homogeneous.

Step 2 — Apply the substitution $y = vx$: Then $\frac{dy}{dx} = v + x\frac{dv}{dx}$, and the equation becomes

$$v + x\frac{dv}{dx} = 1 + v \implies x\frac{dv}{dx} = 1.$$

Step 3 — Observe the separation: This reduces to $dv = \dfrac{dx}{x}$, whose variables are now fully separated, confirming that $y = vx$ is the correct reducing substitution (it integrates to $v = \ln|x| + C$, i.e. $y = x\ln|x| + Cx$).

Why other options are wrong:

- (B) $x = vy^2$ is not the standard substitution for a homogeneous equation and does not separate the variables here.
- (C) $y = v + x$ is a translation, not a scaling by x; it fails to exploit the $\frac{y}{x}$ structure.
- (D) $x + y = v$ replaces a sum by a single variable, which does not reduce this equation to separable form.

Final Answer: use $y = vx \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept — A standard inverse-trigonometric integral: Integration reverses differentiation, so to integrate $\dfrac{1}{1+x^2}$ we look for a function whose derivative is exactly this. Since $\dfrac{d}{dx}\left(\tan^{-1} x\right) = \dfrac{1}{1+x^2}$, the antiderivative is $\tan^{-1} x$ plus an arbitrary constant. This is one of the standard results memorised for calculus, and it is worth distinguishing carefully from the look-alike integrals that produce a logarithm or an arcsine.

Step 1 — Recall the relevant derivative:

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}.$$

Step 2 — Reverse it to get the integral:

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C.$$

Step 3 — Verify by differentiating the answer: Differentiating $\tan^{-1} x + C$ returns $\dfrac{1}{1+x^2}$, the original integrand, confirming correctness.

Why other options are wrong:

- (A) $\log(1+x^2)+C$ is the integral of $\dfrac{2x}{1+x^2}$; its derivative is $\dfrac{2x}{1+x^2}$, not $\dfrac{1}{1+x^2}$.
- (B) $\frac{1}{2}\log(1+x^2)+C$ is the integral of $\dfrac{x}{1+x^2}$, again with an extra x in the numerator.
- (D) $\sin^{-1}x+C$ is the integral of $\dfrac{1}{\sqrt{1-x^2}}$, a different denominator entirely.

Final Answer: $\tan^{-1}x+C \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q22

Q23.

Solution

Concept — Integrating $\sin^2 x$ by the power-reduction identity: The square $\sin^2 x$ cannot be integrated directly, so we first linearise it with the double-angle identity $\cos 2x = 1-2\sin^2 x$, rearranged to $\sin^2 x = \dfrac{1-\cos 2x}{2}$. This converts the integrand into a constant plus a cosine, both of which integrate trivially. The result is a special case of the Wallis formula $\int_0^{\pi/2}\sin^2 x\,dx = \frac{\pi}{4}$.

Step 1 — Apply the identity:

$$\int_0^{\pi/2}\sin^2 x\,dx = \int_0^{\pi/2}\frac{1-\cos 2x}{2}\,dx.$$

Step 2 — Integrate term by term:

$$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2}.$$

Step 3 — Substitute the limits: At $x=\frac{\pi}{2}$, $\sin 2x = \sin\pi = 0$; at $x=0$, $\sin 2x = 0$ as well, so

$$= \frac{1}{2}\left[\left(\frac{\pi}{2}-0\right)-(0-0)\right] = \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4}.$$

Step 4 — Symmetry cross-check: Since $\int_0^{\pi/2}\sin^2 x\,dx = \int_0^{\pi/2}\cos^2 x\,dx$ and their sum is $\int_0^{\pi/2}1\,dx = \frac{\pi}{2}$, each equals half of $\frac{\pi}{2}$, namely $\frac{\pi}{4}$, confirming the answer.

Why other options are wrong:

- (A) $\frac{\pi}{2}$ forgets the overall factor $\frac{1}{2}$ from the identity.
- (C) 1 ignores the π that the integration of the constant term produces.

- (D) $\frac{\pi}{3}$ does not arise from any correct step; it matches no part of the computation.

Final Answer: the integral is $\dfrac{\pi}{4} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q23

Q24.

Solution

Concept — Area under a curve by definite integration: The area of the region bounded by a curve $y = f(x)$, the x-axis, and the vertical lines $x = a$ and $x = b$ (with $f(x) \geq 0$ on the interval) is $\displaystyle\int_a^b f(x)\,dx$. Here the boundary is the line $y = x$ between $x = 1$ and $x = 3$, where $y = x > 0$, so the definite integral directly gives the shaded area. The region is a trapezium, which also allows a geometric cross-check.

Step 1 — Set up the integral:

$$\text{Area} = \int_1^3 x\,dx.$$

Step 2 — Find the antiderivative:

$$\int x\,dx = \frac{x^2}{2}.$$

Step 3 — Apply the limits:

$$\left[\frac{x^2}{2}\right]_1^3 = \frac{3^2}{2} - \frac{1^2}{2} = \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4 \text{ sq. units.}$$

Step 4 — Geometric verification: The region is a trapezium with parallel vertical sides of heights $y(1) = 1$ and $y(3) = 3$ and width $3 - 1 = 2$. Its area is $\frac{1}{2}(1+3) \times 2 = \frac{1}{2} \cdot 4 \cdot 2 = 4$, matching the integral.

Why other options are wrong:

- (A) 2 comes from using 0 as the lower limit incorrectly or dropping a term; it is the area of only part of the region.
- (B) 8 forgets the factor $\frac{1}{2}$ in $\frac{x^2}{2}$ (i.e. evaluates $9 - 1$).
- (D) 9 evaluates $\frac{x^2}{2}$-type expression only at $x = 3$ and ignores the lower limit.

Final Answer: area $= 4$ sq. units $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q24

Q25.

Solution

Concept — Definite integral of the exponential function: The exponential function is its own derivative and its own antiderivative, $\int e^x\,dx = e^x + C$. By the Fundamental Theorem of Calculus, the definite integral over $[a, b]$ equals the antiderivative evaluated at the upper limit minus its value at the lower limit. The only subtlety is remembering that $e^0 = 1$, not 0.

Step 1 — Write the antiderivative:

$$\int e^x\,dx = e^x.$$

Step 2 — Apply the limits 0 and 1:

$$\int_0^1 e^x\,dx = \left[e^x\right]_0^1 = e^1 - e^0.$$

Step 3 — Simplify using $e^0 = 1$:

$$= e - 1.$$

Step 4 — Sanity check numerically: $e \approx 2.718$, so $e - 1 \approx 1.718$, a positive number, as expected for the area under a positive curve from 0 to 1.

Why other options are wrong:

- (A) e forgets to subtract $e^0 = 1$ at the lower limit.
- (B) 1 swaps the roles of the limits or evaluates e^0 only.
- (C) $e + 1$ uses a plus sign instead of the subtraction demanded by the Fundamental Theorem.

Final Answer: the integral is $e - 1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q25

Q26.

Solution

Concept — Intercept form of a line: The equation $\frac{x}{a} + \frac{y}{b} = 1$ is the intercept form of a straight line, in which the denominators a and b are precisely the x- and y-intercepts. This is because setting $y = 0$ leaves $\frac{x}{a} = 1$, i.e. $x = a$ (where the line crosses the x-axis), and setting $x = 0$ leaves $\frac{y}{b} = 1$, i.e. $y = b$ (where it crosses the y-axis). So the intercepts can be read off directly without rearranging.

Step 1 — Match to the standard form: The given line $\frac{x}{4} + \frac{y}{3} = 1$ has $a = 4$ and $b = 3$.

Step 2 — Find the x-intercept (set $y = 0$):

$$\frac{x}{4} = 1 \implies x = 4.$$

Step 3 — Find the y-intercept (set $x = 0$):

$$\frac{y}{3} = 1 \implies y = 3.$$

Step 4 — State the result: The line crosses the axes at $(4, 0)$ and $(0, 3)$, so the x- and y-intercepts are 4 and 3 respectively.

Why other options are wrong:

- (B) 3 and 4 swaps the two intercepts (assigns the y-value to the x-axis).
- (C) $\frac{1}{4}$ and $\frac{1}{3}$ takes reciprocals of the denominators, confusing them with slopes/coefficients.
- (D) -4 and -3 uses the wrong signs; both intercepts here are positive.

Final Answer: intercepts 4 and $3 \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q26

Q27.

Solution

Concept — Tangent to a circle at a point on it: For the circle $x^2+y^2 = a^2$ centred at the origin, the tangent line at a point (x_1, y_1) lying on the circle is obtained by the "replacement rule": replace x^2 by xx_1 and y^2 by yy_1, giving $xx_1+yy_1 = a^2$. This works because the tangent is perpendicular to the radius at the point of contact, and the formula encodes exactly that perpendicularity. The point $(3, 4)$ here lies on the circle since $3^2 + 4^2 = 25$.

Step 1 — Identify the data: The circle has $a^2 = 25$ and the point of contact is $(x_1, y_1) = (3, 4)$.

Step 2 — Apply the tangent formula:

$$xx_1 + yy_1 = a^2 \implies 3x + 4y = 25.$$

Step 3 — Verify the point lies on the tangent: Substituting $(3, 4)$:

$$3(3) + 4(4) = 9 + 16 = 25,$$

which satisfies the line, so $(3, 4)$ is indeed on it.

Step 4 — Verify perpendicularity: The radius to $(3, 4)$ has slope $\frac{4}{3}$; the tangent $3x + 4y = 25$ has slope $-\frac{3}{4}$. Their product is -1, confirming the tangent is perpendicular to the radius, as it must be.

Why other options are wrong:

- (A) $4x + 3y = 25$ swaps the coefficients; it does not pass through $(3, 4)$ $(12 + 12 = 24 \neq 25)$.
- (C) $3x - 4y = 25$ has the wrong sign; $9 - 16 = -7 \neq 25$, so $(3, 4)$ is not on it.
- (D) $3x + 4y = 5$ uses $a = 5$ instead of $a^2 = 25$ on the right-hand side.

Final Answer: $3x + 4y = 25 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q27

Q28.

Solution

Concept — Parabola from focus and directrix: A parabola is the locus of points equidistant from a fixed point (the focus) and a fixed line (the directrix). When the focus is $(a, 0)$ and the directrix is the vertical line $x = -a$, the vertex sits at the origin and the curve opens rightward with standard equation $y^2 = 4ax$. The parameter a is the focal distance (distance from vertex to focus). One can derive this by equating the distance from a point (x, y) to the focus and to the directrix.

Step 1 — Read off the focal parameter: The focus $(2, 0)$ matches $(a, 0)$ and the directrix $x = -2$ matches $x = -a$, so

$$a = 2.$$

Step 2 — Substitute into the standard form:

$$y^2 = 4ax = 4(2)x = 8x.$$

Step 3 — Verify with the definition: Take the vertex point $(0, 0)$: its distance to the focus $(2, 0)$ is 2, and its distance to the directrix $x = -2$ is also 2. Equal distances confirm the vertex lies on the parabola, consistent with $y^2 = 8x$.

Why other options are wrong:

- (A) $y^2 = 4x$ takes $4a = 4$, i.e. $a = 1$, which would put the focus at $(1, 0)$, not $(2, 0)$.
- (B) $y^2 = 2x$ takes $4a = 2$, i.e. $a = \frac{1}{2}$, again the wrong focal distance.
- (C) $x^2 = 8y$ is a parabola opening *upward* about the y-axis, inconsistent with a focus on the x-axis.

Final Answer: $y^2 = 8x \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q28**

Q29.

Solution

Concept — Direction cosines from direction ratios: Direction ratios a, b, c specify the *direction* of a line but not a unique length; direction cosines l, m, n are the cosines of the angles the line makes with the coordinate axes and satisfy $l^2 + m^2 + n^2 = 1$. To convert ratios to cosines, divide each ratio by the magnitude $\sqrt{a^2 + b^2 + c^2}$, which scales the direction vector to unit length. The normalisation is what guarantees $l^2 + m^2 + n^2 = 1$.

Step 1 — Compute the magnitude of the direction vector:

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3.$$

Step 2 — Divide each ratio by the magnitude:

$$l = \frac{1}{3}, \qquad m = \frac{2}{3}, \qquad n = \frac{2}{3}.$$

Step 3 — Check the unit-sum identity:

$$l^2 + m^2 + n^2 = \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = \frac{9}{9} = 1,$$

confirming these are genuine direction cosines.

Why other options are wrong:

- (B) $1, 2, 2$ are the raw direction ratios, left un-normalised; they fail $l^2 + m^2 + n^2 = 1$ $(1 + 4 + 4 = 9)$.
- (C) $\frac{1}{9}, \frac{2}{9}, \frac{2}{9}$ divides by 9 (the magnitude squared) instead of 3.
- (D) $\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, \frac{2}{\sqrt{5}}$ uses an incorrect magnitude $\sqrt{5}$ rather than $\sqrt{9} = 3$.

Final Answer: $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q29

Q30.

Solution

Concept — Line perpendicular to a plane: A plane is characterised by its normal vector, whose direction ratios are a_2, b_2, c_2. A line is perpendicular to the plane exactly when it points along that normal, i.e. when the line's direction vector is *parallel* to the plane's normal vector. Two vectors are parallel precisely when their components are proportional, so the perpendicularity-to-plane condition becomes a proportionality of direction ratios. (Contrast this with a line lying *in*, or parallel to, the plane, where the line is perpendicular to the normal and the dot product is zero.)

Step 1 — Translate "line $\perp$ plane" into "line $\parallel$ normal": The line direction (a_1, b_1, c_1) must be parallel to the normal direction (a_2, b_2, c_2).

Step 2 — Write the proportionality (parallel) condition:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

Step 3 — Sanity note: If, say, $(a_1, b_1, c_1) = k(a_2, b_2, c_2)$ for some scalar k, all three ratios equal k, exactly the stated condition.

Why other options are wrong:

- (A) $a_1a_2 + b_1b_2 + c_1c_2 = 0$ is a zero dot product, which makes the line *parallel to* (lying in) the plane, the opposite situation.
- (B) $a_1a_2 = b_1b_2 = c_1c_2$ equates products, which is not a valid parallelism condition.
- (D) $a_1 + b_1 + c_1 = a_2 + b_2 + c_2$ equates sums of components, which has no geometric meaning for direction.

Final Answer: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q30

Q31.

Solution

Concept — Scalar projection along a direction: The scalar projection (component) of a vector $\vec{a}$ along a unit vector $\hat{u}$ is the dot product $\vec{a}\cdot\hat{u}$. It measures how much of $\vec{a}$ points in the direction of $\hat{u}$. When $\hat{u}$ is one of the coordinate unit vectors, the dot product simply picks out the corresponding component of $\vec{a}$, because $\hat{i}\cdot\hat{i}=1$ while $\hat{j}\cdot\hat{i}=0$. Thus the component along $\hat{i}$ is just the coefficient of $\hat{i}$ in $\vec{a}$.

Step 1 — Take the dot product with $\hat{i}$:

$$\vec{a}\cdot\hat{i}=(3\hat{i}+4\hat{j})\cdot\hat{i}=3(\hat{i}\cdot\hat{i})+4(\hat{j}\cdot\hat{i}).$$

Step 2 — Use orthonormality: Since $\hat{i}\cdot\hat{i}=1$ and $\hat{j}\cdot\hat{i}=0$,

$$\vec{a}\cdot\hat{i}=3(1)+4(0)=3.$$

Step 3 — Interpret: The component 3 is exactly the $\hat{i}$-coefficient of $\vec{a}$, as expected for a projection onto the x-axis direction.

Why other options are wrong:

- (A) 4 is the component along $\hat{j}$ (the y-direction), not $\hat{i}$.
- (C) 5 is the magnitude $|\vec{a}|=\sqrt{3^2+4^2}=5$, not a single-axis projection.
- (D) 7 wrongly adds the two components $3+4$.

Final Answer: component $=3\Rightarrow\boxed{\text{B}}$

Answer: (B) Go Back to Q31

Q32.

Solution

Concept — Cross products of the standard unit vectors: In a right-handed coordinate system the unit vectors obey the cyclic rule: the cross product of two consecutive vectors in the cycle $\hat{i}\to\hat{j}\to\hat{k}\to\hat{i}$ gives the next one. Explicitly $\hat{i}\times\hat{j}=\hat{k}$, $\hat{j}\times\hat{k}=\hat{i}$, and $\hat{k}\times\hat{i}=\hat{j}$. Each result is a different unit vector, so summing all three produces $\hat{i}+\hat{j}+\hat{k}$ rather than any cancellation. (Reversing the order would flip the sign of each term.)

Step 1 — Evaluate each cross product:

$$\hat{i}\times\hat{j}=\hat{k},\qquad \hat{j}\times\hat{k}=\hat{i},\qquad \hat{k}\times\hat{i}=\hat{j}.$$

Step 2 — Add the three results:

$$\hat{\imath} \times \hat{\jmath} + \hat{\jmath} \times \hat{k} + \hat{k} \times \hat{\imath} = \hat{k} + \hat{\imath} + \hat{\jmath}.$$

Step 3 — Reorder:

$$= \hat{\imath} + \hat{\jmath} + \hat{k}.$$

Step 4 — Note the distinctness: Because the three products land on three different axes, none cancels another, so the sum is the full diagonal vector $\hat{\imath} + \hat{\jmath} + \hat{k}$, not the zero vector.

Why other options are wrong:

- (A) $\hat{\imath} + \hat{\jmath}$ drops the $\hat{k}$ contribution from $\hat{\imath} \times \hat{\jmath}$.
- (B) $\vec{0}$ would require the terms to cancel, but $\hat{\imath}, \hat{\jmath}, \hat{k}$ are linearly independent and cannot.
- (C) $3\hat{k}$ wrongly assumes all three products give $\hat{k}$, ignoring the distinct cyclic results.

Final Answer: $\hat{\imath} + \hat{\jmath} + \hat{k} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q32

Q33.

Solution

Concept — Cyclic property of the scalar triple product: The scalar triple product $[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$ represents the (signed) volume of the parallelepiped formed by the three vectors. A cyclic rotation of the vectors ($\vec{a} \to \vec{b} \to \vec{c} \to \vec{a}$) leaves both the parallelepiped and its orientation unchanged, so the value is unchanged: $[\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$. However, a single *swap* of two vectors reverses the orientation and flips the sign.

Step 1 — Apply one cyclic rotation: Moving each vector one place forward in the cycle gives

$$[\vec{a}\,\vec{b}\,\vec{c}] = [\vec{b}\,\vec{c}\,\vec{a}].$$

Step 2 — Confirm via the determinant view: Writing the triple product as a 3×3 determinant of the components, a cyclic permutation of the rows is an even permutation (two row swaps), which preserves the determinant's value, matching the equality above.

Why other options are wrong:

- (B) $[\vec{b}\,\vec{a}\,\vec{c}]$ swaps just $\vec{a}$ and $\vec{b}$ (one transposition), so it equals $-[\vec{a}\,\vec{b}\,\vec{c}]$, not $+$.
- (C) $-[\vec{c}\,\vec{a}\,\vec{b}]$ negates a cyclic term; but $[\vec{c}\,\vec{a}\,\vec{b}] = +[\vec{a}\,\vec{b}\,\vec{c}]$, so this gives the wrong sign.
- (D) 0 holds only when the three vectors are coplanar, which is not given in general.

Final Answer: $[\vec{b}\,\vec{c}\,\vec{a}] \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q33

Q34.

Solution

Concept — Range as a measure of dispersion: The range is the simplest measure of spread in a data set, defined as the difference between the largest and smallest observations, range $=$ maximum $-$ minimum. It depends only on the two extreme values and ignores everything in between, which makes it quick to compute though sensitive to outliers. To find it, we first sort or scan the data to locate the extremes.

Step 1 — Scan the data for extremes: The set is $7, 12, 3, 20, 8, 15$. The largest value is 20 and the smallest is 3.

Step 2 — Subtract minimum from maximum:

$$\text{range} = 20 - 3.$$

Step 3 — Simplify:

$$\text{range} = 17.$$

Step 4 — Quick check: Ordering the data as $3, 7, 8, 12, 15, 20$ makes the extremes obvious at the two ends, confirming $20 - 3 = 17$.

Why other options are wrong:

- (A) 20 is only the maximum value, not the difference.
- (B) 3 is only the minimum value.
- (D) 23 *adds* the extremes $(20 + 3)$ instead of subtracting.

Final Answer: range $= 17 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q34

Q35.

Solution

Concept — "At least one" via the complement: Events of the form "at least one" are most efficiently computed through their complement, because "at least one head" is the exact opposite of "no heads at all." By the complement rule, $P(\text{at least one head}) = 1 - P(\text{no head})$. For two tosses of a fair coin the eight-free sample space is $\{HH, HT, TH, TT\}$, each equally likely with probability $\frac{1}{4}$, and "no head" is the single outcome TT.

Step 1 — List the sample space: Two tosses give four equally likely outcomes:

$$\{HH,\ HT,\ TH,\ TT\}, \qquad P(\text{each}) = \frac{1}{4}.$$

Step 2 — Probability of no head: The only outcome with no head is TT:

$$P(TT) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

Step 3 — Apply the complement rule:

$$P(\text{at least one head}) = 1 - \frac{1}{4} = \frac{3}{4}.$$

Step 4 — Direct count cross-check: The favourable outcomes HH, HT, TH number 3 out of 4, giving $\frac{3}{4}$ directly, in agreement.

Why other options are wrong:

- (A) $\frac{1}{4}$ is $P(\text{no head})$, the complement, not the required event.
- (C) $\frac{1}{2}$ counts only "exactly one head" (HT, TH), omitting the HH case.
- (D) 1 assumes a head is certain, ignoring the TT outcome.

Final Answer: $P = \frac{3}{4} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q35

Q36.

Solution

Concept — Recovering n from binomial mean and variance: For a binomial distribution $B(n, p)$ the mean is $\mu = np$ and the variance is $\sigma^2 = npq$, where $q = 1 - p$ is the failure probability. Given numerical values of μ and σ^2, dividing variance by mean isolates q (because the common factor np cancels), and from q we get p; substituting back into $np = \mu$ then yields n. This two-step elimination is the standard route.

Step 1 — Form the variance-to-mean ratio:

$$\frac{\sigma^2}{\mu} = \frac{npq}{np} = q = \frac{2}{4} = \frac{1}{2}.$$

Step 2 — Find p: Since $q = \frac{1}{2}$,

$$p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}.$$

Step 3 — Solve for n using the mean:

$$np = 4 \implies n \cdot \frac{1}{2} = 4 \implies n = 8.$$

Step 4 — Verify the variance: With $n = 8,\ p = q = \frac{1}{2}$, $\sigma^2 = npq = 8 \cdot \frac{1}{2} \cdot \frac{1}{2} = 2$, matching the given variance, so $n = 8$ is consistent.

Why other options are wrong:

- (A) 4 is the mean np, not the number of trials.
- (B) 2 is the variance npq, not n.
- (C) 16 would arise from forgetting to halve, i.e. using $p = \frac{1}{4}$; it doubles the correct n.

Final Answer: $n = 8 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q36

Q37.

Solution

Concept — Complementary-angle (co-function) identity: Angles that add to 90° are called complementary, and the sine of an angle equals the cosine of its complement: $\sin(90^\circ - \theta) = \cos\theta$. This reflects the fact that in a right triangle the sine of one acute angle equals the cosine of the other. Recognising that 70° and 20° are complementary ($70^\circ + 20^\circ = 90^\circ$) lets us rewrite the numerator so it cancels with the denominator.

Step 1 — Express 70° as a complement of 20°:

$$\sin 70^\circ = \sin(90^\circ - 20^\circ) = \cos 20^\circ.$$

Step 2 — Substitute into the given expression:

$$\frac{\sin 70^\circ}{\cos 20^\circ} = \frac{\cos 20^\circ}{\cos 20^\circ}.$$

Step 3 — Simplify:

$$= 1.$$

Step 4 — Numerical check: $\sin 70^\circ \approx 0.9397$ and $\cos 20^\circ \approx 0.9397$; their ratio is ≈ 1.000, confirming the identity.

Why other options are wrong:

- (B) 0 would require the numerator to vanish, but $\sin 70^\circ \neq 0$.
- (C) $\frac{1}{2}$ ignores the equality $\sin 70^\circ = \cos 20^\circ$ and assumes some half-value.
- (D) $\tan 20^\circ \approx 0.364$ does not equal the ratio, which is 1; it misapplies the co-function rule.

Final Answer: the value is $1 \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q37

Q38.

Solution

Concept — Compound-angle formula for cosine: A non-standard angle like $15°$ can be evaluated by writing it as a difference of two standard angles and applying $\cos(A - B) = \cos A \cos B + \sin A \sin B$. The natural choice is $15° = 45° - 30°$, since both $45°$ and $30°$ have known exact trigonometric values. Substituting and rationalising yields the exact surd form of $\cos 15°$.

Step 1 — Decompose the angle:

$$\cos 15° = \cos(45° - 30°) = \cos 45° \cos 30° + \sin 45° \sin 30°.$$

Step 2 — Insert standard values: Using $\cos 45° = \sin 45° = \frac{1}{\sqrt{2}}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$:

$$\cos 15° = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}.$$

Step 3 — Combine and rationalise:

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{(\sqrt{3}+1)\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}.$$

Step 4 — Numerical check: $\frac{\sqrt{6}+\sqrt{2}}{4} \approx \frac{2.449+1.414}{4} \approx 0.966$, which matches $\cos 15° \approx 0.966$.

Why other options are wrong:

- (A) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ is the value of $\sin 15°$ (uses a minus sign), not $\cos 15°$.
- (B) $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$ is the unrationalised form of $\sin 15°$, again the wrong function.
- (D) $\frac{1}{2}$ is $\cos 60°$, unrelated to $15°$.

Final Answer: $\cos 15° = \dfrac{\sqrt{6}+\sqrt{2}}{4} \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q38

Q39.

Solution

Concept — Principal value of $\sin^{-1}$: The inverse sine function returns only its principal value, which by definition lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. So $\sin^{-1}(\sin\theta)$ equals θ only when θ already lies in that range. Here $\frac{2\pi}{3} = 120^\circ$ lies in the second quadrant, outside the principal range, so we must first evaluate the sine and then identify the unique angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ with the same sine value.

Step 1 — Evaluate the inner sine using the supplementary-angle identity:

$$\sin\frac{2\pi}{3} = \sin\left(\pi - \frac{2\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

Step 2 — Apply the principal-value inverse sine: We need the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is $\frac{\sqrt{3}}{2}$:

$$\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}.$$

Step 3 — Confirm it is in range: $\frac{\pi}{3} = 60^\circ \in [-90^\circ, 90^\circ]$, so it is the correct principal value.

Step 4 — Note the key point: The answer is $\frac{\pi}{3}$, *not* the original $\frac{2\pi}{3}$, precisely because $\frac{2\pi}{3}$ falls outside the allowed output interval.

Why other options are wrong:

- (A) $\frac{2\pi}{3}$ is the inner angle, but it lies outside the principal range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and so cannot be the output.
- (C) $\frac{\pi}{6}$ has sine $\frac{1}{2}$, not $\frac{\sqrt{3}}{2}$.
- (D) $-\frac{\pi}{3}$ has sine $-\frac{\sqrt{3}}{2}$ (wrong sign); the value here is positive.

Final Answer: $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \dfrac{\pi}{3} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q39

Q40.

Solution

Concept — Heights and distances with two angles of elevation: When the angle of elevation of a tower's top is measured from two points in line with its base, the tangent of each angle equals (height)/(horizontal distance) for the right triangle formed. Letting x be the distance from the *nearer* point to the foot of the tower, the farther point is at distance $x + 20$. Writing the two tangent equations and eliminating h gives one equation in x; back-substituting yields the height. This is the standard two-station method.

Step 1 — Set up the tangent equations: Let h be the height and x the distance from the nearer point ($60°$) to the base. From the nearer point and the farther point ($30°$, which is 20 m farther):

$$\tan 60° = \frac{h}{x}, \qquad \tan 30° = \frac{h}{x+20}.$$

Step 2 — Express h from each: Using $\tan 60° = \sqrt{3}$ and $\tan 30° = \frac{1}{\sqrt{3}}$:

$$h = \sqrt{3}\,x, \qquad h = \frac{x+20}{\sqrt{3}}.$$

Step 3 — Eliminate h and solve for x: Equate the two expressions and multiply through by $\sqrt{3}$:

$$\sqrt{3}\,x = \frac{x+20}{\sqrt{3}} \implies 3x = x + 20 \implies 2x = 20 \implies x = 10.$$

Step 4 — Find the height:

$$h = \sqrt{3}\,x = \sqrt{3} \cdot 10 = 10\sqrt{3}\text{ m}.$$

Step 5 — Verify with the far station: $\dfrac{h}{x+20} = \dfrac{10\sqrt{3}}{30} = \dfrac{\sqrt{3}}{3} = \dfrac{1}{\sqrt{3}} = \tan 30° \checkmark$, so the solution is consistent.

Why other options are wrong:

- (A) $20\sqrt{3}$ comes from wrongly taking $x = 20$ instead of solving $2x = 20$.
- (B) 20 ignores the tangent ratios and just reuses the walking distance as the height.
- (C) $\frac{20}{\sqrt{3}}$ misplaces the 20 m, dividing instead of using it in $2x = 20$.

Final Answer: height $= 10\sqrt{3}$ m $\Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q40**

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	C	4	D	5	C
6	A	7	B	8	D	9	A	10	C
11	B	12	D	13	A	14	B	15	C
16	D	17	A	18	C	19	B	20	D
21	A	22	C	23	B	24	C	25	D
26	A	27	B	28	D	29	A	30	C
31	B	32	D	33	A	34	C	35	B
36	D	37	A	38	C	39	B	40	D