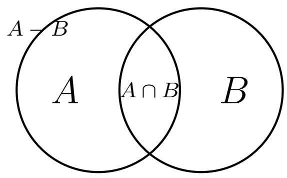
SRMJEEE Mathematics Sample Paper – 8

Duration: 47 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of **SRMJEEE** (SRM Joint Engineering Entrance Examination).
- Each correct answer carries **+1 mark**. There is **no negative marking**; an unattempted or wrong answer scores 0.
- Only **one** option is correct. Choose carefully.
- The actual SRMJEEE is a **computer-based test** conducted in remote-proctored online mode, with all sections sharing a common time window and no per-section limit.
- Personal calculators, mobile phones, log tables and other electronic gadgets are strictly prohibited.

Q1. For two sets A and B, $n(A) = 28$ and $n(A \cap B) = 11$, as shown in the Venn diagram. The number of elements in A but not in B, that is $n(A - B)$, is:



(A) 39

(B) 11

(C) 17

(D) 28

Q2. Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. The relation $R = \{(1, 4), (2, 5), (3, 6)\}$ from A to B is:

(A) a function

(B) not a function (one input has two outputs)

(C) not a relation

(D) a function only if $A = B$

Q3. The range of the real function $f(x) = \dfrac{x}{1+|x|}$ is:

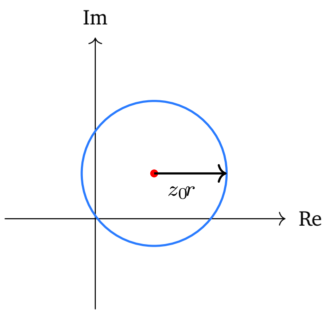
(A) $[-1, 1]$

(B) $(-1, 1)$

(C) $\mathbb{R}$

(D) $[0, 1)$

Q4. In the Argand plane, the equation $|z - z_0| = r$ (with $r > 0$) represents:



(A) a straight line

(B) a single point

(C) an ellipse

(D) a circle of radius r centred at z_0

Q5. For $n \geq 2$, the sum of all the nth roots of unity is:

(A) 0

(B) 1

(C) n

(D) -1

Q6. The quadratic equation whose roots are 3 and -2 is:

(A) $x^2 + x - 6 = 0$

(B) $x^2 - x - 6 = 0$

(C) $x^2 - x + 6 = 0$

(D) $x^2 + 5x + 6 = 0$

Q7. If α, β are the roots of $x^2 - 3x + 2 = 0$, then the equation whose roots are α^2 and β^2 is:

(A) $x^2 - 3x + 2 = 0$

(B) $x^2 - 9x + 4 = 0$

(C) $x^2 - 5x + 4 = 0$

(D) $x^2 + 5x + 4 = 0$

Q8. If $\begin{bmatrix} x+1 & 2 \\ 3 & y-1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}$, then the values of x and y are:

(A) $x = 4,\ y = 5$

(B) $x = 4,\ y = 6$

(C) $x = 2,\ y = 4$

(D) $x = 3,\ y = 6$

Q9. The inverse of the matrix $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ is:

(A) $\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

(D) $\begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix}$

Q10. In the determinant $\begin{vmatrix} 2 & 3 & 1 \\ 4 & 1 & 2 \\ 0 & 5 & 6 \end{vmatrix}$, the cofactor of the element 3 (first row, second column) is:

(A) 24

(B) -24

(C) 20

(D) -20

Q11. Using the determinant form, the equation of the line passing through the points $(1, 2)$ and $(3, 6)$ is:

(A) $y = 2x + 1$

(B) $x = 2y$

(C) $y = 2x$

(D) $y = x + 1$

Q12. In how many ways can the letters of the word "CHAIR" be arranged if the first and last positions are fixed as C and R respectively?

(A) 24

(B) 120

(C) 12

(D) 6

Q13. The number of triangles that can be formed by joining 7 points lying on a circle (no three collinear) is:

(A) 35

(B) 21

(C) 42

(D) 210

Q14. In how many arrangements of the letters of the word "ORANGE" (all letters distinct) do the two vowels O and A never sit next to each other?

(A) 720

(B) 240

(C) 480

(D) 120

Q15. If α, β, γ are the roots of $x^3 - 7x^2 + 14x - 8 = 0$, then the value of $\alpha\beta + \beta\gamma + \gamma\alpha$ is:

(A) 7

(B) 14

(C) 8

(D) -14

Q16. If α, β are the roots of $x^2 - 6x + 8 = 0$, then the quadratic equation whose roots are $\alpha + 2$ and $\beta + 2$ is:

(A) $x^2 - 6x + 8 = 0$

(B) $x^2 - 10x + 8 = 0$

(C) $x^2 + 10x + 24 = 0$

(D) $x^2 - 10x + 24 = 0$

Q17. The value of $\lim_{x \to 0} \frac{a^x - 1}{x}$ (with $a > 0$) is:

(A) a

(B) 1

(C) $\ln a$

(D) 0

Q18. If $y = a^x$ (with $a > 0,\ a \neq 1$), then $\frac{dy}{dx}$ equals:

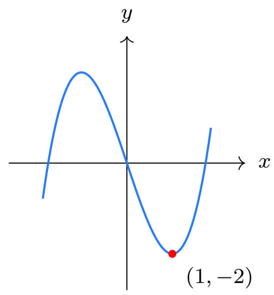
(A) $a^x \ln a$

(B) $x\,a^{x-1}$

(C) a^x

(D) $\dfrac{a^x}{\ln a}$

Q19. For the curve $f(x) = x^3 - 3x$, whose graph is shown, the point $x = 1$ corresponds to:



(A) a point of inflexion

(B) a local minimum

(C) a local maximum

(D) a discontinuity

Q20. Using differentials, the approximate value of $\sqrt{25.1}$ is:

(A) 5.1

(B) 5.05

(C) 4.99

(D) 5.01

Q21. The solution $y = Cx$ of the differential equation $x\dfrac{dy}{dx} = y$ (where C is an arbitrary constant) is called:

(A) a singular solution

(B) a particular solution

(C) the general solution

(D) not a solution

Q22. $\int \frac{1}{\sqrt{1-x^2}}\,dx$ equals:

(A) $\cos^{-1} x + C$

(B) $\sin^{-1} x + C$

(C) $\tan^{-1} x + C$

(D) $\frac{1}{2}\ln(1-x^2) + C$

Q23. Using even symmetry, the value of $\int_{-1}^{1} x^2\,dx$ is:

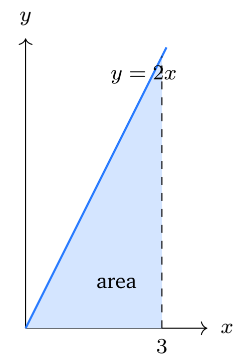
(A) $\frac{2}{3}$

(B) 0

(C) $\frac{1}{3}$

(D) 2

Q24. The area of the region bounded by the line $y = 2x$, the x-axis and the line $x = 3$ (shaded) is:



(A) 6 sq. units

(B) 18 sq. units

(C) 3 sq. units

(D) 9 sq. units

Q25. The value of $\int_{0}^{\pi/2} \sin x\,dx$ is:

(A) 0

(B) -1

(C) 1

(D) $\frac{\pi}{2}$

Q26. The equation of the straight line passing through the points $(2, 3)$ and $(4, 7)$ is:

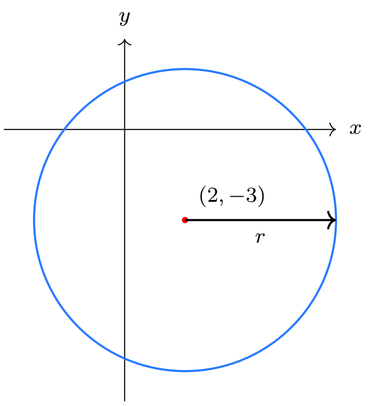
(A) $y = 2x - 1$

(B) $y = 2x + 1$

(C) $y = x + 1$

(D) $y = 2x - 3$

Q27. The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$, whose centre is shown at $(2, -3)$, is:



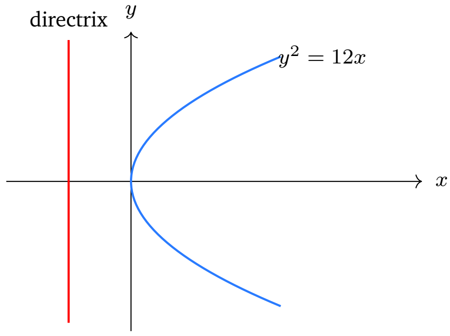
(A) 12

(B) 5

(C) 4

(D) 25

Q28. The equation of the directrix of the parabola $y^2 = 12x$ (which opens to the right, as shown) is:



(A) $x = 3$

(B) $y = -3$

(C) $x = 12$

(D) $x = -3$

Q29. A line makes angles 60° and 45° with the x-axis and y-axis respectively. If n is its direction cosine with the z-axis, then n^2 equals:

(A) $\frac{3}{4}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

Q30. The intercepts made by the plane $2x + 3y + 4z = 12$ on the x-, y- and z-axes are, respectively:

(A) $2, 3, 4$

(B) $12, 12, 12$

(C) $6, 4, 3$

(D) $3, 4, 6$

Q31. If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, then $|\vec{a}|$ (using $\vec{a} \cdot \vec{a} = |\vec{a}|^2$) equals:

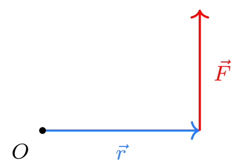
(A) 3

(B) 9

(C) $\sqrt{5}$

(D) 5

Q32. A force $\vec{F} = 3\hat{j}$ acts at the point whose position vector is $\vec{r} = 2\hat{i}$. The moment $\vec{r} \times \vec{F}$ about the origin is:



(A) $\vec{0}$

(B) $5\hat{k}$

(C) $-6\hat{k}$

(D) $6\hat{k}$

Q33. The value of the scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}]$ for $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + \hat{k}$, and hence whether they are coplanar, is:

(A) 2, not coplanar

(B) 0, coplanar

(C) 1, not coplanar

(D) -2, coplanar

Q34. The standard deviation of the five observations $7, 7, 7, 7, 7$ is:

(A) 7

(B) $\sqrt{7}$

(C) 0

(D) 1

Q35. For two mutually exclusive events A and B with $P(A) = 0.3$ and $P(B) = 0.45$, the value of $P(A \cup B)$ is:

(A) 0.135

(B) 0.75

(C) 0.15

(D) 1

Q36. A random variable X takes the values $0, 1, 2, 3$ with probabilities $k,\ 2k,\ 3k,\ 4k$ respectively. The value of k is:

(A) $\frac{1}{4}$

(B) $\frac{1}{5}$

(C) $\frac{1}{6}$

(D) $\frac{1}{10}$

Q37. The value of the expression $5\sin^2\theta + 5\cos^2\theta - 2$ is:

(A) 3

(B) 5

(C) 0

(D) -2

Q38. If $\tan\theta = \frac{1}{3}$, then the value of $\tan 2\theta$ is:

(A) $\frac{2}{3}$

(B) $\frac{3}{4}$

(C) $\frac{1}{4}$

(D) $\frac{2}{9}$

Q39. The principal value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is:

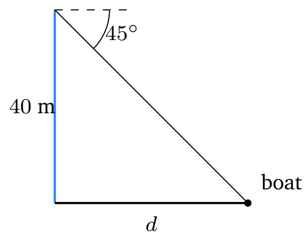
(A) $\frac{7\pi}{6}$

(B) $\frac{\pi}{6}$

(C) $\frac{5\pi}{6}$

(D) $-\frac{5\pi}{6}$

Q40. From the top of a cliff 40 m high, the angle of depression of a boat on the sea is 45°, as shown. The horizontal distance of the boat from the foot of the cliff is:



(A) 20 m

(B) $40\sqrt{3}$ m

(C) $40\sqrt{2}$ m

(D) 40 m

Detailed Solutions

Q1.

Solution

Concept — Set difference and the structure of a Venn diagram: For any two sets A and B, the set A is split by B into two disjoint pieces: the part of A that also lies in B (the overlap $A \cap B$) and the part of A that lies outside B (the difference $A - B$). Because these two pieces share no element and together make up all of A, their counts must add to $n(A)$. This gives the fundamental relation $n(A) = n(A-B) + n(A \cap B)$, which rearranges to $n(A-B) = n(A) - n(A \cap B)$. The diagram shows exactly this split, with $A - B$ as the left-hand crescent and $A \cap B$ as the lens-shaped overlap.

Step 1 — Read the data from the diagram: We are told $n(A) = 28$ (everything inside circle A) and $n(A \cap B) = 11$ (the lens where the two circles meet). The unknown is $n(A - B)$, the crescent of A lying outside B.

Step 2 — Apply the difference formula: Substitute the known counts into $n(A - B) = n(A) - n(A \cap B)$:

$$n(A - B) = 28 - 11.$$

Step 3 — Simplify:

$$n(A - B) = 17.$$

Step 4 — Cross-check by reassembling A: The two disjoint regions of A must sum back to $n(A)$. Indeed $n(A-B) + n(A \cap B) = 17 + 11 = 28 = n(A)$, confirming the answer is consistent.

Why other options are wrong:

- (A) 39 comes from *adding* the two given numbers, $28 + 11 = 39$; that double-counts the overlap and even exceeds $n(A)$, which is impossible for a subset of A.
- (B) 11 is simply $n(A \cap B)$, the overlap region, not the crescent outside B.
- (D) 28 is $n(A)$ itself, the whole circle A, which still includes the part inside B that we were asked to remove.

Final Answer: $n(A - B) = 17 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q1

Q2.

Solution

Concept — When a relation is a function: A relation R from A to B is any subset of the Cartesian product $A \times B$, i.e. any collection of ordered pairs (a, b) with $a \in A$ and $b \in B$. Such a relation is promoted to a *function* precisely when two conditions hold: (i) every element of the domain A appears as a first coordinate (existence of an image), and (ii) no element of A is paired with more than one element of B (uniqueness of the image). In short, each input must have exactly one output.

Step 1 — List the inputs and their images: The relation is $R = \{(1,4),(2,5),(3,6)\}$. Reading first coordinates, the inputs used are $1, 2, 3$, which is exactly the whole domain $A = \{1, 2, 3\}$.

Step 2 — Check existence: Every element of A does appear as a first coordinate: $1 \mapsto 4$, $2 \mapsto 5$, $3 \mapsto 6$. No domain element is left without an image.

Step 3 — Check uniqueness: Scan for any repeated first coordinate paired with different second coordinates. Each of $1, 2, 3$ occurs once only, so no input is sent to two different outputs.

Step 4 — Conclude: Both defining conditions hold, so R is a function (in fact a one-to-one correspondence, since the images $4, 5, 6$ are all distinct).

Why other options are wrong:

- (B) claims an input has two outputs, but no first coordinate repeats; the "two outputs" situation never occurs here.
- (C) is false because *every* set of ordered pairs from $A \times B$ is by definition a relation, so R certainly is one.
- (D) invents a condition that does not exist; functionhood depends only on the input-output rule, never on whether A and B are the same set.

Final Answer: R is a function $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q2

Q3.

Solution

Concept — Range by analysing the formula: The range of a function is the set of all output values it actually attains. For $f(x) = \dfrac{x}{1+|x|}$ the denominator $1+|x|$ is always positive, so f is defined for every real x and is continuous. The key idea is to rewrite f on each sign of x so we can see what values it sweeps through, and to use the oddness of f (since $f(-x) = -f(x)$) to handle the negative half cheaply. A second, faster idea is to solve $y = f(x)$ for x and ask which y admit a real solution.

Step 1 — Rewrite for $x \geq 0$: Here $|x| = x$, so

$$f(x) = \frac{x}{1+x} = \frac{(1+x)-1}{1+x} = 1 - \frac{1}{1+x}.$$

As x runs from 0 to $+\infty$, the term $\dfrac{1}{1+x}$ falls from 1 down to 0, so $f(x)$ rises from $f(0) = 0$ up toward (but never reaching) 1. Thus on $x \geq 0$ the outputs fill $[0,1)$.

Step 2 — Use oddness for $x < 0$: Since $f(-x) = \dfrac{-x}{1+|x|} = -f(x)$, the function is odd. Reflecting the interval $[0,1)$ through the origin gives the outputs $(-1,0]$ for $x \leq 0$. The negative values therefore fill $(-1,0)$.

Step 3 — Combine the two halves:

$$\text{Range} = (-1,0) \cup \{0\} \cup [0,1) = (-1,1).$$

The endpoints ± 1 are approached as $x \to \pm\infty$ but never attained.

Step 4 — Verify by inverting: Solve $y = \dfrac{x}{1+|x|}$ for $x \geq 0$: $y(1+x) = x \Rightarrow x = \dfrac{y}{1-y}$, which is a valid nonnegative real exactly when $0 \leq y < 1$. The odd symmetry then forces $-1 < y \leq 0$ on the other branch. A real x exists iff $-1 < y < 1$, confirming the range $(-1,1)$.

Why other options are wrong:

- (A) $[-1,1]$ wrongly includes the endpoints ± 1; from $x = \dfrac{y}{1-y}$ the value $y = 1$ has no solution (division by zero), and similarly $y = -1$ is unattained.
- (C) $\mathbb{R}$ ignores the bound $|f(x)| < 1$ that follows from $|x| < 1 + |x|$.
- (D) $[0,1)$ keeps only the $x \geq 0$ branch and discards all the negative outputs produced when $x < 0$.

Final Answer: range $= (-1,1) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q3

Q4.

Solution

Concept — Modulus as distance: On the Argand plane a complex number $z = x + iy$ is plotted as the point (x, y). The modulus of a difference, $|z - z_0|$, equals the straight-line (Euclidean) distance between the point representing z and the fixed point representing z_0. This is because if $z_0 = x_0 + iy_0$ then $|z - z_0| = \sqrt{(x - x_0)^2 + (y - y_0)^2}$, which is exactly the distance formula. Equations built from $|z - z_0|$ therefore describe purely geometric loci.

Step 1 — Translate the equation into geometry: The condition $|z - z_0| = r$ says: the distance from the moving point z to the fixed point z_0 is always the constant $r > 0$.

Step 2 — Recall the definition of a circle: A circle is, by definition, the set of all points at a fixed distance (the radius) from a fixed point (the centre). The condition in Step 1 is word-for-word this definition with centre z_0 and radius r.

Step 3 — Confirm algebraically: Writing $z = x + iy$ and $z_0 = x_0 + iy_0$ and squaring,

$$|z - z_0|^2 = (x - x_0)^2 + (y - y_0)^2 = r^2,$$

which is the standard Cartesian equation of a circle of radius r centred at (x_0, y_0). The diagram matches: a circle drawn about the marked centre z_0 with the radial arrow of length r.

Why other options are wrong:

- (A) a straight line arises from *equal* distances to two points, $|z - z_1| = |z - z_2|$ (a perpendicular bisector), not from a single fixed distance.
- (B) a single point occurs only in the degenerate case $r = 0$; here $r > 0$ is given.
- (C) an ellipse is the locus where the *sum* of distances to two foci is constant, $|z - z_1| + |z - z_2| = 2a$, which is a different equation entirely.

Final Answer: a circle of radius r centred at $z_0 \Rightarrow$ $\boxed{D}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — nth roots of unity and Vieta's formula: The nth roots of unity are the n complex solutions of $x^n = 1$, i.e. of the polynomial equation $x^n - 1 = 0$. They are evenly spaced on the unit circle at the points $e^{2\pi ik/n}$ for $k = 0, 1, \ldots, n-1$. By Vieta's formulas, for a monic polynomial $x^n + c_{n-1}x^{n-1} + \cdots + c_0$, the sum of all roots equals $-c_{n-1}$, the negative of the coefficient of x^{n-1}.

Step 1 — Identify the coefficient of x^{n-1}: Write the polynomial fully as $x^n + 0 \cdot x^{n-1} + 0 \cdot x^{n-2} + \cdots + 0 \cdot x - 1 = 0$. The coefficient of x^{n-1} is 0.

Step 2 — Apply Vieta: The sum of the roots is $-c_{n-1} = -0 = 0$.

Step 3 — Verify with the geometric-series view: The roots are $1, \omega, \omega^2, \ldots, \omega^{n-1}$ where $\omega = e^{2\pi i/n} \neq 1$. Summing the geometric series,

$$1 + \omega + \omega^2 + \cdots + \omega^{n-1} = \frac{\omega^n - 1}{\omega - 1} = \frac{1-1}{\omega - 1} = 0,$$

since $\omega^n = 1$. The two methods agree. Geometrically, the roots are symmetric vectors about the origin, so they cancel out.

Why other options are wrong:

- (B) 1 confuses the sum with one particular root (the root $k = 0$ is 1), or with the *product* when n is even $(= -1)$ versus odd $(= 1)$.
- (C) n merely counts how many roots there are; it is not their sum.
- (D) -1 is the constant term $\times(-1)^?$ confusion, but Vieta gives the sum as 0, not -1.

Final Answer: the sum is $0 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q5

Q6.

Solution

Concept — Reconstructing a quadratic from its roots: If a quadratic has roots α and β, it can be written in factored form as $(x-\alpha)(x-\beta) = 0$. Expanding gives $x^2 - (\alpha+\beta)x + \alpha\beta = 0$. Hence the monic quadratic with given roots is $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$. Note carefully the minus sign in front of the sum.

Step 1 — Compute the sum of the roots: With $\alpha = 3$ and $\beta = -2$,

$$\alpha + \beta = 3 + (-2) = 1.$$

Step 2 — Compute the product of the roots:

$$\alpha\beta = (3)(-2) = -6.$$

Step 3 — Assemble the equation: Substitute into the template:

$$x^2 - (1)x + (-6) = 0 \implies x^2 - x - 6 = 0.$$

Step 4 — Verify by direct substitution: Check $x = 3$: $9 - 3 - 6 = 0$. Check $x = -2$: $4 + 2 - 6 = 0$. Both roots satisfy the equation, confirming it is correct. (Equivalently, $(x - 3)(x + 2) = x^2 - x - 6$.)

Why other options are wrong:

- (A) $x^2 + x - 6 = 0$ has $+x$, i.e. it uses sum $= -1$; its roots are -3 and 2, not 3 and -2.
- (C) $x^2 - x + 6 = 0$ keeps product $+6$ instead of -6; its discriminant $1 - 24 < 0$, so it has no real roots at all.
- (D) $x^2 + 5x + 6 = 0$ factors as $(x + 2)(x + 3)$, giving roots $-2, -3$.

Final Answer: $x^2 - x - 6 = 0 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q6

Q7.

Solution

Concept — Building the equation of transformed roots: When the new roots are functions of the old ones, we express the new sum and new product in terms of the old sum $S = \alpha + \beta$ and old product $P = \alpha\beta$, which we read from the original quadratic by Vieta. For new roots α^2, β^2 we need their sum and product. The product is easy: $\alpha^2\beta^2 = (\alpha\beta)^2 = P^2$. The sum uses the algebraic identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = S^2 - 2P$, which avoids ever having to find α, β individually.

Step 1 — Read S and P from the original equation: For $x^2 - 3x + 2 = 0$, Vieta gives

$$S = \alpha + \beta = 3, \qquad P = \alpha\beta = 2.$$

Step 2 — New sum of roots:

$$\alpha^2 + \beta^2 = S^2 - 2P = 3^2 - 2(2) = 9 - 4 = 5.$$

Step 3 — New product of roots:

$$\alpha^2\beta^2 = P^2 = 2^2 = 4.$$

Step 4 — Assemble the quadratic: Using $x^2 - (\text{sum})x + (\text{product}) = 0$,

$$x^2 - 5x + 4 = 0.$$

Step 5 — Verify with the actual roots: The original factors as $(x-1)(x-2)$, so $\alpha = 1, \beta = 2$ and $\alpha^2 = 1, \beta^2 = 4$. A quadratic with roots $1, 4$ is $(x-1)(x-4) = x^2 - 5x + 4$, matching exactly.

Why other options are wrong:

- (A) $x^2 - 3x + 2 = 0$ is the original equation, with roots $1, 2$, not their squares.
- (B) $x^2 - 9x + 4 = 0$ takes the sum as $9 = S^2$, forgetting the $-2P$ correction in $\alpha^2 + \beta^2$.
- (D) $x^2 + 5x + 4 = 0$ has the wrong sign on the linear term, giving roots $-1, -4$.

Final Answer: $x^2 - 5x + 4 = 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q7

Q8.

Solution

Concept — Equality of matrices: Two matrices are equal if and only if they have the same order (same number of rows and columns) and every pair of corresponding entries is equal. Here both matrices are 2×2, so equality reduces to four separate scalar equations, one per position. We only need the positions that contain unknowns.

Step 1 — Confirm the orders match: Both matrices are 2×2, so entrywise comparison is valid.

Step 2 — Equate the $(1,1)$ entries: The top-left entries give

$$x + 1 = 4 \quad \Longrightarrow \quad x = 4 - 1 = 3.$$

Step 3 — Equate the $(2,2)$ entries: The bottom-right entries give

$$y - 1 = 5 \quad \Longrightarrow \quad y = 5 + 1 = 6.$$

Step 4 — Check the remaining entries for consistency: The off-diagonal positions already match on both sides: $(1,2)$ gives $2 = 2$ and $(2,1)$ gives $3 = 3$. No contradiction, so $x = 3,\ y = 6$ is the full solution.

Why other options are wrong:

- (A) $x = 4, y = 5$ reads off the right-hand entries 4 and 5 directly, ignoring the $+1$ and -1 shifts.
- (B) $x = 4, y = 6$ solves y correctly but takes $x = 4$ instead of $x + 1 = 4$.
- (C) $x = 2, y = 4$ fits neither equation: $2 + 1 = 3 \neq 4$ and $4 - 1 = 3 \neq 5$.

Final Answer: $x = 3,\ y = 6 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — Inverse of a 2×2 matrix: For an invertible matrix the inverse is $A^{-1} = \dfrac{1}{|A|}\,\text{adj}\,A$, valid only when the determinant $|A| \neq 0$. For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ there is a memorable shortcut for the adjugate: *swap* the diagonal entries a and d, and *negate* the off-diagonal entries b and c, giving $\text{adj}\,A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. The determinant is $|A| = ad - bc$.

Step 1 — Compute the determinant: With $a = 2, b = 1, c = 5, d = 3$,

$$|A| = ad - bc = (2)(3) - (1)(5) = 6 - 5 = 1.$$

Since $|A| = 1 \neq 0$, the inverse exists.

Step 2 — Form the adjugate: Swap diagonal, negate off-diagonal:

$$\text{adj}\, A = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}.$$

Step 3 — Divide by the determinant:

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}.$$

Step 4 — Verify $AA^{-1} = I$:

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 6-5 & -2+2 \\ 15-15 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

confirming the inverse is correct.

Why other options are wrong:

- (B) is just A itself; multiplying $A \cdot A$ does not give I, so it cannot be the inverse.
- (C) $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ forgets to negate the off-diagonal entries.
- (D) $\begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix}$ negates the wrong entries (the diagonal instead of the off-diagonal).

Final Answer: $A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Minors and cofactors: The minor M_{ij} of an entry is the determinant of the smaller matrix left after deleting the entry's whole row i and column j. The cofactor attaches the chessboard sign $(-1)^{i+j}$ to the minor: $C_{ij} = (-1)^{i+j}M_{ij}$. The sign is $+$ when $i+j$ is even and $-$ when $i+j$ is odd, so the sign pattern across a 3×3 matrix is $\begin{smallmatrix} + & - & + \\ - & + & - \\ + & - & + \end{smallmatrix}$.

Step 1 — Locate the element: The element 3 sits in row 1, column 2, so $i = 1,\ j = 2$.

Step 2 — Form the minor M_{12}: Delete row 1 and column 2 of $\begin{vmatrix} 2 & 3 & 1 \\ 4 & 1 & 2 \\ 0 & 5 & 6 \end{vmatrix}$, leaving the rows/columns $\begin{vmatrix} 4 & 2 \\ 0 & 6 \end{vmatrix}$. Evaluate:

$$M_{12} = (4)(6) - (2)(0) = 24 - 0 = 24.$$

Step 3 — Attach the cofactor sign: Since $i + j = 1 + 2 = 3$ is odd, the sign is $(-1)^3 = -1$:

$$C_{12} = (-1)^{1+2} M_{12} = -(24) = -24.$$

Why other options are wrong:

- (A) 24 is the minor M_{12}; it drops the required negative cofactor sign at an odd position.
- (C) 20 and (D) -20 both come from misreading the deleted submatrix (for example using $\begin{vmatrix} 4 & 2 \\ 0 & 5 \end{vmatrix}$ from the wrong column), giving 20 instead of 24.

Final Answer: cofactor $= -24 \Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q10**

Q11.

Solution

Concept — Collinearity determinant: Three points (x, y), (x_1, y_1), (x_2, y_2) are collinear exactly when the determinant $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$ (this determinant is twice the signed area of the triangle they form, which is zero only when the “triangle” is degenerate). Treating (x, y) as a variable point, setting the determinant to zero forces (x, y) to lie on the line through the two fixed points, so the expanded determinant *is* the line’s equation.

Step 1 — Insert the two given points: With $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (3, 6)$,

$$\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0.$$

Step 2 — Expand along the first row:

$$x\,(2\cdot 1-1\cdot 6)-y\,(1\cdot 1-1\cdot 3)+1\,(1\cdot 6-2\cdot 3)=0.$$

Step 3 — Simplify each bracket:

$$x(2-6)-y(1-3)+1(6-6)=-4x+2y+0=0.$$

Step 4 — Solve for y:

$$2y=4x \quad\Longrightarrow\quad y=2x.$$

Step 5 — Verify both points lie on it: At $(1,2)$: $2=2(1)$. At $(3,6)$: $6=2(3)$. Both check out, so $y=2x$ is the required line.

Why other options are wrong:

- (A) $y=2x+1$ fails at $(1,2)$ since $2(1)+1=3\neq 2$.
- (B) $x=2y$ reverses the roles of x and y; at $(1,2)$ it gives $1=4$, false.
- (D) $y=x+1$ has slope 1, but the slope between the points is $\dfrac{6-2}{3-1}=2$.

Final Answer: $y=2x\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q11

Q12.

Solution

Concept — Counting with fixed positions: The multiplication (fundamental counting) principle says the number of arrangements is the product of the number of independent choices at each slot. When certain slots are pinned to specific letters, those slots offer only one choice each and effectively drop out of the count; only the free slots, filled by the leftover distinct letters, are permuted. With k free slots holding k distinct letters, the number of arrangements is $k!$.

Step 1 — List the letters and slots: "CHAIR" has 5 distinct letters C,H,A,I,R in 5 ordered positions. The condition fixes position $1=C$ and position $5=R$.

Step 2 — Identify the free letters: Removing the pinned C and R leaves the three letters H,A,I to fill the three middle positions $2,3,4$.

Step 3 — Count the permutations of the free letters: Three distinct letters in three slots can be ordered in

$$3! = 3 \times 2 \times 1 = 6$$

ways.

Step 4 — Multiplication-principle check: Position 1: 1 way (C). Position 5: 1 way (R). Positions $2, 3, 4$: $3, 2, 1$ choices respectively. Total $= 1 \times 3 \times 2 \times 1 \times 1 = 6$, matching Step 3.

Why other options are wrong:

- (A) $24 = 4!$ corresponds to fixing only one letter, leaving four to permute.
- (B) $120 = 5!$ ignores the constraint entirely and permutes all five letters freely.
- (C) 12 does not equal any relevant factorial here and has no valid counting basis.

Final Answer: $3! = 6 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Triangles as unordered triples of points: A triangle is determined by choosing 3 of its vertices, and the order in which we name those vertices does not matter, so this is a *combination* problem, not a permutation. The only way a chosen triple fails to form a triangle is if the three points are collinear; but on a circle no three distinct points are collinear, so every choice of 3 points genuinely makes a triangle. Hence the count is $\binom{n}{3}$.

Step 1 — Set up the combination: With $n = 7$ points we need

$$\binom{7}{3} = \frac{7!}{3!\,(7-3)!}.$$

Step 2 — Simplify:

$$\binom{7}{3} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = \frac{210}{6} = 35.$$

Step 3 — Sanity check that no triples are wasted: Since the 7 points lie on a circle and no three are collinear, all 35 triples are valid triangles; none must be subtracted.

Why other options are wrong:

- (B) $21 = \binom{7}{2}$ counts pairs of points, i.e. chords/line segments, not triangles.
- (C) 42 is 2×21, with no combinatorial meaning for triangles here.
- (D) $210 = {}^7P_3 = 7 \cdot 6 \cdot 5$ counts *ordered* triples, overcounting each triangle by $3! = 6$; dividing back by 6 recovers 35.

Final Answer: $\binom{7}{3} = 35 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept — "Never adjacent" via the complement: Counting arrangements where two items are *never* together is awkward directly, so we count the opposite and subtract. The complement of "never together" is "always together," which is handled by the *gluing* (block) method: treat the two letters as a single fused unit, arrange everything, then multiply by the internal orderings of the unit. The required count is therefore (all arrangements) − (arrangements with the pair together).

Step 1 — Total unrestricted arrangements: "ORANGE" has 6 distinct letters O, R, A, N, G, E, so

$$6! = 720.$$

Step 2 — Arrangements with O and A together: Glue O, A into one block, leaving 5 items to arrange: the block plus R, N, G, E. These give $5!$ orders, and the block itself can be OA or AO, i.e. $2!$ internal orders:

$$5! \times 2! = 120 \times 2 = 240.$$

Step 3 — Subtract the "together" count:

$$720 - 240 = 480.$$

Step 4 — Cross-check by the gap method: Arrange the other 4 letters R, N, G, E in $4! = 24$ ways; this creates 5 gaps (including the two ends) into which O and A may be placed so they are not adjacent, choosing 2 of the 5 gaps and ordering them: ${}^5P_2 = 5 \cdot 4 = 20$. Total $= 24 \times 20 = 480$, matching Step 3.

Why other options are wrong:

- (A) 720 is the unrestricted total $6!$, before removing the adjacent cases.
- (B) 240 is exactly the "together" count we were meant to subtract, not the answer.
- (D) 120 is only $5!$, the block arrangements without the internal $2!$ and without subtracting from 720.

Final Answer: $480 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q14

Q15.

Solution

Concept — Vieta's formulas for a cubic: For a monic cubic $x^3 + px^2 + qx + r = 0$ with roots α, β, γ, Vieta's relations give $\alpha + \beta + \gamma = -p$, $\alpha\beta + \beta\gamma + \gamma\alpha = +q$, and $\alpha\beta\gamma = -r$. The middle symmetric function (sum of pairwise products) equals the coefficient q directly, with a $+$ sign. These follow from expanding $(x - \alpha)(x - \beta)(x - \gamma)$ and matching coefficients.

Step 1 — Match the equation to the template: Comparing $x^3 - 7x^2 + 14x - 8 = 0$ with $x^3 + px^2 + qx + r$ gives

$$p = -7, \qquad q = 14, \qquad r = -8.$$

Step 2 — Read off the required symmetric function: The sum of products two at a time is

$$\alpha\beta + \beta\gamma + \gamma\alpha = q = 14.$$

Step 3 — Verify with the actual roots: The cubic factors as $(x - 1)(x - 2)(x - 4)$, so the roots are $1, 2, 4$. Then

$$\alpha\beta + \beta\gamma + \gamma\alpha = (1)(2) + (2)(4) + (4)(1) = 2 + 8 + 4 = 14,$$

confirming the value.

Why other options are wrong:

- (A) 7 is $\alpha + \beta + \gamma = -p$, the sum of the roots $(1 + 2 + 4)$, not the pairwise sum.
- (C) 8 is the product $\alpha\beta\gamma = -r$ $(1 \cdot 2 \cdot 4)$.
- (D) -14 attaches a wrong sign; Vieta gives $+q$ for this symmetric function.

Final Answer: $\alpha\beta + \beta\gamma + \gamma\alpha = 14 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q15

Q16.

Solution

Concept — Translating roots by substitution: If the original equation $f(x) = 0$ has roots α, β, and we want a new equation whose roots are each shifted by $+2$ (that is, $\alpha + 2, \beta + 2$), let $X = x + 2$ denote a new root. Then the old root is $x = X - 2$, so substituting $x \to X - 2$ (renamed back to x) into f produces the polynomial with the shifted roots. Equivalently, one can recompute the new sum and product by Vieta; both methods are shown.

Step 1 — Substitute $x \to x - 2$: Replacing x by $x - 2$ in $x^2 - 6x + 8 = 0$,

$$(x - 2)^2 - 6(x - 2) + 8 = 0.$$

Step 2 — Expand each term:

$$(x^2 - 4x + 4) - (6x - 12) + 8 = 0.$$

Step 3 — Collect like terms:

$$x^2 - 4x + 4 - 6x + 12 + 8 = x^2 - 10x + 24 = 0.$$

Step 4 — Cross-check via Vieta: The original has $\alpha + \beta = 6$, $\alpha\beta = 8$ (roots $2, 4$). The new roots are $4, 6$, with sum $(\alpha + 2) + (\beta + 2) = \alpha + \beta + 4 = 10$ and product $(\alpha + 2)(\beta + 2) = \alpha\beta + 2(\alpha + \beta) + 4 = 8 + 12 + 4 = 24$. So the new equation is $x^2 - 10x + 24 = 0$, matching Step 3.

Why other options are wrong:

- (A) $x^2 - 6x + 8 = 0$ is the unshifted original (roots $2, 4$).
- (B) $x^2 - 10x + 8 = 0$ updates the linear term but keeps the old constant 8 instead of 24.
- (C) $x^2 + 10x + 24 = 0$ has the wrong sign on the linear term, giving roots $-4, -6$.

Final Answer: $x^2 - 10x + 24 = 0 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — A standard exponential limit: The expression $\frac{a^x - 1}{x}$ has the indeterminate form $\frac{0}{0}$ at $x = 0$, since $a^0 - 1 = 0$. Its limit is the standard result $\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a$. This is in fact the derivative of a^x evaluated at $x = 0$, because $\frac{d}{dx} a^x \Big|_0 = a^0 \ln a = \ln a$.

Step 1 — Rewrite the base using e: Any positive base satisfies $a^x = e^{x \ln a}$. Let $k = \ln a$, so $a^x = e^{kx}$ and the expression becomes $\frac{e^{kx} - 1}{x}$.

Step 2 — Use the core exponential limit: Using $e^u = 1 + u + \frac{u^2}{2!} + \cdots$ with $u = kx$,

$$\frac{e^{kx} - 1}{x} = \frac{kx + \frac{(kx)^2}{2!} + \cdots}{x} = k + \frac{k^2 x}{2!} + \cdots .$$

Step 3 — Take the limit: As $x \to 0$ every term after the first vanishes, leaving

$$\lim_{x \to 0} \frac{a^x - 1}{x} = k = \ln a.$$

Step 4 — Confirm with L'Hopital: Differentiating top and bottom of the $0/0$ form: $\frac{d}{dx}(a^x - 1) = a^x \ln a$ and $\frac{d}{dx}(x) = 1$, so the limit is $a^0 \ln a = \ln a$, the same answer.

Why other options are wrong:

- (A) a confuses the base with its natural logarithm.
- (B) 1 is correct only in the special case $a = e$, where $\ln e = 1$; it is not general.
- (D) 0 wrongly treats the $0/0$ form as just 0, ignoring that numerator and denominator vanish at the same rate.

Final Answer: the limit is $\ln a \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q17

Q18.

Solution

Concept — Differentiating an exponential with constant base: The function a^x has a *constant* base a and a variable exponent x, so the power rule $\frac{d}{dx}x^n = nx^{n-1}$ (which is for a variable base and constant exponent) does *not* apply. Instead we rewrite the base in terms of e and use the chain rule. The result is $\frac{d}{dx}(a^x) = a^x \ln a$.

Step 1 — Express a^x with base e: Using $a = e^{\ln a}$,

$$a^x = \left(e^{\ln a}\right)^x = e^{x \ln a}.$$

Step 2 — Differentiate with the chain rule: The outer function is e^u and the inner is $u = x \ln a$, where $\ln a$ is a constant. Then $\frac{du}{dx} = \ln a$, and

$$\frac{d}{dx}e^{x \ln a} = e^{x \ln a} \cdot \frac{d}{dx}(x \ln a) = e^{x \ln a} \cdot \ln a.$$

Step 3 — Convert back: Since $e^{x \ln a} = a^x$,

$$\frac{dy}{dx} = a^x \ln a.$$

Step 4 — Consistency check: The special case $a = e$ gives $\ln e = 1$, so $\frac{d}{dx}e^x = e^x \cdot 1 = e^x$, the familiar result, confirming the formula behaves correctly.

Why other options are wrong:

- (B) $x\,a^{x-1}$ wrongly applies the power rule, treating the exponent as fixed and the base as variable.
- (C) a^x omits the factor $\ln a$; this is right only when $a = e$.
- (D) $\frac{a^x}{\ln a}$ divides by $\ln a$ instead of multiplying.

Final Answer: $\frac{dy}{dx} = a^x \ln a \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q18

Q19.

Solution

Concept — The second-derivative test: A smooth function can have a local extremum only where its first derivative vanishes (a critical point). To classify such a point we look at the concavity there: if $f'' > 0$ the curve is concave up, forming a valley, so it is a local *minimum*; if $f'' < 0$ the curve is concave down, forming a peak, so it is a local *maximum*. If $f'' = 0$ the test is inconclusive and the point may be an inflexion.

Step 1 — First derivative and critical points: For $f(x) = x^3 - 3x$,

$$f'(x) = 3x^2 - 3.$$

Setting $f'(x) = 0$ gives $3x^2 = 3$, so $x^2 = 1$ and $x = \pm 1$. The two critical points are $x = -1$ and $x = 1$.

Step 2 — Second derivative: Differentiate again:

$$f''(x) = 6x.$$

Step 3 — Classify $x = 1$: Evaluate $f''(1) = 6(1) = 6 > 0$. Since the second derivative is positive, $x = 1$ is a local minimum.

Step 4 — Match the marked point: The minimum value is $f(1) = 1^3 - 3(1) = 1 - 3 = -2$, so the turning point is $(1, -2)$, exactly the red dot shown on the graph, where the curve bottoms out and turns back upward.

Why other options are wrong:

- (C) the local maximum sits at the *other* critical point $x = -1$, where $f''(-1) = -6 < 0$.
- (A) an inflexion would require $f'' = 0$, i.e. $6x = 0$ at $x = 0$, not at $x = 1$.
- (D) $f(x) = x^3 - 3x$ is a polynomial, continuous (and differentiable) everywhere, so there is no discontinuity.

Final Answer: $x = 1$ is a local minimum $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q19

Q20.

Solution

Concept — Approximation by differentials: Near a point, a smooth curve is well approximated by its tangent line. The linear (tangent-line) approximation is $f(x+\Delta x) \approx f(x)+f'(x)\,\Delta x$, where the change in y is estimated by the differential $dy = f'(x)\,\Delta x$. The trick is to pick a nearby base point x where $f(x)$ is exact and easy (a perfect square here), and let Δx be the small leftover.

Step 1 — Choose the function and base point: Let $f(x) = \sqrt{x}$, with $x = 25$ (since $\sqrt{25} = 5$ exactly) and $\Delta x = 0.1$ so that $x + \Delta x = 25.1$.

Step 2 — Compute the derivative at the base point:

$$f'(x) = \frac{1}{2\sqrt{x}} \quad \Longrightarrow \quad f'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{2 \cdot 5} = \frac{1}{10}.$$

Step 3 — Apply the approximation:

$$\sqrt{25.1} \approx f(25) + f'(25)\,\Delta x = 5 + \frac{1}{10}(0.1) = 5 + 0.01 = 5.01.$$

Step 4 — Cross-check against the true value: Squaring the estimate, $5.01^2 = 25.1001$, which is essentially 25.1 (the tiny excess 0.0001 reflects the small quadratic error of the tangent approximation). The estimate 5.01 is accurate.

Why other options are wrong:

- (A) 5.1 adds the whole $\Delta x = 0.1$ to 5, forgetting the factor $f'(25) = \frac{1}{10}$.
- (B) 5.05 would use $f' \approx \frac{1}{2}$, the wrong derivative value.
- (C) 4.99 subtracts the increment; but $25.1 > 25$, so the root must *increase* above 5.

Final Answer: $\sqrt{25.1} \approx 5.01 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

Q21.

Solution

Concept — General, particular, and singular solutions: The order of a differential equation is the highest derivative it contains; the general solution of an nth-order equation contains exactly n independent arbitrary constants. Assigning specific numerical values to those constants yields a *particular* solution. A *singular* solution, by contrast, is an extra solution not obtainable from the general family for any choice of the constants. The equation here is first order, so its general solution carries exactly one arbitrary constant.

Step 1 — Determine the order and constant count: The equation $x\dfrac{dy}{dx} = y$ involves only the first derivative, so it is first order and its general solution must contain exactly one arbitrary constant.

Step 2 — Verify $y = Cx$ solves it: If $y = Cx$ then $\dfrac{dy}{dx} = C$, and

$$x\frac{dy}{dx} = x \cdot C = Cx = y,$$

so the equation is satisfied identically for every value of C.

Step 3 — Classify: Because $y = Cx$ satisfies the equation and contains exactly the one arbitrary constant C that a first-order equation requires, it is the *general* solution; the whole family of lines through the origin is captured.

Why other options are wrong:

- (B) a particular solution would have C pinned to a number, e.g. $y = 2x$; here C is left arbitrary.
- (A) a singular solution lies outside the family $y = Cx$; this expression is the family itself, not an exception to it.
- (D) Step 2 shows by direct substitution that it genuinely is a solution.

Final Answer: the general solution $\Rightarrow$ $\boxed{\text{C}}$

Answer: (C) Go Back to Q21

Q22.

Solution

Concept — Recognising a standard inverse-trig integral: Integration is the reverse of differentiation, so to integrate $\frac{1}{\sqrt{1-x^2}}$ we ask which function differentiates to it. The known derivative $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ answers this directly, giving $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$. The constant C is mandatory for an indefinite integral.

Step 1 — Identify the form: The integrand $\frac{1}{\sqrt{1-x^2}}$ matches exactly the derivative of $\sin^{-1} x$, so the antiderivative is

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C.$$

Step 2 — Verify by differentiating back: Differentiating $\sin^{-1} x + C$ returns $\frac{1}{\sqrt{1-x^2}} + 0 = \frac{1}{\sqrt{1-x^2}}$, the original integrand, confirming the result.

Why other options are wrong:

- (A) $\cos^{-1} x + C$ differentiates to $-\frac{1}{\sqrt{1-x^2}}$, which has the wrong sign (though it differs from the answer only by a constant, $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$).
- (C) $\tan^{-1} x + C$ is the antiderivative of $\frac{1}{1+x^2}$, a different integrand.
- (D) $\frac{1}{2}\ln(1-x^2) + C$ differentiates to $\frac{-x}{1-x^2}$, unrelated to the given integrand.

Final Answer: $\sin^{-1} x + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q22

Q23.

Solution

Concept — Symmetry shortcut for definite integrals: Over a symmetric interval $[-a, a]$, parity simplifies the integral. If f is even ($f(-x) = f(x)$) the graph is mirror-symmetric about the y-axis, so the two halves contribute equally: $\int_{-a}^{a} f = 2\int_{0}^{a} f$. If f is odd the two halves cancel and the integral is 0. Recognising parity first can save work and guard against sign errors.

Step 1 — Check the parity of the integrand: For $f(x) = x^2$, $f(-x) = (-x)^2 =$

$x^2 = f(x)$, so f is even and the even-function rule applies.

Step 2 — Fold the interval:

$$\int_{-1}^{1} x^2\,dx = 2\int_{0}^{1} x^2\,dx.$$

Step 3 — Integrate and evaluate:

$$2\left[\frac{x^3}{3}\right]_0^1 = 2\left(\frac{1^3}{3} - \frac{0^3}{3}\right) = 2\cdot\frac{1}{3} = \frac{2}{3}.$$

Step 4 — Verify without symmetry: Computing directly, $\int_{-1}^{1} x^2\,dx = \left[\frac{x^3}{3}\right]_{-1}^{1} = \frac{1}{3} - \frac{-1}{3} = \frac{2}{3}$, the same value.

Why other options are wrong:

- (B) 0 is the value for an *odd* integrand like x or x^3, not for the even x^2.
- (C) $\frac{1}{3}$ is $\int_0^1 x^2\,dx$, the half-integral, forgetting the factor 2 (equivalently, integrating only over $[0, 1]$).
- (D) 2 would arise from integrating 1 rather than x^2 over $[-1, 1]$.

Final Answer: the integral is $\frac{2}{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q23

Q24.

Solution

Concept — Area under a curve as a definite integral: The area between a curve $y = f(x)$ (lying above the x-axis), the x-axis, and the vertical lines $x = a$ and $x = b$ is $\int_a^b y\,dx$. Geometrically the integral sums infinitely many thin vertical strips of height y and width dx. Here the region is bounded by $y = 2x$, the x-axis, and $x = 3$; it starts at the origin because $y = 2x$ passes through $(0, 0)$.

Step 1 — Set up the integral: The strips run from $x = 0$ (where the line meets the x-axis) to $x = 3$ (the given boundary), with height $y = 2x$:

$$\text{Area} = \int_0^3 2x\,dx.$$

Step 2 — Integrate: The antiderivative of $2x$ is x^2, so

$$\int_0^3 2x\,dx = \left[x^2\right]_0^3.$$

Step 3 — Evaluate at the limits:

$$\left[x^2\right]_0^3 = 3^2 - 0^2 = 9 \text{ sq. units}.$$

Step 4 — Geometric cross-check: The region is a right triangle with base 3 (along the x-axis) and height $y = 2(3) = 6$ (the value of the line at $x = 3$). Its area is $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 3 \times 6 = 9$, matching the integral.

Why other options are wrong:

- (A) 6 is just the height $y = 2(3)$ at the edge, not an area.
- (B) 18 is base×height $= 3 \times 6$, forgetting the factor $\frac{1}{2}$ for a triangle.
- (C) 3 uses only the base length, ignoring the height.

Final Answer: area $= 9$ sq. units $\Rightarrow$ $\boxed{\text{D}}$

Answer: (D) Go Back to Q24

Q25.

Solution

Concept — The Fundamental Theorem of Calculus: A definite integral is evaluated by finding any antiderivative F of the integrand and computing $F(\text{upper}) - F(\text{lower})$. For the sine function the antiderivative is $-\cos x$, because $\dfrac{d}{dx}(-\cos x) = \sin x$. Care with signs at the endpoints is the main pitfall.

Step 1 — Find the antiderivative:

$$\int \sin x\,dx = -\cos x\ (+C).$$

Step 2 — Substitute the limits:

$$\int_0^{\pi/2} \sin x\,dx = \left[-\cos x\right]_0^{\pi/2} = \left(-\cos\tfrac{\pi}{2}\right) - \left(-\cos 0\right).$$

Step 3 — Evaluate the cosines: Using $\cos\frac{\pi}{2} = 0$ and $\cos 0 = 1$,

$$= \big(-0\big) - \big(-1\big) = 0 + 1 = 1.$$

Step 4 — Reasonableness check: On $[0, \frac{\pi}{2}]$ the curve $y = \sin x$ rises from 0 to 1 staying positive, so the area beneath it is a positive number less than the bounding rectangle of area $\frac{\pi}{2} \approx 1.57$. The value 1 sits sensibly below that bound.

Why other options are wrong:

- (A) 0 would require equal antiderivative values at both ends, which is false here.
- (B) -1 drops the leading minus sign of the antiderivative, flipping the result; but $\sin x \geq 0$ on this interval, so the area cannot be negative.
- (D) $\frac{\pi}{2}$ is merely the length of the interval, not the integral of $\sin x$.

Final Answer: the integral is $1 \Rightarrow$ C

Answer: (C) Go Back to Q25

Q26.

Solution

Concept — Equation of a line through two points: A line is fixed by its slope and any one point on it. The slope between two points is the rise over the run, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, and the point-slope form $y - y_1 = m(x - x_1)$ then gives the equation, which we rearrange into slope-intercept form $y = mx + c$.

Step 1 — Compute the slope: With $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, 7)$,

$$m = \frac{7-3}{4-2} = \frac{4}{2} = 2.$$

Step 2 — Apply point-slope form: Using the point $(2, 3)$,

$$y - 3 = 2(x - 2).$$

Step 3 — Simplify to slope-intercept form:

$$y - 3 = 2x - 4 \quad \Longrightarrow \quad y = 2x - 1.$$

Step 4 — Verify both points satisfy it: At $(2, 3)$: $2(2) - 1 = 3$. At $(4, 7)$: $2(4) - 1 =$

7. Both points lie on $y = 2x - 1$, so it is correct.

Why other options are wrong:

- (B) $y = 2x + 1$ has the right slope but wrong intercept; at $(2, 3)$ it gives $5 \neq 3$.
- (C) $y = x + 1$ has slope 1, but the slope between the points is 2.
- (D) $y = 2x - 3$ fails at $(2, 3)$ since $2(2) - 3 = 1 \neq 3$.

Final Answer: $y = 2x - 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q26

Q27.

Solution

Concept — General equation of a circle: The general second-degree circle $x^2 + y^2 + 2gx + 2fy + c = 0$ has centre $(-g, -f)$ and radius $r = \sqrt{g^2 + f^2 - c}$. This comes from completing the square in x and y to reach the standard form $(x+g)^2 + (y + f)^2 = g^2 + f^2 - c$. The radius is real only when $g^2 + f^2 - c > 0$.

Step 1 — Read off g, f, c: Compare $x^2 + y^2 - 4x + 6y - 12 = 0$ with the template. Matching $2g = -4$ gives $g = -2$; matching $2f = 6$ gives $f = 3$; and $c = -12$.

Step 2 — Confirm the centre: Centre $= (-g, -f) = (2, -3)$, exactly the point marked in the figure, which is a good sign we read the coefficients correctly.

Step 3 — Compute the radius:

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{(-2)^2 + 3^2 - (-12)} = \sqrt{4 + 9 + 12} = \sqrt{25} = 5.$$

Step 4 — Cross-check by completing the square:

$$(x^2 - 4x) + (y^2 + 6y) = 12 \Rightarrow (x - 2)^2 - 4 + (y + 3)^2 - 9 = 12,$$

so $(x - 2)^2 + (y + 3)^2 = 25 = 5^2$, confirming centre $(2, -3)$ and radius 5.

Why other options are wrong:

- (A) 12 is $|c|$ taken as the radius, ignoring the $g^2 + f^2$ contribution and the square root.
- (C) 4 uses $\sqrt{g^2 + f^2} = \sqrt{13}$ rounded or drops the $-c$ term entirely.
- (D) 25 is r^2, forgetting to take the square root.

Final Answer: radius $= 5 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q27

Q28.

Solution

Concept — The standard parabola $y^2 = 4ax$: A parabola of the form $y^2 = 4ax$ (with $a > 0$) has its vertex at the origin and opens to the right along the positive x-axis. Its focus is the point $(a, 0)$ and its directrix is the vertical line $x = -a$, placed symmetrically on the opposite side of the vertex. Every point on the parabola is equidistant from the focus and the directrix.

Step 1 — Match to the standard form: Compare the given $y^2 = 12x$ with $y^2 = 4ax$:

$$4a = 12.$$

Step 2 — Solve for a:

$$a = \frac{12}{4} = 3.$$

Step 3 — Write the directrix: For $y^2 = 4ax$ the directrix is $x = -a$, so

$$x = -3.$$

This is a vertical line, consistent with the figure (a right-opening parabola has a vertical directrix to its left).

Step 4 — Sanity check with the focus: The focus is $(a, 0) = (3, 0)$, inside the curve, and the directrix $x = -3$ is the same distance 3 on the other side of the vertex; this symmetry about the vertex confirms the value.

Why other options are wrong:

- (A) $x = 3$ is the x-coordinate of the *focus*, not the directrix.
- (B) $y = -3$ is horizontal; a parabola opening along the x-axis must have a *vertical* directrix.
- (C) $x = 12$ uses $4a$ directly instead of a.

Final Answer: directrix $x = -3 \Rightarrow$ D

Answer: (D) Go Back to Q28

Q29.

Solution

Concept — Direction cosines of a line: If a line makes angles α, β, γ with the x-, y- and z-axes, its direction cosines are $\ell = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$. These are the components of a unit vector along the line, so they satisfy the identity

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \ell^2 + m^2 + n^2 = 1.$$

This single relation lets us find the third direction cosine once the other two angles are known.

Step 1 — Substitute the known angles: With $\alpha = 60°$ and $\beta = 45°$,

$$\cos^2 60° + \cos^2 45° + n^2 = 1,$$

i.e. $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + n^2 = 1.$

Step 2 — Evaluate the squared cosines:

$$\cos^2 60° = \frac{1}{4}, \qquad \cos^2 45° = \frac{1}{2}.$$

Step 3 — Solve for n^2:

$$\frac{1}{4} + \frac{1}{2} + n^2 = 1 \Rightarrow n^2 = 1 - \frac{3}{4} = \frac{1}{4}.$$

Step 4 — Consistency check: Since $0 \le n^2 \le 1$, the value $\frac{1}{4}$ is admissible (it gives $\cos\gamma = \pm\frac{1}{2}$, i.e. $\gamma = 60°$ or $120°$, both valid directions of the same line).

Why other options are wrong:

- (A) $\frac{3}{4}$ is the sum $\cos^2 60° + \cos^2 45°$, i.e. $1 - n^2$, not n^2 itself.
- (C) $\frac{1}{2}$ would come from using $\cos^2 45°$ alone, ignoring the $\cos^2 60°$ term.
- (D) 1 would require the first two contributions to vanish, contradicting the given angles.

Final Answer: $n^2 = \dfrac{1}{4} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q29

Q30.

Solution

Concept — Intercept form of a plane: A plane written as $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ cuts the axes at $(a,0,0)$, $(0,b,0)$, $(0,0,c)$, so a,b,c are its x-, y-, z-intercepts. To use this we must first force the right-hand side to be 1 by dividing through, after which the reciprocals of the coefficients reveal the intercepts. Equivalently, the x-intercept is found by setting $y=z=0$ and solving for x, and similarly for the others.

Step 1 — Normalise the right-hand side: Divide $2x+3y+4z=12$ by 12:

$$\frac{2x}{12}+\frac{3y}{12}+\frac{4z}{12}=1.$$

Step 2 — Reduce each fraction to intercept form:

$$\frac{x}{6}+\frac{y}{4}+\frac{z}{3}=1.$$

Step 3 — Read off the intercepts: Matching with $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ gives $a=6$, $b=4$, $c=3$.

Step 4 — Verify by setting two variables to zero: Put $y=z=0$: $2x=12\Rightarrow x=6$. Put $x=z=0$: $3y=12\Rightarrow y=4$. Put $x=y=0$: $4z=12\Rightarrow z=3$. All three confirm the intercepts $6,4,3$.

Why other options are wrong:

- (A) $2,3,4$ are the *coefficients*, not the intercepts; the intercept is 12 divided by the coefficient.
- (B) $12,12,12$ uses the constant term for every axis, ignoring the coefficients.
- (D) $3,4,6$ has the correct numbers but in reversed order (x and z intercepts swapped).

Final Answer: intercepts $6,4,3\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q30

Q31.

Solution

Concept — Magnitude of a vector via the dot product: The dot product of a vector with itself gives the square of its length: $\vec{a}\cdot\vec{a} = |\vec{a}|^2$. In components, $\vec{a}\cdot\vec{a} = a_1^2 + a_2^2 + a_3^2$, which is just the three-dimensional Pythagoras theorem. Hence $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$, the square root of the sum of the squared components.

Step 1 — Identify the components: For $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, the components are $a_1 = 2$, $a_2 = -1$, $a_3 = 2$.

Step 2 — Form $\vec{a}\cdot\vec{a}$ (sum of squares):

$$\vec{a}\cdot\vec{a} = 2^2 + (-1)^2 + 2^2 = 4 + 1 + 4 = 9.$$

Step 3 — Take the square root:

$$|\vec{a}| = \sqrt{9} = 3.$$

Step 4 — Note the sign handling: The middle component -1 contributes $(-1)^2 = +1$, not -1; squaring removes the sign, which is why magnitudes are always nonnegative.

Why other options are wrong:

- (B) 9 is $|\vec{a}|^2$ (the dot product), not the magnitude; the square root is still needed.
- (C) $\sqrt{5}$ comes from dropping one term, e.g. using $2^2 + (-1)^2 = 5$ and forgetting the third component.
- (D) 5 adds the components $2 + (-1) + 2 = 3$ wrongly, or sums without squaring.

Final Answer: $|\vec{a}| = 3 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q31

Q32.

Solution

Concept — Moment (torque) as a cross product: The moment of a force $\vec{F}$ acting at a point with position vector $\vec{r}$, taken about the origin, is $\vec{M} = \vec{r} \times \vec{F}$. The cross product is bilinear, so scalar multiples factor out, and it relies on the right-hand unit-vector rules $\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\hat{\jmath} \times \hat{k} = \hat{\imath}$, $\hat{k} \times \hat{\imath} = \hat{\jmath}$ (cyclic, with a sign flip if reversed). The magnitude is $|\vec{r}||\vec{F}| \sin\theta$ and the direction is perpendicular to both $\vec{r}$ and $\vec{F}$.

Step 1 — Write the vectors: $\vec{r} = 2\hat{\imath}$ and $\vec{F} = 3\hat{\jmath}$ (the figure shows $\vec{r}$ along the x-axis and $\vec{F}$ vertical, so they are perpendicular).

Step 2 — Factor out the scalars:

$$\vec{r} \times \vec{F} = (2\hat{\imath}) \times (3\hat{\jmath}) = (2)(3)\,(\hat{\imath} \times \hat{\jmath}) = 6\,(\hat{\imath} \times \hat{\jmath}).$$

Step 3 — Apply the unit-vector rule: Since $\hat{\imath} \times \hat{\jmath} = \hat{k}$,

$$\vec{r} \times \vec{F} = 6\hat{k}.$$

Step 4 — Magnitude check: With $\vec{r} \perp \vec{F}$, $\theta = 90^\circ$ and $|\vec{M}| = |\vec{r}||\vec{F}| \sin 90^\circ = 2 \cdot 3 \cdot 1 = 6$, matching $|6\hat{k}| = 6$; the direction $+\hat{k}$ (out of the page) follows the right-hand rule from $\vec{r}$ to $\vec{F}$.

Why other options are wrong:

- (A) $\vec{0}$ would require $\vec{r} \parallel \vec{F}$; here they are perpendicular, so the moment is maximal, not zero.
- (B) $5\hat{k}$ adds the magnitudes $2 + 3$ instead of multiplying them.
- (C) $-6\hat{k}$ uses $\hat{\imath} \times \hat{\jmath} = -\hat{k}$, which has the wrong sign (it would be $\hat{\jmath} \times \hat{\imath}$).

Final Answer: $\vec{r} \times \vec{F} = 6\hat{k} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q32

Q33.

Solution

Concept — Scalar triple product and coplanarity: The scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$ equals the determinant whose rows are the components of the three vectors. Its absolute value is the volume of the parallelepiped they span. Three vectors are coplanar precisely when that volume collapses to zero, i.e. when the determinant is 0; a nonzero value means they are not coplanar.

Step 1 — Build the determinant from the components: With $\vec{a} = (1, 1, 0)$, $\vec{b} = (0, 1, 1)$, $\vec{c} = (1, 0, 1)$,

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}.$$

Step 2 — Expand along the first row:

$$1\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}.$$

Step 3 — Evaluate the 2×2 minors:

$$1(1 \cdot 1 - 1 \cdot 0) - 1(0 \cdot 1 - 1 \cdot 1) + 0 = 1(1) - 1(-1) = 1 + 1 = 2.$$

Step 4 — Interpret: The value is $2 \neq 0$, so the parallelepiped has nonzero volume and the three vectors are *not* coplanar.

Why other options are wrong:

- (B) 0, coplanar: would require the determinant to vanish, but it equals 2.
- (C) 1, not coplanar: the value is correct in conclusion but the number is wrong (the minor expansion gives 2, not 1).
- (D) -2, coplanar: both the sign of the determinant and the coplanar verdict are wrong.

Final Answer: $[\vec{a}\ \vec{b}\ \vec{c}] = 2$, not coplanar $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q33

Q34.

Solution

Concept — Standard deviation measures spread: The standard deviation $\sigma = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2}$ quantifies how far the data scatter from their mean $\bar{x}$. It is built from the squared deviations $(x_i - \bar{x})^2$, each of which is zero only when an observation equals the mean. If every observation is identical, there is no spread at all, so σ must be 0 regardless of the common value.

Step 1 — Compute the mean: For $7, 7, 7, 7, 7$,

$$\bar{x} = \frac{7+7+7+7+7}{5} = \frac{35}{5} = 7.$$

Step 2 — Compute the deviations: Each $x_i - \bar{x} = 7 - 7 = 0$, so every squared deviation is $0^2 = 0$.

Step 3 — Apply the formula:

$$\sigma = \sqrt{\frac{1}{5}(0+0+0+0+0)} = \sqrt{\frac{0}{5}} = \sqrt{0} = 0.$$

Step 4 — Interpret: A standard deviation of 0 correctly signals that the data are perfectly constant, with no variability about the mean.

Why other options are wrong:

- (A) 7 is the common value (and the mean), not a measure of spread; identical data have zero spread.
- (B) $\sqrt{7}$ has no basis here; it would need nonzero, unequal deviations.
- (D) 1 assumes some variation, but all five observations coincide.

Final Answer: $\sigma = 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q34

Q35.

Solution

Concept — Addition rule and mutual exclusivity: In general $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where the overlap is subtracted to avoid double counting. Two events are *mutually exclusive* when they cannot occur together, i.e. $A \cap B = \varnothing$ and so $P(A \cap B) = 0$. The addition rule then simplifies to $P(A \cup B) = P(A) + P(B)$.

Step 1 — Use mutual exclusivity: Since A and B are mutually exclusive, $P(A \cap B) = 0$, so the overlap term drops out.

Step 2 — Apply the simplified rule:

$$P(A \cup B) = P(A) + P(B) = 0.3 + 0.45.$$

Step 3 — Add:

$$P(A \cup B) = 0.75.$$

Step 4 — Validity check: The result 0.75 lies in $[0, 1]$, as any probability must, so it is a legitimate answer.

Why other options are wrong:

- (A) 0.135 *multiplies* the probabilities (0.3×0.45), which is the rule for *independent* intersection, not a union of exclusive events.
- (C) 0.15 subtracts ($0.45 - 0.3$), which has no justification here.
- (D) 1 ignores the given values and wrongly assumes the events are exhaustive.

Final Answer: $P(A \cup B) = 0.75 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q35

Q36.

Solution

Concept — Normalisation of a probability distribution: For a discrete random variable, the probabilities of all its possible values must be nonnegative and add up to exactly 1 (the total probability axiom). When the individual probabilities are given as multiples of an unknown k, we impose $\sum P(X = x_i) = 1$ to solve for k, then check that the resulting probabilities are all valid (between 0 and 1).

Step 1 — Write the total probability: The values $0, 1, 2, 3$ carry probabilities

$k, 2k, 3k, 4k$, so
$$k + 2k + 3k + 4k = 1.$$

Step 2 — Combine like terms:
$$10k = 1.$$

Step 3 — Solve for k:
$$k = \frac{1}{10}.$$

Step 4 — Verify the distribution: The probabilities become $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}$, i.e. $0.1, 0.2, 0.3, 0.4$. Each lies in $[0, 1]$ and their sum is 1.0, so the distribution is valid.

Why other options are wrong:

- (A) $\frac{1}{4}$ gives a total $10k = 2.5 \neq 1$.
- (B) $\frac{1}{5}$ gives $10k = 2 \neq 1$.
- (C) $\frac{1}{6}$ gives $10k = \frac{10}{6} \neq 1$. Only $k = \frac{1}{10}$ makes the probabilities sum to 1.

Final Answer: $k = \dfrac{1}{10} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q36

Q37.

Solution

Concept — The Pythagorean identity: The fundamental trigonometric identity $\sin^2\theta + \cos^2\theta = 1$ holds for every angle θ (it follows from the unit-circle definition, where a point $(\cos\theta, \sin\theta)$ lies on $x^2 + y^2 = 1$). Expressions mixing $\sin^2\theta$ and $\cos^2\theta$ with equal coefficients can therefore be collapsed to a constant, independent of θ.

Step 1 — Factor out the common coefficient: The two squared terms share the factor 5:
$$5\sin^2\theta + 5\cos^2\theta - 2 = 5\big(\sin^2\theta + \cos^2\theta\big) - 2.$$

Step 2 — Apply the identity: Replace the bracket by 1:
$$= 5(1) - 2.$$

Step 3 — Simplify:
$$= 5 - 2 = 3.$$

Step 4 — Numerical spot-check: Take $\theta = 0$: $5\sin^2 0 + 5\cos^2 0 - 2 = 5(0) + 5(1) - 2 = 3$. Take $\theta = \frac{\pi}{4}$: $5 \cdot \frac{1}{2} + 5 \cdot \frac{1}{2} - 2 = 5 - 2 = 3$. The value is constant, as expected.

Why other options are wrong:

- (B) 5 stops at $5(\sin^2\theta + \cos^2\theta) = 5$ and forgets to subtract the 2.
- (C) 0 would treat $\sin^2\theta + \cos^2\theta$ as 0, contradicting the identity.
- (D) -2 ignores the $5(\sin^2\theta + \cos^2\theta)$ term entirely.

Final Answer: the value is $3 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q37

Q38.

Solution

Concept — Double-angle formula for tangent: The tangent of a doubled angle is $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$, derived from the sum formula $\tan(\theta+\theta)$. It expresses $\tan 2\theta$ purely in terms of $t = \tan\theta$, so once $\tan\theta$ is known the rest is arithmetic; the only care needed is simplifying the compound fraction.

Step 1 — Substitute $\tan\theta = \frac{1}{3}$:

$$\tan 2\theta = \frac{2\cdot\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2} = \frac{\frac{2}{3}}{1-\frac{1}{9}}.$$

Step 2 — Simplify the denominator:

$$1 - \frac{1}{9} = \frac{9-1}{9} = \frac{8}{9}, \qquad \text{so} \qquad \tan 2\theta = \frac{\frac{2}{3}}{\frac{8}{9}}.$$

Step 3 — Divide the fractions: Dividing by a fraction means multiplying by its reciprocal:

$$\frac{2}{3} \times \frac{9}{8} = \frac{18}{24} = \frac{3}{4}.$$

Step 4 — Plausibility check: Since $\tan\theta = \frac{1}{3}$ is small (about 18.4°), 2θ is roughly 36.9° with $\tan \approx 0.75 = \frac{3}{4}$, matching the computed value.

Why other options are wrong:

- (A) $\frac{2}{3}$ is just the numerator $2\tan\theta$, ignoring the denominator $1-\tan^2\theta$.
- (C) $\frac{1}{4}$ comes from squaring or mishandling, not the correct division.

- (D) $\frac{2}{9}$ multiplies $\frac{2}{3}$ by $\frac{1}{3}$ instead of dividing by $\frac{8}{9}$.

Final Answer: $\tan 2\theta = \dfrac{3}{4} \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q38

Q39.

Solution

Concept — Principal value of $\cos^{-1}$: The inverse cosine returns angles only from its principal range $[0, \pi]$. So $\cos^{-1}(\cos x)$ equals x only when x already lies in $[0, \pi]$; otherwise we must find the unique angle in $[0, \pi]$ that shares the same cosine value. Here $\dfrac{7\pi}{6}$ (which is $210°$) lies outside $[0, \pi]$, so a reduction is needed.

Step 1 — Evaluate the inner cosine: The angle $\dfrac{7\pi}{6}$ is in the third quadrant, where cosine is negative, with reference angle $\dfrac{\pi}{6}$:

$$\cos\frac{7\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}.$$

Step 2 — Find the principal angle with this cosine: We need $\theta \in [0, \pi]$ with $\cos\theta = -\dfrac{\sqrt{3}}{2}$. Negative cosine puts θ in the second quadrant, and the reference angle $\dfrac{\pi}{6}$ gives

$$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

Step 3 — Verify: $\dfrac{5\pi}{6} \in [0, \pi]$ and $\cos\dfrac{5\pi}{6} = -\dfrac{\sqrt{3}}{2}$, which matches the inner value, so

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \frac{5\pi}{6}.$$

Why other options are wrong:

- (A) $\frac{7\pi}{6}$ lies outside the principal range $[0, \pi]$, so it cannot be a $\cos^{-1}$ output.
- (B) $\frac{\pi}{6}$ has cosine $+\frac{\sqrt{3}}{2}$, the wrong sign.
- (D) $-\frac{5\pi}{6}$ is negative, again outside $[0, \pi]$.

Final Answer: $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \dfrac{5\pi}{6} \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q39

Q40.

Solution

Concept — Angle of depression and the right triangle: The angle of depression is measured downward from the horizontal line of sight at the observer's eye. Because the observer's horizontal and the sea surface are parallel, the angle of depression from the cliff top equals the angle of elevation from the boat (alternate interior angles). This gives a right triangle whose vertical leg is the cliff height and horizontal leg is the distance to the boat, so $\tan(\text{angle}) = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{\text{height}}{\text{horizontal distance}}$.

Step 1 — Set up the tangent relation: With height 40 m, horizontal distance d, and angle $45°$,

$$\tan 45° = \frac{40}{d}.$$

Step 2 — Substitute the known value: Since $\tan 45° = 1$,

$$1 = \frac{40}{d}.$$

Step 3 — Solve for d:

$$d = 40 \text{ m}.$$

Step 4 — Geometric check: A $45°$ angle makes the right triangle isosceles, so the horizontal leg equals the vertical leg; with height 40 m the distance is indeed 40 m, confirming the result.

Why other options are wrong:

- (A) 20 m halves the height for no valid reason.
- (B) $40\sqrt{3}$ m corresponds to $\tan 30° = \frac{1}{\sqrt{3}}$, i.e. $d = 40\sqrt{3}$, the wrong angle.
- (C) $40\sqrt{2}$ m is the hypotenuse (line of sight) length, not the horizontal distance.

Final Answer: $d = 40 \text{ m} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	B	4	D	5	A
6	B	7	C	8	D	9	A	10	B
11	C	12	D	13	A	14	C	15	B
16	D	17	C	18	A	19	B	20	D
21	C	22	B	23	A	24	D	25	C
26	A	27	B	28	D	29	B	30	C
31	A	32	D	33	A	34	C	35	B
36	D	37	A	38	B	39	C	40	D